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THE INSTITUTE OF
CHARTERED ACCOUNTANTS OF INDIA

STUDY MATERIAL F.S.P. (N) SA & DP—7

FINAL COURSE (N)

MANAGEMENT INFORMATION & CONTROL SYSTEM
SYSTEM ANALYSIS & DATA PROCESSING
STUDY – VII
COMPUTER BASED MIS

This Study, though not compulsory, is recommend for SA & DP Students

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Suggested Reading : *Information System for Modern Management by Murdick and Ross, Prentice—Hall (India).*

1. AN OVERVIEW OF THE STUDY :

This study is concerned specifically with computer-based MIS. Though it is not absolutely necessary for MIS, yet it is the computer which has made a reality of the fundamental concepts of MIS. Its assets are speed, accuracy and freedom from errors. It however lacks judgement for which the human beings can complement it, therefore both man and computer capabilites with regard to MIS are discassed in this study. Computerised MIS can be viewed from several angles :user's view, physical structure, functional composition, 3 tier planning and control activity etc. The structure of MIS is detailed separately from these angles and later these views are synthesised to depict the overall structure of MIS.

Basically, MIS is supposed to support planning, control, and decision-making activities of the management. Now, when we say "project control" project planning is also implied. Likewise, when we say production planning it may also include production control What we want to say that often "planning" or "control" are used to broadly designate three activities : forecasting (without which no plan can be made) planning, and control (without which no plan can be implemented). It is the purpose of this study paper to discuss in a little detail the three activities sparately Furthermore, in manual systems it is not unusual to forecast, plan and control intuitively, but in computerised systems, it becomes essential to build models for each forecast, each plan and each control Generalised modelling structure for a business organisation is presented The structure is the same whatever the organisation but models within it may vary from organisation to organisation. Referring to bottom-up approach (SADP study III for MIS development), once the individual files have been fused into a data base, the organisation can think of advancing by incorporating this modelling structure in its MIS This would generally require the creation a model base in addition to the data base both of which, however, are inter-linked, Once MIS has been thus installed (first conscious effort in MIS development is its installation even though MIS must be existing in some rudimentary from already) and advanced by embedding the modelling structure in it, models may subsequently be refined as the planning, forecasting,control and modelling faculties of the organisation improve with time, or models may be deleted or modified or expanded etc. to respond to changes in the organisational environment. Besides, projects triggered by the systems maintenance or audit phase of systems development (Ref : SADP Study III), would have to be incorporated in MIS. As distinct from installation of MIS, then, the refinement and revision of the models and systems audit are the problems of operating MIS.

It should be obvious that fusion of files into a data base requires magnetic disc (or drum) as the file media Data base enables the efficient use of data which has come to be recognised as a resource in itself. Planners, forecasters, supervisors, programmers, modellers and systems designers also become aware of the huge amount of data (hitherto vivisected departmentwise) available from directories circulated by the data base administrator ; therefore, obviously, they can perform their functions much better. All these functionaries have a great and complex

deal of interactions in discharging their functions, and, thereforee, one of the general objectives of MIS ought to be to harmonise their interactions.

Problems and opportunities do arise wihin the plans. Build up of backlog is a problem within the production planning function, for example. Therefore. MIS should provide support for intelligence on problems or opportunities. Once a problem has been analysed there comes the question of design of and choice amongst alternatives. This is what has been termed as decision making. MIS should then provide support for decision-making too, and this matter has been also discussed.

Finally, limitations of MIS are discused.

1 1 Introduction : Although is not absolutely essential for MIS, yet it is the computer that has translated the MIS concept into the present day reality For both accurate computations and transaction proceesing, computer is far more quick than man. The latter, however, has his own capabilities such as taking into cognizance a great many factors. Thus man and machine are complementary to each other and the question really is to what extent the computer should be employed. Also, the systems approach is its basis. In the following section, we discuss the man-machine capabilities (information processors) that need to be harmoonised in the design of MIS. The following definition of MIS should set the framework of discussion ahead.

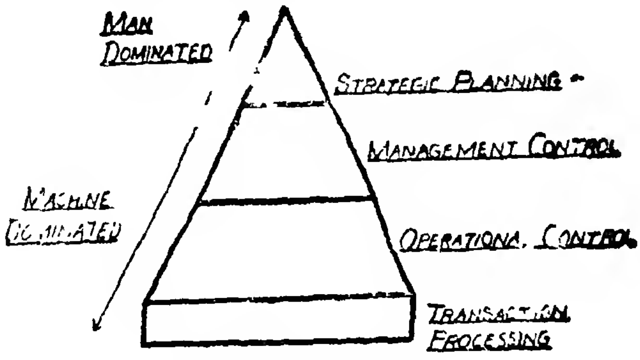
> "MIS may be defined as an integrated man-machine system in producing information to support the operations, management planning and control and decision-making function in an organisation "

The pyramid structure of the figure on page 4 depicits a view of MIS in respect of three-tier planning and control activity discussed in study 1 At the bottom there is the ubiquitous transaction processing system of handling such transactions as customers's orders, payroll, inventory transactions (receipts, issue and stock taking adjustments). Upwards, there are operational control management control and strategic planning, in this order. Transaction system is a fountain head of data. Various types of data are classified and aggregated in a suitable manner to serve the purpose of operational control, management control and strategic planning. ' Strategic planning may, however, also require data on external environment. Aside from the three tier planning and control activity we shall discus MIS from several other views.

It is to be noted that at the bottom MIS is machine dominated as it is very much structured here. Towards the top it tends to be more and more unstructured and man dominated.

MIS supports the planning and control and decision making activities of the management. The terms 'planning' and 'control' are usually loosely and narrowly understood. Expectations, i e. forecasts a re essential inputs to the planning process. And planning without control is unthinkable Thus when we say management control forecasting and planning are also implied. Likewise, when we say strategic planning, forecasting is involved though control, in this particular case, is virtually non-existent. In manual systems, often estimations have served the purpose of forecasts and planning has been done by judgement intuition etc. Though it would be desirable to carry out the three activities explicitly even in the manual

systems, computerised systems simply cannot work without plans, forecasts and controls being precisely specified by means of models. Modelling, therefore, is a common denominator of forecasting and control.



PYRAMID STRUCTURE OF MIS

Now it is to be carefully noted that all forecasting, planning and control involve decision-making Which forecasting model to use and what value of its parameters and constants to employ, what alternative plan out of the several to select and what changes in the implementation process to make i e effect control, are points of decision-making.

2. HUMAN BEINGS AS INFORMATION PROCESSORS :

The sensory receptors consist of eyes nose, ears, etc which pick up the signals and transmit them to the brain consisting of storage and mental processing unit There is also a filter that filters the inputs passing through the channel. Whenever number of inputs exceeds a limit, known as the information load point, filteration is effected i e. some of the inputs are disregarded The implication of this is that human beings have limitation for information processing and whenever the inputs are less than those for the information overload point all of them will be processed ; but beyond, some of them are liable to be ignored. The best example is provided by a telephone operator who, faced with too many messages to handle, will ignore some of them Regarding which inputs are disregarded for processing, the following points are worth noting :—

(i) Frame of reference of the subject i.e., his experience, education, customs etc.

(ii) *Decision procedure* depending upon which the subject may ignore data that is not specified as relevant to decision making.

(iii) Decision under stress. A production manager faced with series of objectives to optimise is liable to ignore some of them while choosing a particular course of action. Such overloading is particularly probable when the manager is also under some other stresses viz; production crisis. Likewise, as discussed in study, III, a wide variety of marketing reports are possible but they may overwhelm the marketing manager who may resort to filteration and ignore some of them.

Another point of interest is that different persons would naturally have different frames of reference. As a consequence, the writer of a report may mean something and the reader may follow another thing which may be designated as perception errors.

Miller summarised the results of his empirical research by the pharse "The magical number seven, plus or minus two". What he means is that the human beings can process symbols of length 5 to 9 most effectively. This therefore, requires that the length of the various codes, quantity and other such data fields is kept between these limits.

Some other human limitation are discussed below briefly :—

2·1 Noticeable Difference : Human beings have a limitation in discerning difference in shades of colours, brightness of illumination, loudness of sound etc. Weber has formulated a law in this regard. "The difference that is noticable is a constant proportion of the physical dimensions of stimulus." Mathematically,

$$\frac{\Delta C}{C} = K \text{ for all, C and K being constants.}$$

The other symbols would be more clear by means of an example. Consider two roughly 10 kg weights. Assuming $K = \frac{1}{25}$, ΔC may be computed as below :

$$\frac{\Delta C}{10} = \frac{1}{25}$$

$$\Delta C = 100/25 = 4$$

ΔC means that the difference in the two weights must at least be 4 kg to be significant.

Now, if there is another set of two roughly 50 kg weights, $\Delta C = 50/25 = 2$ kg indicating that the difference has to be 24kg at least to make it discernible by human beings. The implication of Weber's law is that human beings do not discern the difference by their absolute size but in relation to a base. A 30% variation against the budgeted sale of Rs. 1,00,000 would as much surprise the executive as a 30% variation in selling expenses of Rs. 10,000.

2·2 Probablistic Difference : H G Wells prophesied that statistics will be a part of common sense in the 21st century. But researches in psychology point out differently Human beings have serious limitations in grasping probablistic situations. Consider two students A and B who answer 10 and 100 questions (of same standard) respectively. A answers 1 correctly and B answers 10 correctly. It would be a human tendency to rate the two equal since percentage wise they have scored equally but surely B has been more thorougly examined Likewise 2 rejects in a sample of 10 do not tell the same thing as 20 rejects in a sample of 100. Sampling variance decreases in proportion to sample size and human beings inherently fail to recognise this at once.

We cited example on nonsense correlation in an intermediate study on Statistics that rising production of both pig iron and pigs in India does not mean that the two are correlated. But human beings do tend to jump to conclusions with regard to cause/effect relationships and correlations. On the other hand. several correlations and casualties may not be discerned because the organisational events occur at a very low rate.

Human beings are particularly deficient in perceiving and assigning probabilities to events, and are liable to give way to a variety of factors leading to bias here.

Human beings lack consistency in exercising choice or making decisions For example, subjects presented with various gambling situations will demonstrate inconsistent choice.

Human beings tend to resort to *anchoring and adjustment*. Budgets, for example are set by establising the previous budget as the anchor and then effecting adjustments to it. This happens all too often in pricing and planning too. This has its pros and cons. Time is saved since each situation is not reviewed *ab-initio* but also important differences in circumstances are liable to be missed. And when it comes to making probabilistic estimates (viz probability of market share surpassing 10% etc.) there is hardly anything to anchor on.

Another point that has a particular relevance in drafting the format of reports is that human beings tend to use the information only in the form it is available. They do not tend to search for more data or manipulate the existing data.

Regarding summarised data a research study indicates that better decisions are made with summarised data than with raw data ; but at the same time, the decision maker is less confident in the former case. And the executives may have different predilections in this regard as borne out by Orlicky:"President Elisenhover, for instance, evidenced a very high information threshold as he preferred problems to be summarised for him very succiently, preferably on one page. Former Secretary of Defence McNamara on the other hand wanted his data in considerable detail and sometimes would dig deep into subsidiary reports before deciding an issue."

We discussed Hertz's simulation experiment for deriving the NPV distribution in OR Study VI. His idea was to provide the executives not only with mean NPV but also its standard deviation, shape of its probability distribution. Anderson conducted a study by presenting three setes of data to executives on the lines of Hertz's model as shown below :

1. Mean value
2. Mean value and disperstion
3. Mean value, dispersion and shape of probability distribution.

Subjects presented with all three data items were more confident about their decisions but less consistent.

Accumulation of unused Data : There is a tendency to store data that has no/little relevance to the future. It is uneconomic but the user feels more confident with all the data in store and perhaps has vague notion that it may be used some day. But cost/benefit analysis must override this phyholigical value of the data. At least, such data should be consigned to low cost storage.

Below, we compare and contrast human capabilities with computer capabilities.

2.4. Man (versus) Machine capabilities :

Major Machine Capabilities

1. Repetitive operations.
2. Accuracy of calculations.
3. Speed of calculations.
4. Deductive Reasoning (Given $A<B$ and $B<C$ the machine can conclude that $A<C$).
5. Operation in parallel. The human processing system operates in what may he called serial mode. That is human being can process only one information at a time. Machines can operate in a parallel fashion. For example humans would add digits one by one but the computer can do several bits simultaneously
6. Well defined data search.
7. Following orders precisely (right or wrong).
8. Consistency.

Major Human capabilities

1. Unusual operations.
2. Operations with incomplete or ambiguous data.
3. Inductive reasoning, besides deductive reasoning.
4. Filtering of information : The capacity of the human beings to accept inputs and produce output is limited. Upto a limit each input would result into an output. But beyond that performance deteriorates. The humans however reduce the inputs to manageable proportions by a filtering selection process. Unimportant inputs are blocked.
5. Initiative and planning.
6. Judgement/Intuition.

2.5. Below, we dichotomise information by several criteria with a view to see which type is most suited for computerisation.

Information can be dichotomised by several attributes. This is mainly with a view to automate as much of information handling as possible.

Action/Non-action. Such information as receipt of a customer's order, receipt of goods and the advice note against an outstanding order on a vendor ; placement of works order etc. would trigger action shortly if not immediately. But an acknowledgement of receipt of payment against a vendor's invoice would possibly be filed without any action whatever and even and subsequent referencing.

Recurring/Non-recurring. All special reports are non-recurring in nature. An example would be studies and summaries for a sitting of the top management on strategic planning. Most of accounting, production and inventory control, sales and purchasing transactions are of recurring nature,

Documentary/Non-documentary. Several activities are carried out informally and may not be documented. Also, most of the superior subordinate communications remain undocumented. whereas, all information on paper, paper tape, punched cards, magnetic strip, disc and tape is obviously documented.

Internal External. The information may come in from outside (including sister plants) and usually include customer orders, enquiries, acknowledgements, telephone calls and telex messages, various kinds of literature and bulletins, quotations, advice notes, etc., from the suppliers, This is all external information. The other sub-class, comprising internal information, would be from one department

to another and up or down from the subordinate and superior respectively. Examples of this sub class would be reports, minutes of meetings, instructions, data contained in pre printed forms such as copies of goods receipt notes, copies of challans and despatch notes etc :

Historical/projected. The past sales is the historical information whereas its extrapolation into the future is projection.

The action/recurring/documentary/internal/historical information is most susceptible to automation though not exclusively, The other desirable characteristics for information to be programmable are :

(i) *A number of interacting variables*—most of the mathematical/operations research models (discassed elsewhere) fall into this category.

(ii) *Speed*—various statements and summaries, viz., stock valuation, ABC analysis. Although such statements can be compiled manually, computer does it far more quickly, accurately and comprehensively.

(iii) *Accuracy*—Updating stock balances, routine forecasting, payroll etc.

3. STRUCTURE OF AN MIS :

I **Physical Structure** comprises hardware, software, files, proeedures (bound in a booklet or manual) and brainware.

II **Processing Functions** : An MIS can also be described in terms of the processing functions it performs : transaction processing, file updating, report preparation, inquiry handling and 'interaction' with the human user. The last of these, 'interaction' perhaps, requires some elaboration A manager may pose a "what if" question as to what happens if the advertising budget is slashed by 10%. The mathematatial relationship between the magnitude of the budget and its consequenees would have been embedded in ths computer program, and the computer would provide the answer to the manager who would scrutinise the result. He may not be satisfied with the result and may pose another question. Thus, the user and computer interact together to iterate down to a satisfactory solution to the given problem.

III User's view of an MIS The users tend to view computerised MIS as a black box They provide the requisite inputs and expect the following outputs as per their status (Ref Management Information Systems by Davis) :

User	*Uses*
Clerical personnel	Handle transactions, process data, and answer inquiries
First-level managers	Obtaining operations data Assistance with planning, scheduling, identifying out of control situations and making decisions.
Staff specialists	Information for analysis, Assistance with analysis, planning and reporting (Model building for forecasting. planning, control and decision-making in particular).
Management	Regular reports Special retrieval requests. Special analysis and reports. Assistance in identifying problems and opportunities Assistance in decision-making analysis.

IV. 3-Tier MIS has been discussed in Study I. We shall here discuss the 3-tiers with regard to support from computer based MIS,

Operational Control : Only the internal data generated from transactions is used to support operational control. Inquiries of the supervisors are handled and such reports as listings of orders behind schedule, shortages list, requisition lists etc. are prepared for the superviser. Transaction processing, inquiry handling and report preparation usually involve programmable decision-making which means that decisons making is repetitive in nature, requires little human judgement and is expressed by mathematical models and, if appropriate programs are prepared these can be entrusted to the computer for routine execution. An example would be computations of EOQ's for thousands of item. Programmable decision-making however is discussed later in more detail.

Management Control : At this level the managers require information regarding their departments and profit centres for (i) measuring performance (data would be obtained from the transaction processing system and suitably aggregated or compressed viz. for computing ROI, productivity etc., (ii) formulation of new decision rules to be applied by operational control (viz. for changing the value of I, the rate of return, for computing EOQ's (iii) Allocation of resources (viz, application of linear programming to problems of advertising mix, product mix, etc. and (iv) forecasting, planning and control as discussed in subsequent sections.

Strategic Planning : would require data for evaluation of current capabilities (internal data), capability to project this data by means of forecasting models discussed in a subsequent section and external data on market and competitors.

(V) **Functional MIS** : MIS may also be viewed as divided by function such as marketing, production, purchasing, technical, finance and accounting. Various functions however can share the same data base,

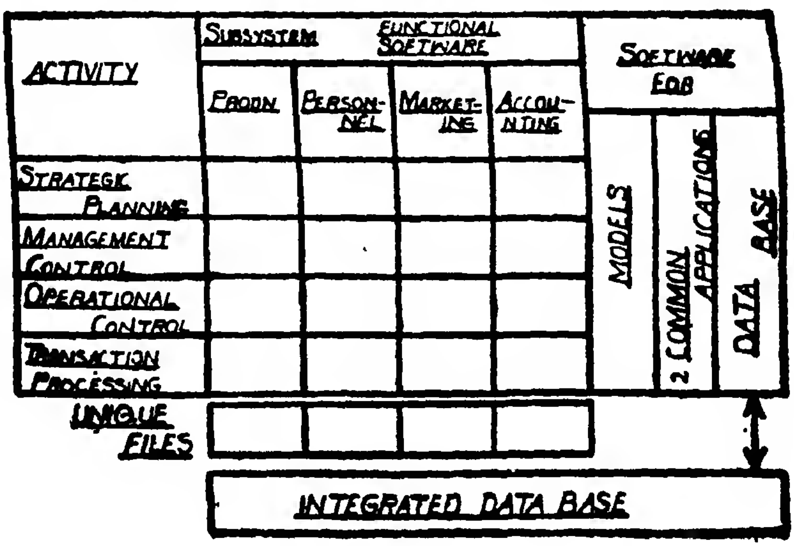


Figure 2

Synthesis of MIS Structure : The 3-tier and functional views of MIS are synthesised in the figure above. MIS is seen as a federation of sub-systems. Each functional system is divided into strategic planning, management control, operational control and transaction processing sub-systems. Each functional sub-system may have some files unique to it ; but they do share a common data base also. Data base requires a special software discussed in Section 4.6 and is known as the Data Base Management System. A model base (for details, see Section 4.4) is also shown. There is also software for common applications. For example, a common input edit routine may be used for all application's. Such common applications may be in modules (Ref : Modularisation SADP Study III) to be assembled in the application programs.

4 SIX LEVELS OF MIS

These are listed below and discussed in detail from Section 4 1 to 4.6.

Planning & Control	) by managers
Foreasting & Modelling	) by management specialists
Computing & Data Base Administration	) by information specialists

4.1 Planning : Planning is an extremely important function. Without planning not a wheel turns in the factory The strategic plan that is handed over to the planner merely (i) sets the course for the organisation viz. switching over to a new technology, expanding markets, diversification of product line etc, (ii) makes the following trade offs -

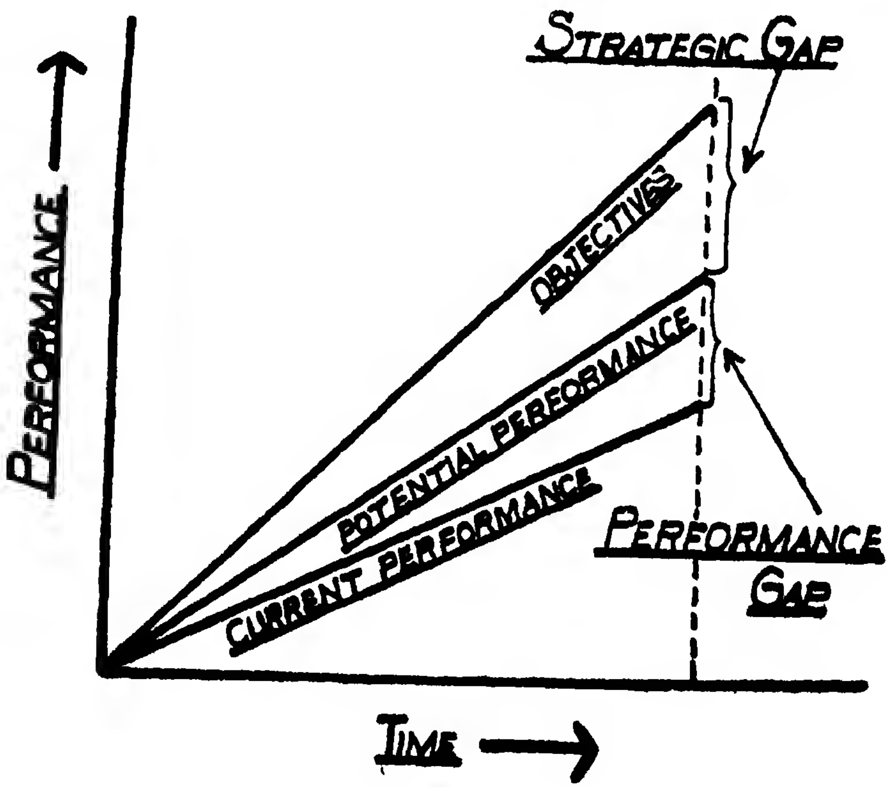


Figure 3

	Short term	*Versus*	*Long term*
1.	Short term profits		Long term growth
2.	Profit margin		Competitive position
3.	Direct sales efforts		Market development effort
4.	Penetration of existing market		Development of new Markets
5.	Related growth opportunities		Unrelated growth opportunities

How these strategic objectives are to be met is left to the planner who compiles resource development plants to fill the strategic gap and the performance gap depicted above. The objectives and performance are measured in terms of ROI The current performance can be improved upto the potential performance by accelerating the use of existing resources but to improve upon the potential performance extra resources would have to be employed.

The performance desired of the organisation should have an element of challenge lest the people tend to be complascent. It should also be not too difficult as this would discourage the people. This incidentally. is true at all levels as investigated by Hosfstede who, in his book, The Game of Budget Control, likens goal setting to games

Thus the legacy of the strategic plan is to develop the resources to fill the two gaps depicted above. The resource development planning is the instrument for doing so and it yields long range plans since their horizon extends from 3 to 5 years in the future. These are :

Market (Product market portfolio) planning : Diversification of product line and entering new product markets. Also, it includes expanding the present customers base by serving new market segments, increased use of selling outlets and/or distribution to sell the firm's products, change in pricing policies, considerations of new advertising media for more effective penetrration of the market, etc.

R and D & Engineering planning is geared towards creating new or improved products for established markets and for those markets in which the organisation does not participate or has only a negligible share.

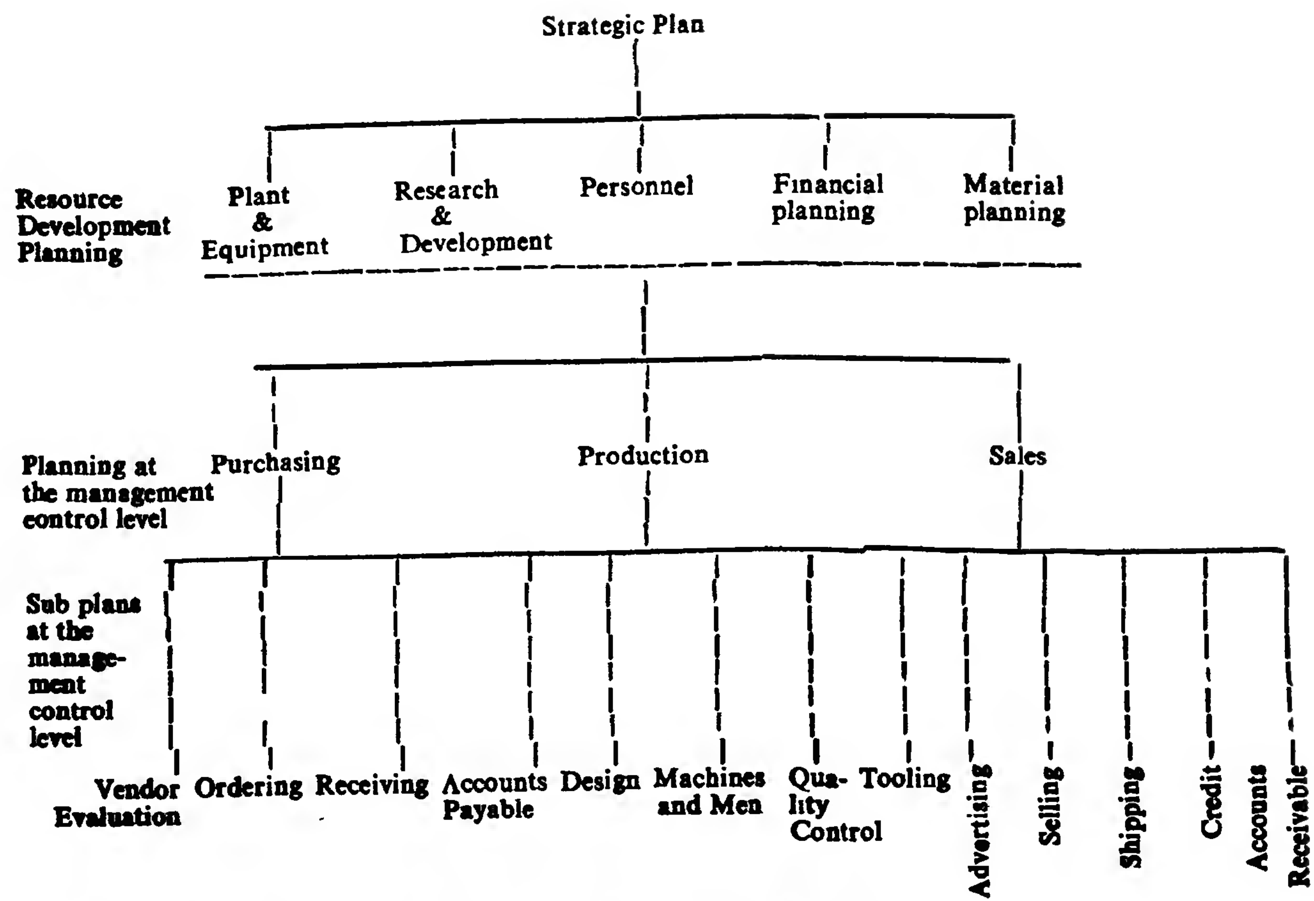
Manpower planning : Project manpower requirements for key management personnel and production labour considering turnover and future growth.

Material planning : Improvements in the materials used, development of new and better materials and scheduling the flow of materials. Materials planning is particularly important for process industries.

Financial planning is directed towards projected sales by product lines, gross profit by product lines, sales and general administrative expenses, net profit by product lines, sales and working capital needs, ROI on product lines, and comparative financial analyses and ratios. Financial planning is particularly important. It provides a framework for summary planning.

Planning at the management control level involves planning the expenditures and revenues for the coming year. Under expenditures are included the costs of all resources in terms of money and time. The allocation of resources and scheduling is done by functional classification of the purchasing, production and sales.

The chart on page 12 depicts the hierarchy of the plans.



There would be a series of plans at the operational control level too.

The planning Loop :. It is depicted in the figure below. Plan preparation consits of 4 phases :

1. Create alternative plans
2. Evaluate plans
3. Formalise and sell plans
4. Establish control procedures.

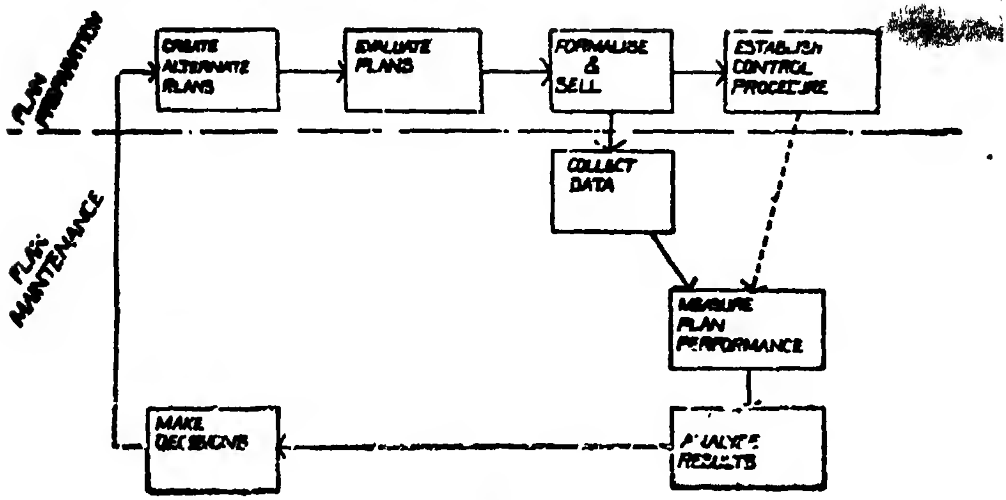


Figure 4

1. Create Alternative Plans : Plans are derived to meet the objectives set by the higher level plans and are based on expectations and forecasts of the variables of interest and the environment. Planning is thus inextricably mixed with forecasting. After all planning is concerned wih the future. The forecasting sub-system provides inputs to the planning sub-system. Consider derivation of the annual production plan. It is derived from the sales programme which is based on sales forecasts and disregards the man machine capacity on which it is liable to impose fluctuating load. Several (infinite, in theory) plans can be derived that meet the sales requirements

2. EvaluatePlans : The alternative plans are to be evaluated against one or more objectives. In case of production planning, the objective might well be to minimise the sum of (i) inventory carrying costs, (ii) change of production level costs, and (iii) change of work force level costs.

Management, may, however, impose such constraints as sub-contracting only to the tune of 1 lakh rupees is possible. Likewise, there could pervail such external constrains regarding firing of personnel (to change level of work force). From the narrowed set of plans the one that best meets the objectives has to be selected. In case of production planning process tends to be simple. The total cost for each plan is computed and the one that has the least total cost is selected. The problem can become complicated if there are several objectives with varying importance. For example, set of marketing plans may have to be evaluated against the following three objectives with inconsistent units.

Net Profit in rupees
Market share in precentage
Service time redution in days

Nevertheless, this problem can be resolved with operations research techniques.

3. *Formalise and Sall Plan* : After the optimal plan has been picked up it has to be formalised and sold to the management. The annual production plan, for example, would have to be detailed in a matrix form showing products in rows and months-wise production in columns. This plan, though theoretically optimal, may be subjected to extensive criticism and negotiations by the departmental heads in meetings to modify it so that it is viable from practical point of view Besides, the management has to be convinced. They usually have broader and overall perspective and can make important suggestions and even endeavour to relax some constraints The formalisation of the plan would require explicit statement of (i) objectives, (ii) action, (iii) measures of performance, and (iv) limits of acceptable performance. *Objectives* should be set forth explicitly and quantitatively, e.g., a plan would capture 13% of the market, diminish service duration from 10 to 4½ days and ensure a net profit of Rs 5½ laks, etc. Actions needed to support the plan should be listed. In the case of the production plan these may be :.

1. Sub-contracting schedule.
2. Facilities expansion/divestment schedule.
3. Material procurement schedule.
4. Work centres to be put on overtime.
5. Extra shifts etc.

Measures of performance may include, for production plan for expamle, units completed for some products and tonnes of production for others. It may also be desired to appraise performance by cost and the relevant techniques of standard costing may be specified. Remembering, however that everything happens in a probabilistic environment it is necessary to lay down *limits* within which performance is acceptable. Wherever possible unilateral limits should be preferred. Revenues, profits etc, if above the limits laid down, should be accepted as alright. Likewise, lower limits only for costs etc. should be stipulated.

With formlisation of the plan the task of plan preparation is over and the plan is implemented. Now arises the task of maintenance of the plan. This may be divided into 5 phases : 1. Establish control procedures 2 Collect data, 3. Measure plan performance. 4 Analyse results, and 5. Make Decisions.

Establish Control Procedures : This requires specification of variables to be measured and the frequency of Measurement. The performance against the annual production plan may, for example, be measured monthly. Whether it is going to be semi automatic control or status control, anticipatory control would have to be specified.

Collect Data : Data for measuring performance is usually collected from the transaction system and is then suitably compressed or aggregared. For resource development and strategic planning however, a great deal of external data may have to be collected. For example, for the long range market plan, the data on the following may have to be obtained annually :

(1) Market Share (%)
(2) Distributors signing (%)
(3) Extent of product awareness.

Measure Plan Performance : Actual results against the plan are presentend to the executive. Presentation should conform to statistical rules for the tabulation and charting, etc. Such complex hierachical measures as ROI should not only be presented in the summary figures but data on the entire hierarchy shold be presented so that the manager can spot reasons for shortfalls.

Analyse Result : This is executive's task. But he should not be complascent over fulfilled plans. High sales revenue may be attributable to boom in economy and to effort of the sales staff. It may be desired to perform statistical analysis to ascertain what part of the achieved results was produced by the plan, the forcerst, management and external forces.

Make Decision : Contrary to the usual notion that changes need to be made in the implementation process the plan itself may also be revised, or the forecast upon which the plan depends may be modified and even an entirely new plan may be formulated. Such decisions are the direct result of the 'analyse results' step above.

4.2 Forecasting : Forcasting is the projection of the past data into the future. There are numerous techniques of forecasting many of which are known to the student on the basis of Intermediate Statistics Study VII. We shall however. be more concerned here with forecasting types according to their relationships with planning and control

Strategic Forecasting is concerned mainly with market, technological, economic and social and political aspects. The marketing system may be considered as embedded in the technological system which in turn is embedded in the economic system which in turn is embedded in the political and social system. Market place is the battleground for an organisation where direct, indirect and emerging competitors are the enemies. Forecasting the future state of market and market share requires, therefore, forecasting of technological, economic and social and political factors. Besides, the needs and wants of the consumer must be forecast.

Technological Forecasting : Emergent competitors with different technologies are usually the most dreaded. Nylon destroyed silk and was itself taken over by terrylence. Plastics have replaced china crockery, straw chair caning, glass bottles jute goods, steel and other metal utensils, etc. This requires forecasting for all technologies that may have effect on the organisation. Such forecasting would be based on own R & D efforts, and perhaps, intelligence information on emergent R & D's efforts. Incidentally, developments of computer technology itself should be carefully watched.

Economic Forecasting : (Please refer to your studies in Economics) Statistics and projections of the following factors are usually available in governmental journals and publications ;

1. Employment and unemployment.
2. Production, income, consumption and trade.
3. Fixed capital investment.
4. Inventories and inventory investment.

5. Prices, costs and profits.
6. Money and Credit.
7. Foreign trade and payments

The cencept of lead, lag and coincident series is particularly useful here. It is also possible to predict economic depression and booms by splitting economic series into trend, seasonnal, cyclical and erratic components.

Political and Social Forecasting are the broadest and perhaps the most difficult, Siegel offers the following two sets of factors that may have to be forecast to support strategic planning. At a more mundane level however is tax forecasting

Political Influence for change.

1 Increased discrepancy between poor and rich countries
2 Revolution in human rights.
3 Government control of innovation.
4 Government control of ethics.
5 Government control of data.
6 More officiency in government

Social influences for change

1. Increased emphasis on ecology.
2. Increased population
3. Increased domination by intellectuals
4. Increased life span.
5. Emphasis on arts or leisure activities.
6 Changing morals and nature of marriage.

Resource Development Forecasting provides input to the resource development planning system If resources are growing in the environment more will be available for the firm. The following resources are to be considered

Plant and Equipment: Included in this category are the furture availability of plant sites and organisations for merger

Materials What materials will be available ? In India, the non-availability of steel and cement is the major constraint on planning for most of the industry Therefore, future projections for these and other such material in short supply should be made. What materials will be needed, for the future plans to be executed ?

Personnel: People of what skill and qualifications would be required ? Would they be available i e , would the institutions and training centres be turning out such people to the desired extent ? What inducements would be needed to recruit key managers ?

Finance: How will the interest rate vary ? What will happen to the price of our shares ? What means of borrowing will be available?

R & D: What developments are to be expected ? What is the scope of emergent technologies ?

At management control and operational control levels forecasting would be needed to support sales, production, inventory and purchasing and budgeting for various departments.

The forecasting loop is depicted in the figure below. The student should be able to exemplify these steps on the basis of his studies in Intermediate Statistics

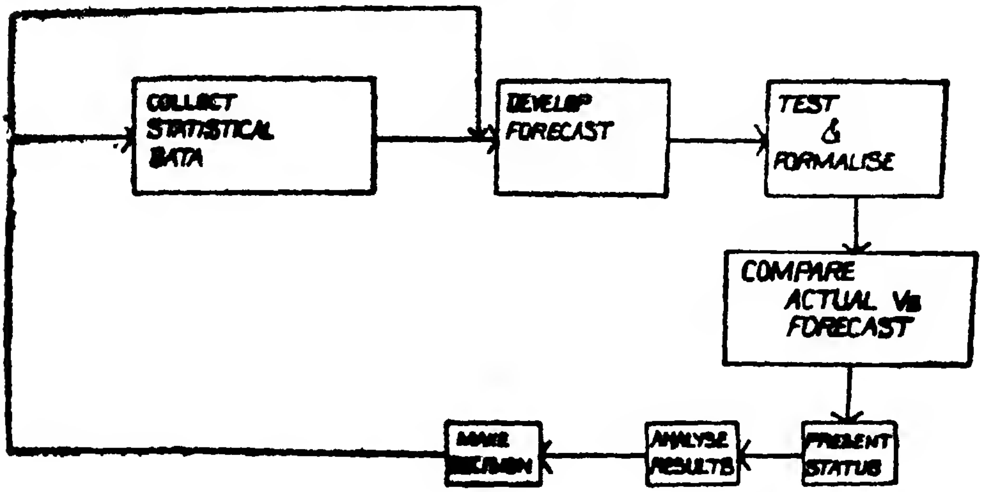


Figure 5

4.3 **Control** : Control means constraining the events to follow the plans. This requires periodic (or even continuous where possible and feasible) measurement of progress against plans, computations of shortfalls, variances or deviations. Based on the variances corrective action is initiated in the implementation process, the implication being that some of the variables of the implementation process can be manipulated by the supervisor

There is a hierarchy of plans, with hierarchy of associated controls too. Planning process, however, is unstructured. i.e., it varies from problem to problem. This fact is well brought out by Caroll when he says, "Planning is characterised by criteria problems, iteration and reiteration of search procedures, and a look of perplexity on the faces of the practitioners". But this is not so with control which is more or less structured i.e., whatever the plan it is the same. Nevertheless, the following four gradations of control can be distinguished.

Automatic Control : It is rarely used in MIS. An example, however is given to elucidate the gradations of the other controls Numerically controlled machines are the latest technical development in machining. These machines are capable of cutting complex and different contours on components. Once the co-ordinates of the contour are fed into the computer the program monitors the tool along the contour, i.e., there is no human intervention whatsoever.

Semi automatic control is usually possible to exercise at operational control level. The computer can execute the decision rules of the control process, the implication being that the decision rules are programmable (Reference : Section No. 6-2-2). Consider an example in inventory control. It is to be noted that inventory (planning and) control is slightly different from other controls in that here planned inventory levels are varied periodically in sympathy with the

fluctuation in demand i e , there is no control over the demand and all that can be done is to revise the stock levels. This is accomplished by revising the reorder levels and E O Q's periodically To clarify this point the fixed order quantity stock control system is briefly discussed below. It is, however, discussed at much greater length in OR Study II. The fixed order quantity system is controlled by two parameters which can be varied in sympathy with the demand fluctuations : order size and reorder level. The stock fluctuations are depicted in the diagram below

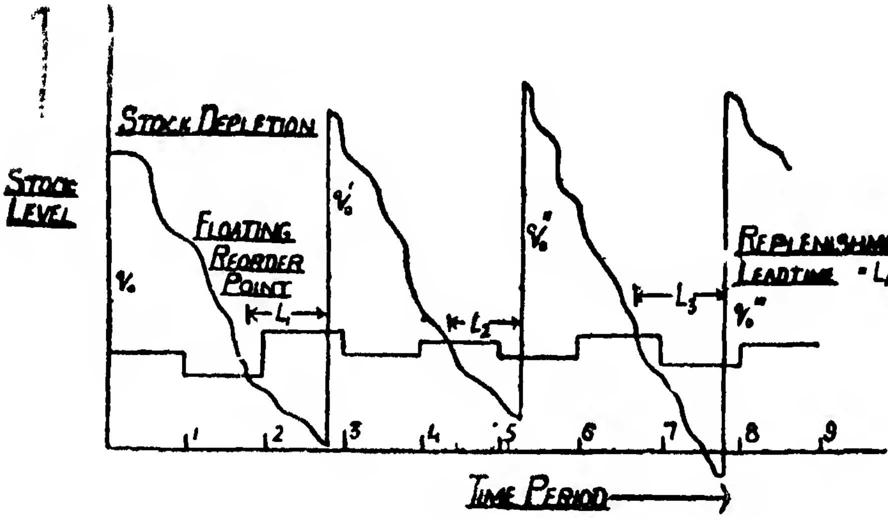


Figure 6

It can be seen that the reorder level is revised periodically. The mathmatical details of this are discussed in OR Study II. Even the EOQ may be revised periodically or at the time of placement of the replenishment order The decision rules for recomputing ROL and EOQ every period against fluctuating demand can be embedded in the computer program and the control, then, is semi automatic, because occasionally the decision rules themselves may have to be revised in the face of ever-changing circumstances. As explained in OR Study II reorder level is a function of the desired service level to the customer. In a fiercer competition the management may want to improve the service level, i.e., change the decision rules for execution by the computer,

Another version of semi-automatic control is explained by means of an example in production control Consider an assembly of 50 components scheduled for production on the shop floor If, even a couple of the components are delayed beyond the schedule, despite genuine efforts, the other 48 components would have to be postponed accordingly. Not only this, even the other products scheduled alongwith would have to be rescheduled to exploit the capacity created by the delay in the assembly under consideration. This amounts to revising the schedule i e, plan, and being a very complex process; it can be done by the computer only. It would be forbiddingly expensive and time consuming manually. Thus the computer is assisting the supervisor in initiating the corrective action. It is to be

carefully noted, however, that the components had been manually expedited before the computer action. This then is another version of the semi-automatic control for which the computer, according to the decision rules built into its program, assists the supervisor for initiating corrective action.

Yet another example in semi-automatic control is provided by statistical quality control. This, again is at the operational control level. Quality of any production process is inherently variable owing to scores of such minor causes as humidity, tool wear, etc. Being variable it can be described by one of

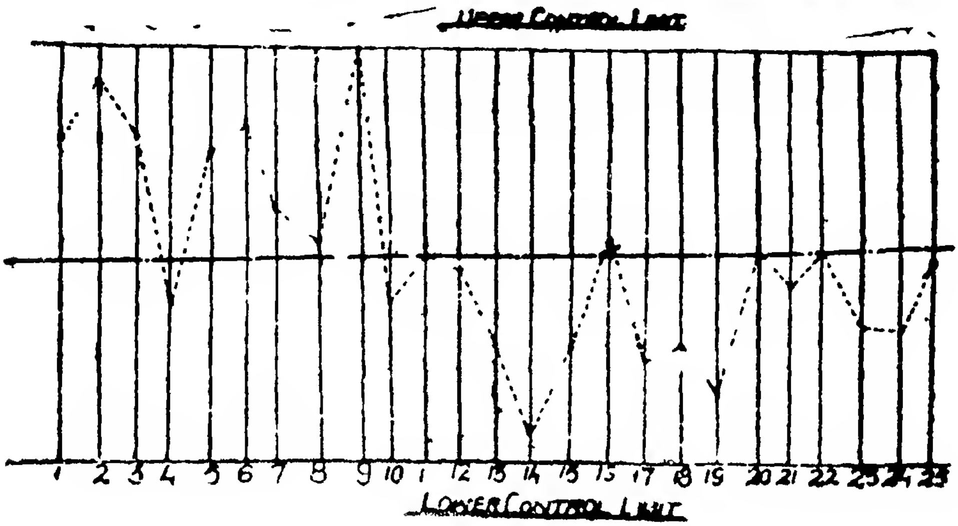


Figure 7

such distribution as normal, Poisson or binomal—3 sigma limits around the central quality level can then be statistically established and plotted in the form of control chart depicted above. Batches of output are then taken periodically, the average quality of the batch is measured and a point for it is plotted on the control chart. The UCL & LCL represent the planned levels If the point lies beyond these, this is a signal for the deteriorating process i e some abnormal causes (like negligence in lubricating the machine) are operating which should be investigated and the process set right Considering that (i) there could be hundreds of components each with several dimensions for quality control and (ii) a great deal of computations (but with derivable statistical decision rules) computer can carry out (semi-automatic)control

Status Control : All operational plans, however, cannot be controlled by means of the computer. For example, daily or weekly cost variances have to be analysed by the supervisor himself Computer cannot do this. Likewise, at the management control level semi-automatic control is inapplicable. There are always some data not in the machine, some considetations that are non-quantifiable and some constraints that are subjective. Here then status control is applicable. Status of progress is measured periodically and then compared with the plan. Computer can, however, provide assistance in aggrregating the data in a

variety of ways. Data representing achieved performance are gathered from transaction system and compared against the performance specified in the operational plan The supervisor analyses the variances and initiates corrective action in the implementation process.

Anticipatory Control is generally exercised at the resource development planning level. The predicted performance is measured against the resource development plan This process is called trend measurement. The supervisor initiates corrective action on the basis of predicted variances.

The nature of Control : The idea of control may be represented in elementary form as in figure 8. Control comprises the following components :

The System which can be defined as an assemblage of objects or functions united by some regular interaction and interdependence.

Input consisisting mainly of parameters, *parameters* are those variables which can be varied at will by a controller.

The Output . Consisting of list of other variables which can only be altered indirectly by varying the input and remeasured at comnenient intervals of time and points

The feed-back · The measured information is fed back to a controller who, in the light of this information against the standards or plans, alters input parameters In the feedback we are interested in the error or divergence or variance from what we planned or expected. The controller will make correction in the opposite direction to bring operation of system back to what is required Hence, what is required is negative feedback. The length of time the feedback process

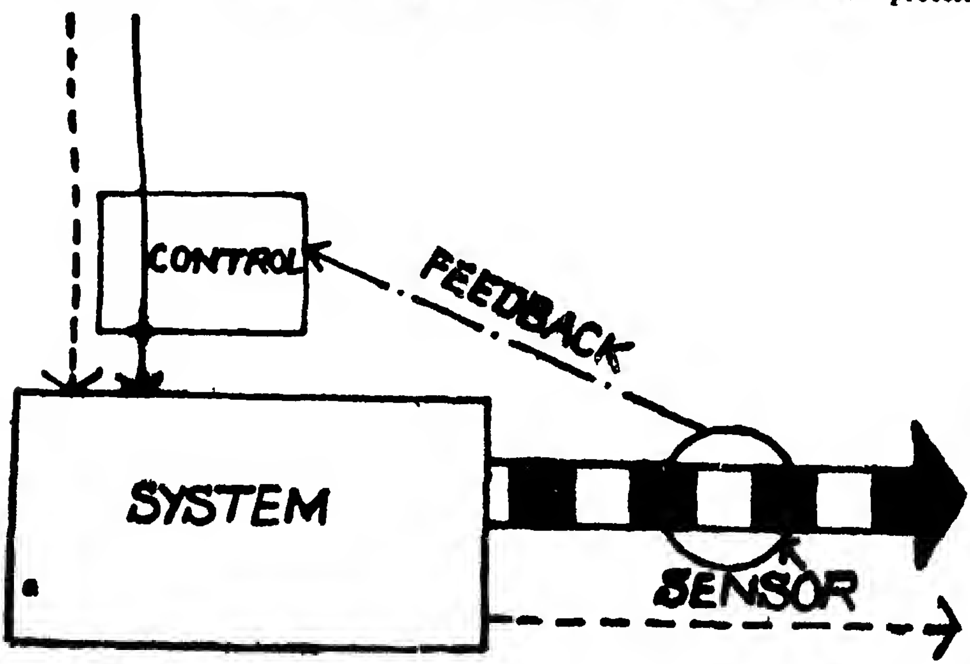


Fig. 8

takes is *important for, until correction of input takes place, the system will be producing inaccurate output,*

The measuring device to measure the progress of the system is known as a *sensor* which being a fitting nomanclature in the context of machanical systems. It requires a unit of measurement for the outputs. Frequently, it is not possible to devise appropriate measures of some outputs. For example, the machine shops of the Heavy Engineering Corporation at Ranchi produce machined components varying greatly in size, shape and weight. The output, therefore cannot be measured statisfactorily and all the small or large parts are merged together for this purpose, measurement unit being tonnes of production Some other outputs may either not be possible or too costly to measure Such variables would have obviously to be ignored in control. Having selected variables to control comes the question of establishing the periodicity of measurement which, in the context of MIS, is usually in the form of classified data aggregated in the form of reports For example, at the management control level the progress may be measured monthly and at the operational control level the progress may be weekly, daily and even hourly basis Occasionaly progress may also be measured continuously. Also it is desirable to check the sensor periodically to ascertain if it is defective or biased or recording false information in some other way. Regarding bias, consider an example in reporting of production. The production superintendents usually tend to report progress on the items put to inspection whereas actually progress is the items okayed by inspection. This introduces bias in the sensor Also sometimes the collection of information may seriously affect the working of the system itself and hamper it. For example, this may be so if long and complicated forms have to be filled in by the production workers

4 4. **Modelling** . A scientific model is a representation of some subject of inquiry. The subject may be objects, events, processes or system. The model may be used for purposes of description, prediction and control. The model affords easy manipultation of some of its aspects to determine the effects of such a manipulation on other aspects or the whole. This, then, can be generalised for the modelled entity. The real system of the entity is not, therefore, disturbed during such experimentation. It would be either impossible or extremely difficult and expensive to experiment with the real system. Industrial organisations are quite complex and to determine the effects of such changes as scheduling rules or inventory policies on the organisation directly would be too time consuming Not only that, such experimentation is liable to upset the entire working of the organisation into chaos. It will be absurd for the Reserve Bank, for example, to try different discount rates to assess their impact on the national economy.

Three types of models below may be distinguished (Ref. OR Study I) 1. Iconic Models, 2 Analogue Models and 3 Symbolic Models. For forecasting and planning symbolic models are mostly used. Analogue models find extensive use in control

Models and Planning : Modelling is common denominator of forecasting, planning and control, all the three of which are inextricably mixed. Planning without forecasting and control is unthinkable. When we say management control or operational control forecasting and planning are also implied. Likewise, when

we say strategic planning, forecasting too is implied—in this particular case though control is virtually non-existent. The purpose of discussing planning, forecasting and control separately has been to set forth the management planning and control activity as explicitly as possible since this is what has to be supported by MIS Now each forecast, each plan and each control is derived from a model Control models are mostly analogue in nature but forecasting and planning models are usually symbolic. (*Below (p 23) is given the proposed modelling structure for a business organisation*) Models bring out the information requirements most explicitly. Once each of the models in this structure is clearly defined information requirements for planning and control are completely and comprehensively determined. The need to define models is particularly acute in computerised systems since computer would only hum if something is left undefined In manual systems, the manager can exercise judgement and intuition but is not modelling the basis of judgement and intuition there as well ?

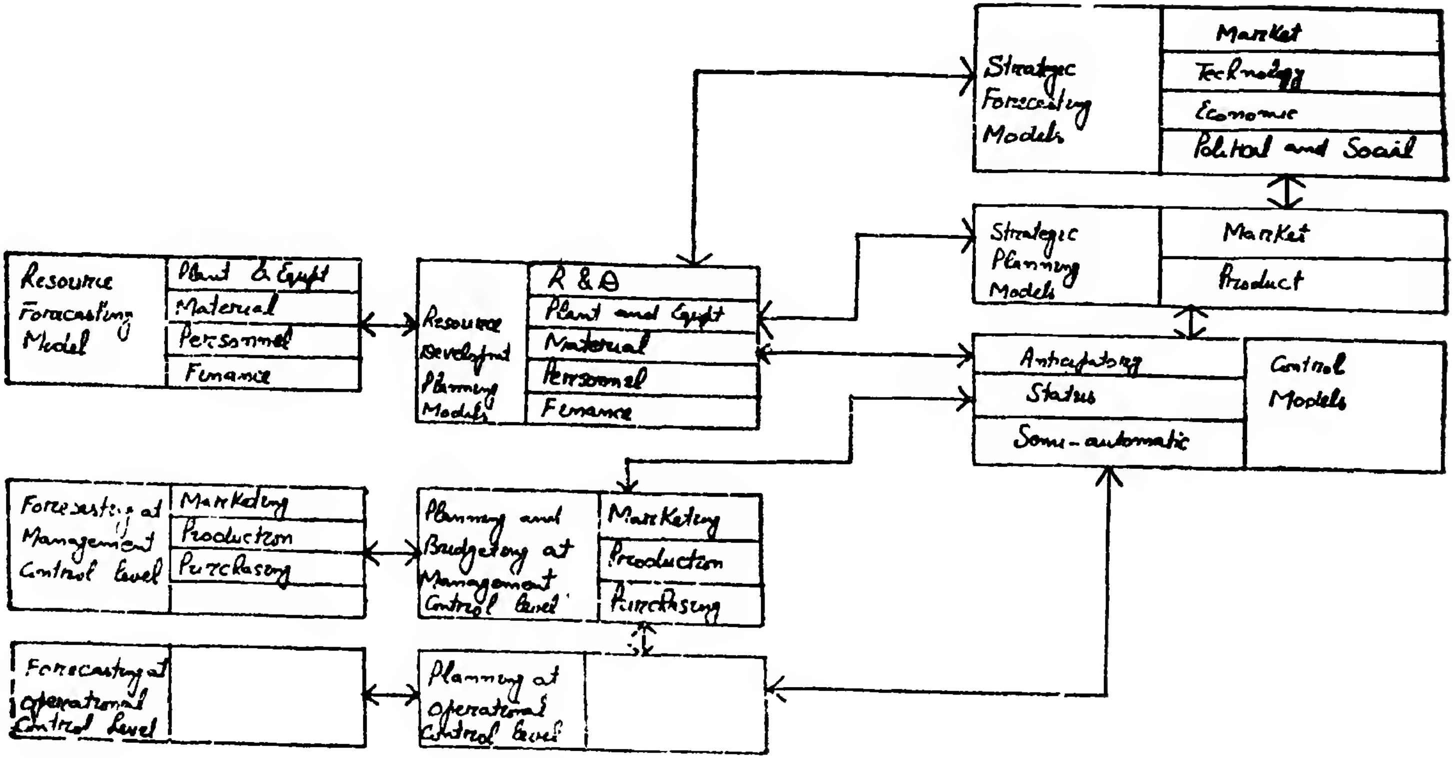
Simulation models (Ref, OR Study VI) are a special class of symbolic models They relate probabilistic variables by essentially executing a series of events at random, but in such a way as to conform to the given probability distribution Often we employ a conglomerate of models i e. several models interacting with one another. An example on such a conglomerate is to be found in OR Study II where exponential smoothing forecasting and E O Q. models have been combined Simulation is particularly useful in testing if such models in combination work satisfactorily

The student may also refer to OR Study I for phases of model building.

4 5 **Computing** Computing is systems analysis and programming. The various phases of systems analysis are discussed in detail in SADP Study III. They are listed below :

I Area Selection
II Feasibility Study
III Compilation of Master Development Plan
IV Systems Analysis
V Systems Design
VI Systems Implementation

A few words here on systems analysis and MIS are necessary The two are practically synonymous in U S A and mainly the latter term is used. In U K., especially in the past, systems analysis has been a more prevalent term though MIS is steadily finding use. Systems Analysis has the connotation of a one-shot activity i e. system is designed, computer installed and the systems analysts team disbanded to be engaged again only upon need of any major systems overhaul and advancement or dismantling of the existing computer installation. This though, was fairly widely practiced in the past, is increasingly being abandoned in favour of continuous systems development of projects for computerisation as continually trigered by the systems maintenance (Syn : audit) activity. Organisational objectives, and hence the MIS objectives, do undergo continual change via strategic planning amidst everchaning world. Therefore, systems analysis has to be a continuous activity Continuity, besides being essential, also, permits sophistication of planning, forecasting, control and modelling abilities with time,



GENERALISED MODELLING STRUCTURE

Fig 9

We derived the modelling structure to support planning and control activity as the gist of discussion on planning control and forecasting Where does it fit then ? In discussion on compilation of the master development plan in SADP Study III, we mentioned the bottom up approach It requires what we do not jump to a highly sophisticated computerised system but approach it in stages the description of which is reproduced from SADP Study III below for quick ref.rence

Bottom up Approach It consists of the following five steps :—

1 Individual functional applications are planned separately and they consist of transaction processing, updating of files and simple reports

2 The files of the various functional applications are integrated by means of indexing and chaining into a data base On line inquiries can now be handled by means of the random access disc storage The information requirements of the operational level are thus completely met at this stage

3 Such decison making and planning models for the various functions as follows are then added to operate on the data base at the management control level production planning by linear programming, etc,

4. Integration of models into a model base having a wide variety of analysis, decision and planning models The model and data bases may be enlarged to include data for the expanded use. The ROI model, for example, straddles over different funcions and it would not by possible to compute ROI in one stroke under the scheme of step 3 (See Study Paper No. 1 in Advanced Accounting).

5 Strategic planning data and planning models are added to the information system. The data requirements for strategic planning involve new environment data which should be added to the two bases in this final stage"

Consider now stage 2 of the bottom-up approach. It calls for development of a data base which is discussed more fully in the following section It is the stage 3 and beyond where tuning in of the modelling structure of figure 9 is called for The modelling structure may stay the same but surely the variable, parameters, and constants and even structure of the individual models would undergo change. Such changes would be brought to the analyst's notice during the systems maintenance phase Thus initially the structure would have to be built into MIS but subsequent appropriate changes would be continually incorporated in it.

4.6 Data Base Administration : In a small organisation, data processing functions are performed by a clerk who would have a set of thick and well bound ledgers in front of him. He processes an order in entirety starting from updating the stock ledger and placing (if necessary) a replenishment order, updating the order book and billing the customer Likewise he can answer queries of the owner, customers and vendors by flipping through appropriate ledgers He would balance the books at the end of day and if costing figures were needed, he could maintain separate records for them too. In a way, then he has all the needed infor-

motion at his finger tips. As the organisation grows in size, the number of ledgers and the clerks to maintain them rise, division of labour takes place Each clerk specialises in a particular task. Data processing may reach the threshold where unit record equipment or computer is considered more economical To make efficient use of such mechanised systems batch processing is usually adopted. Different kinds of transactions are processed in different machine setups in sizeable batches. But this does away with flexibility of the early-day methods of the single clerk. Records are out of date by a week or whatever the processing period. Furthermore, whether or not there was transaction against an item, it would be written out afresh (viz on the magnetic tape) And inquiry handling is virtually impossible. For example, ROI data staddles several functions and if the manager wanted its value at particular date, several files may have to processed to compress the raw data into this information. With the increasing size and complexity of the business such inquiries become everyday occurrence. Thus, batch processing may ensure good utilisation and productivity of the computer but it cannot fulfil the varied information needs of the business. The more advanced system of organising the data in a data base offers considerable flexibility. In a data base, all the data items, no matter to which department they belonged originally in the manual system, are pooled together, usually, in the magnetic disc storage Just as a clerk can flip through several files promptly to collect and aggregate data for presenting such information as ROI, the data base can also be quickly and directly searched To enable this search, however, the various data items in the hierarchy of RCI are suitably linked Therefore, to embed the necessary linkages such information needs as ROI have to be conceived during the initial stage of organising the data base Since, however, businesses operate in a dynamic environment the information needs are likely to undergo change The data base should therefore be restructurable to cater to the changing needs Compression or aggregation of data however is not the only facility provided by the data base. It also offers the possibility of experimentation with data. Executives often need information on such matters as effects of slashing the advertising budget by, say 10% or the personnel implications of the marketing decisions or the impact on production of a new distribution strategy or the labour costs associated with higher sales, etc Such types of experiments can be conducted on the data base and the necessary information supplied to the executive, For these reasons, data is increasingly being viewed as a basic resource As with other resources, professional management and organisation of the data are needed. The importance of making efficient use of data will become so great in computerised systems that this will have a major impact on the growth and survival of corporations. Data base ensures both efficiency of storage and efficiency of processing, the former stemming from elimination of redundant data fields *See the following charts intended to bring out extent of redunpancy.* The processing efficiency not only results from reduced file storge but because the logically related data which, otherwise, would be maintained in separate files, is now found linked in a single file This has already been illustrated by means of an example on information for ROI Sorting, comparisons and merging which are often required in batch processing are annihilated The data base improves efficiency for retrieval of information because all the files are found logically related in single place.

The difficulties of standardising and linking the various data fields and maintaining the base can be a formidable task even in a moderate size organisation and

indeed is a Herculean task in corporations with several plants and divisions, hence the need of the data in charge known as the data base administrator. Like a librarian who is the custodian of the library the data base administrator (D,B.A.) is the custodian of the data base. He controls the overall structure of the data base and it indeed is an onerous task considering that he should not only be an expert in data base technology but should be well informed on organisational policies.

DATA ITEMS	RECORDS AND DOCUMENTS: ORDER FROM CUSTOMER	CUSTOMER FILE	WAREHOUSE ORDER	WAREHOUSE TICKET	INVOICE	WEEKLY TOTAL SHIPMENT REPORT	WEEKLY ITEM SHIPMENT REPORT	YEAR-TO-DATE SHIPMENT REPORT	YEAR-TO-DATE ITEM REPORT	ITEM FILE	WEEKLY ITEM INVENTORY REPORT	INVENTORY FILE	BRANCH FILE	MONTHLY BRANCH SUMMARY	YEAR TO DATE SALES REPORT	ITEM ON-ORDER REPORT	TOTAL ON-ORDER REPORT	MONTHLY CUSTOMER REPORT	CUSTOMER PAYMENT	ACCOUNTS RECEIVABLE	OVERDUE ACCOUNTS	OVERDUE NOTICES
CUSTOMER NUMBER	X	X	•		•													•	•	•	•	•
CUSTOMER ORDER NUMBER	•		•		•														•	•	•	•
ITEM NUMBER	•		•	•	•		•		•	•	•	•				•		•				•
ITEM TYPE AND SIZE	•		•	•	•		•		•	•	•	•				•		•				•
CORPORATE ORDER NUMBER			•	•	•		•											•		•	•	
INVOICE NUMBER					•														•	•	•	•
BRANCH OFFICE NUMBER	•				•								•	•	•		•	•		•	•	•
CUSTOMER NAME	•	•			•									•				•	•	•	•	•
ITEM NAME	•		•	•	•		•		•	•	•	•			•	•		•				•
DATE OF INVOICE					•															•	•	•
QUANTITY ORDERED	•		•	•	•				•		•	•				•		•				•
QUANTITY SHIPPED				•	•	•	•	•										•		•	•	•
QUANTITY OUT OF STOCK				•	•	•	•	•	•		•							•				
SHIPPING INSTRUCTIONS			•	•	•																	
CODE FOR SHIPPING			•	•	•																	
CUSTOMER ADDRESS		•			•																	
BRANCH OFFICE NAME	•	•			•								•	•	•		•	•				•
BRANCH OFFICE ADDRESS	•				•								•									•
PRICE					•	•	•	•	•	•									•	•	•	•
INVOICE LINE VALUE					•															•	•	•
DISCOUNT RATE AND QUALIFN					•					•										•	•	•
INVOICE LINE DISCOUNT					•																	•
C.O D OR CREDIT CODE	•	•			•													•				•
WEEKLY TOTAL OF ITEM SHIPPED						•	•	•	•		•											
WEEKLY VALUE OF ITEM SHIPPED						•	•	•	•													

The same data items are used in multiple records and documents.
Figure 10 (Source : Principles of Data-Base Management by Martin J)

It is to be noted however that like the custodian of lockers in a bank the D.B A. is merely a custodian and not the owner of data. He is responsible for safekeeping and controlling the data. Data may be used by anyone who is entitled. Also just as the locker custodian knows that such and such person owns such number locker but is ignorant of its contents the D B A too, knows, for example, that the PAYROLL record has a SALARY data item and if it has to be expanded from 6 to 8 digits only the D.B A. would do it. The D B A. encourages standardisation of data items and establishes the right data structures and layouts Therefore, if a programmer wishes to amend a record or data items he must seek permission of the D.B.A. since the latter has an overall view of the system and will make the right arrangement. Since data base administration requires understanding of the data

RECORDS OR DATA GROUPINGS	APPLICATIONS: ACCOUNT ORIGINATION	ACCOUNT UPDATING	TRANSACTION PROCESSING			CREDIT APPLICATION	CREDIT EVALUATION	LOAN TRANSACTION PROCESSING	STATEMENTS			CASH POSITION MANAGEMENT	PROFIT PLANNING	RESOURCE/LIABILITY FORECASTING	CUSTOMER PROFITABILITY REPORTING	MANAGEMENT FINANCIAL REPORTING		
CUSTOMER INFORMATION	•	•	•			•	•	•	•						•			
CHECKING ACCOUNTS		•	•			•	•		•									
SPECIAL ACCOUNTS		•	•			•	•		•									
SAVINGS		•	•			•			•									
DEMAND DEPOSITS		•	•			•			•									
REAL ESTATE LOANS						•		•										
COLLATERAL LOANS								•										
MISCELLANEOUS LOANS								•										
TIDEOVER								•	•									
AUTHORIZATION I.D	•	•	•															
CREDIT SUPPORT	•					•	•											
ACCOUNT HISTORY							•								•			
⋮																		
⋮																		
ACCOUNTS RECEIVABLE												•	•	•		•		
GENERAL ACCOUNTING												•	•	•		•		
GENERAL LEDGER																		
READY RESERVE												•	•	•		•		
CENSUS																		
CASH INVENTORY												•	•	•		•		
⋮																		

Source : Principles of Data Base Management by Martin J.
Figure 11. The same files or data grouping are used in multiple applications.

		User Group	System Analysis And Application Programmers	Staff Data Base Administration	Corporate Data Administration
1	Corporate Information Strategy	C	C	P	Prime Responsibility
2	Project Selection	P	P	P	Prime Responsibility
3.	Project Feasibility Study	P	Prime Responsibility	P	P
4	Selection of Data Base Software		P	P	Prime Responsibility
5.	Data-Base Design and development	C	P	Prime Responsibility	P
6.	Application Design	P	Prime Responsibility	P	C
7.	Application Programming and Testing	P	Prime Responsibility	P	
8.	System Implementation and Testing	P	Prime Responsibility	P	
9	System Maintenance	P	Prime Responsibility	P	
10	Data-Base Maintenance	C	C	Prime Responsibility	P
11.	Data-Base Accuracy and Security	C	C	Prime Responsibility	P

P=Participating Responsibility

C=Consulting when Required

Recommended breakdown of data-base responsibilities

Source Principles of Data Base Management by J. Martin. Prentice-Hall)

base, its organisation, its economics, its design criteria requirements of numerous users the D.B.A is not usually one person but a department or a group. The chart on page 28 shows the responsibilities of the users (planners, forecasters, supervisors and modellers) : systems, analysis, application, programmers, D.B A. and corporate D.B.A The corporate D B A. is also known as the data administrator and supervises the D B.A.'s of several data bases to be found in large corporations.

Users and Suppliers of Data Base : Users include the application programmers, planners, forecasters, modellers and supervisors (including managers). With complete directory or dictionary (syn · schema) of the data items the users know what data is being maintained in the data base and can forecast plan, control, model and program much better than was ever possible with individual appiications. Also, the user will find much easier access to data because of the better retrieval and report generation capabilities. The programmers are however restricted in that they can not now alter data fiels without D.B.A 's permission. Likewise the users would be issued passwords for accessing only the data they are entitled to. Thus all the users have to sacrifice some of the facilities available with individualised applications to reap the benefits of the data base. The suppliers of the data (viz sales department for customer's orders and remittance advices, goods receiving section for goods received, etc) would find that there is greater emphasis upon the integrity of data since an error now in the input data may effect several users. The data quality standards will be enforced by the D.B A.

The Schema : The schema is the information that describes the data base. Especially one can think of schema as a dictionary which describes all the data items by name, length, etc. i.e. all the data items and relationships amongst them. For example, the PURCHASE ORDER record is connected to the PURCHASE ITEM records of which the former is composed. Likewise the SUPPLIER RECORD is connected to the QUOTATION RECORDS indicating the parts that the supplier can supply and the relevant prices, discounts, terms etc. Such relationships are suitably portrayed in the schema.

The sub schema : The sub-schema refers to the programmer's view of the schema of the data item types and records types he uses Thus sub schema is a subset of the schema which the programmer puts to use in his application program. Consider a stock item's record as schema below.

Item No. .... Description .... Specification .... Date ......
Status Units ABC category Cost Service factor
ROL ....EOQ ....Leadtime......Supplier. ....

Receipt/Issue			*Balances*				*Forecast & Re-order Data*			
Voucher No.	Date	Qty.	Physical	Potential	Total Outstanding	Total Allocation	Average	Trend	MAD	ROL

The sub-schema for an application programmer for updating stock records for goods received would consist of Item No. and Balances. The sub-schema for a programmer wishing to write a program for revising forecasts and re-order levels periodically would consist of the service factor and forecast and reorder data. This sub schema is programmer's glimpse of the globe that is the schema. It is to be noted that schema depicts only the logical relationship between records and not physical storage. Thus the following 3 data descriptions emerge ;

The sub-schema : A chart of a portion of the data which is oriented to the needs of one or more application programmers (application programmers' view)

The schema · A chart of the entire logical data base (D.B.A.'s view).

The physical Data Base Description · A chart of the physical layout of the data on the storage devices (system designer's and systems programmer's view).

Logical and Physical Records

A punched card (containing. say, 4 transactions) constitutes one logical but 4 physical records. Likewise. a customer name and address record card followed by 9 cards, one each for the 9 items he ordered constitutes one logical record but 10 physical records

In the data base context, the data fields of a record may be dispersed in different files i.e physically they are not contiguous. Yet from the application programmer's point of view these diepersed fields pertaining to this record constitute one logical record.

Data languages are needed to represent the achema, sub-schemas and the physical layout of the data with pointers, chains etc The same language may be used for each of these as the example IBM's IMS or different languages like COBOL etc may be employed.

Referring to figure 12 program 1 carries complete like description of the files A and B on which it operates. But in figure 10 this program has just the sub-schema incorporated This requires less storage space for the program.

Now data base ought to be restructurable, i.e , if data base is reorganised to suit the modifications in one application program other programs should remain valid, i.e. they should not require modifications. This problem is an advanced topic and the student need not worry over the technical details.

Data base management system is a special software that does translation between the logical data organisations and whatever physical organisation gives efficient performance. It is interposed between the application programs and data base to permit restructurability.

Conclusion : In SADP Study III, one of the stated general objectives of MIS is to harmonise the interactions amongst forecasters, planners, supervisors, system analysts, programmers and DBA This should now be obvious from this discussion above that these people have a great and complex deal of interactions and ought work in harmony.

5. **Role of Computer in MIS** ; Computer's rolr in MIS, as should be apparent from the discussion on six levels of MIS, is to be appraised on the following

counts ; forecasting planning, control, modelling, systems analysis and facilities offered by the data base, as below.

1. Forecasting : (A) Computer can perform such historical analyses as below speedily which may be difficult or even impossible manually.

(i) *Date smoothing* by means of moving averages or weighted moving averages (including exponential smoothing) to disentangle the underlying trend from amdist irregularities.

(ii) *Seasonal Analysis* : Economic activity varies with seasons viz. sales of ice cream. The seasonal analysis can be used to compute the seasonal indices.

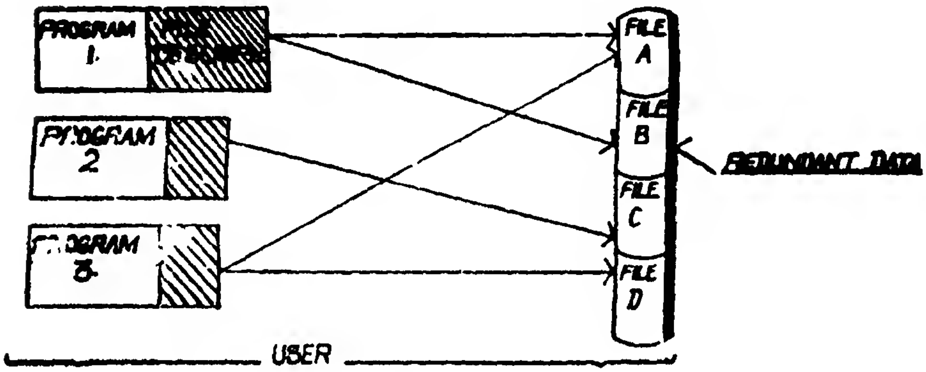


Figure 12

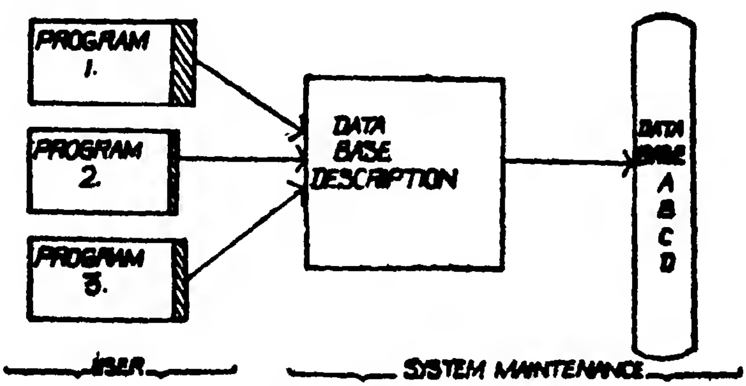


Figure 13

iii) *Cyclical Analysis* : Cycles of booms and depressions can be discerned by factoring an economic time series into trend, seasonal, cyclical and error components.

(iv) *Correlation and regression analysis* : Such relationships as the variance of sales is proportional to the mean sales etc. can be discovered. Multiple regression analysis involves a great deal of calculation and computer would be particularly useful here.

(v) *Auto-correlation Analysis* : There are some variables that have a time-delay relationship, viz sales of repair parts in a period is a function of the sales in the preceding period. Such relationships can be established.

(vi) *Data Description* : Various frequency distributions can be tried for a fit in the given data. Later, the goodness of the fit tests can be performed. Such summary statistics as mean, mode, median, SD, MAD, skewness, etc can be computed to describe the given data.

(vii) Optimal parameters of the selected forecasting model can be derived; for example, in simple exponential smoothing model what value of the smoothing coefficient to use ? Different values are tried on the historical data and the one that yeilds the least cumulative squared error is selected for implementation.

(B) For routine forecasting too, computer provides considerable assistance, For example demand forecasts for thousands of inventory items may have to be prepared every week, month, etc. which would be impossible manually but quite within the reach of the computer. Besides, such sophistications as adaptive response and tracking signals can be incorporated to monitor the forecasts.

Planning :

(A) Planning can be done more frequently and even continously. Because of constraints on computational and data processing abilities in manual systems, it used to be the practice to set reorder level annually Important demand fluctuations were thus ignored resulting in higher stocks and poorer service than is possible now in computerised systems where reorder levels can be revised much more frequently. Eilon reports 40% savings in stockholding for the same service by periodically revising reorder levels on the basis of his extensive simulation experimentation.

(B) Sophisticated models for planning can be employed. For example, linear programming, etc, can be used for production planning. In particular, project planning and control (both physical and cost-wise) and resource allocation can be excellently carried out by means of CPM/PERT on the computer.

(C) Simulation can speed up evaluation of alternative plans. Simulation is a method of reproducing the major elements of a dynamic situation within the foreshortened time available through programming a computer, Thus, instead of waiting for 2 years to see the results of a capital investment project, the 2 year's experience may be simulated on the computer,

(D) The user can interact with the computer to do better planning. For example, he may pose a question as what happens if the advertising budget is slashed by 15%. The computer, according to the mathematical model embedded in its program, would provide him with the consequences with which the user may not be satsisfied. Based on the answer, however, he may structure another question and so on until the man and machine have iterated down to a satisfactory solution with a

fine blend of their capabilities. The planner, together with this type of interactive computer, can probe for data items, compare data, analyse relationships, test for various conditions, evaluate, execute, and produce answers to many 'what if' questions. The interactive computer thus amplifies human intellect,

(E) For complex planning situations heuristic rules can be derived. Such a rule in production scheduling might be to schedule late items first, etc,

3 **Control** :

(A) More comprehensive, varied, accurate and frequent reports can be produced where necessary,

(B) Statistical quality control techniques can be used to monitor marketing and financial ratios,

(C) Variance analysis plus other analyses which might assist in understanding both the reasons for variances and also the courses of action that will be corrective can be undertaken

4. **Modelling** :

(A) Models can be tested for robustness simply or in conglomeration against historical or hypothesised data by simulation or otherwise *Sensitivity analysis* can be performed to see to what extent the input variables affect the output variables. Computer can also assist in model building to sort out relationships.

5. **Systems Analysis** :

(A) Several alternative MIS designs can be evaluated by means of simulation experimentation

6. **Data base facility** :

(A) Data is no longer vivisected departmentwise. All, planner forecaster, supervisor, systems analyst, and modeller, can discharge their functions much better.

6 2 2 Programmed vs. Non programmed decision : A Computer program is defined as a plan for the automatic solution of a problem and the nomenclatures 'programmed' and 'non-programmed' decisions are based on this definition The ultimate goal of MIS ought to be to automate as much decision-making as possible to relieve the human beings for exercising judgement intuition and deliberation on more complex non-programmed decision-making. Such decisions as are repetitive and routine are usually amenable to programming Examples would include deriving E O Q's, setting ROP's, making payroll deductions, appraising credit worthiness of customers, etc. Non-programmed decisions are usually new complex, non-structured, elusive or involve major commitments. Examples would include advertising budgets, introduction of a new product, pricing policies, etc. Now. whenever programmed decisions are based on statistical rules (viz setting safety stocks) there is no need for any extraneous human judgment over and above the decision rules derived initially Statistical decision rules would, in the long run provide the desired protection against unfavourable outcomes Thus, if decision rules for setting safety stocks have been derived for 5% disservice to the customer, their application on routine and repetitive basis will ensure this *in the long run*. There have been several instances in stock control circles where the junior clerks have reacted to unusually high or low demands (ignoring the decision rules by adjusting the reorder levels accordingly by judgement) This only resulted into fluctuations in the inventory levels that were wilder than the demand fluctuations.

Making programmed decisions is standardisation of information in the same way as we standardise materials for production of products. In a survey, it was concluded that about 90% of the decisions the managers make can be programmed Decision-making is thus a lucrative area for automation Computer can make programmed decisions accurately (according to mathematical decision rules embedded in its programs) and consistently as against the human beings who do so by habit and hunches and are neither accurate nor consistent. Just as production processes can be automated to great many advantages so also decision-making in MIS This revolution can be brought about by management science (operations research and associated mathematical and statistical tools), computer and recognising situations for the application of management science and computer. It is however, to be noted that there is considerable scope for programmed decision making in manual systems too The decision rules for computing E O Q 's may be entrusted to clerks for application, for example they can use E O Q slide rules nomograms etc as aids just as interest tables are used in banks In manual system, then, programming decision making would also cut short the training period of the new entrants

The following procedure may be adopted for programmed decision [illegible] computerised systems

Analyse the problem by means of management science approach [illegible] a decision-rule that solves all applications Appendix I should give [illegible] breadth of areas over which management science or OR models are [illegible] an example however, the following E.O Q. formula is a decision [illegible] mal order size from amongst an infinite alternatives (i e , order size)

$$E\,O\,Q = \sqrt{\frac{2CaA}{IC}}$$

Where Ca = Acquisition Cost/order   A = annual demand
I = Rate of Return   C = unit cost

2. Program the decision rules for the computer In the E O Q example, this may require writing the necessary instructions for calculating E O Q and embodying them as a sub-routine in the computer program

3. Design the input of the computer information for automatic decision by the computer In the master stock control file, the purchase card for each item should then carry data fields for C and A., Ca and I (for bought-outs may, however, be put in the program if they are common to items. For all made-ins Ca would be derived as below :

Ca (f) [setup time X cost/hour of the operation] Ca (f) is the fixed component of Ca and reflects the costs of the drawings works order etc released for each order. The variable component varies with item. It is summation of the product of setup time for each machine and the rate (in Rs) of the operation. This would require that necessary data fields are included in the master records

It might appear from the discussion above that decision rules once derived are valid for ever. This is not necessarily and usually so E.O Q 's may not, for example, be applicable for a new year The demand of goods rises and the E O Q's computed according to original rules may violate the total storage space and/or capital that can be released for them. This would require modification of the original decision rule. Likewise the RBI may change the interest rate and the

banks would have to incorporate the changed rate in their programs. Thus an occasional human intervention may be needed to attune the decision rules to changing circumstances.

Now refer to figure 1. It is to be seen that lower levels of MIS are computer dominated i e, there is a great scope for automating decision making for accounts receivable, payroll, inventory control, order placement (vendor selection etc.) customer billing, shipping schedules etc but as we proceed upwards non-programmed decision-making assumes greater importance.

Non-programmed decisions are ill-structured and in the nature of one shot. Managers attain maturity in making non-programmed decisions with training and experience. Training may be imparted by means of case studies, and most business schools have adopted this mode.

7 LIMITATION OF MIS

(1) In the past several computerised MIS's failed because the old ways were being automated. But it is also dangerous to venture upon a much more ambitious MIS The management must study the systems of the companies with regard to failures and successes. One should take advantage of their accomplishments and mistakes and should go only slightly further MIS, at present, is only in adolescent stages and it is also very fragile,

(2) There have been talks of complete integration and very hostile retaliation to the idea of integration Integration implies equality of advancement in each sub-system and the facts are contrary to this notion. One slacking sub-system would throw the entire integrated MIS out of gear. That complete integration is possible is a myth or at best far off from now.

(3) The managers should involve themselves in the systems analysis phase and come out with their information requirement explicitly as otherwise the technicians would decide what information the manager needs The systems analysts should never be given the commanding position in MIS planning. Much depends therefore on the manager analyst interaction. Since the interaction is human versus human conflicts are liable to occur and the personalities matter a great deal. Obviously this leads to development differentials in different sub-systems. Since there are frequent shuffling of the managers in any organisation MIS is inherently fragile or highly sensitive and requires continuous monitoring

(4) The hardware developments have been spectacular We have witnessed four generations of computers in less than 20 years. Software developments have been rather slow and catching up with hardware development is naturally expensive The success of MIS hinges on the quality of software for which there is scarcity of the brainware i e systems analysts, systems designers and programmers

(5) Operations research and mathematical models have been oversold It is to be noted that several simplifying assumptions are made in construction of these models Obviously this reduces their utility. Less simple models would be difficult to construct as well as serve. Jay Forrester of MIT has convincingly shown how valid sub-models can give wrong results when they are aggregated in one model Such fields as marketing are not so amenable to mathematical modelling In inventory control the independent demand items and dependent demand items stand in the ratio of 1 : 100 The papers published on these are just in the reverse ratio 100 1. The student may note that only independent demand items are susceptible to mathematical modeling

Contd. on page 38

Major Business Functions (Subsystems)	*Allocation*	*Assignment*	*Competition*	*Decision Theory*	*Dynamic Programming*	*Inventory*	*Optimizati*
Corporate Planning	Analyze wage, salary, and dividend policies	Assign firm resources to growth areas	Assess corporate policies, competition	Analyze long-range planning policies	Assess multistage corporate strategies	Assess the work force, assets, etc.	Optimize th resources of the firm
Marketing	Determine distribution handling	Assign salesmen on an optimum basis	Assess effects of price changes	Analyze timing of new products	Evaluate long-term marketing trends	Analyze warehousing needs	Determine optimum selling prices
Research and Development	Allocate manpower funds to projects	Determine allocation of funds to research projects	Evaluate alternative R & D designs	Assess life expectancy of new products	Determine reliability to be designed into products	Determine size of staff and the reserve of projects	Determine areas for R & D projects
Engineering	Allocate engineering projects	Assign engineers to projects	Select the better engineering projects	Control long-range engineering projects	Assess multistage engineering projects	Determine purchasing assignments	Optimize resources for engineering produ
Manufacturing	Determine best allocation of production facilities	Schedule job shop operations	Assess production changes	Analyze for stabilization of production and employment	Determine optimum production sequences	Schedule operations, considering current stock levels	Ox th for new machines
nventory	Allocate proper space for inventory	Assign optimal number of personnel to inventory	Assess one product versus another	Analyze future inventory movements	Determine multistage inventory policies	Utilize economic order quantity policies	Optimize for lowest inventory cost
urchasing	Assess in-house manufacture versus outside purchase	Assign buyers in an optimal manner	Assess competitive vendor behavior	Analyze projected long-range prices	Assess multistage buying policies	Develop optimum buying practices	Determine optimum buying periods
ysical tribution	Determine location and sizes of warehouses, retail outlets, etc.	Space allocation for stock accessability	Assess distribution policy	Determine company-owned stores versus franchises	Plan alternative distribution policy	Determine multilevel inventory system	Determine optimum warehousing size
counting	Allocate overhead and expenses on a logical basis	Schedule assignment of accounting jobs	Assess accounts-receivable policies	Analyze bad debts in the long run	Assess multilevel accounting policies	Analyze periodic inventory reports	Optimize net profits for the firm
ance	Determine need for funds	Assign optimum credit policies	Select from competing sources of funds	Evaluate long-run investments	Assess long-range dividend policies	Assess investment in inventory	Optimize credit policies
onnel	Select personnel to reduce labor turnover	Assign proper skills to jobs	Measure employee performance	Determine feasibility of accelerating automation	Assess recruiting policies	Evaluate causes for absenteeism	Optimize incentive schemes

FIGURE 2A-1. Major classification of operations research models.

Queuing	Replacement	Routing	Search	Sequencing	Simulation	Transportation
Staff service function	Rotate or replace company personnel	Determine best routing of company products	Determine business strategies	Control integrated programs	Simulate, using corporate planning models	Analyze transportation costs for firm
Staff and assign prices to products	Replace or rotate sales personnel	Determine accounts to be handled by salesmen	Locate customers and/or competitors	Analyze sales strategies	Simulate allocation of advertising budget	Determine policy on freight out
Schedule sales calls	Replace analytical testing equipment	Route R & D projects in an optimum manner	Search for R & D data	Analyze and control R & D projects	Simulate basic and applied R & D projects	Determine effectiveness of R & D activities
Schedule engineering work	Replace engineering equipment	Analyze activities that may delay engineering projects	Retrieve and process engineering projects	Establish and control engineering projects	Simulate engineering projects	Evaluate movement of engineering projects
Schedule equipment and machinery maintenance	Replace old equipment with new	Route production orders in an optimum manner	Retrieve production information	Establish production schedules	Simulate the production process	Determine the best movement of in-plant goods
Analyze stockouts at queues	Consider two-bin inventory policy	Determine best route for inventory stock	Analyze for best physical placement of inventory	Establish sequence of inventory usage	Simulate for lowest cost inventory levels	Determine lowest cost handling procedures for inventory
Schedule to interview salesmen	Assess replacement policies	Determine best routing of incoming goods	Retrieve data on vendor performance	Control flow of purchased parts	Develop rules for buying under varying prices	Determine lowest incoming freight costs
Schedule outgoing traffic	Replace costly warehouses	Analyze for best logistics and supply system	Search and analyze physical distribution data	Determine physical distribution schedules	Simulate physical distribution flow	Determine low-cost distribution channels
Schedule accounting functions	Evaluate present data processing equipment	Determine best routing of in-plant accounting data	Retrieve on-line data for audit check	Evaluate [illegible]	Simulate sales and costs in budget analysis	Evaluate incoming freight costs
Schedule cash flow	Evaluate capital projects	Determine best method of obtaining funds	Retrieve important financial information	Determine long-range capital requirements	Simulate operations for cash needs	Allocate funds to projects on a timely basis
Evaluate queues of personnel and equipment	Assess personnel replacement policies	Route new employees through plant for indoctrination	Search inputs for needed skills	Allocate personnel development programs	Simulate for future personnel needs	Transport workers to work

Contd. from page 35.

(6) The systems analyst has to be proficient in so many diverse areas to satisfactorily design MIS. Obviously, it would be difficult to secure such a super analyst A team and highly balanced one at that is needed

(7) It is a fact that computers have turned out piles of outputs for the manager Most emphasis in devising MIS goes to pointers, indexes and chaining in the data base and literature is virtually devoid of discussion on saving the manager from over abundance of information or in Ackoff's words there is paucity of literature on filteration and condensation of information The manager suffers from information overload. One of the possible reasons is that the manager is not fully conscious of the process of his decision making and his data requirements. Hence the manager who does not understand the phenomenon he controls plays safe and with respect to information wants ' everything". And the analyst understands it still less and is willing to provide everything

(8) Information problems of each company are unique to itself Therefore during the development stages. it is extremely difficult to estimate with any certainty what time it takes to develop the programs with a given pool of analysts and programmers.

(9) Budgeting for MIS is extremely difficult. Some of the questions that arise for budgeting are .—

Is the program new or a modification of an existing system ? If new, is it similar to any previous work ?

What degree of complexity is involved with the input/output, files as well as calculations in programming ?

What is the size of the program and the expected storage problems ?
What is the level of the programming language to be used ?
If the system to be mechanised exists, is it well documented ?

(10) The data base concepts have been realised by means of the magnetic disc media. Since the updating on discs is carried out by overlaying the previous records are lost If therefore, there is any corruption of data it is difficult to trace. Owing to possibility of such an information common to many files such corruption asrumes high proportion The fear of a major systems failure therefore is always lurking The information on magnetic disc can be periodiclly dumped on magnetic tapes to facilitate tracing of corrupted information but this is quite expensive.

(11) Decision making at higher levels still depends upon the "feel", intuition and guts of the executives because all the data and cost (specially, about the competitors and exerternal environment) cannot be collected with reasonable time and at reasonable expense. Management Science and computers can help solve some of the problems at this level but is only about 10% of the ingredients of decision making : the rest 90% has to be supplied by the executives.

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F. III. B-I

THE INSTITUTE OF CHARTERED ACCOUNTANTS OF INDIA

STUDY MATERIAL

F.S.P. (N) O.R.—1
Combination 'B'

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FINAL COURSE (N)
OPERATIONS RESEARCH & STATISTICAL ANALYSIS
STUDY—I

Contents :

Suggested Reading : Fundamentals of Operations Research *by Ackoff and Sasieni* Wiley International Edition.

Overview of the Study Material

This study material deals with the evolution of Operations Research in the context of development of the industrial and business organisations, to the present day complexity ; the scope and purpose of O R. are outlined. Being a relatively new science no universally acceptable definition has been offered as yet, and therefore, some of the O.R. definitions are critically appraised with respect to the current practice.

The methodology of O.R. is dealt with in the following phases :—

1. Formulation of the O.R Problem
2. Construction of the O.R models.
3. Solution of the O.R. models.
4. Testing, implementation and control of O.R. models.

It is suggested that as the student goes through the following studies on O.R. he should interpret the various O.R., problems in the above phases. In practice, however, a clear distinction between any two phases may not obtain and the phases may well be overlapping.

N.B *The student is also advised to first read through SADP Study paper-III and O.R Study paper-I to VI and then read this Study note carefully again.*

An Overview of Operations Research*

This section is intended to provide a sort of layman's introduction to operations research before we proceed with the technicalities in a more rigorous form.

It is rather a new science and its origin, in a significantly discernible form, dates back only to 1940. In early days of industrialisation the enterprise was fairly small and manageable by a single person, usually the owner. He did the supervision of production as well as managed all the associated functions, which now-a-days have distinct entities, e.g , planning, procurement, marketing, finance, personnel, etc. With the impetus provided by mechanisation and automation, the enterprise rapidly grew in size, complexity and geographical spread. It was no longer possible for a single person to perform all the required functions ; there arose the necessity of assistance by functional or departmental managers. The owner was replaced by the board of directors who were rather remote from the actual production. The intermediary managers were specialists in their fields ; Design, Production, Marketing, Personnel, Purchasing, Public Relations, Engineering, etc. etc., Continued mechanisation necessitated further sub-divisions of each function. For example, production was sub-divided into supervision of production, inspection and quality control, planning, storing etc. This, together with multiple plant operation horizontally and vertically and geographically, contributed further to the hierarchy of the managers.

*Operations Research is synonymous with Operational Research (the nomenclatures in U.S.A. and U.K. respectively. We shall use the former, the more common one throughout.

Concomitant with the sub-divisions of the total management function into components was the effort devoted by scientists towards specialisation and enhancing functional efficiency. Departmentalisation, though essential for conducting the operation of the organisation, did away with the broad (and possibly vague) perspective of the objectives of the enterprise of the early day single owner. Each department developed its own sub-objectives which might conflict with those of other departments and may not conform to the overall objectives. O.R. (abbreviation of Operations Research), in a laymans' language, seeks, to be the substitute of the single owner's overall perspective.

In these terms, each time a managerial function is broken down into a set of sub-functions, a new task is created that of integrating the diverse sub-functions so that they serve efficiently the interest of the whole. This integrating task is the executive function of management To carry out the executive fuuction, it is necessary to lay down objectives and methods and measures for achieving these objectives. The company executives usually have the following objectives for the main functions of the business :

Production : To minimise the production cost and maximize the production volume.

Marketing : To maximize the sales volume and minimize the sales cost

Finance : To minimize the capital investment.

Personnel : To maintain morale and high productivity amongst the employees.

These functional objectives may be inconsistent with each other and, therefore, hard to pursue in practice. Consequently, when each department pursues. its own objectives conflicts may arise among the various departments. The inventory policy of a company provides a good example. The sales department would want generous inventories over diverse range of the product so that the customer can be promptly serviced. Finance, however, would tend to keep the investment to the minimum and that means maintaining the minimum inventory. Thus sales and finance conflict over the inventory policy. The aforesaid objectives of sales would require the production department to be flexible enough to fill even the special orders at a short notice ; but production would be interested in long, uninterrupted runs to minimise the production cost. The personnel department would like to stabilise labour which can be accomplished when goods are produced for inventory during slack periods, etc.

The departmental objectives have, therefore, to be suitably reconciled and the problem is then to devise an inventory policy that is best or optimal for the organisation as a whole. This is an example of the executive type of problem that is solved by Operations Research. In the early days, such a problem did exist but only in a very rudimentary form and was intuited out by the owner himself. For the present day industry, however, this would require an extensive research of the operations of the enterprise in a systematic and scientific way to resolve the conflicts suitably.

Continuing our example of inventory policy, the problem has deeper roots, in fact Even within the production department the conflicts in objectives are to be encountered. The production supervisor would wish to have as long an uninterrupted run for a given machine set-up as possible. This tends to minimise the production cost. This implies that he would like to have as big lots or batches of each item as possible ; but the warehousing may be restrictive here in view of the limited storage space. The depth of the problem brings one towards the scope of O.R.

What has been said about the industrial and business organisations applies equally well to other human organisations, e.g., the military. Incidentally, the origin of O.R. lies in the military context in the II War. We shall, however, continue our attention to industry and business.

Scope of O.R.

Management of a business firm involves a large complex of activities involving analysis, decision, communication, leadership, motivation, measurement and control. O.R. focusses on the fundamentals of analysis and decision making. An objective of O.R., as it emerged in the context of discussion on evolution of industrial organisations, is to provide the management with a scientific basis for solving problems involving the interaction of components of the organisation in the best interest of organisation as a whole. A decision that is best for the organisation as a whole is called an optimum solution or policy. The problem, then, is to derive the optimal solution though estabilishing criterian for an optimum decision is usually complex.

O.R. purports to find the best solution relative to as large a segment of the organisation as possible. For example, in attempting to solve a maintenance problem in a factory O.R. tries to consider the effects of alternative maintenance policies on the production department ; also, if possible, repurcussions on other departments and on the entire business in which the factory operates.

In the inventory policy cited above, the stock levels and batch sizes may be established individually ; but the two interact and the O.R. expert may wish to pursue the matter further by considering the two jointly for an optimal policy. But he may not be able to do so to this depth because of limitations of financial and other resources and want of analytical means.

Let us consider the scope of O.R. and its various definitions in a slightly more rigorous manner.

Operation Research, rather simply defined, is the research of operations ; though obvious, yet a useful way to start with. What is an operation then ? An operation is a set of acts required for the accomplishment of a desired outcome. It is not a single act but a complex of inter-related acts performed simultaneously or in a sequence, which leads to the accomplishment of some predetermined objectives or outcomes. Such complex, inter-related acts can conceivably be performed by four types

of systems ; Man, Machine, Man-machine unit, and any organisation of these. we shall soon conclude O.R. is concerned with the operations of the last categor the systems.

When a man runs for a train, for example, myriads of his nerves and phys limbs are set into action ; they act in harmony, however, i.e. in an organised way the man pushes through his way from amongst crowd, etc. and reaches his object of catching the train. But, it is to be emphasised that all these components/ner and limbs act in an organised way. In contrast, consider a running mad person. nerves and limbs may neither act in a co-ordinated fashion nor towards any goal.

Likewise, when a machine has been set in motion, its components, which m vary from a handful to millions from machine to machine, also perform inter-relat acts, towards some purpose, viz., the railway engine drives the train.

Man-machine units, like a worker running a machine, also perform such int related acts, the components of both the man and the machine together act in harmo towards the end of producing a job at the required quality and quantity standards.

However, these three categories are not our concern here. Man as a system the subject of psychologists, physiologists, biologists, etc. The machine as a syste is the concern of systems engineers. And the man-machine unit as a system is t subject of industrial engineers though we shall see some of these in Study-V, viz. t cyclegraphs and multiple activity charts compiled by the industrial engineers. Oper tions research is the research into the operations of organisations of men, machin and man machine units which we shall see more rigorously soon. Prior to that let note the fact that the conflicting objectives is not unique to Operations Researc Systems engineers also encounter such conflicts. For example, the design of a car h such conflicting objectives as cost, maintainability, maneouverability, esteem feature speed, etc.

O.R. is applicable to such man machine systems which involve :

1. A large number of parts (e.g., men and machines).
2. Parts interacting with one another in varying degree ; of complexity
3. Parts involving feed back, control, e.g., depending upon the progress plan may be amended.

These are incidentally the systems that may be called organisations, with th following essential characteristics :

(a) Some of its components are human beings.

(b) Responsibility of choices from the set of acts is divided amongst two o more individuals or groups of individuals :

(i) by function (Production, Research, etc.)

(ii) by area (plants of the same organisation in Calcutta, Delhi and Bangalore).

(iii) by time (A plant after construction and commissioning passed on to a Production Manager).

The individuals, however, may not themselves carry out the actions they select. They may employ machines for the purpose :

(c) The functionally distinct groups are aware of each other through communication or observation (operators in a factory in touch with their supervisors or amongst themselves).

(d) A subgroup (or the total group) of individuals in the system has a control function

If any of the conditions (b) to (d) are not satisfied by a group, it is unorganised. O R seeks to study possibilities of changes within the aforesaid characteristics and evaluates outcomes :

1 *Organisation content* An organisation may not function well because of the inefficiency of its personnel. The situation may be improved by training or motivating the personnel suitably.

2 *Organisation structure* : Reorganisation and or hiring and firing is a common method of enhancing organisational efficiency.

3. *Communication* : This would require analysis (followed by improvements) of information generation and processing in an organisation.

4 *Control* : An organisation with good personnel and equipment may function inefficiently because its operations are not controlled properly and need to be improved.

Characteristics of O.R.

The student might wonder at this stage where lies the difference between O.R. and systems analysis since both are concerned with the improvement of the type of man-machine systems delineated above. The essential characteristic of O.R. is that it uses mathematical or quantitative techniques. This, however, is a matter of slant only and systems analysis includes O.R. modelling to be discussed shortly, Just as, as mentioned in SADP Study I, the computer is used mostly at lower levels of MIS so is the case with O.R. too.

At lower levels decision making is programmable and is carried out at a large scale, viz. setting E.O.Q's and R.O.L.'s of thousands of items so frequently. O.R. can be usefully employed to derive decision rules once and for all. Such decision-making is termed programmable since either the computer can be programmed to carry out such decisions on a routine basis or the junior staff provided with slide rules, nomograms, tables, etc. for executing such decisions. These decisions are structured in the sense that all the variables of interest are known to be related by means of some mathematical formula. However, they are also simple. Programming such decision-making by O.R. modelling has the advantage that the repetitive decision-making is made consistently,

accurately and robustly. However, forecasting, planning (and to some extent control, too) at higher levels have to be carried out in the face of a highly uncertain and complex external (and to some extent internal) environment. Either the variables are far too numerous or they are difficult to be related by means of some mathematical equation. For example. to what extent the sales would increase by improving the quality or would the internal staff successfully absorb the new technology, etc, are questions that defy resolution by Mathematics. Here, the top executives have to exercise judgment and expertise. In other words, the problems for decision making at higher levels are highly ill-structured and usually in the nature of one-shot i.e. they are not repetitive. Yet, O R. models could be employed sometimes even for such problems to lay bare some aspects of the reality so that the executive is in a better position to exercise his judgment or intuition Thus at lower levels of MIS OR can make the decisions for the managers, at upper levels it can aid in managerial decision making.

Another characteristics of OR is systems approach. Putting in a simple way, OR is the quantitative part of systems analysis. The underlying systems approach consists of the attitude, 'Let us, stand back and look at the system as a whole." And an operating manager, even though with his vast and indispensible expertise and knowledge in his function is not likely to stand back. This is because he is knee-deep in his own function and as such he is liable to be coloured or biased in favour of his own function The marketing manager, for example, is likely to consider all the good for the company in the marketing function alone. He might tend to take greater risks in the market place than would be desired by the manager of finance department who would be more conservative, preferring to rely on currently successful products to expand sales The marketing manager may also tend to advocate broad, deep and multiple products and this could be frustrating to the production manager, who is charged with keeping labour costs and capital investments at a minimum. It is often the case that the operating managers make it a game of bargaining with the systems analysts and tend to extract 'benefits' for their departments. It is not so much the intention here to prove that the operating manager cannot be a good systems analyst as to emphasise the need to "stand back" or to forget the routine and porblems of the day, and view the system as a whole. The problem of declining sales may not turn out to be a problem but merely a symptom of the problem. Rewards or punishments to the salesmen may do no good to the sales if the real problem lies in quality control or costing. Likewise the frequent rejections may be no reflection on the skill of the workers and supervision of the foreman if the tolerances prescribed by the design are a bit too tight. Problems usually spawn many functions or departments and this necessitates viewing the system as a whole and going to the roots of the problem. The proposed solution this way would therefore, naturally have factory-wise repurcussions, if not even industry-wise. The effects of the change on all the departments would have to be taken into account in making the proposal.

Consider, for example, the proposal by the production department of a firm to switch over to the continuous production from batch production with a (narrow) view to

decrease the unit production costs. This is a major change and would effect all the departments in one way or the other. Under the batch production system the batches of raw materials and components would be to scheduled that they arrive just at time of a need (i e, assembly) or production, minimising thereby the raw material and component inventories With continuous production these would still have to be acquired in batches but they would be consumed continuously (i.e slowly) and this would mean continuously maintaining inventories of raw materials and components with all the attendant problems and costs. Reverse would be the case with the inventories of the finished goods. Under batch production the ffnshed goods were delivered into the stores in lots but sold continuously i e , there was considerable stoking. Under the continuous production, the delivery into the stores and withdrawals would be both continuous, meaning far less finished goods inventories Also, if the demand rate matches the production rate inventories would be minimal but if the former exceeds there would be shortages

The continuouts production would necessitate special purpose equipment and different category of material handling equipment : (like belt conveyors, gravity hoppens etc Also less skilled workers would be alright with continuous production. The quality of the finished goods may be different under the two methods, affecting marketing via sales

Before a final decision is made, the net efiect and accompanying indirect changes will have to be assessed as a part of the overall saving produced by the project. In brief, the essence of the systems approach is to trace, for each proposed change, all significant and indirect effects on all parts of the system, and to evaluate each action in terms of the effect for the system as a whole. This is quite in consonance with the difinition of an operation earlier as a set of inter-related acts. OR tries to establish these inter-relationships in terms of mathematical symbols and models. However the 'system as a whole' would vary in size with problems, or the resources at the disposal of the researcher. The above proposal for change over to continuous production may well have industry-wise repurcussions on the competitors as well as several societal implications. Though the researcher may want to pursue the problem to full depth he may be restrained by considerations of money and time for the research. In other cases the sponsors may restrict the scope of the system for considerations best known to them. Under such circumstances the researcher must satisfy himself that the restricted optimisation does not seriously jeopardise the success of the project. Often, however. it is a shrewed move to improve the system in part so that all concerned are motivated for a full scale research.

Reverting to the example of switchover to continuous production, the accounting system here would merely be content with providing the net effects of all the changes in monetary units to the management who prompted the accounting system to do so and the management themselves got the idea of the switch over from their judgement and expertise. O.R. would not be content with monetary figures alone. However,

more important, perhaps, is the fact that O.R endeavours to quantify such heretofore intangibles as service to the customer, employee's morale, prestige of the organisation, etc. These intangibles, though vaguely very much in the minds of the management during their deliberations, did not enter explicity in the accounting figures. And often these intangibles represent the most compelling reasons for the switch-over More concretely, that the switch-over reduces/increases the stockouts, say, from 50 to 100 per week would not be reflected in the accounting figures ; yet this is a highly significant variable which can have serious effects both in the short run and the long run. Thus O.R. can treat the problem as a multi-dimensional one with numerous variables. Also the accounting system, without the aid of O.R. is likely to lose sight of significant interactions. In Study III, the student might notice that the technique of linear programming, in a way, the sophisticated extension of the breakeven analysis. The latter is concerned with merely one product decisions on pricing and fixed costs. But when more than one products are being produced they contend for the limited resources and treating them independently and separately for break even analysis may lead to bad decisions. The products ought to be considered together We are then back to the definition of O.R. Accounting function treats the components, the products separately anc may ignore the important interactions which O.R. does not do.

The accounting function would also presume a deterministic environment, whereas many of the variables in the system, e.g. demand and leadtimes are probablistic that would be treated as such by O.R. Another variable of interest is the labour's attitude to change which, perhaps, is more a matter for the systems analyst and not the O.R team although, it must be pointed out that the distinction between O.R, and systems analysis can at best be blurred one only because in reality it does not exist. Anyway, concerning ourselves with the distinction between O R and the accounting function, the latter works with "givens," e g., the management would 'give' to the accounting function the intended production rate after switch-over. But these very "givens" for the accounting function would have either to be derived by means of O.R. or by the judgment and intuition of the management. In fact, the crux of the matter lies here. O.R., in a way, is the scientific substitute for judgement and intuition, not wholly but substantially. O.R can assist in building a mathematical model (to be taken as a set of equations for the time being) that relates the rate of production to various variables, monetary, non-monetary and many of them (heretofore intangibles) suitably quantified in monetary terms as to be explained shortly. O.R. allows the manager to experiment with this model, to manipulate the values of various variables which might provide interesting insights and results. The "givens" for the accounting function would thus be generated. However, this discussion might also inadvertently overemphasise the impoitance of O.R. Neither all the intangibles can be quantified nor it is possible to ascertain all the probability. Yet, however, O.R. can provide "a bad solution to problems where worse is expected" which in fact is a definition of O.R. to be discussed several pages latter. There have been cases where the executives had made a decision for such problems on the basis of their judgement and intuition and yet engaged the O.R. team only to be fully satisfied that O.R. verified this.

The Inter-disciplinary Team Approach is yet another characteristic of O.R. Once upon a time, there was only one discipline known as philosophy Modern physics owes much of its origin to the work of Newton who in his day was known as a natural philosopher The nomenclature, "physics", was coined later. Upto the end of the 17th century much of what would be called to-day the scientific knowledge could be learnt and retained by one man. But since then this knowledge has been proliferating at a phenomenal rate. The natural philosophy was soon to be subdivided into Physics and Chemistry. Biology and Psychology were also, not late is being recognised as disciplines. This continued subdivision of the scientific knowledge has led to as many as about 100 disciplines today under the label of science, each of these having its own name. The universities have so instituted these names it would appear as if the nature itself is presented to man in this classified fashion, which, however is untrue. Nature is wholesome and it is the disciplines which view it from different angles. The physicist is likely to review his laws of motion upon seeing a bus accident. The civil engineer would perhaps look at the condition of the roads, the mechanical engineer would suspect something with the engine and a psychologist would wonder over the state of health of the driver. It is not infrequent to come across patients turning from one specialist to another, each specialist treating the patient as per his speciality ; yet the patient remains patient, Problems usually spawn many disciplines and often what is a problem at the first thought is only a symptom and the problem is usually buried deep Consider the stagnant productivity in a firm. Each specialist may have his own pat theory and solution for it, some right and some wrong. The ergonomist may attribute it to poor illumination and workseat design, the social psychologist to the inherent lack of inertia of the labour population, the mechanical engineer to the inferior machines, the industrial engineer to irrational incentive schemes, and so on. Sometimes, the pat thoery of one of these may be a brilliant solution for the problem. Sometimes, the views of two or more may have to be synthesised. It is, therefore, highly desirable to employ a team of operations researchers so that the problem is viewed from different angles In O.R. this approach is particularly useful because different disciplines have developed their own specific mathematical tools and techniques which can often be applied with or without some modifications and refinement to the business problem. Much of the application of forecasting techniques to inventory control, for example, is the contribution of control engineers. Queuing, theory was originated and developed by a communication engineer. The randomly distributed lateness of trains in a situation in India was reminiscent of the novement of gas molecules as envisaged in physics under the Kinetic theory of gases and the physics model, suitably modified, was usefully applied here. The origins of the technique of linear programming are often traced to the work of Leontif, an economist on input/output analysis. It is also not to be forgotten that O.R. itself is originally a military science.

Therefore, it is common to find an O.R. team comprising experts of different disciplines. If some of them are not physically and whole-time in the team may be consulted when required. Also, as the research proceeds some of them may leave after having done their job and others may join.

Purpose of O.R.

Much of this though is self evident in the discussion above yet we shall spell it out more clearly. As stated earlier, the managerial function has been sub-divided many times. Obviously, then, one man looking after the entire system cannot be the same thing as several managers looking after their respective sub-systems. Conflicts are bound to arise amongst the managers beause each one of them is liable to pursue his sub objectives so far th .t it impairs the working of other sub-systems. Consider the product decision, or example. Below, we list the sub-system goals or sub-objectives and the desired product policies of three functions together with those of the general management (Ref. : Production & Operations Management by Chase & Aquilano) to bring the conflicts into a sharp focus :

Sub-system	*Sub-objectives*	*Desired product policy*
Production	1. Ease of manufacture 2. Cost minimisation. 3. Stability of demand and output. 4. Maintenance of quality standards	1. Few products 2. Similar products. 3 Products which are easily and inexpensively manufactured.
Marketing	1. Product innovation 2. High sales volume 3. Increasing market shares. 4. Flexibility in market place. 5. New markets. 6. Revenue maximisation. 7. Consumer orientation 8. Consumer research.	1. Broad product lines 2. Introduction of new products. 3. Frequent modification of existing products and lines. 4. Balanced product lines.

Sub-system	*Sub-objectives*	*Desired product policy*
Finance	1. Ensure liquidity. 2. Maximise profit. 3. Assure corporate survival and growth. 4. Eliminate expenses not directly contributing to profits. 5. Minimise risks to the firm.	1. Market products which yield immediate profits. 2. Select most highly profitable products. 3. Eliminate low profit products.
General Marketing	1. Maximise difference between revenue and cost.	1. Market products with proven profitability.

2. Maximise long run profit.	2. Balance highly profitable products with short lives against products which earn lower profits initially but exhibit excellent long run profit potential.
3. Ensure corporate survival and growth.	3. Provide for ease in managing and co-ordinating the firm's activities both internally and in the market place.
4. Secure and increase profits over time.	

It is obvious that sub-objectives conflict with each other as also with those of the general management. Three types of conflicts are often encountered by the operations researcher.

1. Conflict across time.

This conflict may exist between the sub-objectives of a sub-system or different sub-systems. In the above list, for example, general management's "Maximise long-run profit" conflicts with finance's "Ensure liquidity." The top management is usually faced with the following conflicts whilst formulating the strategic plan :

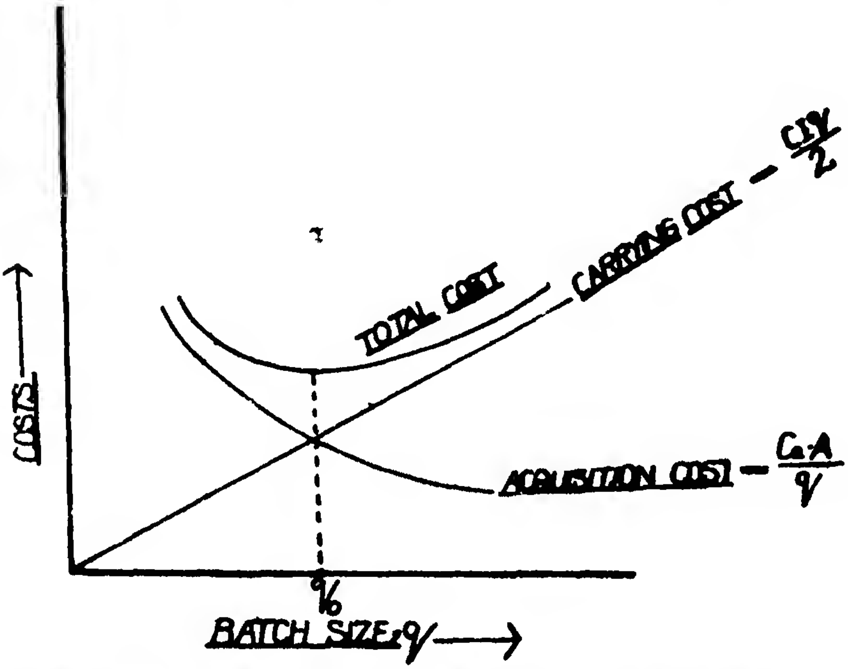
Short-term	*Versus*	*Long-term*
1. Short term profits		Long-term growth
2. Profit margin		Competitive position.
3. Direct sales efforts.		Market development effort.
4. Penetration of existing markets		Development of new markets.
5. Related growth opportunities.		Unrelated growth opportunities.

Compromises have thus to be made over time. Soon we shall see how O.R. resolves conflicts, sometimes accurately and sometimes rather crudely in the sense of someone's remarks "O.R. provides bad answers where worse would be expected otherwise".

2. Conflict between subsystems (including within a subsystem).

Numerous examples can be picked up from the list of p. 11. Consider, however, the case of economic ordering quantity where there is a conflict between the acquisition cost and the inventory-carrying cost. (Ref. ; Study II) with regard to the batch size. The former decreases with the increasing batch size and the latter increases with the increasing batch size. This is depicted below. The total cost curve (cup shaped) is also shown. This cup shape as such or reversed is bound to obtain whenever there are conflicting costs or conflicting gains. The purpose of O.R. is to minimise total cost *i.e.* find the mimima of the cup shaped curve or maxima of the reversed cup

shaped curve in case of gains. Such minimisation or maximisation is known as optimisation of cost or profit or any other effectiveness measure. The curve may be continuous or stepped. In this case one can think of a simple application of differential calculus to find the minimum or the optimal cost but in other cases more advanced mathematical techniques or trial and error or iterative approach may have to be applied. Another simplicity of this problem is that both the conflicting objectives : acquisition cost and carrying cost are in the same (monetary) units. In other problems



the units may not be the same. For example, in the case of setting safety stock (Ref. : Study II compromise has to be made between the carrying costs and number of stockouts. Frequently, it is possible to express such a variable as stockouts in monetary units and then resolve the conflict but it may not be possible always. Other problems may also be more complex because many more objectives have to be resolved jointly.

Continuing with this inventory example, we optimised the following twos at a time (in Study II) :

1. Acquisition cost vs. 2. Carrying cost	for E.O.Q.	1. Carrying cost vs. 2. Stockout	for safety stock

If, however, the three (1. acquisition cost, 2. carrying cost, 3. stockouts) were optimised jointly we would have got a superior solution in the sense that the total cost now would be less than that of the two separate optimisation. Strictly, therefore,

these two separate optimisations would be called sub-optimisation. Thus EOQ. is most economic only if the safety stock problem is ignored, which is frequently done i.e the two are determined independently by 2 sub-optimisations because optimising the three objectives together requires the use of intricate mathematics i.e. more calculations or cost. Thus, though optimisation is theoretically possible we do not do so because of more effort entailed in it. This is termed as *the principle of bounded rationality*. As per this we may not always pursue for complete optimisation either because it is not possible owing to limitations on time, data, etc. or because the incremental cost in so doing is not fully offset by the benefit.

As another example of the principle of bounded rationality, the top management faced with a highly uncertain external environment about which little data exists or can be extracted may not be able to derive strict optimisation amongst such conflicting objectives as profit, corporate survival and growth, productivity etc. In fact at this (top) level O.R. cannot be of much help and the top management may have to resolve the conflict intuitively.

Consider, however, the hierarchy of the production function objectives

Output Objectives		*Cost Objectives*	
Volume Objectives	*Performance Objectives*	*Explicit Costs*	*Implicit Costs*
Production rate	Time schedules	Material input, scrap.	Stockouts
Inventory levels	Quality	Direct and indirect labour	Grievances
Work force level	Efficiency of work-force equipt. facilities	Maintenance	Late deliveries, etc.

In this case O. R. can be applied in various places but the attempts at overall optimisation may again be futile. Thus optimal solution is not always possible for a variety of reasons, some of which follow :

1. All alternatives and states of nature (which may run into myriads) have to be considered.
2. All the data may not be available or too much data has to be researched.
3. Conditions change rapidly and the optimal solution now may be irrelevant for even the near future.

Thus, a satisfactory compromise is often welcome and as such this approach, as against precise optimisation, is known as the *satisficing approach*.

Consider yet another example on resource allocation in network planning (Ref. : Study IV). The network activities easily run into hundred and permuting these with so many resources leads to myriads of alternatives. Even with the computer, it would be very time-consuming and costly to pursue the optimal solution i e. activity start and finish schedule, and even if it could be found it would soon be invalidated by the erratic progress against the schedule. Under such circumstances where data is not a constraint but it is only myriads of alternatives that have to be considered for optimisation one can instead derive such heuristics or rules of thumb as : 'The higher the float the less urgent the activity" etc. and carry out scheduling according to these rules which will yield near optimal solution. O R can frequently help in finding these quasi-scientific rules or heuristics

Thus the purpose of O R is to resolve the conflicts resulting from sub-divisions of the managerial functions in an optimal, near optimal or satisficing way. This applies to the conflicts of all the three types

3. **Conflicts between a sub-system and the system**

A division may tend to buy its materials at a cheaper rate from an outside vendor rather than from another sister division In such conflicts, too, O R. could be useful.

Problem-solving and Dicision-making

The two are often taken as synonymous There is however, a basic difference. Problem-solving means seeking answer to a question Decision-making means picking up one answer (or one set of answer) from amongst many for implementation. Whereas the former facilitates understanding of a situation the latter is action oriented for alleviating the situation.

Usually, not always, what is a problem at the first sight is only a symptom. This is as much true in management situations as in medical science.

A management problem generally does not fit neatly in one functional jurisdiction i.e. a management problem is not very likely going to be just the purchasing management problem It is apt to spawn many functions, customers, vendors, government, etc. and even society at large.

A problem consists of clusters of sub-problems "Why our machines are idle so much of the time" may, perhaps be attributed to poor planning or poor co-ordination or dissatisfied labour or scarcity of a particular steel or combinations thereof. The scarcity sub-problem may lead to another cluster of sub-problems, viz. poor buying policies, or inflexibility of the engineering department to come up with substitutes. A problem is thus a host of sub-problems which have no regard for departmental jurisdictions. In a negative way, though, this should also suggest the need for systems view. And the clusters should also suggest that they can be set right by research into the operations of the organisation which, upon implementation, would set into motion a sequence of inter related corrective acts.

Also, then, it would not just be one decision but a series of decisions that have to be carried out to alleviate the situation. Thus decisions may have to be made horizontally or up and down the organisational hierarchy.

It is not difficult to appreciate that any organisation will be faced with myriads of problems (and opportunities which, too, are problems for our purpose because the organisation *has* to be directed to exploit these opportunities) at any time. However, we shall here be concerned with executive type of problems which arise because of segmentation of the managerial task. Thus O.R is concerned mainly with those problems that involve conflicting interests.

Components of a Problem

There must be an individual or a group who feel that there is a problem in a particular environment where they are not satisfied with the state of things. They may be clear as to what the state *should be* or they may be hazy about it but dissatisfaction with the existing state anyway, is there. What the state *should be* is best expressed in terms of one or more objectives. Thus one of the task of the operations researcher is to carefully ascertain the objectives from this individual or group. Improvement on the existing state to attain these objectives would obviously be made either by accelerating the use or value of existing resources or employing more resources. In other words, the resources would be varied. Thus, resources are variable. If they cannot be varied, as is the case sometimes, the problem, then, is insoluble. These variables of interest may be continuous or discrete. If they are continuous, infinite alternatives for manipulation would exist in theory. If they are discrete, for the problem to be soluble, at least two alternatives must exist

The purpose of this brief discussion was to see the problem in parts. A problem has four components which have to be studied by the operations research team. These are :

(i) The individual or the group [decision maker (s)] who face a problem and sponsor research

(ii) The environment wherein the problem is supposed to lie. It is not unlikely that the roots of the problem are traced to a different area ; therefore, it is wise to call the entire organisation as the second component.

(iii) Objectives.

(iv) Alternatives or courses of action that affect the objectives. Usually, it is not easy to list these alternatives directly by the study of the organisation since it is far too complex. It becomes necessary to make a model of the slice of the reality of interest. Usually, in business situations, this model takes the form of a mathematical equation(s) that express the relationship in variations in the variables of interest. Modelling will be discussed at a greater length soon. For the time being, the breakeven equation may be taken as an example of such models. Such variables as selling prices, vari-

able costs, *quantity to be manufactured*, etc and the profit objective (dependent variable) are thus related by means of this break even model or equation. Models permit a great deal of manipulation of the input or independent variables or resource values or quantities to ascertain the effects on objective(s). It would not be easy or even possible to play about the input variables like this in the real-life systems directly ; therefore modelling

Another component of a problem.

(v) *Constraints* are also usually to be encountered in business situations There could be for example, governmental constraints on both the types of the variables, objectives (dependent) or alternative courses of action which restrict action Likewise, there could be constraints imposed by union or society at large etc

More on these components later.

Types of Problems and Decision making

Problems are usually divided into various types by the amount of information available regarding the liklihood of occurence of outcomes of the various alternative courses of action. Accordingly they have different criteria and methods for arriving at the appropriate decision

1 *Decision making under Risk* is supposed to occur when the probabilities of the various outcomes of the different courses of action are known Referring to part II of Study II which is exclusively concerned with this type of problems, the set of available courses of action, [C , C_2, .C_m] and the outcomes constitute the states of nature, [Q_1. Q_2 .Q_n] It is assumed that in this type of problems, the values of the various probabilities with which the states of nature occur as also all the values in the payoff matrix are known We have entered hypothetical (monetary) payoff figures

State of Nature	*Probability*	*Strategies* C_1	C_2 ... C_m
Q_1	P_1	400	
Q_2	P_2	800	
Q_3	P_3	500	
:	:	:	
Q_n	P_n	400	

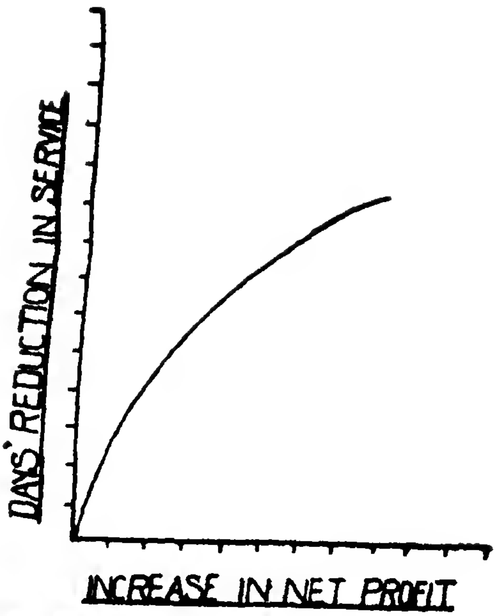
for the first course. These figures represent the attainment of the objective with C_1 as per probability distribution of col. 2. The expected payoff can be computed for each course and the course with the highest payoff would be the optimal. Thus decision making under risk would be carried out by the expected payoff or expected regret or expected cost criteria.

The above table is simple because there is only one objective of maximising the payoff But in other problems there could be more than one objective in inconsistent units Consider, for example, an inventory situation in which there are two objectives, (i) Reduction in days of service and (ii) Decrease in inventory costs. The former objective has days as the unit and latter has rupees as the unit. As such the following 'payoff' table would result, the upper position having payoffs in days and the lower in rupees and as such it cannot be used for the application of the payoff criteria

States	*Probability*	*Strategies* C_1 C_2 .. .. C_m
Q_1 Units, D Q_2 i.e. days' reduction Q_n	p_1 p_2 p_n	Figures in days (to be converted into rupee figures by means of the trade-off curve discussed below or by weighting the objectives)
Q_1' Units, Rs. Q_2' Q_n'	p_1 p_2' p_n'	Figures in rupees
More objectives could be had here, if necessary		

How to resolve this problem of inconsistent units ? This is done by means of what have come to be known as trade-off curves which are highly useful for handling multiple objectives in inconsistent units as is the case with the example on hand

We can convert the days' reduction in equivalent monetary figures either on the basis of past historical records that "2 days' reduction in service is worth Rs. 70,000 to use," etc This process is similar to that of the utility curves discussed in Study VI. As a result the curve like the one above can be obtained By means of this curve all the 'payoff' figures in days can be converted to their equivalent figures in rupees. And now the expected payoffs can be computed for all the courses in monetary figures.



2. Decision-making under uncertainty

In this type, the probabilities of occurrence of various states or outcomes are not known Such problems do arise in business situations, e.g. introduction of a new product where the demand probability distribution cannot be ascertained. It is assumed, however, that payoffs for various combinations of courses (rates of production) and states (demand) can be estimated i.e. the payoff table as the one below can be constructed

For such problems *maximax, *maximin (for payoff tables) and minimin, minimax (for cost tables) criteria can be applied.

States of Nature	*Probabilities unknown*	*Strategies (Rates of Production* C_1 C_2 ——— C_m
Q_1 Q_2 | | Q_m		

However, it is to be noted that one is rarely confounded with complete ignorance of the probability distribution of the states of nature of outcomes. Often mean or mean and S.D. may be known. If only mean is known the payoff table will collapse into just one row for the 'mean state of nature', and the problem is easily solved. If, however, the knowledge of the S.D. is also to be incorporated the use of

*These two criteria were first suggested by Hurwichz and Wald respectively and they are, therefore, also known by their respective names.

Tchebbycheff's inequality can be made, which, however, is a statistical technique beyond the scope of our syllabus and the student need not delve into its technical details.

3. Decesion-making Under Certainty.

Each course of action is presumed to have just one outcome which is deterministic For example, in the case of the E.O Q. model, the total cost for each value of the batch size is known (or presumed to be known) In this case, then, though it is not necessary to construct the payoff table as it reduces to just one row and finding the best course is all too simple However, since the variations are continuous maximisation or minimisation by differential calculus would be in order here

Where the variations are not continuous, e.g in the case of replacement problems (Study VI) where running cost vary discretely with the age the mathematical technique of finite differences would be used for optimisation.

The E.O Q. model can be handled by ordinary calculus but more complex mathematical functions would require numerical solution which are explained later. However, far more complexity, which cannot be handled, when the variables are far too numerous may require tedious iterative techniques, e.g., the simplex method. Thus even though there is no uncertainty regarding the states of nature it is not to be believed that decision-making under certainty is always simple

4. Decision making Under Competition

Please see Study VI (Theory of Games) : These problems also can lead to enormous complexity in decision-making For example, although in the game of chess all the possible courses of action can be enumerated in theory (astronomically large they would be) the game has not lost its flavour even after having been played for thousands of years

Methodology of Operations Research

We have already tried to bring out the difference between O.R. and Systems Analysis. So to say, O R. is quantitative part of systems analysis The phases of systems analysis and operations research are listed below .

Systems Analysis	*Operations Research*
1. Area Selection 2. Feasibility Study 3. Master Development Plan 4. Systems Analysis	1 Formulating the problem
*5. Systems Design	2 Constructing the Model 3 Deriving Solution 4. Testing the Model 5. Establishing Controls
6. Systems Implementation	6. Putting solution to work.
7. Systems Maintenance.	

As briefly noted in SADP Study III O.R. models are a tool of Systems Analysis and Design.

Now when O R. modelling is a part of the overall systems development effort the O R. problems would be formulated during the systems analysis stages 1 and 4 and stages 2 and 3 would take into account the costs and benefits of O.R. modelling and the scheduling aspects Stage 5 of systems analysis which is systems design would also include construction of O R models, deriving solution, testing the model, establishing controls In addition, systems design of course is also concerned with input/output layouts, codes design, etc. which are covered in SA & DP Study III

But if an O R. study is to be conducted on a one-shot basis then the 6 stages of O R as listed on the R.H S. of the above table would be gone through independently. However, some amount of systems analysis and design in the sense of SA & DP Study III will still be done, e g in an exclusive O.R study for quantifying inventory control stock cards format would still have to be designed Systems Analasis and O R. in fact, are inseparable Just as the phases of Systems Analysis are iterative so are the O.R. phases too

(i) **Formulating the Problem** : Any research begins with a problem which may be set out explicity by the consumer, the sponsoring organisation in the case of the O R study ; or may have to be formulated by the O R team. In O.R. it is not uncommon that the study may commence with tentative formulation of the problem and the problem may be re formulated over and again during the study. It is, however, to be preferred to formulate the problem fairly elaborately before embarking upon the research The procedure of formulating the problem is generally lengthy and requires considerable time and ingenuity , but is quite justified in view of the fact that, unlike other r searches, O.R is a research into the operations of an organisation. Therefore, it must consider the economics of its own operations. As mentioned earlier, O R. team usually draws experts from different disciplines, and may also include members from the sponsoring organisation The number of the members of the O.R. team need not be fixed over the period of the study It may decrease or increase depending upon the turns the study takes The first requisite of this stage of formulating the problem is establishment of rapport between the O R. team the consumer organisation. The O R. team studies the various aspects of the organisation with regard to content, structure, communication, and controls. This enables the O R team to assess the organisation and its problems before it undertakes commitment on a specific problem. The same opportunity is afforded to the sponsoring organisations They can take appropriate administrative measures to meet the conditions which the O R team may specify

Objectives : Direct questioning of the decisions maker may not elicit all the objectives. The O R team could compile a rough list of the possible outcomes and ascertain the decision-maker's reactions. He may not, for example, wish to base the factory on a particular site owing to his justifiable apprehensions of labour situation in the area. Such probing is necessary as the decision maker is himself usually vague about the objectives and the outcomes. Similarly, the objectives should include not only the ones which are desired to be attained but also the existing ones which are required to be retained, *e.g.*, employment level or family control. Many such objec-

tives as stable employment, reducing imports, good relations with the community, leadership of the product. enhancing public image, not to change the company's cash position etc. may be desired to be retained by the decision maker.

Organisation : The O.R team would naturally want to study and understand the organisation content, that is, men, machines, materials, associated consumers, suppliers, competitors and the Government and public Obviously, all the content may not play important role in organisation's problem This would determine the depth of the O.R study and the type of the problem. In a problem of price setting, for example, competitors would figure importantly whereas in a production scheduling problem the Government and the public may perhaps be ignored

Allternative Courses of Action ; It is desirable to make an exhaustive list of the alternative courses of action The team may check each alternative against :

Does switching over to the alternative of change in organisational content, structure communication, control and environment affect the efficiency of system ?

Any alternative which does influence the efficiency should be included in list These are what may be called the existing alternatives On the other extreme, the sponsors may require the development of a course of action, *e g.*, a new weapon in the military and new product in industrial contexts respectively Each developed alternative, then, would have to be compared with the available course of action towards fulfilment of the objectives

Further still, counteractions may be listed down, e g , what competitors will do if the decision-maker adopts a particular course of action. Such counteractions are easy to determine by personal interviews of the workers and consumers, if involved ; but in the case of competitors such a possibility does not exist A history of counteractions may be available in that the decision-maker would recall or subjectively assess the competitor's reactions or counteractions.

Once the lists of objectives and courses of actions (and counteractions) are ready, the team has understood the organisational content and structure and identified the decision maker, the team can convert the decision-maker's problem into research problem.

Editing the objectives and courses of actions : The rough list of objectives and courses of action may turn out to be far too lengthy. It is desirable, therefore, to reduce the number of each of the two components of the O.R. problem. The list of objectives may be shortened by screening out :

(i) Intermediate objectives : An intermedtate objective is the one which is not an end in itself but a means to another already in the list. Reducing production cost and enhancing profit are in this sequence. The formar is necessary for attaining the latter and may therefore well be dropped out ; and

(ii) The objectives which are inert to each course of action, i e , these objectives are unaffected whatever the course of action These may have been included inadvertently and need to be dropped out, and

(iii) The list of the objectives is usually compiled by probing the various participants in the O.R. study or those concerned with it The parties could well have the same objective Both manufacturers and the consumer may be interested in high quality and low cost, for example The list should, therefore, be screened for duplication of such objectives

Similarly, screening may be carried out for the courses of actions Past experience may rule out certain courses of action as unpractical. Legal and social obligation and constraints may dictate exclusion of a few other courses of action

Models and Modelling

The process of modelling is ubiquitious and has been consciously or unconsciously carried out at all times in almost all human endeavours Its meaning is also evident though now a days there is lot more to it, than what we persume the way the layman understands it Also there is a great deal of conscious effort in modelling not only in business situations but also in military, economics, social sciences, astronomy, etc Well, yes, in the past the model meant merely a physical construct either scaled down or up of such objects as earth or an atom for pedagogical purpose. Now a days, however, particularly in the case of O R. and business situations, a model is *any* (not necessarily physical) representation of one or a few aspects of reality and as such diverse items as a map, a multiple activity chart, an autobiography, C.P M. network breakeven equation, etc. are all models since each one of them represent a few aspects of a real life situation. We shall emphasise "a few aspects" just as the map is intended to represent mainly physical boundaries and may, for example ignore the heights of various places, above the sea level The long list of models exemplified above should suggest that the models can be categorised in many ways Incidentally, by means of this listing we tried to bring out the widespreadness and characteristic of models and therefore, this listing itself is modelling. In fact, wherever you come across "for example" or "as an instance" rest assured modelling is being undertaken by way of the example that is intended to bring out some characteristics of the reality. Thus there is some correspondence or equivalence either physically or conceptually or by analogy between the system or reality being modelled and the model However, this correspondence may neither be exact nor complete In fact, it has not to be complete since the intention is to focus on a few aspects of the system or the reality. Many details are, therefore, deliberately left out and the aspects of interest only appear in the model. To what use are the models put ? One of the uses is that they can act as descriptive devices. The organisation chart or the pie chart or the layout diagram describe the features of their respective systems.

A model, in a literal sense, can also be something to strive for or emulate, viz. the physiolgist's concept of a healthy person or an engineer's ideal design, a politician's view of an ideal democratic system, the total information systems increasingly being

envisioned in management information circles, the most economic order quantity to manufacture or procure are all normative models in this sense

Models can also be used to explain or predict the behaviour of a system. With the exponential smoothing forecasting model for example, we can forecast the sales. In the case of the breakeven model, we can know the effects on profit of changes in fixed costs or prices or advertising budget, etc Such manipulations may either be impossible or forbiddingly expensive to perform on their respective real entities

We shall now discuss the various classification schemes of Models.

1 **By degree of abstraction**

With the aforesaied liberty in the definition of a model (i e , it may or may not be a physical construct, whatever we speak or write or read is after all a model Surely when we speak or write we describe some event or whatever which, though, we cannot do perfectly well because of our mastery of the language and the limitations of the language itself There could, in fact be other limitations too For example, in the case of a cricket match commentary the commentator who is modeling the play for his audience is usually under time limitations All such models are *language models.* Business case studies are such models in our context.

Language models are far more abstract than the *concrete model* like a globe of the earth or the model planes mounted in wind tunnels since they (the concrete models) are at once suggestive of the shape or properties or characteristies sought after of the modelled entity However, even more abstract than the language models are the mathematical models (viz the break even equation or linear programming formulation of the product mix problem) Because to get the idea of the real-life situation they represent requires mathematical training and on the top of that considerable concentration

2 **By function**

(a) **Descriptive models.** They decribe some aspects of a situation.
Examples Multiple – activity chart in Work Study (Ref Study V).
Map of the world.

(b) **Perdictive models.** They can answer "what if" questions.
Examples Exponential Smoothing Forecasting Model. Break even equation.

(c) **Normative models.** Economic order Quantity.

3. **By Structure**

(a) **Iconic models.** An iconic model looks like what it represents. A photograph, and a painting are iconic models of persons or objects. The toy automobile is an iconic model of a real automobile. A globe is an iconic model of the earth. An iconic model is said to be scaled down when the dimensions of the model are smaller than those of the real object ; for example, a globe representing the earth. A model is said to be scaled up when it is bigger than its real entity, viz., the physicist's model of an atom or the sketch of an insect.

Commonly, an iconic model represents a static event. Characteristics that are not considered relevant are not included. Another limitation of the iconic model is that it is not capable of incorporating several dimensions.

(b) **Analogue Models**. Here there is no look like a gadget, even if a strong correspondence of basic characteristics between the partern examined and phenomenon observed is present. For example, the seasonal demand may be represented by a plot in time-demand axes To the extent a model represents one set of properties by another set of properties that model is an analogue model. In graphs, we use distance to represent such properties as time, number, age, profit, demand, etc , therefore, they are analogue models. In the analogue computers, we represent quantities by voltages. Analogue computers are, therefore aptly termed analogue Analogue models can thus represent dynamic realities. Also, they are easier to change than the iconic models.

By transforming properties into analogous properties we can frequently increase our ability to make changes. Usually it is simpler to change an analogue model than to change an iconic models and not as many changes are required. Analogue models can represent dynamic situations and they are customarily used more than iconic models because of their vaster capacity to depict the characteristics of the phenomenon or event under consideration. For instance, a network of pipes through which water flows could be used as a parallel for understanding the distribution of electrical currents The breakeven equation is a symbolic model discussed below but the break-even chart is an analogue models.

(c) **Symbolic Models**. (Syn. Mathematical Models) are by far the most commonly employed in an O.R. study because of the great deal of complexity associated with an organisation It is so easy to change them (merely changing the symbols) or to manipulate them i.e., experiment with them. Just substitute different values of the variables towards experimentation. They may be further categorised by the nature of environment.

4 **By Nature of the Environment.**

(a) **Deterministic Models**. E.O.Q. model. Here the effect of changes in the batch sizes on the total cost is known.

(b) **Probablistic Models**. As an example, the forecasts by the exponential smoothing model do not purport to be true, they are most probable.

(c) **Games**. The competitors have conflicts of interests.

5. **By the Extent of Generality.**

(a) **General Models**. Linear progrmming model can be used for all the functions (viz, marketing, production, etc.) in an organisation.

(b) **Specific Models**. Sales response as a function of advertising in the form of a curve or an equation is specifically usable in the marketing function alone.

Properties of a good model : A model does not always has the characteristic of being a yardstick—it can be explanatory rather than merely descriptive. Following are chief characteristics that a good model should have :

(a) A good model should be capable of taking into account new formulations without having any significant change in its frame.

(b) The number of assumptions made should be as small as possible.

(c) It should be simple and coherrent. Number of variables used should be small

(d) A good model should be open to a parametric type of treatment. Such situations are often faced when response to an advertising campaign or the customer acceptance of a new product is studied.

Advantages of a model :

(a) Through a model we can have a good grip over the problem.

(b) It provides some logical and systematic approach to the problem.

(c) It indicates the limitations and scope of an activity.

(d) Through models we are able to incorporate useful tools which help in eliminating duplication of methods applied to solve specific problems.

(e) Models help us finding avenues for new research and improvements in a system

Limitations of a model :

(a) Models are only an attempt in understanding an operation and should never be considered absolute in any case.

(b) The validity of any model with regard to the corresponding operation can only be verified by carrying on experiment and relevant data characteristics

Constructing the Model .

It is not unusual for the construction of a mathematical model to take hundreds of hours of the model builders (the O.R. team) and the concerned manager. It is a highly creative activity. It is not, therefore, possible to lay down a manual for model building, which even, if it could be done, would only stifle the creativity of the model builders. Nevertheless, some guidelines as accumulated over the years of modelling experience as also the pitfalls to be avoided can save many man-hours.

To illustrate the thinking involved in model building process consider a well known marketing model known as Reilly's Law which we persume does not exist and feign to discover it.

Two cities attract trade from a town located somewhere between them and wonder what fetches them this trade i.e. what are the factors that have a bearing on the magnitude of trade they each capture. Two factors at once strike the mind. The larger the city (in fact the trade activity) the more the trade it is likely to attract from the town. And greater its distance from the town the less the trade it would attract.

In other words, the trade a city attracts is directly proportional to its size and inversely proportional to its distance. We use the following symbols to denote the variables of interest.

T_A=Trade (in Rs) captured by city A

T_B = " " B

P_A=Population of A (supposedly a measure of its size)

P_B=Population of B " "

D_A=A's distance from the town.

D_B =B's distance from the town.

Then the following model is obtained for this situation.

$$\frac{T_A}{T_B}=\left(\frac{P_A}{P_B}\right)^x\left(\frac{D_B}{D_A}\right)^y$$

x would be 1 if the trade increases linearly with the increasing population. Likewise y would be 1 if the trade decreases linearly with the increase in distance. Anyhow their values remain to be determined. It is reasonable to assume $x=1$ but intuitively it does not appeal that if the distance is halved the trade would be doubled *i.e.* $y=1$. Now Reilly used $x=1$ and determined y by plugging in $x=1$ and other variable values in several such triplets of cities and found that y varies between 1.5 to 2 5. He, then, recommended the following model for general application :

$$\frac{T_A}{T_B}=\left(\frac{P_A}{P_B}\right)\left(\frac{D_B}{D_A}\right)^2$$

This is an abstract mathematical model. Its language model counterpart would go as, "Two cities attract retail trade from an intermediate town in the vicinity roughly in direct proportion to the populations of the two cities and in inverse proportion to the square of distances from these two cities to the intermediate town."

Several points emerge from this modelling

1. Newton, a renowned physicist, discovered the gravitational Law, "Every object in this universe attracts the other with a gravitational force which is directly proportional to the product of their masses and inversely proportional to the distance between them". It is not difficult to appreciate the similarity between the two laws in widely different walks of life The situations are analogous. This should suggest how the mathematics of astronomy or physics can be usefully applied in marketing or business problems.

2. It is assumed in this modelling that the trade activity of a city can be approximated by the size of population. This assumption may not be always valid. If both A and B were equally populated A may still draw more trade because it is highly industrialised and it carries out high value transactions with the town. However, it is possible to refine this model by incorporating a few indices of the trade activity. But that would make modelling expensive, difficult to understand and appreciate by the operating staff, etc. Likewise distance alone may not be sufficient. The condition of

the roads or means of transport, etc. are also the factors to be reckoned with. What we are trying to drive home is that a model is built after great many simplifying assumptions and it just cannot be grafted on to another situation without scrutinising the underlying assumption For example, this model built in U.S A may be inappropriate in India A model is a model only and it should not be allowed to be confused as the reality itself. We say so because models are often epitomised as reality. Even the aforesaid famous Newton's Law supporting the edifice of scientific inquiry was challanged by Einstein for its inapplicability in outer space. *Miller and Starr condemn this thought process by exemplifying the child who said "It is a good thing they call pigs "pigs" because they are such dirty animals"

Also, both the models are not to be discarded for their simplicity as useless Both of them have practical utility howsoever limited or unlimited, and if nothing else at least, they do throw light on the underlying theory. The famous E O.Q model has often been abused for similar reasons but its sharp focus on cost variations with the batch size is a discovery of a high order. Thus the utility of a model not only lies in the extent of its application but also the principles it exposes or explains.

3. We considered only a tripet (two cities and a town) in isolation. But there could be other cities or towns in the vicinity which may distort these proportionities significantly. For example, another city in the close proximity of A may make the town people rarely to look to B since all their demands can be met in one direction. Thus, what are the boundaries of the system or the reality to be covered in modelling is a delicate matter to be decided before undertaking the construction of the model or such interactions may be discovered during the process of construction or model building and the initial model may have to be modified to account for these. Thus model building is an iterative process. In inventory situations the independently computed E.O.Q.'s may violate the storage space limitations in a similar way.

4. We said that $y=2$ was derived by Reilly after trying this model in several such triplets. The O R. models may have likewise to be tested against past historical data, etc., in the same way.

5. T_A, T_B, P_A, P_B, D_A, and D_B above are variables. But x and y are the parameters. Not entirely jokingly they may be called variable constants because as we said, in U.S.A., their values are 1 and 2, in India, they may be different. Or if today their values are 1 and 2 after a lapse of a few years they may change because of changing habits, of demographic movements, etc. Therefore, it could be neccssary to verify their values periodically

6 This example very forcefully brings out a point to be picked up later that counting for the variables could be a tough task. Estimation of populations of the two cities would not be easy and as straightforward as it looks to be.

What if y were 9 and not 2 ? That would mean that the model is highly sensitive to the changes in the distance variable. The total cost curve of the E.O.Q., model

*Executive Decisions and O.R., (PHI).

(figure of page 13) is rather flat at the bottom, meaning that it is insensitive to slight changes in the E.O.Q. Thus if $q_o=122$ was computed it would increase the total cost negligibly upon rounding it off to 125. Incidentally, the protagonists of the E.O.Q model use this argument as a hedge against inaccuracies in the estimation of Ca, I etc.

Solution of the Model

The solution from the model is a solution to the model alone and may not be optimal solution for the real life situation because of simplifying assumptions or other inaccuracies to be discussed soon

Three approaches for solving the model exist.

Analytical Solution. Ordinary or advanced mathematics can be applied to a variety of mathematical models for deriving the solution. The E O Q, model is an example A simple application of differential calculus yields the optimal solution. In the case of several replacement problems mathematics of finite differences is put to use. Frequently, some restrictions (like storage space availability in the case of E.O Q. model) complicate the solution procedure Nevertheless, there exists a technique of Lagrangian multipliers to solve such models, too. The range of the techniques to use is as wide as that of the models. In view of this it is to be desired to include a mathematician in the O R team However, some models, though possible to handle mathematically, are more easily solved numerically.

Numerical Solutions are arrived at by the process of trial and error. The student is supposedly familiar with this approach for determining the internal rate of return for an investment proposal Many of the differentials, integrals, matrices to be inverted, etc. may either be impossible or difficult to solve analytically and the numerical methods are quite handy here As a simple example, consider the following quadratic equation which the student should be able to solve readily analytically. However, we propose to solve it numerically as an example

$$x^2 - 3x + 4 \quad 40.$$

Try $x=2$ L H S $=2^2+2\times3+4=14<$R.H.S ; therefore.

try $x=5$ L H S $=5^2+5\times3+4-44>$R.H.S. ; therefore

try $x=4$ L H S $=4^2+4\times3+4=32<$R.H.S. ; therefore.

try $x=4\frac{1}{2}$

Proceeding in this way we can find highly accurate solution. Our analytical techniques would fail us in higher order polynomials ; but the numerical approach can be usefully applied there As another example the square root program flowchart of fig. 6 of SADP Study II is based on the numerical approach.

Monte-Carlo Solution can be obtained when the above 2 methods are of no help to us. This method makes use of random numbers and is introduced in Study VI. However, it is like using a brute force to solve a problem since it requires a great deal of computer time and, therefore, it should be used as a last resort only.

Steps in Simulation (The student may interpret these in term of the Hertz's model for capital investment discussed in FM Study V).

1 Sample observations are taken to choose some appropriate model for the system For accurate results, use of correct sampling procedure is necessary Here, we determine the distributions for the variables of interest such as in quenuing theory (or in Hertz's model, those of the 9 input or exogenous variables)

2 Convert the exogenous variables probability distributions to cumulative distributions

3 Establish structural relationships between the exogenous and the output or endogenous variables

4 Select a sequence of random nos from the exogenous cumulative distributions

5 Perform calcutations according to the structural relationships and determine a value of the output variable

6 Repeat step 4 and 5 to obtain the distribution of the output variable

Classes of O R Models

The mathematical analogue models usually to be encountered in O.R fall into certain classes depending on the types of process to which they are applicable In most of the cases these processes have acquired their own names Following is a brief survey of these (6 in number)

(a) **Theory of allocation** Problems of allocation arise whenever there are a number of activities to perform but limitations on either the amount of resources or the wav they can be spent, prevent us from performing each separate activity in the most effective way conceivable In such situations. we wish to allot the available resources to the activities in a way that will optimise the total effectiveness The simple case here is the one where the objective function and the restrictions are stated in terms of linear functions of the allocations The analysis of these situations is in the domain of linear programming dealt with in Study III However, there have been recent advancements upon linear programming , integer programming where the variables assume discrete values, parameteric programming which is a sort of sensitivity analysis on the LPP solution in that it investigates its behaviour as a result of predetermined linear variations in the parameters of the problem, non-linear programming, geomecric programming where variables are multiplicative unlike linear programming in which these are additive.

(b) Replacement Theory (Ref. Study VI) is concerned with the following situations .

(i) For equipment of which efficiency worsens witn time,

(ii) equipment which "dies" or fails suddenly ; and

(iii) staff replacement problems

(c) **Inventory processes.** Here two or all of the following costs may be in conflict : (i) acquisition procurement or set up costs, (*ii*) inventory carrying cost and (iii) disservice cost. These are discussed in Study II.

(d) **Queuing Processes.** Decision situations frequently arise in which units arriving for service must wait before they can be serviced. Arriving units may form one queue and be serviced through only one station, as in an accountant's office ; they may form one line and be serviced through several stations as against tea (vending machines) or they may form several lines and be served through as many stations as at the checkout counter of a supermarket. As the costs (though apparently intangible) are associated with waiting times, If therefore the laws governing arrivals, service times, and the order in which arriving units are taken into account i e. arrival (service probabilities) the nature of this waiting situation can be studied and analysed mathematically *i.e.*, an optimum decision can be taken with regard to opening of new service facilities, closing of the existing one. Some elementary queues are discussed in Study II.

(e) **Competitive.** Processes are characterised by the fact that two or more individuals (or groups) are making decisions in situations that involve conflicting interests and in which the outcome is controlled by the decisions of all the parties involved. These are discussed in Study VI.

However, all the probabilistic models, too can also be taken as games against the nature and as such they are discussed in Part II of Study II. Their discussion by the utility criteria and in the form of decision trees is embodied in Study VI.

However, with the advancement of O.R. in future more processes would have been added to this list. Also, the distinction between them is not so clear. Often the inventory processes, for example, can be treated as queuing processes, the work-centres being the service facilities and the batches of components to be processed being queues. Also such O.R. tools as simulation, which can be utilised in solving the models of such process and others legitimately fall in the domain of O.R.

(ii) **Testing the model.** Model is, after all, a partial representation of a situation. A large number of assumptions go into the construction of a model To give an example, consider the Economic Order Quantily Model When an item is procured from a vendor the item having a fairly regular usage, the problem arises . should one just procure the immediate week's or month's etc. requirements, or combine requirements of more than one period immediately ahead ? On the one hand, there are costs associated with the placement of an order and its further processing and on the other, are inventory carrying costs if more than the immediate requirement is sought to be stocked. The balance has to be struck between the two and the size of the order is thereby determined which optimises the sum of the two costs. The ordering costs usuall comprise several elements, *e.g.* cost of writing and typing the peperwork of the order copies, cost of expediting through reminders or by telephone calls and even personal

visits, costs of maintaining the progressing diaries and the files, costs of receiving the goods by the Goods Receipt Notes, sampling or complete inspection of the deliveries, updating stock records, handling vendor's invoices etc. From ordering point of view, therefore, the fewer the orders, the cheaper it is for the receiving organisation Or inversely, this implies as large batch sizes as possible. The stocks in excess of the immediate requirements, however, are an expense or burden in the sense that the storage space is required, there is a loss owing to deterioration or obsolescence and expenses of the insurance charges and. above all, the investment tied up. Theoretical modelling of the situation leads to the following formula for computing q, the optimal batch size

$$q = \sqrt{\frac{2 \times \text{cost of placing and getting an order} \times \text{Annual usage in units}}{\text{Annual cost of holding a unit in stock per year}}}$$

It purports to minimise the sum of ordering and inventory carrying costs. But a host of assumptions (see Study II) are made in its mathematical derivation, some of them are given below :

1 Items are replenished instantaneously. i.e, the replenishment leadtime is zero Furthermore, items are delivered in a single batch In practice, neither the leadtimes are close to zero nor deliveries are always made in a single batch. Although, theoretically, it is possible to refine the model to take into account both of these shortcomings, this would make the model rather cumbersome to operate routinely.

2. The demand of the item is regular Certainly the formula provides sufficiently near optimal solution if the demand is not very much irregular ; but, in practice, there will be great many possibilities of the demand behaviour and subsequent stock depletion The batch may 'be consumed the moment it arrives or a substantial portion may be consumed at a time later.

3 The optimal batch size is computed in isolation and the batch sizes aggregated over several items may violate the storage space, machine capacity (for the made-ins) and the total capital restrictions. Although the formula is amenable to mathematical refinements here again they have further complications in their trail

4. The aforesaid cost elements which enter into the explicit solution for q, as enumerated above, are very diticult to measure or may only be hazily defined. The ordering costs are supposed to be linear with the number of orders, whereas this may not really be the case The associated inventory carrying costs have been a topic of a long controversy and there exist two difinite schools of thought in stock control circles. Also, many practitioners have summarily dismissed the formula as impractical.

This model was intended to give an example of the likelihood of a distorted representation of the system. Furthermore, it is not just a single model that the O.R. team might end up with. There is usually a 'conglomerate' of models. (We shall have such cases in Study II). There is a great need. therefore, to thoroughly test the model. Aside from simplifying assumptions, which are necessitated to handle the problem mathematically. there are a few other possibilities of distortion in representation. Some of these are discussed below :

1. The model may assert the dependence of the effectiveness of the system on one or more independent variables, which as a matter of fact do not influence system's effectiveness. In an exponential smoothing model, for example, demand may be wrongly presumed as seasonal whereas it has just but one peak during Christmas.

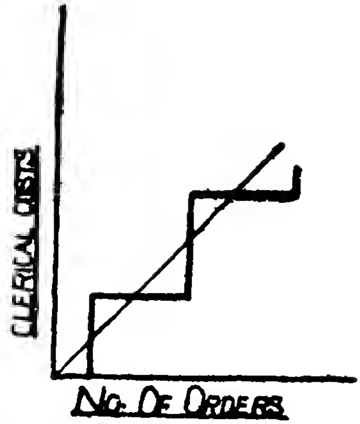


Fig. 1

2 The model may fail to include a variable which does influence the effectiveness of the system In a production model, for example, set up and inventorying costs may have been well balanced but the important cost of changing the labour force might have been ignored

3 The model may inaccurately express relationship between the measureo f effectiveness, E and one or more of the pertinent variables e g , the forecast error is usually assumed to be normal , but it may not be so (See Figs 2 and 3). This would lead to services and/or inventory levels contrary to expectations.

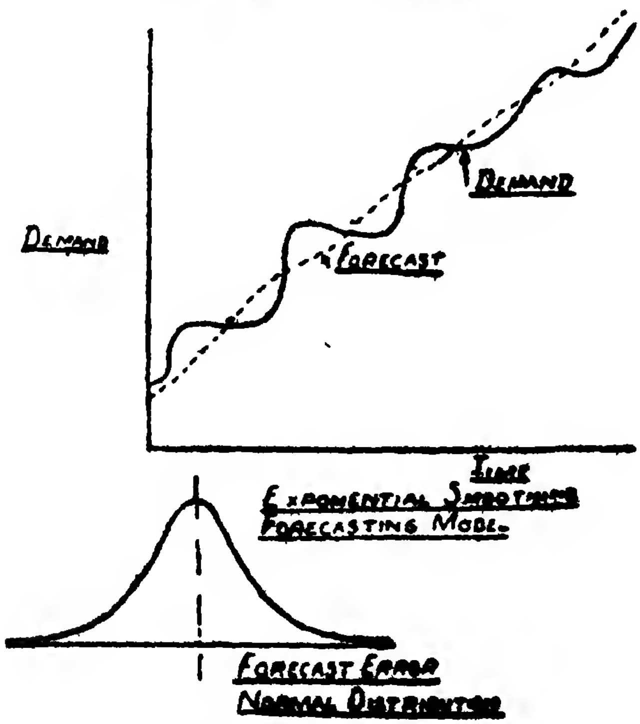


Fig. 2 and 3

It is usual to assume the acquisition costs in stock control as linearly related with the number of orders. In the case of clerical cost element, at least, the relationship is steoped as shown in Fig. 1. This implies that a fixed strength of the concerned staff is capable of handling number of orders in a range, *e.g.*, 600 to 1,000 per year. Putting it other way, clerical costs are not divisible

We stated earlier that *q*, the optimal batch size and safety stocks interact, whene the overall objectives of inventory control system are :

(i) Service to the customer at a tolerable level

(ii) Inventory level at the minimum possible.

(iii) Ordering costs at the minimum possible.

The interaction shows itself significantly when the batch size is large, *e.g.*, a year's requirements Under such circumstances, less than the ordinarily and individually computed safety stock is enough for the desired level of service to the customer. The consequence is over-inventory if the interaction is not taken into account.

In the critical path method (Study IV), the network may not be correct. This could cause confusion and delays during implementation.

4 If the model is sound with respect to the variables and relationships it may still yield poor results if the parameters are wrongly estimated. In an exponential smoothing model, higher (than the optimum) values of α may lead to over estimation or under-estimation of demand which is rather stable. Such a "nervous" forecasting model may either adversely affect service to the customer or increase inventories.

It is almost an established custom to take the inventory carrying cost as 20% of the inventory in calculating optimal batch sizes So mystically universal is the use of this parameter that an expert on stock control, when questioned about the rationality behind 20%, cleverly saved himself from his boss to explain it by the reply. "Now that the inventory control systems are being computerized, 24% is being favoured because, it is simply divisible by 12, the number of months in a year" ! If the various elements of inventory costs are patiently assessed for the given organisation the result may be markedly different from 20% or 24%.

This should underline the great necessity of testing the model.

The evidence or that data against which the performance of the model is to be tested should be gathered in a careful manner. It may be fairly painstaking as the sponsoring organisation may have to be convinced by such tests. It appears to be a straightforward and simple matter but is deceptively crucial. The conditions under which and the procedures by which, the evidence is to be collected should be rigorously defined. The actual procedures of testing however, would differ from situation to situation. If the O.R. team is fairly confident about the validity of the model they may start testing the model as a whole. If, there is defficiency they may then pursue for the faulty variables, parameters or relationships. If however, the team is not very

confideut, they may proceed the other way round beginning with individual variables, parameters and relationship, finally ending up which testing the model as a whole.

For the purpose of collection of evideace, the variables may be put into two categories : Enumerative and Metric variable.

Enumerative Variables : These are the countable variables. Counting is considered to be too simple to deserve special attention ; but experience suggest it to be too difficult to perfrom accurately It is frequently the case that one counting the number of members in a party ignores himself ! As another example, the student may find that in different trials of count of the number of vowels in the following couple of pages come out to be different. Therefore, it is necessary to devise the best possible counting scheme, and cross checks. If, for example, it is required to count the various stockitems falling in the following classes of the usage value (the range of the classes, first of all, should be clearly stated) a tally role scheme may be quite useful. See figure 4.

Metric Variables. Unlike the enumerative variables where, basically, the emphasis was on "yes" or "no" regarding item *i.e.*, whether an item possessed certain attribute, here. with metric variable, one is concerned with "how much" aspect of the variables. A rigorous set of specifications regarding the things or events to be measured, the environment and the changes in the environment under which measurements are to be made, operations to be performed, instrutments to be used nnd readings to be taken should be set. The student is referred to study matererial on Work Study for a more detailed discussion on such topics.

Sampling and Experiment Design : The need of sampling arises either because of great difficulty or impossibility of complete enumeration on economic considerations. These general considerations are as applicable here as in any other situation.

USAGE VALUE	TALLY ROLL	TOTAL
0 < 500		8
500 < 1500		18
1500 < 3500		15
3500 < 6000		14

Fig. 4

With experimental design, it is possible to ascertain the effeciveness of a function by considering several variables at time, This permits an analyis of the possible interaction between the various variables. The technique, an extension of analysis of variance, is highly elegant and versatile.

Reduction of data : *The data collected may require collation, editing, coding punching, etc before it is usable The student is familiar with these aspects on the basis of his study of statistics The ultimate form to which the data is to be reduced will either be an estimate of the value of a parameter or an inferential statistice For example, in testing the optimal batch size formula we may first evaluate cost variables in order to compute the total cost predicted by the model. These predctions may then be compared with the observed values A statistical measure can then be derived to escablish if the model predicts well*

The statistical measures of the variable computed from the collected data may be tested by means of significance tests, or otherwtse for the following :

(i) Whether a variable should not be included.

(ii) Whether the form of an analytic function is linear or some other type.

(iii) Whether the form of a probability function is normal or some other type.

(iv) Whether the model has failed to include a variable that ought to have been included

Testing the Solution · The test itself usually consists of a comparison of actual effectiveness with what would have been obtained if the solution derived had been used. For example, the testing may be performed retrospectively In a forecasting model, as an instance, forecasts of the (say) immediately preceding 6 months may be compared with the actual demand that had materialised during those months. Some times, there may elapse considerable time before the data for testing accumulates For a new product the forecasts may be made on the basis of demand of another item having similar characteristics in this respect But one would have to wait for the future demand to materiaise so that it can be compared with the hypothesised forecasts. It is to be noted that collation, collection, editing the past data for comparison may be quite painstaking sometimes. An example is to find the weekly or monthly demand of an item over the past one year Demand is constituted by the custmoer's orders for which adequate records may not exist. Therefore, it may be thought that demand could be substituted by withdrawals (issues). Dates of issues may not have been properly recorded One would have also to be cautious that an unusually high and low demand or issues might have occurred with some assignable and exceptional causes. Such data should be omitted Illegibility of the handwriting may frequently be disgusting. If the stock control office has the custom of keeping not more than one card aside from the existing stock cards for each item and destory the past ones, this might lead to distortion. A slow moving item would have records on its two cards ; current and the previous, dating back to several years Whereas a fast moving item would have records on two cards for just a few months Paradoxically the interest may be in longer records for the fast moving items. The example should give an idea of how formidable the task of data collection may be :

(v) Control and Implementation of the Solution : Many of the O.R. projects deal with management decisions that are recurrent. Forecasts may have to be calculated every time an order is placed, especially with a computerized systems. Many of the production scheduling rules are similarly routinely applied. The systems to which O.R. is generally applied are dynamic in nature. The model may be outdated sooner or later by changes in the values of parameters and the number of variables. Even structural relationships for computing the measure of effectiveness, are liable to undergo change under such dynamism. Demand patterns may change for various abrupt or gradually bulit up reasons. Demand depicting simple linear trend may assume a peak or seasonality. This would call for a change of the model. In less severe cases, the device of adaptive response and tracking signals (dealt in O.R. Study II in more detail) may be embedded in the model right at the time of implementation of the forecasting model. The adaptive response refers to automatic change in the value of the smoothing coefficient in sympathy with changes in the "magnitude" of fluctuations in demand. The adaptive response element is put into operation when an accumulated statistical measure exceeds the limits. This indeed is continuous monitoring or control of the O.R., solution. There are many cases where the control may have to be exercised at discrete intervals of time. As we shall discuss in slightly more detail, some of demand patterns are not serially correlated, *i.e.* the demand is not regular. If in one month the demand is 13 the next month it may be 100 and later drop to 2. Some of these patterns may admit description by some of the standard, mathematically easily manipulative distributions, *e.g.*, Poisson or Lognormal. Instead of time series forecasting models, the probability distribution models are used. Now the probability distribution model may change in its value of parameters, *e.g.*, the mean (Fig. 5). The mean demand level has, therefore, to be periodically checked and if there is a significant change the probability distribution model has to be suitably amended. Standard deviation of the distribution may also undergo a significant change and even the form of distribution.

Therefore, if the O.R. solution is to be operative on repetitive basis suitable control system should be designed and embedded in it or put in black and white as an easily understandable procedure. With the existing state of the art, this may not always be possible to devise optimal control ; but nevertheless, control procedures can be devised that are better than if the system were to be left as such.

Control has to be designed with regard to the following possibilities :

(i) A variable becomes irrelevant or another, hitherto not included, becomes a candidate for inclusion. The student could imagine such possibilities in the context of regression models.

(ii) The parameters change in value ; as was the case above with the illustrative example of the probability distribution models for forecasting the leadtime demand.

(iii) Change in the structural relationship in the regression model as cited above.

But the change in the value of a parameter should be significant in the statistical sense. As will be discussed later in this section at a greater length significance entails

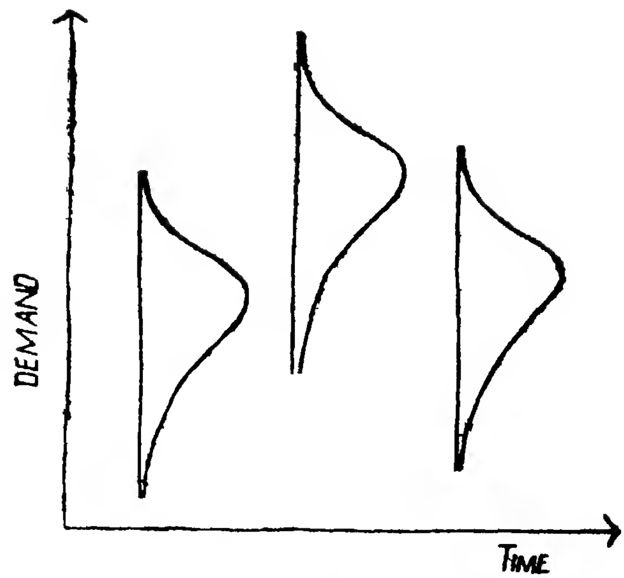


Fig. 5

risks of two types Briefly a significant change may be ignored or an insignificant change may be mistaken as important. Obviously, then, the variables, parameters and relationships that are susceptible to change and any variable likely to be included after a few years' operation should be listed and, where possible procedures for detecting significant changes laid down, the necessary remedial measures for adjustment specified and the concerned persons in the organisation suitably apprised of such contingencies.

Control of the Parameters : Obviously, the first step would be to list all the variables if a change in their values is likely to influence the system's effectiveness. It may not, however, be so obvious as to which of the variables not included in the solution may gain importance in effectiveness owing to the changed circumstances in which the system is operating. A re-examination of the variables discarded whilst screening during the formulation of the problem stage may be carried out. It may also be advisable to "explode" a variable into more elementary components *e.g*, acquisition cost In this way, it is easier to discern the possibility of change in the value of a variable The variables may be categorised as :

1. Variables whose values during the period covered by a decision can be known in advance, *e.g.*, number of models of a particular product.
2. Measures whose values cannot be known in advance such as arrival rates of trucks, demand, labour turnover ; etc.

It is not a difficult matter to devise control scheme for former category, but the latter is problematical. First of all it has to be established if the value of a parameter should be adjusted to a new level or the procedure for estimating a variable should be overhauled. Acquisition cost, again, may be suitably amended and incorporated in the Inventory Control Model, but if the demand behaviour abruptly changes, *e.g.*,

it tends to be seasonal the forecasting model itself would have to be revised. Now the determination of the change to be significant or insignificant is quite important. This determination consists of testing the hypothesis that no change has occured in the variable or the characteristics of its estimates, which are themselves variables.

The student may recall Type I and Type II errors inherent in significant testing.

	H_0 True	H_1 True
H_0 Accepted	Correct Decision	Type II Error
H_1 Accepted	Type I Error	Correct Decision

In the following Study Material we shall discuss exponential smoothing forecasting model's reaction to a sudden change in demand. Here the control system is familiarly known as tracking signal The purpose is that if the change is significant the computer signals out for manual assistance. The person concerned, upon such a signal of an abrupt change, would hunt for the causes He may ultimately be able to assign the abrupt change to such a cause as a competitor shutting down or a political event, etc. But there is also a likelihood that the abrupt change was merely a chance occurrence and he would have unnecessarily expended efforts in tracing the assignable causes which, in fact, do not exist, Let H_0=No real change in the demand level. If H_0 is true and there is signal it is a false signal and constitutes what is called, Type I error If, however, H_0 is not true, *i.e*, there is an abrupt and real change but there is no signal this constitures Type II error and is equal to α where $(1-\alpha)$ is the significance level. Here, the control procedure of tracking signal has to be so devised that Type I and Type II errors are at an acceptable level. *i.e.*, false signals and un-signalled true changes should be within tolerable limits. False signals entail unnecessary effort in hunting for the assignable causes Unsignalled true changes result in, as should be more clear from the following study material, either more inventories or poorer service to the customer. Incidentally, as an exercise, the student may consider : -

(i) frequency of false signals,

(ii) poor service to the customers,

(iii) increase in inventories

as a problem of weighting the objectives for O.R. study of devising control mechanism to be built in the forecasting model for detecting abrubt changes in demand level.

Detection of and adjustment for significant changes : Ideally, in the design of control system for a variable, six inter-dependent decisions should be made if possible :—

(i) The frequency (*i.e.*, period between) of control checks.

(ii) The number of observations per control check, if more than one is possible.

(iii) *The way items should be selected for observation (i.e.,) the sample design if more than one observation is specified.*

(iv) *The statistical testing procedure to be used to determine whether or not a* value has changed.

(v) The specific decision rule based on the test.

(vi) The action to be taken if the test indicates that a parameter's value has changed.

Again, ideally, these decisions should be made in such a way as to minimize the sum of the following costs :

(i) The cost of taking the observations,

(ii) The cost of performing the test,

(iii) The expected cost of Type I Error,

(iv) The expected cost of Type II Error.

Here again the expected total cost could form the criteria.

Controlling the relationships : The probability distribution function *e g.*, a Lognormal distribution for demand may change its parameters (*e.g.*, mean) over time and may need amendments. Or the form of the probability distribution itself may undergo change Chi-square test of goodness of fit may be performed periodically to keep the necessary control.

In the following Study Material, the student will find a regression curve fitted between the Mean, M and Standard Deviation, S of the past demand of sampled items for inventory control purposes. Initially, the estimate could be.

$$S = KM \text{ where k is a constant.}$$

It may undergo change in value of its parameter, k or even the straight line regression equation may be no longer applicable and a curve may be more appropriate.

Implementation : It is the final phase in the Operation Research methodology and the most important for the sponsoring organization but, perhaps, least developed by the O.R. theoreticians.

The solution the O.R. team concludes should be written down in the form of an operating manual, especially when the decision rules are to be applied routinely. This may require answers to questions below :

1. Who should do what ?
2. When ?
3. What information and facilities are required to do it ?

The people and the department responsible for taking each action need to be pinpointed. The implementation phase requires continuous co-operation and communication between management and operations researchers. Each person entrusted with an action should be suitable instructed. The tools required for taking the action

should be made available to him. These should be simple and quick to use, *e g*, a nomogram. The *tools* may require considerable simplification even to the extent of some loss in system's effectiveness. This should be obvious since the mathematics employed in an O.R study *usually tends to be quite complex* ; *but the* operating personnel *would have either* no background or time to understand and apply.

Self-Examination Questions

1. Support or refute that the following problems are suited for operations research by giving reasons in defence, briefly—

(i) Stockpiling of crackers prior to Dewali.

(ii) Decision by a contractor to replace trucks after about 8 year's usage even though some of them may be in working condition.

(iii) Reinforcing the strength of the maintenance crew by Air India's repair base in Bombay.

(iv) A newspaper boy estimating how many copies of the Times of India he should procure each day.

(v) Allocation of plant capacity amongst the 4 products of ABC Co., Ltd, Sometown.

(vi) A scooter, its driver and his companion towards office.

(vii) Opening another payment counter by a commercial bank.

(viii) Reinforcing the strength of the unloading equipment and facilities by the port authorities of Madras.

(ix) Setting up the minimum levels for a particular shoe in the Bata warehouse.

(x) Statistical forecasting of the sales of ice-cream

(xi) Derivation of the most economic lot sizes for procurement by the super market.

(xii) Establishing dependency relationships of the inter-related activities entailed in introduction of an originally new product.

(xiii) Modification in the design of an originally new product.

(xiv) Adapting the exponential smoothing forecast model for fluctuations in demand.

(xv) Embedding tracking signals in a forecasting model for caution upon abrupt decline in the demand of the transistorised radios.

(xvi) Determination of the recruitment policy for air-hostesses in view of 25% reinforcement owing to opening of some new airways.

(xvii) Finding the number of bomb hits required to destory enemy's leading gasoline plant.

(xviii) Considering group replacements to replace the existing policy of individual replacements of tubes along a segment of the Grand Trunk road.

(xx) Choice of an optimal policy to meet a spurt in demand by a combination of hiring and machines, stocking and protracting delivery period to the customers

(xxi) Improving means of written communication to provide better feedback for control of a jobbing workshop

Model answers to selected problems

It constitutes an O R problem. The objective here is to maximise the profit. Courses of action would be stockpiling various volumes of crackers. Overstocking would result in virtually complete wastage of the unsold crakers since there would hardly be any sales after the fair is over. Understocking would mean regret or the opportunity loss An empirical sales distribution may be compiled from the past historical data or by judgment such as most optimistic sales, average sales, etc The courses of actions can then be evaluated in terms of the probable sales and associated net profit. If desired, one of the maximax, maximin or the expected payoff criteria may be applied for choosing the optimal policy sales volume The timing of marketing the crackers is also important Too early marketing would mean tie up of capital as well as risk of deterioration of the crackers. Too late marketing would imply regret later

(vi) Although the driver and his companion are most likely driving with a purpose in view but there is hardly any conflict amongest the three. Furthermore, there is no other recognisable course of action but to drive. Not an O.R problem.

(vii) It is queuing type of problem. The management may set some limit for the average waiting time for the queue length. The number of counters that are required with this objective may then be calculated after having collected statistical data of different periods of the day for a fortnight, month, etc

(viii) It is again a queuing problem. By studying the statistical behaviour of the queue formation of ships waiting for being unloaded the optimal number of equipment and facilities can be determined towards desired rate of service.

(ix) Setting up the minimum level is an inventory type of O.R. problem. The objective would be to minimie the inventory levels for desired service to the customer.

(x) Sales of ice cream are influenced by seasonal effects. Mere subjective guesses may be widely off the mark. They could perhaps be better estimated by employing a forecasting model. The objective would be to maximise the profit. Too high production would result into unsold and perishable ice-cream.

Too low production would entail regret or opportunity loss. Stocking by an appropriately forecasting model would provide an optimal compromise between the two.

Others ; (ii) Replacement (iii) Queuing (iv) Inventory (xi) Inventory (xii) Network (please wait for the following studies) (xiii) Not an O.R. problem (xvii) Forecasting Inventory (xv) Forecasting (xvi) Replacement problem (xvii) Simulation (xviii) Replacement (xix) Production Inventory (xx) Production Scheduling/Control.

2. Suggest suitable models for the following :—

(i) Output measured in units and cost measured in rupees

(ii) Corn production measured in bushels per acre and rainfall measured in inches per season.

(iii) Elements or components of a new product such as an electric coffee maker.

(iv) Members of five man team running of a machinery producing synthetic cloth.

(v) Company profits and share of market and GNP.

Answer

(i) $y = a + bx$ (Symbolic Model)

where y is the output in units.

a is the fixed overheads in rupees

b is the variable cost in rupees/unit of output :

(Alternatively ; Break- even chart).

(ii) $y = a + bx$ (Symbolic model, Linear regression)

$$a = \frac{\Sigma y}{N} \quad \text{and} \quad b = \frac{\Sigma xy}{\Sigma x^2}$$

N is the number of seasons considered.

y is the corn production in bushels per acre.

x is the rainfall in inches per season.

(iii) Engineering Drawings and Bills of Materials : the former being an analouge model and the latter a symbolic model. The bill of material is useful, however only if modelling is sought in the production or assembly context.

(iv) Multiple Activity Chart – Analogue Model.

(v) Time Series plotted with years on the x-axis and the variables under consideration along the y-axis.

3. (a) State the type of the following models by one or more classification scheme.

(i) microfilmed documents.

(ii) flowchart for porcessing a customer's order.

(iii) Exponential Smoothing forecast model without trend correction :

$u_t = u_t + ae_t,$

(iv) A nomogram for computing economic order quantity.

(v) Motion film

(vi) Project network.

(vii) Templates used for manipulation towards best layout.

(viii) Poisson distribution curve.

(ix) A histogram of income of the employees of the Railways.

(x) Economic order quantity.

$$q_o = \sqrt{\frac{2ACs}{IC}}$$

Answer : Iconic : (i), (v), Analogue : (ii), (iv), (vii), (viii), (ix), Symbolic, (iii), (x). [(ii) is iconic/analogue strictly.]

(b) Give 5 examples each of the inconic, analogue and symbolic models preferably from business situations.

Definitions of O.R.

Having discussed the salient features and methodology of O.R. we are now in a position to critically appraise its definitions offered by different authorities. A definition is the most concises and precise model of a subject ! Therefore, there is a need to expatiate on these definitions. However, most of them are not very satisfactory but this should not be surprising for four reasons. First, none of the sciences has been defined to the satisfaction of all its practitioners and researchers. Second, O.R is a relatively new science Third, these definitions have been offered at different times of O.R's development and are bound to emphasise one or the other characteristic. For this reason, with its development more definitions, perhaps more precise than the existing lines should, be forthcoming. [In this connection, we shall make mention of the term "Management Science" which connotes OR in routine use in operation of MIS In fact, when we said earlier that OR is basically inseparable from systems analysis *i.e.*, we could say that as such it is management science but if it is used to sort out a one shot problem (viz., a shipping dock, having no OR staff of its own contacting an OR team to straighten its problem and then disbanding the team.) Anyhow, we would not pursue this semantic nuance any more.] Fourth, OR is inherently intediscplinary in nature with applicatson not only in military or business but also in medicine, engineering, astronomy, government administration the new science of genetic engineering, physics, and so on. Any definition is very likely to be found wanting to some extent. However we shall only be concerned with business situations.

1. "OR is the application of the theories of probability, statistics, queuing games, liner programming, etc to the problems of war, government and industry". This definition tries to bundle all the tools and techniques of OR together perhaps purporting to be comprehensive but unfortunately it fails completely to distil the essence of OR. It has also the merit of further misleading the misled practitioner and giving no indication of the discipline to a layman. How it can reinforce the wrong perceptions of the misled or unimaginative OR practitioner is to be learnt from Ackoff's humour

ous story about one such OR worker who had nothing to do during the weekend and thought of tightening all the screws in his house and as such started doing this with a screw driver. Upon having tightended all of these he encountered a nail, took a file, made a slct in its head and then started applying the screw driver to drive the nail whereas this should have been hammered down into place. The story has a lesson. The OR worker should not wander around the organisatian with the kit of OR tools like linear programming, queuing and hunt problems to suit these. It should be the other way round. He should originally formulate the problem and build the model and then make use of these tools if and where appropriate. Further, there is a host of prababilty theory and statistics applications in business which have no flavour of OR whatever, *e.g.* statistical quality control, Index numbers, etc.

Then, is accounting merely a collection of budgeting, breakeven analysis etc ?

2. "OR is is the study of administrative systems persued in some scientific manner *in which Physics, Chemistry and Biology are studied in natural sciences*" (Italics ours). We think that without the italicised portion the author could have said what he wanted to say in a more foroeful way. Much of the research in physics, chemistry and biology remains of pure nature only with little or no fruits in the short run. The applied research in these three disciplines tend to become at once of engineering or medical interest. Also their research is of highly rigorous and exacting standard. The team approach is not essential there. Mathematical ability is not essential. And no interactions may be there. OR, like engineering or surgery, is at once of practical nature. In its research bank, it does not have much of its own pure research. It adapts pure reserch of other disciplines to its own use. It is not highly exacting or rigorous. A model that works is the best. Whereas the researches in those subjects are based on the most fundamental axioms which are all too obvious, in OR fairly large number of bold assumptions are often permitted. Just as engineering allows approximations fairly so does OR too The element of uncertainly which is almost ubiquitious to OR applications is nearly missing in chemistry at least. The author perhaps intended to say that OR is scientific research into the operations of administrative systems but he has fouled this up by bringing in these three disciplines of pure science. The world "pure" is the crux of the matter. All applied science like psychology or political science or sociology are not pure in this sense since they are all concerned with the inherent vagaries of the variable and not totally predictable human behaviour. OR is more like these applied sciences than pure sciences

3. "OR is the use of scientific methods to provide criteria for decisions regarding man machine systems involving repetitive operations." This definition goes to the heart of the field of application and is commendable to that (but only to that) extent. Because if you replace OR by cost accounting in this definition you have still a very valid phrase Even it can be replaced by quality control chart. Thus this definition is poor because it does not delineate OR from several disciplines or tools.

4. "OR is the application of scientific methods, techniques and tools to operations of a system with optimum solutions to the problems." This definition is offered

by Churchman, Ackoff and Arnoff who are some of the pioneers of OR. Not because of that though, it is a good definition. We would have only wished the qualification of quantitative to the scientific method... *i.e.* the definition could perhaps be improved as "OR is the application of scientific methods; techniques and tools with a quantitative slant to operations of a system with optimum solutions to the problems". Because ergonomics, for example is concerned with optimal design or provision of furniture, workseats, knobs and fixtures, illumination, humidity, etc. and it has nothing much to do with OR. Likewise, many of the industrial engineering techniques like work Study, plant layout can sit well in their original definition. Anyhow, to repeat it is a good definition.

5. Steinhardt defines it as "Research into Operations". In other words, he has simply made RO of OR. Therefore deservingly he has been able to explain nothing whatever with this definition.

6. OR society of U.S.A. offered another good definition in 1952, "Scientific methods for providing executive departments a quantitative basis for decisions regarding the operations under their control."

7. "OR is the art of winning was without actually fighting" is obviously a definition in the context of Warfare alone.

8. "OR is the art of finding bad answers where worse would be expected" is more a proverb, though a good one, than a definition.

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THE INSTITUTE OF CHARTERED ACCOUNTANTS OF INDIA

STUDY MATERIAL

F.S P. (N) O.R.—
COMBINATION-

FINAL COURSE (N)

OPERATIONS RESEARCH & STATISTICAL ANALYSIS

STUDY—II

Contents :

APPLICATION OF STATISTICAL TECHNIQUES IN OPERATIONS RESEARCH
—INVENTORY CONTROL
—QUEUING THEORY

STATISTICAL EVALUATION OF ALTERNATIVE PROGRAMMES AND PROJECTS
STATES OF NATURE AS
—EMPIRICAL DISTRIBUTIONS

Suggested Reading : Part I.

Operations Research *by Goel and Mittal, Pragti Prakashan. Meerut*

Part : II.

Quantitative Analysis for Business Decisions by Bierman et al. Richard D. Irwin (Asian Ed).

Introduction

The study material deals with the following two segments of the syllabus on operations research and statistical analysis and is, therefore, correspondingly divided into two parts :

1. Use of statistical techniques in O.R.

2. Statistical evaluation of alternative programmes and projects.

(The student is advised to revise his Inter. Stat. studies.)

Part 1. Regarding use of statistical techniques, two foremost areas of O.R. have been covered, these being inventory control and queuing theory. It is being left to the student to note for himself other applications of the statistical techniques in the following study materials This, however, does not exhaust the areas where these techniques can be profitably employed or even sometimes are absolutely essential Nevertheless a fair deal of their application of interest to an accountant has been covered.

As against the 19th century of classical mathematics this is the age of probability and statistics. Their application, in general as well as in O R in particular, is bound to grow exponentially. A sound grounding in statistics is, therefore, a necessary pre-requisite for understanding and applying O R. in practice, if not contributing to the development of the state of art. As a matter of fact, O R has mainly developed at the hands of mathematicians and statisticians and to a lesser extent engineers and economists. There are many "gaps", *e.g.*, costs in inventory control where the accountants could bear their expertise.

As a note of caution, the treatment of inventory control and queuing theory is not at all exhaustive. Only areas of extensive application of the statistical techniques have been selected and treated in a rather elementary way. These should provide an insight into labyrinths of O.R.

Inventory Control

Inventory management has emerged as one of the most important, yet preplexing tasks faced by modern managers The investment in inventories for most firms is substantial. Many companies have as much as 10 to 25 per cent or more of their assets committed to inventor: Even more troublesome is the fact that inventories are often the least stable and difficult to manage type of assets. Fads, styles, technological change, and so on, all serve to make part or even all of an inventory obsolete. Soaring interest rates have substantially increased the cost of maintaining inventories. Thus, the problem of inventory control has become even more significant.

Inventories : Their functions.

Most companies carry inventory in one form or the other. Since the manufacturing industry is the most complex of all the organisations concerned with inventory control, we may classify inventories in its context, into the following categories.

1. **Finished goods,**
2. Subassemblies and manufactured components,
3. Raw materials and purchased parts.

The basic function performed by inventory, as indicated earlier, is that of decoupling the production from customers and vendors, and, in more general terms, decoupling the various work-centres so that they can be operated independently with cushions of stock in-between. This leads to considerable simplification of otherwise very complex organisation for manufacture, and reduces the co-ordination effort. Fluctuations in demand or consumption can be absorbed substantially by building up stock during slack periods and depleting these during peaks, thereby maintaining undisturbed production and stable employment rates. Also, inventories may have to be built to meet special sales, viz., for Christmas etc.

Larger lots (Synonymous with batches in the context of the subject) of inventory may be procured to take advantage of discounts on bulk purchases or to take advantage of favourable market speculatively or to keep shipping costs in balance. Buying larger lots would entail less orders and, therefore, less clerical costs and also help in meeting the delivery dates for the long leadtime items. This applies to goods produced within the works itself also. Less orders, as a result of larger lots, would entail less machine set-ups and other associated expenses.

An organisation has to deal with several customers and vendors who are not necessarily close to the works. Inventories have, therefore, to be built to cover at least the requirements during the transit time. In actual practice, an organisation does face competition as well, and early and timely deliveries can fetch more goodwill and orders. This may be nsured by stocking the goods at the finished or appropriate intermediate stages depending upon the intensity of the competition and internal capabilities.

On the other side of the ledger, inventory involve an expense and are not far away from cash in liquidity. The capital could be usefully employed in other ventures as well. Also, with high stocking there is always likelihood of obsolescence and several other costs which we have discussed below elaborately. Thus the basic problem of an inventory policy would be to strike a balance between the costs of stockholding (synonymously, inventory-carrying) and the relaxation achieved in co-ordination and enhancement of service to the customer.

Causes of poor Inventory control

1. Overbuying without regard to the forecast or proper estimates of demand when a shortage occurs or the market is temptingly favourable.

2. Overproduction or early production of goods even before the customer requires them. Also, in an endeavour to keep equipment loaded stocks may accumulate. Unused portions of the materials drawn from the stores may be held in the works with the result that when a shortage occurs a batch is pushed through earlier than otherwise would have been the case.

3. In an endeavour to keep the inventories low, the fast-moving and the slow-moving stocks may be indiscriminately suppressed. This would result in (lower

inventory of slow-moving items than needed and) higher stocks for the fast-moving items than needed.

4. To provide better service to the customer there is always the tendency of the sales department to have inventories. Production department may also be interested in longer runs to cut down production costs.

Besides the above causes which are more or less controllable, stocks may also arise owing to cancellations of orders and minimum quantity stipulations by suppliers.

Inventory costs

The broad objectives of any inventory control system are to minimise inventories and provide reasonable service to the customer. These resolve into the following couple of trade-offs and parameters.

Inventory level versus acquisition cost, i e., cost of procurement/set-up for bought-outs and made-ins respectively.	How much to order? i.e. Order Size
Inventory level versus service to the customer or conversely dis-service to the customer that leads to loss of goodwill and, therefore, monetary loss.	When to order? i.e. Reorder level.

The costs entering into inventory control decisions may then be categorised as : —

1. Stockholding costs (both for how much and when to order)
2. Procurement/Set-up Costs (for bought-outs and made-ins respectively)
3. Shortages Cost (i.e., dis service to the customer).

(A) *Stockholding Costs* : The stockholding costs arise on account of maintaining the stocks and the interest paid on the capital tied up with the stocks.

Let us look more closely at the components comprising the stockholding costs.

(i) *Cost of money or capital tied up in inventories** : Money borrowed from banks for holding inventories may cost anything between 10 and 15%. But it is the usual practice to view the problem in a slightly different way, that is, how much the organisation would have earned, had it invested the capital on an alternative project, e.g., developing a new product or to buy new equipment, etc. Owing to this it is considered that the cost of the tied up capital may be taken somewhere in the region of 15 to 20% a year of the value of the inventories.

(ii) *Cost of storage space* : This would obviously depend upon the volume to value ratio of an item – the extremes, for example, being moulded packing piece made of foamed polystyrene (with a very high volume to value ratio) and gold

*when expressed in%age of the stock value it is known as carrying charge.

arrings or a sparkling diamond. Typical values obtaining in metal working and other industry may vary from 1 to 3%. Besides space expenses this will also include heating, lighting shelves and expenses of any other atmospheric precautionery measures. Regarding these costs there is an argument that if there is unoccupied space what does it cost to fill it? The answer to this lies in that if the store is 50% (or less) full it can be rented or some other activity undertaken e.g., assembly work with or without partitioning the store. If it is around 75% full it is near the optimum because bulk of the space is being utilised towards the intended purpose of storage and the rest of the space is sufficient as a congenial atmosphere for the stores staff. Beyond 75% the store becomes congested resulting into protracted man/material movements and/or difficulty in searching for the right materials. Linear relationship between space requirements and its cost is usually taken as an approximation.

Damage and Deterioration : Some items are subject to deterioration, e g, canned foodstuff and others may require precautions and protection, chemicals may require dust and cans rust-proofing. Fragile items, e g, crockery are liable to damage. The perishables would be withdrawn of FIFO (first in and first out basis) and wherever a new delivery arrives this may call for physical replacement of the perishables already in the stores. Such handling is likely to require more space than the case would have been ordinarility. 0-2% of the stock value may be lost owing to damage and deterioration.

Pilferage : It would depend upon the nature of an item. Valuables are more tempting, an example being gun metal bushes or special, expensive tools. On the other extreme there is a little possibility of a heavy casting or forging being stolen The former category, i.e., valuables may have to be kept under lock and key under the custody of the storekeeper and the latter may be "dumped" often in stockyards.

Obsolescence : It depends upon the technological state of art in the manufacture of a particular material Transistor and computer components are liable to be fast outdated. In other cases, however, design changes, e.g, for a motor, cycle may lead to obsolescence of some components. It is possible in this case, at least, to quantify the %age loss because of obsolescence. Past write-offs and the inventory levels could provide the arithmetic data for such a computation.

Insurance : Most firms get insurance cover and 1 to 2% may be taken as the representative percentage.

Typical overall costs are given below.

Cost of capital tied up	10 to 15%
Storage space	1 to 3%
Damage and deterioration	1%
Pilferage and precautions	1%
Obsolescence	around 5%
Total	around 20%

Example : If the average stock during a year is Rs. 50,000 the stockholding costs, I being 20% equal $50,000 \times \frac{20}{100} =$ Rs. 10,000.

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(B) *Procurement Costs* : As the student is aware through the study of Cost Accounting these consist of two major components · Ordering and receipt costs.

Set-up Costs : Analogous to procurement costs for bought-outs are the set-up costs for the goods to be made within the works. For the made-ins there will be machine set-up costs and scheduling costs as against the requisitioning, ordering and expediting costs for the purchased goods.

Related to the procurement costs are the quantity discounts which are generally quoted in terms of price breaks. Sometimes they may be offered in indirect form, e.g , deferred payments, free goods, gifts or allowances. These basically arise because of the advantages of manufacture of larger lots accruing to the suppliers. The acquisition costs may not bear any simple relationship to the number of orders placed More than one stock item may be ordered on one set of documents. The clerical staff is not divisible and without existing staff increasing or decreasing there may well be considerable scope for changing the number of orders. Nevertheless, to the first approximation it may be possible to represent the acquisition costs by a straight line. In some situations, linear costs relationship may be inadequate and a quadratic or a step function may be more desirable Setting up costs for machines, however, can be determined fairly accurately.

(C) *Shortage Costs* : These are usually hard, though not impossible, to measure and quantify Service to the customer is, therefore, expressed in number of shortages in several manners

As discussed above the inventory control problem usually resolves into the following couple of questions for each item.

1. When to order ?
2. How much to order ?

When to order. This question is related to the leadtime of an item. Lead-time may be defined as the time interval between the placement of an order and the receipt of goods against it. It may be the replenishment order on an outside party or within the works. There should be enough stock for each item so that when a replenishment order is placed the customers' orders can be reasonably serviced from this stock until replenishment. This stock level, known as reorder level has therefore, to be determined for each item.

It is derived by compromising between the expenses of maintaining these stocks and the disservice to the customer if his orders are not filled in due time.

How much to order. Placement of an order, receipt of goods against it and the formalities of closing the order are an expense to the company. These may be called acquisition costs associated with each order. These expenses considered alone would call for as few orders as possible (or equivalently and inversely) orders of large sizes. But large size of orders would imply stocking and, therefore, high inventory carrying costs consisting of storage space they occupy, insurance charges, interest on the capital invested on these extra stocks, etc. This question of how much to order is solved by compromising between the acquisition costs and inventory carrying costs.

Routine operation of the inventory control system would be conducted on the basis of these decision rules of the reorder level and the batch size *i.e.*, number of orders. There are quite a few operating policies. Basically they depend upon

the demand characteristics. Consider a television set as an example. The future demand of the television sets of different models assembled by an organisation may be ascertained by forecasting. Exponential smoothing method is one of the most useful forecasting models. The demand of the television sets emanates from the market directly and is not dependent upon demand of any other item. It may therefore be called independent, in contrast to that of the various components which go into assembly of the television models. The demand of these components is obviously dependent upon demand of the television sets and may be simply and arithmetically computed from the latter. This arithmetic computation of dependent demand is called explosion of the production or assembly programme of the independent demand of the end products, television sets here. The categorisation of demand : independent and dependent is crucial in choosing the appropriate inventory control policy for an organisation. There is a case of mixed demand of the spares. The demand for the spares may emanate directly from the market, but, they may also be required in the assembly of the end-product. Hence the demand of such spares is partly independent and partly dependent.

The dependent demand, as mentioned above, is derived by explosion of the production programme of end-products. We shall leave the discussion on dependent demand here, because it can be ascertained by simple, though, enormous arithmetic.

The independent demand is usually ascertained by extrapolating the past demand history, that is, forecasting. The reorder level can be fixed from the forecasts and the leadtime. Whereas in the case of dependent demand simple arithmetic computations were enough to ascertain requirement of the components, in the case of independent demand items statistical forecasting techniques have to be employed. The reorder level (ROL) is fixed statistically mainly. Fixation of ROL forms the basis of this section on inventory control. The following family tree should give an idea of the various inventory control policies.

Demand

Independent (Statistical Inventory Control Systems)

Dependent (Requirements Planning) Arithmetical Explosion of demand.

Fixed order Size Systems (fully discussed in this section)

Periodic Review Systems (discussed briefly) (more or less similar to the Fixed Order Size System with regard to setting the two decision rules)

Fixed Order Cycle System

Mini-max. System

Order size varies, items are reviewed for order placement at fixed interval : one month, one quarter, one year, etc. Stock levels are fixed somewhat in the manner of setting ROL.

Here in this study, however, we shall discuss mainly the fixed order quantity system.

Fixed order quantity system : The system is depicted in Fig. 1.

When the stock (*being continually depleted by the customers' orders*) reaches the reorder level (*abbreviated as ROL*) an order of fixed size, *q* is placed. The time interval between the placement of an order and receipt of deliveries against it is called the leadtime In the figure, the time elapsing between the event *A* of placement of order and the event *B* of receipt of deliveries of size *q* is the leadtime. *CD* gives another leadtime. Leadtime is usually variable and possible to be described by standard or an empirical probability distribution It may, however, be treated as constant for certain analytical expositions. The ROL is so fixed that the customers can be "reasonably" served from this stock until the replenishment of size, *q* arrives against the order placed So the problem of "when to order" reduces to fixing the ROL with the operating policy that as the stocks cross this level an order is placed (on a vendor or within the works ; of which size is predetermined and fixed).

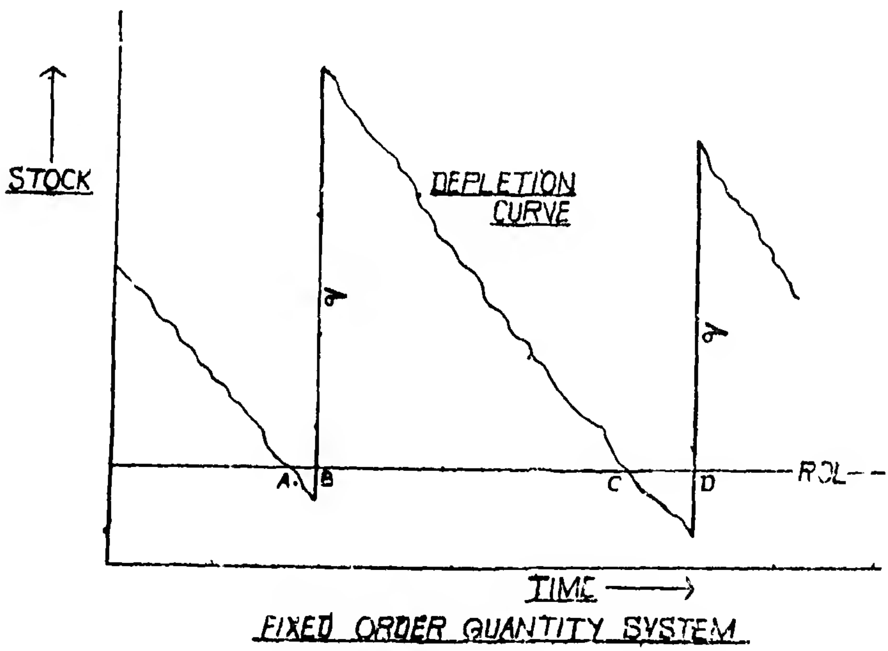


Fig. 1

Fixing the Reorder Level : The student has had introduction to the exponential smoothing method for demand/sales forecasting during the Intermediate Statistical Course. *He may also refer to appendix I of this study.* It may be recalled that this method is particularly suited for short-term forecastings, especially when several hundreds or thousands of items are involved (*e g* , in manufacturing or warehousing situations) and especially, again, in computerised application. Forecasts derived by this method for the next month or week as such are

of no direct interest. The interest lies in extending this monthly or weekly forecast over the span of the leadtime. If, for example, leadtime is 3 months we would have to accumulate the next 3 months' forecasts in order to compute the forecasted leadtime demand (synonymous) with working stock and fix the reorder level therefrom, as below :—

R.O L.=Leadtime demand+Safety Stock.

It should further be noted that the forecasts of the months encompassed by the leadtime are to be accumulated, *e.g* , if the leadtime is 3 months and the forecasts of these three months may be 85, 45 and 95. Such would be the case with demand affected by seasonals. Similarly forecasts may be 50, 55 and 60 in the case of the linear trend. The simple rule for computing the Leadtime demand= leadtime×monthly forecast would, therefore, be valid only in the case of the demand pattern depicting no trend and seasonal variations which is an exception rather than the rule in practice.

For calculating the R.O.L. all the three variables : leadtime, forecasts and safety stock have to be ascertained for each and every item in stock. We shall defer the discussion on the safety stock to a later section.

Periodical Forecasts : Periodical (monthly, weekly or daily) forecasts may be derived from the exponential smoothing model for each and every stock item. The exponential smoothing model may be the simple or elementary one if the demand of an item depicts no trend or seasonal variations, but this is rarely the case in practice. Trend is always likely to be there and it may, therefore, be necessary to incorporate the trend factor in the exponential smoothing model. Not many items are affected for their demand by seasonal variations. Where, however, seasonal variations are significant *e g.*, the sales of ice cream the model, would be further extended to take account of this. In every period, therefore, the forecast would be raised for each stock item by one of the following exponential smoothing models :—

1. Simple with stationary demand. 2. With trend component.

3. With seasonal factors.

The smoothing coefficient may be between 0.1 and 0 3 for different patterns of demand. Items with a more or less stable demand would be put on a low value of α close to 0.1 and those with highly fluctuating demand have α close to 0.3. The student may recall that value of α determines the magnitude of adjustment to be made to the previous forecast upwards or downwards in the light of the forecast error which is the difference between the actual demand that has materialised and its forecast.

Symbolically,

Error, $e_t=(y_t-u_{t-1})$, where y_t is the latest demand,

u_{t-1} is its forecast a period earlier.

or $u_t=u_{t-1}+e_t$.

But there is always a possibility that the pattern of demand hitherto more or less stable starts fluctuating because of, say, certain market changes, consumers behaviour, etc. The model should, therefore, be so designed that it can switch over from one value of α to another depending upon the behaviour of demand. Such

models are called adaptive models because they adapt the changing pattern of demand by suitably changing the value of α. Thus a gigantic forecasting complex is in operation that not only takes into account the declining or growing trend and seasonal nature of demand but also discriminates between stable and fluctuating behaviour and adapts itself suitably. The matter, however, does not end here. Monthly forecast is just one variable in the ROL equation stated above. The forecasting period such as month has to be determined suitably. This again depends upon the demand behaviour Longer period entails less computational task of forecasting. But too long a period may miss important changes in the demand behaviour. The compromise has to be struck between the computational time and expenses on the one hand and tracing the demand behaviour precisely on the other. This may tend to be a complicated matter and simulation may provide the right solution. Quite often, however, the forecasting period is fixed arbitrarily as a convenient unit of time, quarter, month, week, etc. Nevertheless, even in this arbitrarily choice underlies the aforesaid intuitive compromising. The forecast period should never be longer than the leadtime.

Leadtime is simply the time interval between the placement of an order and receipt of goods against it. The leadtime may not be constant and is usually not so. For one batch of stock keeping item a vendor may take 45 days and for the next 50 days and so on. Leadtime itself is, therefore, a stochastic variable. This complicates the problem of accumulating the forecasts over the period encompassed by the leadtime. Leadtime may also be forecasted exponentially as is done with demand. Many theoreticians have ignored the problem in view of the mathematical complications involved and either assume it to be constant or use its expected value, or any reasonable value.

Safety stock : Leadtime demand (forecasts accumulated over the periods in the leadtime) is the stock level, which on the average, is sufficient to fill customers

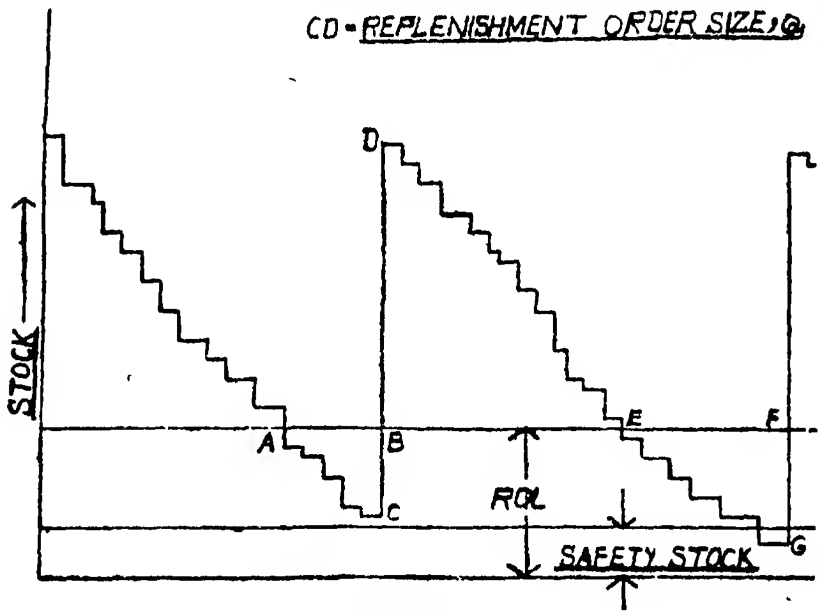


Fig. 2

orders as the stocks are being replenished. "On the average" would mean here that during this period of replenishment 50% of the customer's order can be filled and the rest 50% may either be refused or backordered" to be filled later. The reason for this is obvious. Forecast after all is a point of estimate only. The actual demand that materialises may either be more or less. In the former case of demand being greater than the accumulated forecasts, the customers would not be serviced. In the later case of demand falling short of accumulated forecasts overstocking would occur. When these two variables: stock level and service to the customer (suitably quantified are summed up over thousands of stock items in an organisation it becomes a major policy decision for the management to find the acceptable compromise between the two. Where the ROL=leadtime demand as mentioned earlier, only 50% of the orders are expected to be serviced during the replenishment period. This may not be acceptable to the management who may wish to limit the "disservice" to the customer down to 5% or 10% at the cost of extra-stocking. This extra-stock in excess of the leadtime demand is called the safety stock. Safety stock may be expressed as a percentage of the leadtime demand or as, for example, one month's demand. It may again, either be computed precisely as discussed below or set arbitrarily and intuitively with the aforesaid considerations in mind. The safety stock is also called by other names such as buffer stock, cushion, etc. The stock depletion diagram given earlier is suitably amended in Fig. 2 : ROL as comprising two components : leadtime demand and safety stock. Also more precisely, depletion is shown here to be discrete.

Service to the customer : There are various measures of service to the customer. And these may require definition and quantification unique to a given organisation. Customer's reactions to the order not filled promptly may vary considerably depending upon the type of the item, his needs, its availability elsewhere, possibility of substitutes and so on and so forth. Some customers may therefore, be willing to wait, others may cancel the order, still others may cancel the orders for even the goods available but jointly ordered with the one not available. And a few of the customers may abandon the organisation for future needs even.

Computations for the safety Stock : The following figure shows the actual demand by the full lines and the forecast demand by the dotted lines in a hypothetical case.

As argued earlier, if the demand exceeds the forecast $(y_t > u_{t-1})$ the situation would result in bad service to the customer. If the demand falls short of the forecast $(y_t < u_{t-1})$ the consequence is overstocking depending upon the forecast error.

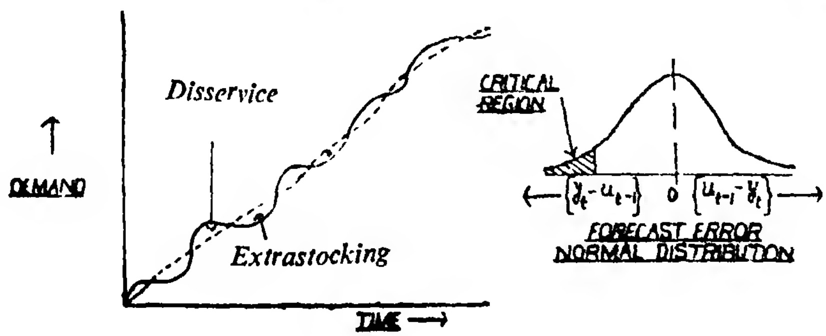


Fig. 3

It is not unreasonable to suppose that the forecast error is normally distributed with zero mean in an appropriately designed forecasting model The student may take this assumption of normal distribution of forecast errors on faith. We would not enter into greater discussion on the topic *lest* he is lost in detail. If, therefore, the standard deviation of the forecast error is computed safety stock may be set with the desired confidence to result into not more than 5% or 10% shortages, etc. All that is needed, then, is to record the data on the error and compute the standard deviation and finally calculate safety stock

Safety stock may be fixed initially and then revised every period in the light of the latest forecast error. Revising the safety stock this way would entail computation of standard deviation for every period which is quite cumbersome especially when thousands of items are being handled. There is another measure of spread, mean absolute deviation popularly known as MAD, which can be computed far more easily and, in fact, routinely exponentially smoothed every period for obtaining far more estimates of the safety stock.

MAD also bears a fixed relationship to the standard deviation. For normal distribution

$$\text{MAD} = \sqrt{\frac{2}{\pi}} \times \text{S D.}$$

Now, Safety Stock=Z. (S.D.)

$$= k \times \text{MAD} \quad \text{where } k = \sqrt{\frac{\pi}{2}}\ Z$$

and k is called the service factor

Here again, the safety stock is for just one period and has to be extended over the lead time The necessary formula is given below :

$$\text{MAD}_{LT} = (0\ 659 + 0.341\ \text{LT}).\ \text{MAD}.$$

The formula has been derived by extensive simulation by a statistician and may be used for other situations as an approximation.

Hence, R O.L.=Leadtime Demand+(0.659+0.341 LT). k.MAD.

Just as safety stock is revised every period so can be leadtime demand. ROL therefore, is not fixed but changes in sympathy with demand. The stock depletion diagram 4 is the version of Fig. No. 2 modified in this regard.

To recapitulate : The problem "of when to Order" is to be solved statistically, the policy being to place an order when the depleting stocks cross a level, ROL. It is therefore, required to fix ROL.

ROL is split into two components :

(*i*) Leadtime Demand and (*ii*) Safety Stock.

Leadtime demand is the accumulated forecast over the periods encompassed by the leadtime. Safety stock is added to ensure service to the customers at a level acceptable to the management.

Safety stock is computed on the assumption of the normality of the forecast error distribution. Standard deviation of this distribution may be computed and

multiplied by Z or alternatively and more conveniently by computing MAD$\times$k. MAD over one period is suitably projected over the leadtime for obtaining the safety stock.

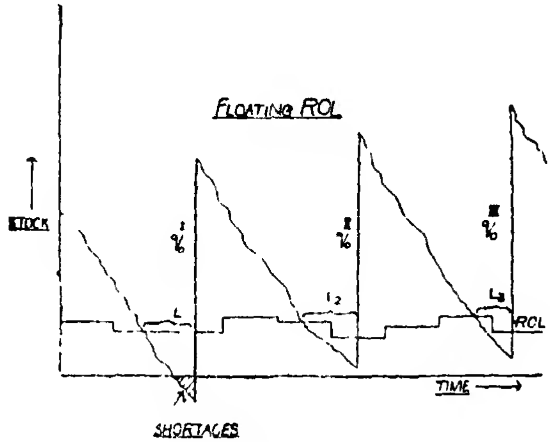


Fig. 4

ROL may be calculated once a year or revised every period. In the latter case, it is called the *floating* reorder point.

We discussed earlier the problem of adapting the forecasting model in case of changes in the demand behaviour But the problem of attuning the model to any

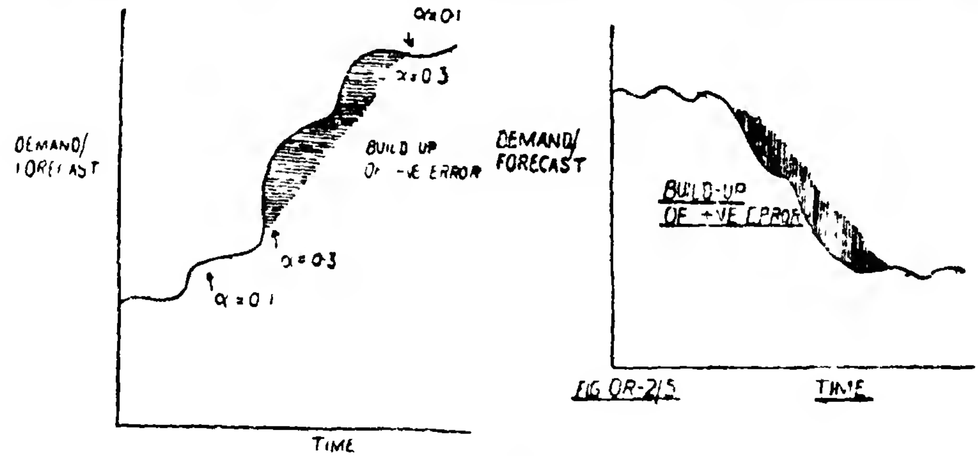


Fig. 5

abrupt changes in demand pattern was left out. This may occur, for example, if a competitor has shut down. By means of the adaptive element α will be increased : but if the change is significantly more (or less) and lingers on, over-estimation or under-estimation of demand will continue to be made for quite some periods despite the change in the value of α. This is depicted above.

The student is familiar with fitting of Normal and Possion Distributions to the given data ; but these very distributions have frequently been found not to provide a good fit. There are quite a few more, *e g.*, Gamma Distribution, Lognormal Distribution, Negative Binomial Distribution, Gram Charlier Series which are usually more appropriate and worth trying.

Once the data admit fitting of a standard distribution and the aforesaid relationship between mean and standard deviation of demand has been established it becomes relatively easy task to set ROL by the following steps. Just suppose that Normal Distribution provides the right fit

(*i*) Find the mean demand for each item periodically, say, after every 6 months, the period depending upon the stability of the average demand.

(*ii*) Derive S simply from the explored relationship.

(*iii*) Seek management's approval on tolerable percentage stockouts.

(*iv*) Set ROL as below

$$\text{Leadtime Demand} = LT \times \text{Mean demand}$$

$$\text{Standard Deviation of Leadtime Demand} = \sqrt{LT} \times S$$

$$ROL = LT \times M + \sqrt{LT} \times S \times Z$$

The same procedure may be adopted for other distributions The leadtime has been taken as constant. This approach, however, even affords handling of the problems where leadtime is a stochastic variable with a known probability distribution Several authors have evolved methods of handling different combinations to the leadtime and demand distributions in setting ROL

How much to order : In the fixed order quantity model considered above, the size of the order is fixed. Briefly, the order size or the batch size is determined on the following cost considerations :—

If the annual requirement for an item is say, 1200 the entire requirement may be bought or made in the works, whatever the case is, in a single batch of 1200. This is one extreme. On the other extreme, an order for one item each may be placed, which implies 1200 orders. There could be numerous (1200 in theory) intermediate possibilities of procuring the requirement of 1200 in 1, 2, 3, 4......... 1200 batches. In choosing the batch size the following cost considerations enter into :—

Case I : *Bought-out items* : With every order placed on a vendor are associated the following cost components ;

(*i*) Cost of typing and paperwork of the copies of the order. If the item has not been procured before there would be costs associated with floating enquiries and screening the quotations.

(*ii*) On some items, there would be expediting effort by means of reminders, telephone calls, personal visits on the vendor's works etc. This entails an expense obviously.

(*iii*) Upon receipt of the goods, there would be inspection on sampling or complete inspection basis. Goods receipt notes would be prepared,

stock records would be updated, vendor's invoices would be processed. This, then, is another cost component.

The aforesaid cost components when summed up may be called the acquisition cost of one batch. It may be a difficult task to assess these components accurately, but the fact stands that they are incurred. It is estimated that in the circumstance obtaining in the Indian industry the acquisition cost per batch, typically, may be from Rs. 20 to 50. These are rough estimates, anyway.

Let us assume Rs. 20 as the acquisition cost per order in the above example of the annual requirement of 1200. The total acquisition cost for a few batch sizes, for example, is given below :—

Annual requirement = 1200,

No. of Orders, N/Yr.	Batch size q=1200/N	Total Acquisition Cost, C_a in Rs.
1	(1200/1)=1200	20
2	(1200/2)=600	40
3	etc. 400	60
4	300	80
5	240	100
6	200	120
10	120	200
12	100	240
20	60	400
100	12	2,000
1200	1	24,000

If we denote annual requirement by A the total acquisition case$=20N=20A/q$. We can, then, say that the acquisition cost varies inversely with the batch size—the greater the batch size the less the acquisition cost and the smaller the batch size the more the acquisition cost.

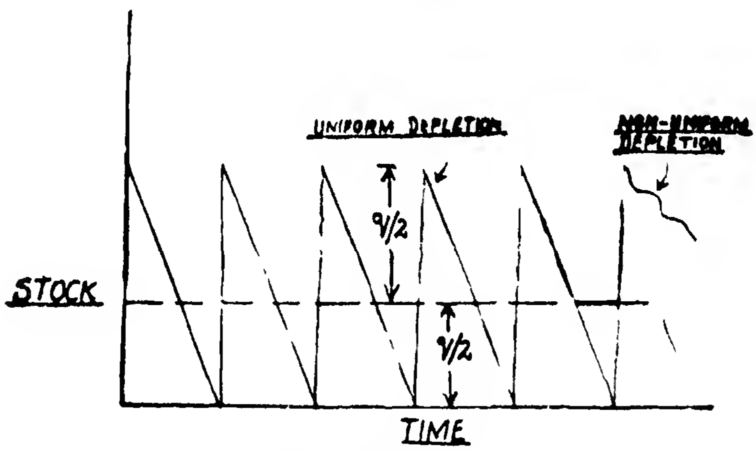
Now, according to the acquisition point of view alone the batch size should be as large as possible and 1200 in our example. But maintaining such a large stock would obviously be expensive in that it requires more storage space, it is liable to obsolescence, would require more insurance charges, but, above all would tie up capital. Interest would have to be paid. From inventory carrying point of view, therefore, the batch size should be as small as possible. Inventory carrying cost is then, directly proportional to the batch size, the larger the batch size, the more the inventory-carrying cost and the smaller the batch size the less the inventory carrying costs.

Therefore, compromise has to be struck between acquisition costs and inventory-carrying costs, *i.e.*, the batch size is to be determined in such a manner as minimises the sum of the inventory carrying and acquisition costs. This batch size is called the economic order quantity. As we shall point out elsewhere also, the mathematical model from which the economic order quantity (E.O.Q.) is determined is a highly controversial model, mainly, from the point of view of the great difficulty in not only measuring but even precisely defining the aforesaid cost components in a given situation. This indeed may be an interesting topic for an accountant and the interested reader is referred to 'Production & Inventory Control' by Magee and Boodman for a thorough treatment of this aspect.

Case II : Made-in Items : Analogous to acquisition costs are the set up costs if a particular batch is made by the company itself. There would be costs of documentation to be supplied to technician, cost of progressing the batches, cost of updating stock records etc., and more, importantly, the time (and therefore cost) expended in setting up the machines for performing the necessary operations. From mathematical point of view, therefore, it is immaterial whether an item is a bought-out one or made-in one. The acquisition cost may be taken as the procurement cost for the bought-outs and set up cost for the made-ins.

Assumptions and mathematical derivation

As mentioned earlier in the chapter the stock items may have to be manufactured or purchased in quantities greater than the immediate requirements in order to compromise between the acquisition and inventory carrying costs. The stock fluctuation in the problem under consideration may be depicted as below. There is no reorder level. This implies an assumption that the leadtime is zero, *i.e.*, items can be had instantaneously. Further assumptions in modelling are noted hereunder :



EOQ MODEL

Fig. 8

1. Demand is deterministic, *i.e.*, known and uniform, say, the annual requirement.

*2. Replenishment leadtime is zero.

3. Replenishment size is constant, say q.

4. No shortage is permitted, *i.e.*, the replenishment of the size q is made the moment the stock falls to zero level.

5. Acquisition and stockholding costs are known.

*For the case when leadtime is non-zero, the model is amended in a later section.

6. Acquisition cost is linearly related to the no. of orders/year, N, *i.e.*,

$$\frac{A}{q} = \text{No. of orders.}$$

7. Stockholding cost is linearly related to the size of batch, q.

Let Ca=acquisition cost/order (Procurement cost for bought-outs and set-up cost for made-ins).

CI=inventory cost/unit/unit time (I=rate of return). Cost variations for this model are depicted in the diagram below ;—

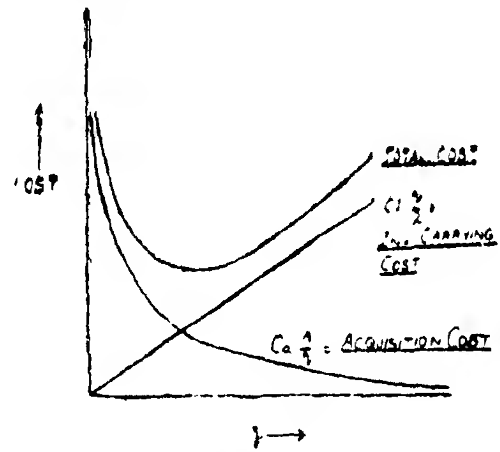


Fig. 9

Average stock= $\frac{q}{2}$.

Inventory carrying cost $= CI.\ \frac{q}{2}$

No. of orders per year $= \frac{A}{q}$

Acquisition cost $= Ca\frac{A}{q}$

Total Cost $T = CI\frac{q}{2} + Ca.\ \frac{A}{q}$. ... ($i$)

∴ $$\frac{dT}{dq} = \frac{CI}{2} + \left\{-\frac{Ca\ A}{q^2}\right\} = 0.$$

∴ $$q = \sqrt{\frac{2ACa}{CI}} = q_0$$

q_0 is then the optimal batch size or E.O.Q.

Now $T = CI\frac{q}{2} + Ca\frac{A}{q}$

Substituting for q_0

$C(q_0) = \sqrt{2ACICa}$

This gives the optimal total cost. The student must be careful in that this formula can be used to compute optimal total cost. For non-optimal batch sizes equation (*i*) of page 19 would be used. See example 4 of p. 24.

It was mentioned elsewhere that the model is highly controversial. It is not only difficult to measure but even to define precisely the components of I and Ca. The model is further criticised for the host of assumption made above.

The most serious objection to the model is that demand may be non-uniform. There exist dynamic E.O.Q. versions for this situation. These are *beyond our scope here, however.*

Yet the model has been successfully modified in various ways and by various theoreticians by removing one or more of the above assumptions.

Some authors have successfully circumvented the computation of these costs. They provide a comparison table between stock level and the total no. of orders, broadly. Each of these represent the aggregates of inventory carrying and acquisition costs for the entire stocks. The management may choose the desired stock *level and the total number of orders. Once the management has done this one can compute the both size for each and every item* based on management's decision. *The problem is thus inverted. Instead of ascertaining CI and Ca for each and every item and computing E O.Q's individually the latter approach enables management to manipulate between stocks level and acquisition costs (represented by the number of orders).*

The model has several modified versions. Where, for example, the vendor provides quantity discounts due consideration is given in building and analysing the model. The model may be modified to take into account investment and/or storage space constraints, by the "inversion" techniques.

This discussion on fixation of E.O.Q. or the optimal batch size should complete the review of the fixed order quantity inventory control system. To summarise: Whereas R.O.L is fixed by optimising the sum of inventory-carrying costs and (monetarily quantified) service to the customer, the batch size is set by optimising the sum of acquisition and inventory-carrying costs. In short, therefore the fixed order quantity inventory-control system is to be designed by minimising the sum of the following three costs—

(*i*) Acquisition Cost

(*ii*) Inventory-Carrying Cost

(*iii*) Shortage (disservice) Cost.

In this section the approach was to have sub-optimisation in the following two pairs :—

(*i*) Acquisition Cost versus Inventory-Carrying Cost,

(*ii*) Shortage Cost versus Inventory Carrying Cost.

There are further savings possible if optimisation is accomplished with the three variables taken together ; but this usually requires application of the simulation technique or rather intricate mathematics.

Solved Examples, set I

Example 1 :

Two items have the following characteristics :

Item	Annual Demand	Unit Price	Order Cost, Ca	Holding Cost I
X	9 Units	Rs. 0.25	Rs. 8 00	25%
Y	27 Units	Rs. 3.00	Rs. 18.00	25%

(A) For each item compute the annual ordering cost, the annual holding cost, and the annual total variable cost for order quantities of 24, 36, 48 and 60 units.

(B) Compute E.O.Q. for each item and verify the computation by computing total variable costs for one unit more than E.O.Q ; and one unit less than E.O.Q.

Solution (A)

Order Qty	Item X			Item Y		
	Ordering Costs	*Holding Costs	Total Costs	Ordering Costs	Holding Costs	Total Costs
24	$\frac{9}{24}\times 8=3$	0.75	3.75	20.25	9.00	29.250
36	$\frac{9}{36}\times 8=2$	1.125	3.125	13.50	13.50	27.000
48	$\frac{9}{48}\times 8=1.50$	1.50	3.000	10.125	18.00	28.125
60	$\frac{9}{60}\times 8=1.20$	1.875	3.075	8.10	22.50	30.600

(B) Item X : $q_0=\sqrt{\dfrac{2\times 8\times 9}{\frac{25}{100}\times 0.25}}=\dfrac{12}{0.25}=48$

X : Let us try the values 47 and 49 respectively.

(i) Order Cost $=\frac{9}{47}\times 8=\frac{72}{47}=1.532$

Holding Costs $=\frac{1}{2}\times 47\times\frac{25}{100}\times 0.25=\frac{47}{32}=1.4687$

Total Cost > Rs. 3.00

(ii) Order Cost $=\frac{9}{49}\times 8=\frac{72}{49}=1.470$

*Syn : Carrying Costs $=\frac{CIq}{2}$

Holding costs $= \frac{1}{2} \times 49 \times \frac{25}{100} \times 0.25 = 1.531$

Total cost > 3

$$Y: qo = \sqrt{\frac{2 \times 27 \times 18}{\frac{25}{100} \times 3}} = 36$$

Let us try the values 35 and 37 respectively

(i) Ordering Cost = 13.886
Holding Costs = 13.125
Total Costs = 27.011

(ii) Ordering Costs = 13.135
Holding Costs = 13.875
Total Costs = 27.010
With E O Q., total cost = 27

Example 2

Calculate the E O Q, in units and total variable cost for the following items, assuming an ordering costs of Rs. 5 and a holding cost of 10%.

Item	*Annual Demand*	*Unit Price*
		Rs.
P	800 units	0.02
Q	400 units	1.00
R	392 units	8.00
S	13,800 units	0.20

Solution ;

$$qo = \sqrt{\frac{2ACa}{CI}} \qquad C(qo) = \sqrt{2C_aCI}$$

$$\text{Item}: P: qo = \sqrt{\frac{2 \times 800 \times 5}{\frac{10}{100} \times 0.02}} = \sqrt{\frac{8,000}{0.002}} = 2,000$$

$$C(qo) = \sqrt{2 \times 800 \times 5 \times \frac{10}{100} \times 0.02} = \text{Rs. } 4$$

$$\text{Item}: Q: qo = \sqrt{\frac{2 \times 400 \times 5}{\frac{10}{100} \times 1.00}} = 200$$

$$C(qo) = \sqrt{2 \times 400 \times 5 \times \frac{10}{100} \times 1} = \text{Rs. } 20$$

$$R: qo = \sqrt{\frac{2\times 392\times 5}{\frac{10}{100}\times 8.00}} = 70$$

$$C(qo) = \sqrt{2\times 392\times 5\times \frac{10}{100}\times 8} = \text{Rs. } 56$$

$$S: qo = \sqrt{\frac{2\times 13{,}800\times 5}{\frac{10}{100}\times 0.20}} = 2{,}627$$

$$C(qo) = \sqrt{2\times 13{,}800\times 5\times \frac{10}{100}\times 0.20} = \text{Rs. } 52.54$$

Example 3 ;

For the above example, compute the E.O Q. in Rs. and years of supply and the E.O.Q. frequency for each of the four items.

E.O.Q., in *Rs.* P : $2000\times 0.02 = 40$

Q : $200\times 1 = 200$

R : $70\times 8 = 560$

S : $2627\times 0.20 = 525.4$

Years of supply P ; $\frac{2000}{800} = 2.5$ year

Q : $\frac{200}{400} = 0.5$ years

R : $\frac{70}{392} = 0.179$ years

S : $\frac{2627}{13800} = 0.190$ years

Economic Ordering Frequency $= \frac{1}{2.5} = 0.4$ orders per year

$\frac{1}{0.5} = 2$ orders per year

$\frac{1}{0.179} = 5.6$ orders per year

$\frac{1}{0.190} = 5.26$ orders per year

Example 4 :

(A) Compute E O Q and the total costs for the following :—

Annual Demand	=25 units
Unit Price	=2.50
Order Cost	=Rs. 4.00
Storage Rate	=1% per year
Interest Rate	=12% per year
Obsolescence Rate	=7% per year

(b) Compute the order quantity and the total variable cost that would result for the item if an incorrect price of Rs. 1.60 were used.

Solution: (A)

$$I=1+12+7=20\%$$

$$q_0=\sqrt{\frac{2\times 25\times 4}{\frac{20}{100}\times 2.50}}=20$$

$$C(q_0)=\sqrt{2\times 25\times 4\times \frac{20}{100}\times 2.50}=\text{Rs. } 10$$

(B) $$q=\sqrt{\frac{2\times 25\times 4}{\frac{20}{100}\times 1.60}}=25$$ *This is non-optimal size.*

Ordering cost $=\frac{25}{25}\times 4=\text{Rs } 4$

Stocking Cost $=\frac{1}{2}\times 25\times \frac{20}{100}\times 2.50^{*}=6.25$

Total cost $=10.25$

Example 5 :

Each item in an inventory of 100 items is ordered every six months. If the total annual demand for the inventory is Rs. 72,000 :

(i) How many orders are placed annually for all the items in inventory ?

(ii) What is Rs. investment in average operating stock for all items in inventory :

Solution :

(i) Ordering frequency of 6 months is equivalent to 2 orders per year. Therefore, the total annual orders for the entire inventory of 100 items=200.

(ii) Rs. investment $=\frac{1}{2}\times\frac{72,000}{2}=18,000$ rupees

---

*For finding stocking cost correct price is to be used. A clerical error does not mean that stock value changes.

Example 6 :

Table A is used in computing the order sizes of 1128 items of inventor detailed in Table B. Calculate the number of orders placed annually and th average operating stocks. Table A is based on ordering cost of Rs. 5.00 and holding rate of 10%.

Table A			Table B		
S. No.	Monthly Demand Rs.	Order Qty. in month M	Range of Monthly Demand in Rs.	No. of items	Value o Monthl Demanc
1	Less than 16.68	12	Rs. 0— 16.68	663	3403
2	16.68— 40 00	6	16.68— 40.00	123	3959
3	40.01— 60.00	5	40.01— 60.00	64	3159
4	60.01—100.00	4	60.01—100.00	72	5835
5	100.01—160.00	3	100.01—160.00	49	6281
6	160.01—240.00	2½	160.01—240.00	32	6366
7	240.01—400.00	2	240.01—400.00	44	13,394
8	400.01—800.00	1½	400.01—800.00	37	20,341
9	800.01—and above	1	800.01—and above	44	100,015
				1128	*162,753*

Solution :

In given Rs. range each item is ordered $\frac{12}{M}$ *times a year. Therefore, the* total numbers of orders may be computed as below :

S. No.	No. of Items	No. of Orders per item	No. of Orders over the range
1	663	12/12=1	663.0
2	123	12/6=2	246.0
3	64	12/5=2.4	153.6
4	72	12/4=3	216.0
5	49	12/3=4	196.0
6	32	12/2½=4.8	153.6
7	44	12/2=6	264.0
8	37	12/1½=8	296.0
9	44	12/1 =12	528.0
	1128		2716.2

Let us denote monthly demand by Am. This is given by the last column of the question.

Average Investment for one item$=\frac{q}{2}C$. Now $q=$Am. M

Thus Av investment for an item$=\frac{M}{2}\ AmC$

Av investment for all items$=\frac{M}{2}\sum AmC$

S. No	M/2	$\sum$ AmC	M/2 $\sum$ AmC=Investment Average in Rs.
1	6	3,403	20,418
2	3	3,959	11,877
3	2.5	3,159	7,898
4	2	5,835	11,670
5	1 5	6,281	9,421
6	1.25	6,366	7,958
7	1.0	13,394	13,394
8	0 75	20,341	15,256
9	0.50	100,015	50,007
			147,899

Solved Example, set II

Example 1 :

A company uses annually 48,000 units of a raw material costing Rs. 1.2 per unit. Placing each order costs Rs. 45 and carrying cost is 15% per year of the average inventory.

(i) Find the E.O.Q.

(ii) Suppose that the company follows the E.O.Q. purchasing policy and it operates for 300 days a year, that the procurement time is 12 days and the safety stock is 500 units, find the reorder point, the maximum, minimum and average inventories.

Solution :

A=Annual Requirement=48,000 units

C=Unit Cost=Rs. 1.2

Ca=Acquisition cost/order Rs. 45

I=0.15

$$q_0=\sqrt{\frac{2CaA}{CI}}=\sqrt{\frac{2\times45\times48{,}000}{1.2\times0.15}}=\sqrt{24\times10^6}$$

=4899 (Answer)

Requirement per day $= \frac{48,000}{300} = 160,$

Leadtime demand 12×160

$= 1920$

$SS = 500$

$\therefore \quad ROL = 1920 + 500 = 2420$

Max. Inventory $= q + SS = 4899 + 500 = 5399$ **(Answer)**

Min. Inventory $= SS = 500$ **(Answer)**

Average Inventory $= \frac{q}{2} + SS = \frac{4899}{2} + 500$

$= 2449 + 500$

$= 2949$ **(Answer)**

Example 2 :

The actual demand on a sample inventory system was 81% of the demand value used for calculating E.O.Q. which was increased by 10% to suit procurement. Find the % error in total cost as a consequence.

(If you find it convenient you may use hypothetical values of annual requirements, I, etc.)

Solution :

Let us use the following hypothetical data for the E.O.Q. that was calculated

$A = 12,000$ units $\qquad Ca = \text{Rs. } 12$

$I = 20\%$ $\qquad C = \text{Rs. } 100$

$$\text{E.O.Q.} = \sqrt{\frac{2CaA}{CI}} = \sqrt{\frac{2 \times 12 \times 12,000}{100 \times 0.20}} = 120$$

Actual order size after rounding off

$$= \frac{110}{100} \times 120 = 132$$

Theoretically correct order size

$$= \sqrt{\frac{2 \times 12 \times 12,000 \times \frac{81}{100}}{100 \times 0.20}} = 108$$

Increase in acquisition cost $= \frac{CaA}{108} - \frac{CaA'}{132}$

$$= 12 \times 12,000 \left(\frac{1}{108} - \frac{1}{132} \right) = \text{Rs. } 242.$$

Decrease in inventory carrying cost

$$=\frac{CIq}{2}-\frac{CIq'}{2}$$

$$=100\times\frac{20}{100}\times\frac{132}{2}-100\times\frac{20}{100}\times\frac{108}{2}=\text{Rs. } 240$$

Net decrease=242−240=Rs. 2

Min. total cost$=\sqrt{2ACaCI}$

$$=\sqrt{2\times12\times12000\times\frac{81}{100}\times100\times\frac{20}{100}}$$

$$=2160$$

%age increase $=\frac{2\times100}{2160}=0.1\%$ approx.

This shows that the total cost is hardly affected (or is not much sensitive) to minor variations in the batch size. This is because the total cost curve of figure 9 is rather flattish at the bottom.

Example 3 :

A scrutiny of past records gives the following distributions for leadtime and daily demand during leadtime.

Leadtime Distribution

Leadtime (days)	0	1	2	3	4	5	6	7	8	9	10
Frequency	0	0	0	2	3	4	4	2	2	2	1

Demand Rate Distribution

Demand units/day	0	1	2	3	4	5	6	7
Frequency	2	4	5	5	4	2	1	2

What should be buffer stock ?

Answer :

(a) Computing average leadtime.

Leadtime (1)	Frequency (2)	Col 1 × Col 2
1	0	0
2	0	0
3	2	6
4	3	12
5	4	20
6	4	24
7	2	14
8	2	16
9	2	18
10	1	10
	20	120

$\therefore$ Av. leadtime $= 120 \div 20 = 6$

Computing average demand rate,

Demand (1)	Frequency (2)	Col 1 × Col 2
0	2	0
1	4	4
2	5	10
3	5	15
4	4	16
5	2	10
6	1	6
7	2	14
	25	75

$\therefore$ Av. demand rate $= \frac{75}{25} = 3.$

$\therefore$ Average leadtime demand $= 6 \times 3 = 18.$

Max. leadtime demand = Max. leadtime × Max. demand rate.

$= 10 \times 7 = 70$

$\therefore$ Buffer stock (100% service level) $= 70 - 18 = 52$ **(Answer)**

Example 4 :

The following information is provided :
Annual use is 24,000 units
Ordering costs are Rs. 120 per order
Carrying costs are 20%
Price of each item is Rs. 20
Leadtime is 10 days.

There are 240 working days per year. Determine the E O Q. and orders per year In the past two years, the use rate has gone as high as 140 units per day. For a reordering system based on the inventory level, what safety stock is required to protect against this higher use rate ? What should be the reorder point at this safety stock level ? What would be the carrying costs for a year ?

Solution :

$$Ca = \text{Rs. } 120$$
$$A = 24{,}000$$
$$C = \text{Rs. } 20$$
$$I = 20\%$$

$$\text{E.O.Q.} = \sqrt{\frac{2CaA}{CI}} = \sqrt{\frac{2 \times 120 \times 24{,}000}{20 \times \frac{20}{100}}}$$
$$= 1200.$$

$$\text{No. of order per year} = \frac{24{,}000}{1200} = 20.$$

$$\text{Average usage} = \frac{24{,}000}{240} = 100 \text{ per day.}$$

Max. usage = 140 per day.

Safety Stock = $(140 - 100) \times 10 = 400$ units.

Av. Leadtime demand = $100 \times 10 = 1{,}000$ units.

ROL = $1000 + 400 = 1400$ units

Inventory would fluctuate from a maximum of $1200 + 400 = 1600$ to a minimum of 400 units.

$$\text{Av. inventory} = \frac{1600 + 400}{2} = 1{,}000 \text{ units.}$$

$$\left(i.e., \ \frac{\text{E.O.Q.}}{2} + \text{S.S.} \right)$$

Inventory carrying costs for a year =

$$1000 \times 20 \times (20\%)$$
$$= 4{,}000 \text{ rupees.}$$

Example 5

For a fixed order quantity system, find the E.O.Q., S.S., ROL. and average inventory for an item with the following data.

$A = 10{,}000$ units		
$C = \text{Re. } 1$	Past leadtimes	15 days
		25 days
$Ca = \text{Rs. } 12$		11 days
		14 days
		30 days
$I = 24\%$		17 days

Solution :

$$\text{E.O.Q.}=\sqrt{\frac{2\times 10{,}000\times 12}{0.24}}=1{,}000$$

$$\text{S.S.}=(\text{Max. leadtime}-\text{Normal leadtime})\times \text{monthly demand}$$

$$=\frac{30-15}{30}\times\frac{10{,}000}{12}=420 \text{ approx.}$$

(N.B. : Leadtimes of 30 and 25 days are presumed to be exceptionally abnormal. From the remaining 4 leadtimes the average is computed as 15 days for the normal leadtime).

$$\text{ROL}=\text{SS}+\text{Leadtime demand}$$

$$=420+\frac{15}{30}\times\frac{10{,}000}{12}\approx 840$$

$$\text{Average Inventory}=\frac{\text{E O Q}}{2}+\text{S.S.}$$

$$=\frac{1000}{2}+420$$

$$=920 \text{ (Answer).}$$

Example 6 :

The average monthly consumption for an item is 200 units and the normal leadtime is one month. If the maximum consumption has been up to 250 units per month and the maximum leadtime is $1\frac{1}{2}$ months, what should be the buffer (safety) stock for the item if the item is controlled by F.O.Q. system.

$$\text{Max. leadtime demand}=\text{Max. leadtime}\times\text{Max. demand}$$

$$=1\tfrac{1}{2}\times 250=375$$

$$\text{Normal leadtime demand}=1\times 200$$

$$=200$$

$$\text{S.S}=375-200$$

$$=175 \text{ (Answer).}$$

Exercise

A factory uses annually 24,000 units of a raw material which costs Rs. 1.25 per unit. Placing each order costs Rs. 25, and the carrying cost is 5% per year of the average inventory. Find the E.O.Q. and the total inventory cost (including the cost of the material).

The factory works for 320 days a year. If the procurement time is 10 days and safety stock 450 units find the reorder point, and the maximum, minimum and average inventories.

Example 7

An airline has determined that 10 spare brake cylinders will give them a stockout risk of 30% whereas 14 will reduce the risk to 15% and 16 to 10%.

It takes 4 months to receive the cylinders from a Kanpur supplier and the airline uses an average of 4 cylinders per month. At what stock level should they reorder, assuming they wish to maintain an 85% service level ?

Solution :

Leadtime demand $= 4 \times 4 = 16$ units

Safety stock for 85% service or 15% disservice $= 14$

$\therefore$ ROL $= 16 + 14 = 30$ **(Answer)**

Example 8

Data on the distribution of leadtime for a pump component were collected as shown. Management would like to set safety stock levels that will limit the stockout risk to 10%.

Leadtime (weeks)	*Frequency of occurrence*
$0 \angle 1$	10
$1 \angle 2$	20
$2 \angle 3$	70
$3 \angle 4$	40
$4 \angle 5$	30
$5 \angle 6$	10
$6 \angle 7$	10
$7 \angle 8$	10

How many weeks of safety stock are required to provide the desired service level ?

Solution :

Leadtime *Col. 1*	*Frequency* *Col. 2*	*Prob.* *Col. 3*	*Cum Prob.*
$0 \angle 1$, ($\frac{1}{2}$)	10	0.05	0.05
$1 \angle 2$, ($1\frac{1}{2}$)	20	0.10	0.15
$2 \angle 3$, ($2\frac{1}{2}$)	70	0.35	0.50
$3 \angle 4$, ($3\frac{1}{2}$)	40	0.20	0.70
$4 \angle 5$, ($4\frac{1}{2}$)	30	0.15	0.85
$5 \angle 6$*, ($5\frac{1}{2}$)	10	0.05	0.90*
$6 \angle 7$, ($6\frac{1}{2}$)	10	0.05	0.95
$7 \angle 8$, ($7\frac{1}{2}$)	10	0.05	1.00
	200		

Col. 1×Col. 2
5
30
175
140
135
55
65
75
680

$$\therefore \quad \text{Average Leadtime} = \frac{680}{200} = 3.4 \text{ weeks.}$$

Hence 6—3.4=2.6 weeks of safety stock would provide the service level of 90%.

Example 9 :

Demand for a product during an order period is normally distributed with mean (μ)=1000 units and standard deviation (σ)=40 units. What% service can a firm expect to offer (*a*) if it provides for average demand only, (*b*) if it carries 60 units of SS.

Solution :

(*a*) Without an SS 50% service would be provided.

(*b*) $^*Z=\frac{SS-0}{\sigma}=1.5.$

*From the normal distribution tables, service level=93.32%.

Example 10 :

A manufacturer of water filters purchases components in E.O Q's of 850 units/order. Total demand averages 12,000 components/year and MAD=32 units/month. If the manufacturer carries a safety stock of 80 units, what service level does this give the firm ?

Solution :

$$\text{S.D.}=\frac{\text{MAD}}{0.8}=\frac{32}{0.8}=40$$

$$\left(\text{N.B.}: \frac{1}{0.8}=\sqrt{\frac{2}{\pi}}\right)$$

$$Z=\frac{\text{S.S.}-0}{\text{S.D.}}=\frac{80-0}{40}=2.$$

From the normal distribution tables, Service level=97.72% (**Answer**).

Exercise 1 :

A firm has normally distributed forecast of usage with MAD=60 units. It desires a service level which limits stockouts to one order cycle per year.

(*b*) How much SS should be carried if the order quantity is normally a week's supply ?

*The student is urged to make himself fluent in the use of "Area Under Normal Curve" table for finding probability against given Z and *vice-versa*. It is given in the Appendix.

TABLE 1.—MOLAR ABSORBANCE AT PEAK WAVE LENGTHS OF DIFFERENT IONS

Ion	Peak wave length (mμ)	Molar absorbance ϵ_m	Ion	Peak wave length (mμ)	Molar absorbance ϵ_m
Cu^{2+}	206.5	1332	$CuY(OH)^{3-}$	206	53650
	800	12.3		750	43.0
CuY^{2-}	191	4617	$CuY \cdot NH_3^{2-}$	193	7905
	730	98.8		730	47.6
$CuYH^{-}$	193	4827			
	745	45.5			
Ni^{2+}	400	4.88	$NiY(OH)^{3-}$	208	16550
	715	2.1		390	18.5
	1150	1.95		590	8.0
				960	20.0
				1010	21.8
NiY^{2-}	195.5	6800	$NiY \cdot NH_3^{2-}$	209	17400
	380	12.3		375	16.8
	590	8.3		590	10.0
	975	32.0		965	21.4
	1150	16.7		1150	12.7
$NiYH^{-}$	191	4600			
	380	14.7			
	590	8.5			
	980	21.7			
	1150	14.5			
Co^{2+}	197.5	97.6	$CoY(OH)^{3-}$	207	15150
	510	4.8		490	15.7
	1150	1.1		570	16.4
				1110	6.2
CoY^{2-}	193.5	6800	$CoY \cdot NH_3^{2-}$	208	17300
	470	13.13		490	16.3
	1085	8.05		500	16.7
				1085	6.55
$CoYH^{-}$	193.5	4120			
	490	15.5			
	510	15.8			
	1085	6.58			

and the value for log $K^{CuY}_{CuY\,OH}$ agrees with that of Bennett and Schmidt (2.24) obtained in 1 M sodium sulphate solution.[9]

The position of H_2O *in the protonated and normal complexes*

Though it was generally believed by the earlier workers that the normal metal–EDTA complexes having high stability constants are completely dehydrated, JORGENSEN[10] assumes that in the normal nickel and copper–EDTA complexes one water molecule is present in the first coordination sphere. It may be interesting to examine these views from the present studies.

[9] M. C. BENNETT and N. O. SCHMIDT, *Trans. Faraday Soc.* **51**, 1412 (1955).
[10] C. K. JORGENSEN, *Acta Chem. Scand.* **9**, 1362 (1955).

TABLE 2.—STABILITY CONSTANTS OF DIFFERENT COMPLEXES

Metal	pH	log K^{M}_{MYH}	pH	log K^{MY}_{MYH}	pH	log K^{MY}_{MYOH}	pH	log K^{MY}_{MY}
Copper	0.7	12.02	2.0	2.90	11.8	2.48	8.8	2.70
	0.8	12.05	2.5	2.86	11.6	2.44	8.6	2.73
	0.9	12.02	3.0	2.86	11.4	2.37	8.3	2.74
	1.0	11.98	3.5	2.77	11.2	2.32	8.0	2.84
	1.1	12.09						
		Av 12.03		Av 2.87		Av 2.40		Av 2.75
					(OH^-)			
Nickel	980 mμ 0.9	11.67	980 mμ 2.5	2.83	0.46	0.60	8.7	2.66
	1.0	11.75	2.7	2.72	0.22	0.50	9.0	2.65
	1.1	11.68	3.1	2.69	0.12	0.58	9.5	2.52
			2.9	2.69	0.17	0.52	10.0	2.40
				Av 2.73		Av 0.55		Av 2.56
	1.2	11.60	380 mμ 3.4	3.01	0.7	0.49	10.0	2.36
	1.3	11.52	3.3	3.08	0.46	0.35	9.5	2.32
	1.4	11.47	3.2	3.27	0.22	0.42	9.0	2.26
			3.0	3.38	0.17	0.37	8.7	2.11
		Av 11.62		Av 3.12		Av 0.41		Av 2.26
					(OH^-)			
Cobalt	1.5	8.65	3.7	2.83	0.22	0.78	10.8	2.00
	1.6	8.60	3.3	3.16	0.172	0.83	10.9	1.99
	1.7	8.62	3.5	2.97	0.122	0.85	10.0	2.11
	1.8	8.75	3.9	2.75	0.076	0.86	9.2	2.21
		Av 8.66		Av 2.93		Av 0.83		Av 2.08

It can be seen from Figs. 2, 3 and 4 that on formation from the normal metal-EDTA complex, of protonated, ammino and hydroxo complexes, similar changes occur in the spectral intensities at major peaks. Thus, in the case of copper the molar absorption at 750 mμ is reduced on formation of mixed complex. For the nickel complexes the absorption at 380 mμ is intensified while that at 980 mμ is reduced. Co(II) is similar. These variations may be attributed, *inter alia*, to structural changes occurring on mixed complex formation. In the case of hydroxo and ammino complexes there can be little doubt about the attachment of OH^- or NH_3 groups to the central atom. Therefore, it is reasonable to assume that the observed intensity changes in the case of the protonated complex have resulted from coordination by H_2O.

JORGENSEN reported[10] the existence of two geometrical isomers of normal nickel-EDTA complex, H_2O being present in the cis- and trans-positions with respect to nitrogen in the two isomers. Apparently, the main evidence for this view is the observed difference in the absorption spectra of the two species. However, such isomers have not been reported so far for CuY^{2-} and CoY^{2-}. The major peaks in the spectrum of NiY^{2-} reported here agree fairly well with that of the α-form reported by JORGENSEN, and the β-form is very much akin to the protonated species. One possibility which has not been excluded by JORGENSEN, is the gradual dehydration of

$NiY\ H_2O^{2-}$ (β) to NiY^{2-} (α, ?) similar to the case observed by SCHWARZENBACH[5] for CoY This argument is in agreement with the present results, since $NiY\ H_2O^{2-}$ will have a spectrum similar to that of $NiYH\ H_2O^-$, as in the case of $CrY\ H_2O^-$ and $CrYH\ H_2O$.[11] Also, it is not then necessary to assume that in the case of CuY^{2-} and CoY^{2-}, where geometrical isomers have not been reported, one water molecule is present in the first coordination sphere

In the case of normal copper and cobalt–EDTA complexes, it is difficult to understand why protonation of an already free carboxyl group should bring about the observed large differences in spectral intensities if we assume coordination by water On the other hand, breaking of one of the metal EDTA bonds with the introduction of new ligands like H_2O, NH_3 and OH^- can be expected to affect the absorption spectra appreciably with a trend similar to that observed here The present results, therefore, do not seem to lend much support to the assumption that one H_2O is attached to the metal atom in the normal complex, except possibly in the case of the β-nickel EDTA complex of JORGENSEN

Comparison of stability constants

The formation constants of mixed complexes (obtained from MY^{2-}) of the type MY X may be expected to depend upon the $1/n \log \beta_n$ value of the MX_n complex The observed results show large deviations from this expectation, especially in the case of hydroxo-complexes of cobalt and nickel, e g, though OH^- is a stronger ligand than NH_3 the stability of the mixed ammino complexes were found to be greater This suggests that, besides the average strength of the metal ligand bond, other factors, such as stereochemical, ligand field stabilization, etc, strongly influence the formation of mixed EDTA complexes KIDA[12] pointed out, from a study of series of complexes, that the formation of mixed complex is favoured when the ligands involved tend to stabilize the same geometrical configuration Since the ammino and EDTA complexes of Co(II) and Ni(II) are nearly octahedral, mixed complexes may be formed easily The hydroxyl ion does not appear to favour the octahedral structure In the case of copper the observed value for the hydroxo-complex is relatively high This may be attributed, as stated by JORGENSEN[10], to possible stabilization of the ground state due to intermixing of the electron transfer states

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(12) S KIDA, *Bull Chem Soc, Japan* **34**, 962 (1961)

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NATURE, STABILITY AND BONDING OF THE PEROXY GROUP IN PEROXY TITANIUM COMPOUNDS

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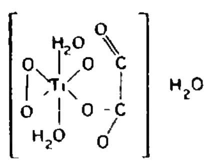
Abstract Some physicochemical properties of peroxy titanium compounds are explained by assigning a strained triangular ring structure to the peroxy titanyl group, with a bent and reduced overlap of the O O bonding orbitals The stability of the peroxy group is found to depend on the stability of the other ligands The decreasing order of stability of the peroxy group in the compounds is as oxalato maleato malonato sulphato peroxide of titanium

ORANGE red peroxy titanium complexes of oxalic,[1] malonic,[2] maleic[3] and sulphuric[4,5] acids were prepared in the solid state, but the corresponding complexes of organic monobasic acids (e g propionic acid) and fumaric and succinic acids could not be prepared because these complexes readily hydrolysed in solution Further, although peroxy titanium perchlorate was prepared in solution, it could not be obtained in the solid state, since, on evaporation even at room temperature, colourless crystals of titanyl perchlorate, $TiO(ClO_4)_2\ 6H_2O$, were obtained [6] The existence of peroxy titanium complexes of oxalic, malonic, maleic and sulphuric acids in solution and the solid state indicates that these acids function as bidentates, occupying two co-ordination positions When peroxy titanium compounds were kept in vacuum, the sulphate complex gave both ozone and oxygen while others gave only oxygen

It is unusual that even in presence of a large excess of hydrogen peroxide or the corresponding acid, the compounds retained one peroxy group and one dicarboxylate ligand Peroxy titanium sulphate, however, lost its orange red colour, when stored over strong sulphuric acid, probably due to the replacement of the peroxy group by a sulphate group The peroxy titanyl complexes may have structures of the type



Peroxy titanyl oxalate

[1] D P KHARKAR and C C PATEL, *Proc Indian Acad Sci* **44 A** 287 (1956)
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[3] G V JERE and C C PATEL, Proc Symposium on Chemistry of Co-ordination Compounds Agra, Pt II, p 117, Natl Acad Sci, Allahabad, India (1960)
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[5] G V JERE and C C PATEL, *Canad J Chem* **40**, 1576 (1962)
[6] C C PATEL and V KRISHNAN, *J Sci Industr Res* (India) **20 B**, 604 (1961)

Solution :

No. of orders (or exposures to stockouts) $=52$

1 stockout in 52 : $\frac{51}{52}=98\%$ *service*

Safety factor, $Z=2.05$ *(from tables).*

$$K=2.05\times\frac{1}{0.8}$$

$$=2.56$$

$SS=2.56\times 60=154$ *units.*

(*b*) How much SS should be carried if the order quantity is 5 weeks' supply ? **(Answer : 96 units)**

If the leadtime is one week and expected usage is 500 units/week, what is the appropriate order point when the order quantity is

(*c*) One week's supply is in (*a*) ? **(Answer :** 654)

(*b*) five weeks' supply as in (*d*) ? **(Answer :** 596)

Exercise 2

A firm has a normal distribution of demand a (constant) leadtime with $\sigma=250$ units and desires to provide 98% service.

(*a*) How much SS should be carried ?

(*b*) If the demand during the leadtime averages 1,200 units, what is the order point ? **(Answer : 512, 1712)**

Exercise 3

The average demand for an item is 120 units per year. The leadtime is 1 month. The demand during leadtime follows normal distribution with average of 10 units and standard deviation of 2 units. If the item is ordered once in 4 months and the policy of the company is that there should not be more than one stockout every two years determine the reorder level.

ABC Analysis

This is a simple and yet useful application of statistical sampling in the context of inventory control. It is based on

$$\text{Usage value}=\text{Cost}\times\text{Requirement}$$

The ABC analysis focusses the attention of the management on those items where the greatest saving can be expected. The items are categorised into three classes, A, B, and C, by their usage value so that close attention is paid to the A items. C items come in the other extreme and can be generously stocked because they require little capital. The B items fall in between. The ABC concept is based on Pareto's Law that is a few high usage value items constitute a major part of the capital invested in inventories whereas a bulk of items having low usage value constitute an insignificant part of the capital. In view of several hundred and even thousand items stocked in most inventory situations the ABC Analysis may be carried out on a sample. Once a random sample has been obtained the following steps may be performed for the ABC analysis :

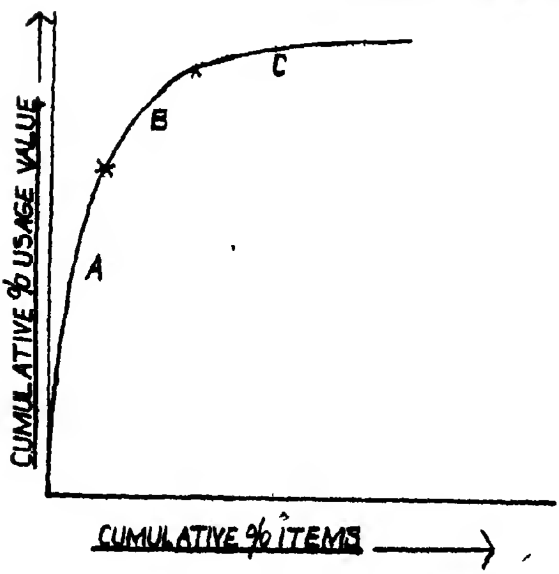
1. Compute the annual usage value of every item in the sample by multiplying the annual requirement by cost/unit.

2. Arrange these items in descending order of the usage value computed above.
3. Accumulate the total of the number of items and the usage value.
4. Convert the accumulated totals of No. of items and usage values into percentage of the grand totals.

This is illustrated in the following :

Usage Value Rs.	*Accumulated Usage*	*Percentage*	*Acc. total No. of Items*
50,000	50,000	$\frac{50,000}{V} \times$	$\frac{1}{N} \times 100$
45,000	95,000	$\frac{95,000}{V} \times 100$	$\frac{2}{N} \times 100$
44,486	139,486	$\frac{139,486}{V} \times 100$	$\frac{3}{N} \times 100$
44,482	183,968	$\frac{183.968}{V} \times 100$	$\frac{4}{N} \times 100$
42,683	226,651	$\frac{226,651}{V} \times 100$	$\frac{5}{N} \times 100$
...	...	...	...
...	...	...	...
...	...	...	...
Grand Total	V		N

5. Plot the two percentages on the graph paper.



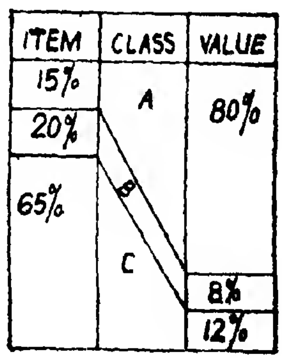


Fig. 10

6. Mark the cut off points X and Y where the curve sharply changes its shape. This provides three segments A, B and C. From points X and Y and tabulated data the usage value classification points A, B and C may be fixed and generalised over the entire population of the stock items. It is usual in most industries to find about % of the items accounting for as much as 80% of the capital in inventories. The ABC analysis may be presented as above.

In practice, the C items are frequently placed on the shop floor and the workers can help themselves without placing a formal requisition which requires time and money. On the other end of the *sale*, the A items are scheduled for procurement and manufacture and are under continual scrutiny of the senior inventory control staff. This leads to selective control. Although the technique is very simple yet few of the manufacturing and warehousing organisations have realised the potential of the technique. By simply concentrating efforts and attention on tight schedules of *A items to be procured in small batch sizes and generously stocking of the C items there is a scope of saving thousands of rupees in investment; aside from other intangible benefits. ABC concept conforms to the considerations implied in the E.O Q model. 'A' items have high inventory carrying costs and should, therefore, be procured in smaller lots. C items require very little capital and therefore have low inventory carrying costs and they should be bought in bigger lots so that there are fewer orders and therefore less acquisition cost. B items are usually placed under statistical stock control.*

Example : Perform *ABC* analysis on the following *sample of 10 items from an inventory.*

Table 1

Col. 1 Item No.	*Col. 2* Annual usage units	*Col. 3* Unit cost Rs.	*Col. 4* Annual usage (Rs.)	*Col. 5* Ranking
140	300	0.10	30	6
142	2,800	0.15	420	1
143	30	0.10	3	9
144	1,100	0.05	55	4
146	40	0.05	2	10
147	220	1.00	220	2
148	150	0.05	7.5	8
149	800	0.05	40	5
151	600	0.15	90	3
152	80	0.10	8	7
	6120			

Solution :

The item are ranked in descending order of the usage value.

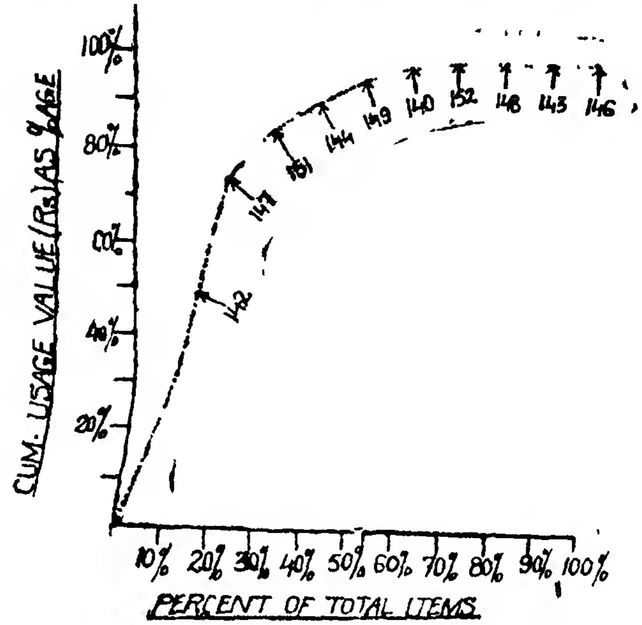


Fig. 11

Table 2

Item No.	*Annual usage Rs.*	*Cum. Annual usage (Rs.)*	*Cum. %*	*Category (got from plot above)*
142	420	420	48.0	A } Cut off points
147	220	640	73.1	A } can be guessed
151	90	730	83.4	B } without the plot
144	55	785	89.6	B } in case like this
149	40	825	94.1	C
140	30	855	97.6	C
152	8	863	98.6	C
148	7.5	870.5	99.4	C
143	3	873.5	99.6	C
146	2	875.5	100.0	C

The plot above could, however, may be avoided where the cut off points can be guess-estimated from the last col. of the 2nd table.

Important Note : We have taken along the horizontal axis the % of total items in the plot *e.g.* upto items 142 it is 10% (*i.e.* just 1 of 10 items) and upto item 144 it is 40% (*i.e.* 4 out of 10 items). Anyhow, the point we want to emphasise is that some authors accumulate the annual usage (Col. 2 of Table 1.) and take the %

ages along the X-axis 6120 as those of total 6120 For example, the item ranked 1 has a usage (in units) of 2800 and they would plot item 142 at $\frac{2800}{6120} \times 100 = 45\%$ along the X-axis This is an inferior way to the one given by us above.

E.O.Q. modified for finite leadtime for made-ins

Deliveries of purchased items are normally delivered in a lot, so that stock level rises by the ordered quantity q in one step.

Deliveries from the factory machine shop may take place, day by day over a period, and if at the same time these items are being withdrawn from stores for use in further stages of assembly, the stock level will rise by less than q ; less, in fact by the amount withdrawn over the period whilst q was being delivered.

The situation when stocks are consumed whilst they are being replenished is depicted below figuratively.

Let d=rate of consumption

p=rate of replenishment *i e.*, production.

It is to be seen from figure below that the average stock is not simply $q/2$ but $(1-d/p)\ q/2$, where the correction factor reflects the amounts withdrawn from stock during the period the item is being produced. The derivation of the economic order quantity amended for the significant leadtime follows exactly the same steps as that of E.O.Q.

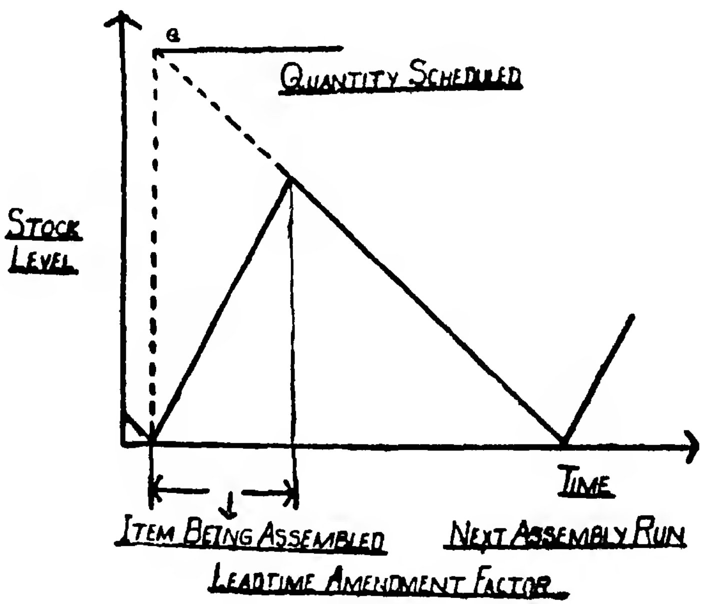


Fig. 12

$$q_0=\sqrt{\frac{2CaA}{CI}}\ \sqrt{\frac{p}{p-d}}$$

and
$$C(q_0)=\frac{ACa}{q_0}+\left(1-\frac{d}{p}\right)\frac{q_0CI}{2}$$

Example : A=24000 units

Ca=Rs. 40

C=Rs. 20

I=30%

p=1000 per week

d=500 per week

E.O.Q. is modified for finite leadtime.

$$=\sqrt{\frac{2CaA}{CI}}\ \sqrt{\frac{p}{p-d}}$$

$$=\sqrt{\frac{2\times 40\times 24000}{20\times\frac{30}{100}}}\sqrt{\frac{1000}{500}}$$

=800 **Answer.** (Explain why higher than usual E.O.Q. is needed).

Find total cost as an exercise.

Economic Order Size in case of price Breaks.

Example : The Conco Company use 12,000 switches per year supplied ordinarily at a price of Rs. 3.00 per item. Carrying costs are 16% of the value of the average inventory, and ordering costs are Rs. 20 per order.

The supplier however offers discounts as per the table below :

Order Size		*Price per item*
Less than	2,000	Rs. 3.00
2000 to	3,999	2.92
4000 or more		2.90

What is the most economic order size ?

Solution. Basic E.O.Q. (Rs. 3.00 per switch as the price)

$$\text{E.O.Q. (Rs. 3.00)}=\sqrt{\frac{2\times 12000\times 20}{3.00\times 0.16}}=1{,}000$$

The problem then is to determine if it is advantageous to increase the order size above the E.O.Q. of 1,000 by availing one of the price discount.

To solve the problem, E.O.Q.'s for all the prices are computed.

$$\text{E.O.Q. (Rs. 2.92)}=\sqrt{\frac{2\times 12000\times 20}{2.92\times 0.16}}=1{,}014$$

$$\text{E.O.Q. (Rs. 2.90)}=\sqrt{\frac{2\times 12000\times 20}{2.90\times 0.16}}=1{,}017$$

We see in the given table that for 2.92 the quantity range is 2000 to 3999 whereas E.O.Q. is 1014. This is, therefore, an infeasible solution. Further explanation will be given later.

Likewise for the price of Rs. 2.90 the quantity range is 4000 or more but the E.O.Q. is 1017 so that this, too, is an infeasible solution

However the price of Rs. 3.00 has the quantity range of 0 to 1999 and the *computed E.O.Q. of 1,000 does fall within this range. The E.O.Q. is thus a feasible solution but we must compare (i) the total cost with E.O.Q =1000 and (ii) the total cost with an order size of 2,000 i.e., the quantity of the following price break, and (iii) at q=4,000.*

		Order Size 1000	2000
Costs	Acquisition cost $= \frac{A}{q} \times Ca$	Rs. 240	Rs. 120
	Carrying cost $= \frac{q}{2}$ CI	Rs. 240	Rs. 467
	Material cost $=$ AC	Rs. 36,000	Rs. 35,040
	Total	Rs. 36,480	Rs. 35,627

Rationale : The figure below shows that total cost curve against the batch size. It is stepped because of the price breaks. If there were no price breaks it would have been of the cup shape as in figure 14.

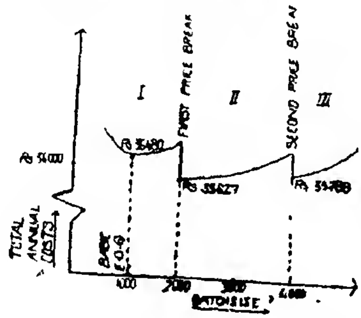


Fig. 13

Concentrating now, however, on figure 13 it has three portions as marked thereon I, II, and III. We see that the curves of steps II and III give monotonously increasing total costs with the rising batch size. This, however, is not so with I which is of cup shape *i.e.*, the total cost first with decreases and then increases. And the minimum occurs at batch size of 1,000 with the total cost of Rs. 36,480. The minimas for II and III occur in their dotted extension to the left and not within II and III themselves. Such monotonously increasing portions as II and III yield what we call *infeasible* portions and portion I is feasible ; but we shall compare the total cost of Rs. 36,480 with the limiting total cost at batch size of the first price break which is 2,000. We found that the total cost with 2,000 is Rs. 35,627 which is less than Rs. 36,480 with 1000. Also, it is compared with that of q=4000 that come 35,788.

In another problem, however, may be portion II was of cup shape. In that case we would compare the min. total cost, (with its E.O.Q.) with limiting total cost at 2000 and 4000.

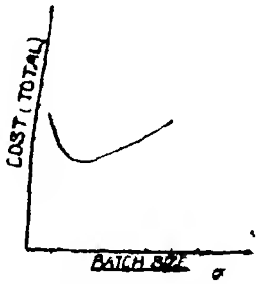


Fig. 14

Exercise

Assume the following price structure.

Units		*Unit Price*
0-199	Rs.	10.00
200-399		9.75
400-599		9.50
600-		9.25

Acquisition cost per order=Rs. 25 ; Cost of the item=Rs. 10 ; Annual Demand=900 units. Carrying Cost=Rs. 2/unit/year. Find the most economic order size. **(Answer : 600)**

Quantity Discounts

Example : Item ; Washers ; Units price Rs. 0.25 ; Annual Requirement= 12,000

	Units	
Discounts :	0-1999	Basic price
	2,000-3999	5% discount
	4,000-5999	10% discount
	over 6,000	15% discount

A=12,000 Ca=Rs. 50 I=20%

No. of orders	q	Stockholding costs	Ordering costs	Purchase price, basic	Total (at basic price)	Purchase price (with discount	Total (with discount)
1.	12,000	300	50	3000	3350	2550	2900
2.	6,000	150	100	3000	3250	2550	2800
3.	4,000	100	150	3000	3250	2700	2950
4.	3,000	75	200	3000	3275	2850	3125
6.	2,000	50	300	3000	3350	2850	3200
12.	1,000	25	600	3000	3625	3000	3625

Thus, 6000 approximately is the economic quantity as against by the simple formula

$$q_0 = \sqrt{\frac{2 \times 12000 \times 50 \times 20}{20 \times \frac{1}{4} \times \frac{1}{100}}} \approx 4{,}900$$

The simple E.O Q. is likely to be invalidated, therefore, where price discounts are offered, since the acquisition cost is stepped rather than linear with respect to the number of orders.

The above quantity of 6,000 has been derived by simple enumeration of a few possibilities of number of orders. Further refinement is possible. Brown (ref. 1) derives that maximum quantity that can economically be ordered to qualify for a discount in unit cost is derived from the following formula.

$$q_1 = \frac{2dS}{r} + (1-d)q_0$$

where the symbols have the following meanings

d=fraction by which the price will be reduced from the basic price if the larger quantity is ordered.

S=Annual Usage in pieces

r=carrying charge (as we have been using)

q_0=E.O Q at basic price.

It is to be noted when there is no discount, $d=0$ the above formula yields the normal E.O.Q. As an instance, consider an oil seal, price being 50 n p. Annual requirement is 12,000. The order processing cost is Rs. 40, I=20%.

$$q_0 = \sqrt{\frac{2 \times 40 \times 12{,}000}{\frac{1}{2} \times \frac{20}{100}}} \approx 3{,}100$$

If the discount is 5% on 3,000 or more, $d=0.05$

The maximum quantity we can order to avail of 5% discount is

$$\frac{2 \times 0.05 \times 12{,}000}{20/100} + 0.95 \times 3100$$

$$= 6{,}000 + 2{,}945 = 8{,}945 \approx 9{,}000$$

For practical purposes, tables have been compiled by several authors which can eliminate the computational task at the expense of slight loss in accuracy. The following table is reproduced from reference No. 2. It is suitably amended from £ to Rs., I=0.26 and Ca≈Rs. 20.

Usage Value per month	Order Qty. in months	% discount needed to justify increasing order to this number of months								
		0.5	1.0	1.5	2	3	4	6	9	12
Rs.										
Over 1000	0.25	0.1	0.5	0.9	1.2	2.0	2.8	4.6	6.6	8.7
250 to 1000	0.5		0.2	0.6	0.9	1.7	2.5	4.0	6 2	8.4
80 to 250	1			0.1	0.4	1.1	1.8	3.3	5.5	7.6
40 to 80	1.5				0.1	0.6	1.3	2.7	4.8	7.0
20 to 40	2					0.3	0.8	2 1	4.2	6.3
10 to 20	3						0.2	1.2	3.1	5.1
5 to 10	4							0.5	2.2	4.0
2.4 to 5	6								0.8	2.3
1.2 to 2.4	9									0.6
under 1.2	12									

An example on the use of this table is in order.

Item : Screw, $4 \times$ P 272 ; unit price, Rs. 0.20, $A = 12{,}000$

Usage value/month $= 12{,}000 \times 0.26 \times 1/12 = 200$; From Column 2

E.O.Q. (roughly) = 1 month's usage *i.e.* $12000/12 = 1{,}000$.

E.O.Q., as the student must know by now, is applicable only if the purchase price is constant *i.e.*, linear w.r.t. number of orders. Quantity discounts upset this assumption. If the quantity is increased we derive benefit of discounts but at the expense of extra stockholding cost. The discount models seek to optimise between the benefits of discounts and extra stockholding costs. If the vendor offers discount of 4% on lots of 1,000 or more we can read from the table (in col. 4% and now containing 1 month's order quantity) that the ordering quantity can be increased to 1.8 month's requirement *i.e.*

$$\frac{12{,}000 \times 1.8}{12} = 1{,}800$$

It is important to note that if the E.O.Q. is greater than the quantity justified by discounts, E.O.Q. is to be used.

Sometimes, discounts may not be as apparent as above. These may be 'had' by purchasing one truck-load or 2 truck-loads etc. of an item. Reasons are obvious ; if the truck is half empty the shipment cost will be same if it were full. Another phenomenon that amounts to quantify discounts is the learning curve. If, for example, 2000 pieces of an item are turned out by an operator he would have 'learnt' *i.e.*, rythmised his bodily motions etc. as discussed in MISC Study. For such cases, a suitable model can be built on the lines of the quantity discount models discussed.

Dynamic Order Quantity : One assumption in our above discussion is that demand is uniform. Even when it is roughly so it practically does not matter.

In practice, we come across with non-uniform demand such as having rising/falling trend and/or depicting seasonal influences. How do we cope with such non-uniform demand, then ? We have already seen in Study 1 as to how floating reorder levels can be fixed that take into account changing behaviour of demand. We shall take up the general method that can be applied to any pattern whether seasonal, irregular, rising or not. The method is based on the E.O.Q. concept. Consider the 12 months' requirements as computed at the commencement of the year and noted below :

Month	1	2	3	4	5	6	7	8	9	10	11	12
Requirement	20	40	10	10	10	2	40	30	40	40	10	20

Set up/procurement cost=Rs. 20 ; unit price=Rs. 5 ; I=24%, *i.e.* 2% per month.

Consider the case when each month's requirement is made separately.
Ordering cost$=20\times 12=$Rs. 240.

If the 2 units required in the month 6 are made a month earlier the set-up cost would come down by Rs. 20 whereas the carrying cost which was non-existent in the previous case would rise to $5\times 0.02=$Rs. 0.10. The 2nd strategy is then more economical than the first by Rs. $(10.00-0.10)=$Rs. 9.90.

To systematise the above method, therefore, we can adopt the following procedure. The first month's requirement has got to be made in the first month at a set up cost of Rs. 20. It can also be verified if the 2nd month's requirement can be batched along *i.e.* made in the first month. This would be the case if the savings on an extra set-up are more than the carrying costs. If the savings on set up outweigh the carrying costs we can try the 3rd requirement also in the same way *i e.* does another saving in the set-ups outweigh the carrying costs for the 2nd month's requirement for one month and for the 3rd month's requirement for two months. We proceed on until the ordering and carrying costs are balanced. The following mathematical procedure may then be adopted.

"For each month n, starting with $(n=1)$, the requirement R_{n+1} is examined to see whether the extra carrying cost $(n.\ R_{n+1}\ I.C)$ in making R_{n+1} in month 1 is less than $\frac{Ca}{n}$, the saving in the set-up cost. As long as the answer is 'yes' n is increased to take the next month into consideration ; but as soon as the answer is 'no' that ends that particular grouping, and the following month is treated as a new month 1 to proceed on in this way".

Let us solve the above example, then methodically below, rewriting the condition as "Is $n^2\ R_{n+1} < \frac{Ca}{IC}$?"

or by substituting the various values $\frac{Ca}{IC}=\frac{20}{0.02\times 5}=200$

Thus the condition is Is $n^2\ R_{n+1} < 200$?

Month	Requirement	n	$n^2 R_{n+1}$	Is n^2R_{n+1} <200	Action
1	20	1	40	yes	Include 40 in month 1.
2	40	2	40	yes	Include 10 in month 1.
3	10	3	90	yes	Include 10 in month 1.
4	10	4	160	yes	Include 10 in month 1.
5	10	5	50	yes	Include 2 in month 1.
6	2	6	1440	No	Set-up again in month 7
	92				
7	40	1	30	yes	Include 30 in 7th month.
8	30	2	160	yes	Include 40 in 7th month.
9	40	3	360	No	Set-up again in month 10
	110				
10	40	1	10	yes	Include 10 in month 10
11	10	2	80	yes	Include 20 in month 10
	20				
12	70				

Thus the optimal plan is 3 set-ups for batch sizes of 92, 110 and 70 respectively though the last step could possibly take up some of the next year's requirements.

Exercise: Compute dynamic E.O.Qs. for the following requirements schedule:

Month	1	2	3	4	5	6	7	8	9	10	11	12
Requirement	50	100	10	170	150	180	1	260	100	80	150	70

(**Answer:** 160, 320, 181, 440, 220)

Periodic Review Systems

In these systems stocks are reviewed periodically (e.g. weekly) unlike the E O Q. system in which stocks are reviewed upon each customer demand transaction. There are two leading types of the periodic review systems: Fixed Order Cycle System and the sS system briefly discussed below.

Fixed Order Cycle System

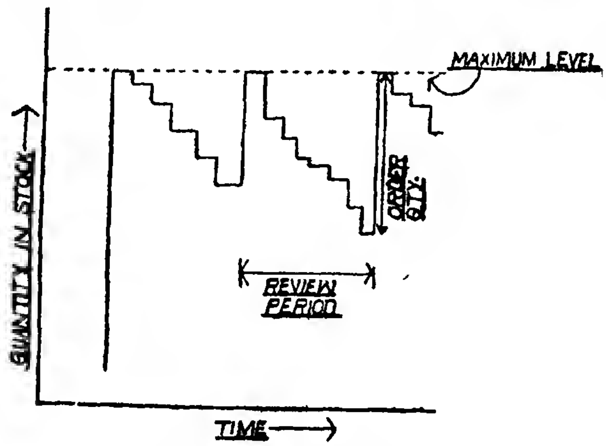
Under this system, stocks are reviewed periodically at fixed intervals, e.g. 7th of every month. To derive the order size stocks are brought to a predetermined level.

The figure below depicts the variations of stock level in this system. Obviously the order size is varying. As against the fixed order quantity systems where fluctuations are absorbed by means of varying time interval of order placement, here the varying order size serves this purpose. The maximum level is set equal to the sum of (i) expected demand during the leadtime which is ordinary leadtime plus review period and (ii) safety stock during the same period. Economic ordering interval is derived like the E.O.Q. and is equal in days to

$$\frac{365 \times \text{E.O.Q.}}{\text{Annual demand}}$$

Another point is worth noting about the cycle review system. An order is invariably placed at the review date unless, of course, there has been no demand during the preceding period. Therefore, when there has not been substantial consumption during the preceding period an order of small amount will have to be placed which is expensive indeed. Therefore, there grew the sS system. The cycle review system is also known as the topping-up system. The nomenclature is quite apt when one thinks of it in terms of such a stock-keeping item as a lubricant. The height of the can in which the lubricant is contained may represent the maximum level. All that the stock controller has to do is to ensure, at every review date, that the can is filled up to brim.

Cyclic review system is particularly useful when a vendor supplies numerous items. Because of the simultaneous review of all these items, joint replenishment orders can be placed. Also the vendor can plan his production better. The maximum level may be changed every period in sympathy with the fluctuations in demand. Such varying maximum level is known as the floating maximum level.

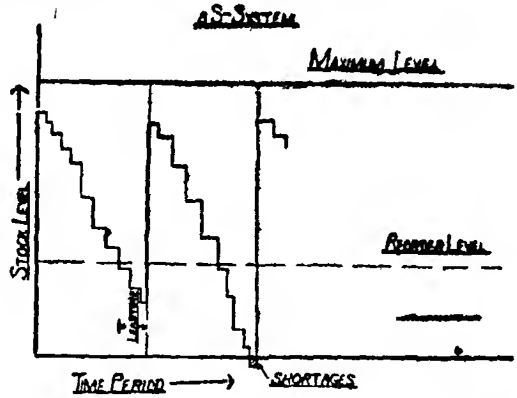


CYCLIC-REVIEW SYSTEM

Size of the order = Predetermined stock level minus stock level at the time of the periodic review.

sS System. As mentioned earlier, in the fixed order cycle system an order is invariably placed even if the demand in the preceding period was very low, i.e.,

the order size would be small which is uneconomic indeed. The sS system stock control system is an improvement in this regard.



Under the sS-system, there are two stock levels which constitute its parameters. The lower stock level, s is equivalent to the topping-up level of the fixed order cycle system The difference between the upper and lower levels that is (S-s) fixes the minimum size of order that will be placed. Obviously, the average order size will be somewhat larger S—s=minimum order size. The average Order size may be taken as

S—s+½ demand during a review cycle.

The fixed order cycle system is controlled just by two parameters : maximum stock level and review period ; maximum stock being set exactly in the way of fixing R.O.L except that the replenishment leadtime also includes the review period. The length of the review period is determined by the same considerations as for the F.O.Q. System *i.e.*, longer review period (as against lot size) for usage value items and shorter review period for high usage value items. The sS system is different in this regard. There are three parameters that control it : maximum level, minimum level and the review period. This makes its mathematical analysis for optimisation fairly complex and ever since its inception it has been the favourite topic for the mathematical minded.

The ordering rules are : (1) Choose two inventory levels S and s. (2) At every review period, compare the available inventory with S and s. (3) If it lies between S and s place no order. If it is below or equal to s, place an order of the size S—I, where I=Inventory level.

When the available stock at any opportunity to order can be actually less than the Order Point, it is more economical to set up sS system rather than the F.O Q. system. This means that if the stock is depleted unit by unit *i.e.*, customer's order size is equal to one always and the stock level would touch the ROL in F.O Q. system ; but would not fall below it. If, however, the order size is greater than 1, which usually is the case in practice, sS system is more economical.

Mathematical basis of Periodic review system

Fixed order cycle system : The analysis is just the same as that for E O Q. Instead of E.O.Q. we are interested here in most economic review period that balances the two costs :—

(i) Acquisition (ii) Stockholding

sS system: We noted that the average order size is S—s+½ demand during a review cycle. Now we know $q_0=\sqrt{\frac{2ACa}{IC}}$

Equating the two (approximation, here)

$$q_0=\sqrt{\frac{2ACa}{IC}}=S-s+\tfrac{1}{2}\,Td$$

where T=length of Review Period

d=expected usage during a period

Thus $$S\approx\sqrt{\frac{2ACa}{IC}}-s+\frac{Td}{2}$$

s is to be calculated just as the maximum level in the fixed order cycle system. That is, in turn, just like ROL except that leadtime=review period plus usual lead time.

Comparison of the fixed order quantity and the periodic review system

1. In the F.O.Q. system there is continual serveillance of the stock level. The moment the reorder level is crossed a replenishment order is triggered. The periodic review systems are at a disadvantage, in contrast. If there is an important transaction of a large size it is not likely to be filed, especially in the closing days of a review period. The replenishment action would be delayed until the review date. But it is to be noted that such exceptionally large size of demand would occur only in a fraction of the total reviews. Where, therefore, demand is fairly uniform; devoid of such exceptional cases, the F.O.Q. System would be unnecessarily costlier.

2. In the F.O.Q. System, the safety stock is set at a level to guard against the demand fluctuations during just the replenishment leadtime; but in the periodic review systems the safety stock is required to cater to the demand fluctuations during the replenishment leadtime plus the review period ; therefore, higher stock levels in the case of the periodic review systems.

3. The fixed order quantity systems are mere sensitive to the demand fluctuations and can respond promptly to any unanticipated demand behaviour. They are liable, however, to change with impulses in demand.

4. The fixed order system is best suited where some continuous monitoring is possible, *e.g.*, in the case of perpetual inventory records. The system is particularly suited, in the case of the bought-outs which do not constitute a major-difficult portion of the vendor's production. This is because the time of placement of orders under the fixed order quantity system is more or less uncertain. Obviously, therefore, the vendor would not be inconvenienced in case the item is an ordinary one.

5. The periodic review systems, especially the fixed order cycle system, are ideally suited for joint replenishment orders. If, for example, a vendor is the regular supplier to the company for a dozen items it is much more cheap to put all the 12 items together on order for replenishment. Aside from the paper costs for raising another eleven orders (were they on the F.O.Q. System) the items would most likely be delivered by the vendor in just one consignment. This would,

therefore, reduce the receiving costs substantially. Furthermore, one item as such may not constitute vendor's major difficult part of production but 12 items might well be so. Joint replenishment facilitates advance and proper planning by the vendor and reduces his production costs which will ultimately be passed on to the receiving organisation. For very expensive items, the fixed order cycle system is quite useful in that frequent reviews are made. Under such circumstances, in fact the system leads to what has come to be known as scheduled ordering which is really not statistical in nature. Under scheduled ordering, an annual (or six monthly etc.) schedule for expensive and difficult (for vender to manufacture) items for procurement is compiled fairly in advance of the commencement of the year. Since such items are not very many the senior managers are supplied with copies of the schedule and continual scrutiny over the progress is maintained. Obviously such items would be crucial to the (ordering) organisation production. These items may also contain a major proportion of the A category items.

SELF-EXAMINATION QUESTIONS

State whether the following statements are true or false. Give reasons briefly.

(i) By ABC analysis, an item costing Rs. 5,000 is most likely an A category item and another costing Rs. 60 only would fall in a lower category.

(ii) Higher the usage value higher is the economic batch size.

(iii) A large batch size is in itself a protection against stockouts.

(iv) Safety stock varies with the level of a demand.

(v) Variable leadtime would call for lower level.

(vi) Forecast as such is of no direct use in inventory control. It is utilised for setting economic order quantity and reorder level.

(vii) Leadtime is the time interval elapsing between the placement of a replenishment order and its receipt in the goods receiving section.

(viii) Leadtime is the time interval elapsing between the posing of the order letter to the supplier and receipt of the last instalment of goods against order.

(ix) If the demand of an item jumps up it would be desirable also to increase the value of the smoothing coefficient.

(x) If the demand of an item declines abruptly it would be desirable to bring down the value of the smoothing coefficient.

(xi) The more stable the demand the less is the value of the smoothing coefficient.

(xii) MAD bears no relationship with the standard deviation of a distribution.

(xiii) MAD is not a prospective statistical measure for use in devising tracking signals.

(xiv) Leadtime MAD$\neq$Leadtime$\times$forecast period.

(xv) It is erratic to assume that the forecast errors conform to normal distribution.

(xvi) Forecast error=forecast for the current period prepared in the beginning of the preceding period and the demand that actually materialised during the preceding period.

(xvii) *Order size and reorder level constitute a good example of interacting variables.*

(xviii) *If the demand behaviour is not time-dependent it may be worthwhile to try to fit some standard distribution to the past historical data for forecasting purpose.*

(xix) *The fixed order quantity system is completely described by the parameters : order size and reorder level.*

(xx) The forecast period should be as small as possible.

(xxi) The forecast period should never be more than a month.

(xxii) Whereas the derivation of E.O.Q, is based on compromise between inventory level and service to the customer the safety stock is set to balance between acquisition costs and inventory level.

(xxiii) ABC analysis is in step with economic ordering regarding the batch size.

(xxiv) Leadtime MAD=(0.659+0 341LT)*k* MAD. Also, interpret the symbols.

(xxv) If desired the reorder level can be made to 'float' in sympathy with demand.

Answer :

(i) No, it is incorrect. It is the usage value that is the criteria of classification into A,B and C categories.

(ii) No, it is the other way round. Higher usage values would entail higher inventory-carrying costs and, therefore, their batch size should be smaller.

(iii) Yes, because it offers only few exposures to stockout. *i.e.*, ordering frequency is small.

(iv) No, safety stock varies with fluctuations of demand.

(v) No, variations in leadtime require higher safety stock to provide the same service and, therefore, reorder level would also be higher.

(vi) That is right.

(vii) No, instead of receipt of goods it is the posting on the stock ledgers that marks the terminal point of the leadtime. There may be considerable period elapsing between these two events, especially if clerical routines are not efficient.

(viii) No, instead of posting the order letter, it is 'recording of the reorder level being crossed that marks the starting point of the replenishment leadtime. Also, it is the first instalment of goods that marks the terminal point of the leadtime.

(ix) Yes, so that forecasting is attuned to higher level promptly.

(x) No, it should still be increased so that forecasting is attuned to the lower level of demand promptly.

(xi) Yes, since this ensures more weightage to past data.

(xii) It does bear a fixed relationship.

(xiii) It is a prospective measure since it provides us with the spread of the forecast errors.

(xiv) *The statement is right since the time series have time dependant demand.*

(xv) *Mostly forecast* errors conform nearly to normal distribution when demand emanates from several customers. *If*, however, one or more customers are dominant for demand, the supposition of a normal distribution is erroneous.

(xvi) That is right. Demand is sought to be estimated by forecasting.

(xvii) That is right. Independently computed they may provide undesirably high service which is expensive. Since a large order size is in itself a protection against stockouts it is desirable to consider the two variables together to yield an optimal total cost comprising acquisition cost, inventory carrying cost and shortage cost.

(xviii) Yes, since if such a distribution provides good fit to the historical data the task of fixing ROL becomes more objective as well as easy and less expensive.

(xix) Yes, fixed order quantity system has only two parameters (there is another statistical system, the, sS system that has three parameters, the minimum level, *s*, the maximum level, S, and period, *t*, for review of stocks).

(xx) No, it might make the computational task too expensive and also time consuming thereby defeating the very purpose of keeping the forecast period small in an endeavour to catch up with all the fluctuation even though they are not significant.

(xxi) That is wrong because if the demand is rather stable why waste time on keeping the forecast period just a month. A quarterly or half-yearly forecasting period may serve the purpose.

(xxii) That is wrong. For E.O.Q. inventory carrying costs and acquisition costs have to be manipulated and for setting safety stocks inventory carrying and shortage costs have to be balanced.

(xxiii) Yes, ABC analysis for example, calls for less attention for C category items which have low usage value and, therefore, higher batch size by economic ordering considerations.

(xxiv) The relationship is right and has been derived for a specific inventory situation by a renowned operations researcher. It may be applied to a different situation. MAD is the mean absolute deviation during the forecast period k is the service factor and LT is the leadtime as a multiple of the forecast period.

(xxv) **Yes, if the ROL is re-computed each time forecast is prepared it wou be floating. As much as 40% reduction in stocks have been achiev by Eilon, a British production and stock control expert by floating RO rather than keeping it fixed.**

Queuing Theory :

One encounters queues in everyday life in the post offices, in the banks, in restaurants, in barber's shop, etc. Less apparent examples are when one waits for the telephone operator to answer, vehicles waiting for the green lights on the cross roads, aeroplanes circling around the airports that are waiting to land, machine operators forming a queue across the tool crib to obtain tools and so on. Obviously a waiting line or a queuing problem arises when either units requiring service or facilities which provide it stand idle, i e., with Operations reasearch one can very effectively analyse such problems but the relevant decision problems must be of sufficient economic significance to warrant the expenditure for undertaking the study.

Queuing System : A queuing system is described by the following three elements :—(1) Input or arrival process of the units or "customers". (2) Queue discipline *e.g.*, first come first served, (3) Service mechanism. For each of these there exist several possibilities.

1. *Input process :*

It is usual to describe input process by *probability distribution* of ;

(*i*) time interval between successive arrivals.

(*ii*) No. of units that arrive during such intervals.

As an example, on the average 1 person/minute may arrive in a bank. Whereas, in the banks the persons usually come singly they may come in groups in a restaurant. In the latter situation, arrival event is called a bulk arrival, Often the successive times are statistically independent and also stationary over a long period of time though this is not necessarily so.

Frequently the population of the units or customers requiring service may be infinite, *e g* , passengers waiting across the booking counters ; but there are situations where the population may be limited such as the number of particular equipment breaking down and requiring service from the factory's maintenance crew.

There may be cases where there is a limit to the maximum number of customers queuing up. Or the customer may walk, i.e., deciding not to join the queue which is quite long already. They, may, however, return later.

2. *Queue Discipline* : FIFO, first come first served is usually the case ; but there may be situations *e g.*, a lift where the last person entering is likely to quit first. It may be random when, for example, a teacher picks up the students for recitation or the discipline may be on priority basis as on the basis of "Ladies First". A customer may get impatient and leave the queue, reneging in the terminology of queuing theory.

3. *Service Mechanism* : Analogous to the input process, here are the probability distribution of service times and number of customers served in an interval. Here, again, the customers may be served singly or in groups. The time required for service may depend upon the nature of the customer, but it may also depend upon the state of the system. The server may rush upon seeing the lengthening queue or may slacken when there are a few customers.

Another characteristic of the service mechanism is the number of service channels. There may be just one channel of service, *e g.*, an air-strip. On the other end of scale there may be a single queue but several channels, *e.g.*, when there are several ticket checkers on the railway stations. Passengers may well change over (or jockey) to another queue or ticket checker. This is an example of service channels in parallel : but there could also be channels in series. An example would be vehicles passing through several crossroads or a component that is processed on several machines.

Obviously then there exist a great variety of queuing problems. We shall just deal with an elementary set of queuing formulas to provide an insight into the method of tackling more complex and practical queuing situations. This, however, is a fact that most queuing problems do not lend themselves to analytical treatment, Simulation is often used under such circumstances.

Arrival of Customers : This can be viewed in two different ways :

1. How many customers will arrive in a given time interval (say, 15 minutes) and the associated probability distribution—obviously discrete.

2. What is the time interval between two successive arrivals ? This will give continuous distribution.

The following formulas (proof omitted) would be utilised in solving the numerical problems. λ denotes average arrival rate and μ denotes average service rate, *e.g.*, 6 arrival per minute and 13 units serviced per minute respectively.

1. P_0=Probability of zero units in the queue

$$=1-\frac{\lambda}{\mu}$$

2. P_n=Probability of n units in the queue

$$=\left(\frac{\lambda}{\mu}\right)^n P_0$$

$$=\left(1-\frac{\lambda}{\mu}\right)\left(\frac{\lambda}{\mu}\right)^n$$

3. Average queue length

$$=\frac{\lambda^2}{\mu(\mu-\lambda)}$$

4. Average length of non-empty queues

$$=\frac{\mu}{\mu-\lambda}$$

5. Average number of units in the system (units in the queue as well as the one being served).

$$=\frac{\lambda}{\mu-\lambda}$$

6. Average waiting time of an arrival

$$=\frac{\lambda}{\mu(\mu-\lambda)}$$

7. Average waiting time of an arrival who waits

$$=\frac{1}{\mu-\lambda}$$

Formulas 3 to 5 pertain to lengths and 6 to 8 pertain to times of queues

8. Average time an arrival spends in the system

$$=\frac{1}{\mu-\lambda}$$

Note : These formulas have been derived on the assumption that probability distribution of inter-arrival and servicetimes are both exponential and FIFO is the queue discipline.

An explanatory Note on the Queuing Formulae

The ratio λ/μ is called the traffic intensity or the utilisation parameter and it is the measure of the degree to which the capacity of the service station is utilised. For example, if customers arrive at a rate of 18 per minute and the service rate is 20 per minute the utilisation of the service facility is $\frac{18}{20}=90\%$. Thus if the service facility is kept 90% of the time busy what is the probability that the service station remains idle ? Clearly this probability symbolised to $P_0=1-\lambda/\mu=0.1$.

As P_0 is defined, it must also be *probability that a customer*, upon the random arrival, *will not have to wait* at all for service. This definition also suggests that the *probability of having* one customer in the waiting line (excluding the one being served) is $P_1=\frac{\lambda}{\mu}P_0$

$$P_2=\frac{\lambda}{\mu}P_1=\left(\frac{\lambda}{\mu}\right)^2\left(1-\frac{\lambda}{\mu}\right)$$

$$P_n=\left(\frac{\lambda}{\mu}\right)^n\left(1-\frac{\lambda}{\mu}\right) \qquad (i)$$

Now the 3 formulae for the queue length and the 3 for waiting time given above have been derived by making use of (i). We shall endeavour to explain what each of these 6 formulae stand for.

3. *Average length of waiting line* $\frac{\lambda^2}{\mu(\mu-\lambda)}$

This stands for the queue length from time to time, the unit being served having to be excluded. For example, consider that a statistician observes the queue against a service facility after every 1 hour and the queue for the 6 observations are as below :

*stands for a unit or customer and —— stand for the service facility.

Observation No.	*QUEUE*	*Service Facility*	*length*
1	* * * * *	*	5
2	* * *	*	3
3	* *	*	2
4		*	0
5	* *	*	2
6		None	0
		Average	12/6=2

The meaning of the aforesaid average queue length=2 should now be clear.

4. *Average length of non-empty queues* $=\dfrac{\mu}{\mu-\lambda}$.

This average would be computed from observations 1, 2, 3 and 5 only since upon observations 4 and 6 the queue was empty (even though upon observation 4 there was a unit being served). For the above situation, then, this average.

$$=(5+3+2+2)/4=3$$

5. *Average no. of units in the system* $=\dfrac{\lambda}{\mu-\lambda}$.

For the above situation, this average

$$=\frac{(5+1)+(3+1)+(2+1)+(0+1)+(2+1)+(0+0)}{6}=\frac{17}{6}$$

6. *Average waiting time of an arrival* $=\dfrac{\lambda}{\mu(\mu-\lambda)}$

Some arrivals will have to wait, others would not have to wait when there is no unit in the system. Assume that the waiting times of 10 units observed were as below :

15, 10, 2, 0, 0, 5, 0, 20, 8, 0

This average, then, is the sum of all these divided by $10 = 60/10 = 6$.

7. *Average waiting time of an arrival who waits* $= \dfrac{1}{\mu - \lambda}$.

In the above 10 times we would ignore zeros for computing this average $(= 60/6 = 10)$.

8. *Average time an arrival spends in the system* $= \dfrac{1}{\mu - \lambda}$.

The above 10 times would not apply here. Now we are interested for each units, waiting time+servicing time and this average would be given by

Σ (waiting time+servicing time)/N, where

N is the number of observations. Here even if waiting time is zero we would not ignore it in computation of this average time.

Solved Examples on Queuing

Example 1

A TV repairman finds that the time spent on his job has an exponential distribution with mean 30 minutes. If he repairs sets in the order in which they come in, and if the arrival of sets is approximately. Poisson with an average rate of 10 per 8-hour day, what is the repairman's expected idle time each day ? How many jobs are ahead of the average set just brought in ?

Solution :

$$\mu = \frac{60}{30} = 2 \text{ service/hour}$$

$$\lambda = \frac{10}{8} \text{arrival/hour}$$

$$P_0 = 1 - \frac{\lambda}{\mu}$$

is the probability of the repairman being idle.

$$P_0 = 1 - \frac{\frac{10}{8}}{2} = \frac{3}{8}$$

Hence the idle time in the 8-hour day.

$$= \frac{3}{8} \times 8 = 3 \text{ hours}$$

Note : If the arrivals in the unit time are Poisson distributed it can be proved mathematically that the inter arrival times follow exponential distribution so that we can make use of formulas given above.

Average no. in the system

$$= \frac{\lambda}{\mu - \lambda}$$

$$= \frac{\frac{10}{8}}{2-\frac{10}{8}} = 1\frac{2}{3} \text{ (Answer)}$$

Example 2

Customers arrive at a box office window, being, manned by a single individual according to a poisson input process with a mean rate of 30 per hour. The time required to serve a customer has an exponential distribution with a mean of 90 seconds. Find the average waiting time of customer.

Solution :

$$\lambda = 30 \text{ per hour}$$

$$\mu = \frac{60 \times 60}{90} = 40 \text{ per hour.}$$

Average waiting time of a customer

$$= \frac{\lambda}{\mu(\mu-\lambda)} = \frac{30}{40(40-30)}$$

$$= \frac{3}{40} \text{ hrs.}$$

$$= 4.5 \text{ min. (Answer)}$$

Example 3

Arrivals of mechanists at a tool crib are considered to be Poisson distributed at an average rate of 6 per hour. The length of time the mechinists must remain at the tool crib is exponentially distributed with the average time being 0 05 hours.

(a) What is the probability that a mechinist arriving at the tool crib will have to wait ?

(b) What is the average no. of mechinists at the tool crib ?

(c) The company will install a second tool crib when convinced that a machinist would have to spend 6 minutes awaiting and being served at the tool crib. By how much time the flow of machinists to the tool crib should increase to justify the addition of a second tool crib ?

Solution :

$$\lambda = 6 \text{ per hr.}$$

$$\mu = \frac{1}{0.05} = 20 \text{ per hr.}$$

(a) $$1 - P_0 = \frac{\lambda}{\mu} = \frac{6}{20}$$

$$= 0.3 \text{ (Answer)}$$

(b) Average no. of mechinists at the tool crib.

$$=\frac{\lambda}{\mu-\lambda}=\frac{6}{20-6}=0.43$$

(c) $\lambda'=?$

Average time an arrival spends in the system

$$=\frac{6}{60}=\frac{1}{\mu-\lambda'}$$

$\lambda'=10$ per hour.

Example 4

Customers arrive at a window drive in a bank according to poissons distribution with mean 10 per hour. Service time per customer is exponential with mean 5 minutes. The space in front of the window including that for the serviced car, can accommodate a maximum of three cars. Other cars can wait outside this space.

(a) What is the probability that an arriving customer car drives directly to the space in front of the window ?

(b) What is the probability that an arriving customer will have to wait outside the indicated space ?

(c) How long is an arriving customer expected to wait before starting service ?

Solution :

$$\lambda=10 \text{ per hour}$$

$$\mu=\frac{60}{5}=12 \text{ per hour}$$

(a) $$P_0+P_1+P_2=\left(1-\frac{\lambda}{\mu}\right)+\frac{\lambda}{\mu}\left(1-\frac{\lambda}{\mu}\right)+\left(\frac{\lambda}{\mu}\right)^2\left(1-\frac{\lambda}{\mu}\right)$$

$$=\left(1-\frac{\lambda}{\mu}\right)\left\{1+\frac{\lambda}{\mu}+\left(\frac{\lambda}{\mu}\right)^2\right\}$$

$$=\left(1-\frac{10}{12}\right)\left\{1+\frac{10}{12}+\left(\frac{10}{12}\right)^2\right\}$$

≈ 0.42 **(Answer)**

(b) $1-P_0-P_1-P_2-P_3$

$$=1-0.42=0.58-P_3$$

Now $P_3 = \left(\frac{\lambda}{\mu}\right)^3\left(1-\frac{\lambda}{\mu}\right) = \left(\frac{10}{12}\right)^3 \frac{2}{12}$

≈ 0.10

$\therefore \quad 1-P_0-P_1-P_2-P_3=0.58-0.10$

$=0.48$ **(Answer)**

(c) Expected waiting time of a customer

$= \frac{\lambda}{\mu(\mu-\lambda)} = \frac{10}{12(12-10)} = 0.417$ **(Answer)**

Example 5

A repairman is to be hired to repair machines which break down at an average rate of 3 per hour. The breakdown follow Poisson distribution. Non productive time of a machine is considered to cost Rs. 10 per hour. Two repairmen have been interviewed—one is slow but cheap, while the other is fast but expensive. The slow repairman charges Rs. 5 per hour and he services breakdown machines at the rate 4 per hour. The fast repairman demands Rs. 7 per hour, and he services at an average rate of 6 per hour. Which repairman should be hired ?

Solution :

$\lambda=3$

Cost of idle machine hour

=Rs. 10.

Slow Repairman :

$\mu=4.$

Hourly charges =Rs. 5

Average no. of units in the *system*

$\frac{\lambda}{\mu-\lambda} = \frac{3}{4-3} = 3.$

Machine hours lost in a 8-hour shift

$=3\times 8=24$ machine hours.

Total cost=hire charges of repairman+cost of idle machines.

$=5\times 8+24\times 10=$Rs. 280.

Fast repairman :

$\mu=6.$

Hourly Charges=Rs. 7.

Average number of units in the system

$$=\frac{\lambda}{\mu-\lambda}=\frac{3}{6-3}=1$$

Machine hours lost in 8-hour shift. $=1\times 8=8$.

Total Cost=Hire charges+Cost of idle machines

$$=7\times 8+10\times 8=136.$$

Obviously, then, the fast repairman should be engaged.

Example 6

Western National Bank is considering opening a drive-in window for customer service. Management estimates that customers will arrive for service at the rate of 15 per hour. The teller whom it is considering to staff the window can service customers at the rate of one every three minutes.

Assuming Poisson arrivals and exponential service, find

1. Utilisation of teller
2. Average number in the waiting line
3. Average number in the system
4. Average waiting time in line
5. Average waiting time in the system.

Solution :

The average utilisation of the teller

$$=\rho=\frac{\lambda}{\mu}=\frac{15}{20}=75\%$$

The average number in the waiting line

$$=\frac{\lambda^2}{\mu(\mu-\lambda)}=\frac{15^2}{20(20-15)}=2.25 \text{ customers}$$

The average number in the system is

$$\frac{\lambda}{\mu-\lambda}=\frac{15}{20-15}=3 \text{ customers}$$

Average waiting time in the line is

$$\frac{\lambda}{\mu(\mu-\lambda)}=\frac{15}{20(20-15)}=0.15 \text{ hours.}$$

Average waiting time in the system is

$$\frac{1}{\mu-\lambda}=\frac{1}{20-15}=0.20 \text{ hours.}$$

Example 7

Arrivals at a telephone booth are considered to be Poisson with an average time of 10 minutes between one arrival and the next. The length of a phone call is assumed to be distributed exponentially, with mean 3 minutes.

(a) What is the probability that a person arriving at the booth will have to wait ?

(b) What is the average length of the queues that form time to time ?

(c) The telephone company will install a second booth when convinced that an arrival would expect to have to wait at least three minutes for the phone. By how much the flow of arrivals be increased in order to justify a second booth ?

Solution :

(a) $\lambda=\frac{1}{10}=0.1$ arrivals per minute.

$\mu=\frac{1}{3}=0.33$ units per minute.

Probability of no (i.e. zero) unit in the queue

$$=P_0=1-\frac{\lambda}{\mu}$$

Probability of one or more units in the queque

$$=1-P_0=\frac{\lambda}{\mu}=\frac{0.1}{0.33}=0.3 \text{ (Answer)}$$

(b) Average length of the queue that form from time to time *i.e.* average length of non-empty queues

$$=\frac{\mu}{\mu-\lambda}=\frac{0.33}{0.33-0.10}=1.43 \text{ (Answer)}$$

(c) Average waiting time of an arrival

$$=\frac{\lambda'}{\mu(\mu-\lambda')}=3.$$

Now find λ'.

Exercise 1

Data have been accumulated at a banking facility regarding the waiting time for delivery trucks to be loaded. The data show that the average arrival rate for trucks at the loading dock is 2 per hour. The average time to load a truck, using 2 loaders, is 10 minutes, so the service rate is 3 trucks per hour.

Find (i) the expected number of trucks in the system,

(ii) the expected number of trucks waiting to be served,

(iii) the expected time that a truck is in the system (including waiting and loading),

(iv) the expected time in the waiting line,

(v) the probability that a truck has to wait for service ; and

(vi) the probability of no units in the system.

Answer : (i) 2 (ii) 1.33 (iii) 1 hr

(iv) 0.667 hr (v) 0.667

(vi) 0.333

Exercise 2

(In continuation to exercise 1) the management is considering hiring another loader at Rs 5 hour to reduce the loading time. Drivers are paid Rs. 4 per hour, and truck utilisation is valued at Rs. 3 per hour. Should the additional loader be hired if an increase in the service rate to 4 trucks per hour would result ?

Answer : Rs. 2 per hr. saving with new loader.

PART—II

STATISTICAL EVALUATION OF PROGRAMMES & PROJECT

In this Part the topic of strategic decisions has been explained mainly by means of numerical examples.

After a few solved example on the *maximin, maximax* and *expected pay off* criteria to refresh the student's knowledge of statistics and build a foundation for further material, this part deals with the empirical and binomial distributions of the "states of natures" facing a decision-maker for choosing the appropriate strategy. There are, therefore, several numerical examples dealing with the expected value of perfect information which is a maximum limit to the costs that a rational decision-maker would be willing to incur on sampling enquiry. The problems are at once of practical nature which usually obtain in business situations. The student, however, should bear in mind that the alternatives suggested by the statistical analysis may be set aside by the decision-maker on extraneous considerations. For example, the decision-maker may also be concerned with the psychological effects of the various strategies.

Introduction. In business situations the decisions have usually to be made under uncertainty ; a choice of action must be made even though its outcome is unknown and determined by chance. The presence of uncertainty and risk makes the decision statistical.

During his Inter-Stat (in Study VIII) the student was familiarised with the statistical decision theory in an elementary manner.

We shall recapitulate that material by the following few examples for proceeding with more practical problems.

Example 1. An independent bicycle repairman has an opportunity to purchase a stock of discontinued bicycles. They were originally supposed to be sold for Rs. 200 each. The repairman is offered all five for Rs. 300 which makes his cost for each bicycle Rs. 60. If he sells them he believes he can get Rs. 150 for each bicycle thereby making Rs. 90.

There are two strategies (1) either to buy all the discontinued bicycles or (2) not to buy them. There are six states of nature ; state of nature representing the consequences of factors beyond the control of the decision-maker. These being the demand for 0, 1, 2, 3, 4, 5, bicycles. The pay-off table is constructed below :—

State of Nature i e.	Strategy	
Demand	1 (Not buy)	II (Buy)
θ	a_1	a_2
0	0	−300
1	0	−150
2	0	0
3	0	·150
4	0	300
5	0	450

The pay-offs for the two strategies having been obtained from the following pay-off equations :

$$Qa_1 = a_1 + b_1 .\ \theta = 0$$
$$Qa_2 = a_2 + b_2 .\ \theta = -300 + 150\ \theta$$

$\begin{cases} a = \text{Initial investment} \\ b = \text{contribution/unit} \end{cases}$

These are the profit pay-offs.

The opportunity loss or regret table can be obtained for the above matrix by the rule ;—

"*For each state, find the pay-off of the optimum strategy. Subtract it from pay-off in that row. Retain the absolute volume of the difference as the regret or opportunity loss for having adopted that particular strategy*".

Regret Table

	a_1	a_2
0	0	300
1	0	150
2	0	0
3	150	0
4	300	0
5	450	0

There are two measures of pay-off either in terms of profit of the opportunity

loss associated with each state end strategy.

The problem may be extended. Let us say the repairman has the option of buying any number of the discontinued bicycles. Now he has six strategies and six states of nature. The pay-off table is constructed below :

θ	a_1	a_2	a_3	a_4	a_5	a_6
0	→0	−60	—120	—180	—240←	—300←
1	0	90	30	—30	—90	—150
2	0	90	180	120	60	0
3	0	90	180	270	210	150
4	0	90	180	270	360	300
5*	0*	90*	180*	270*	360*	*450
Average	0	65	105	120	110	75

The student is familier with the maximin, maximax criteria

Maximin is the pessimistic strategy. In the pay-off table is indicated an arrow against the minimum pay-off of each strategy. The maximin of these minimums is 0 under strategy a_1 which, therefore, is the maximin or the pessimist's strategy. The decision-maker, with the knowledge of this-pay-off table, thinks pessimistically that the demand would be at its worst that is 0 and he goes in for a_1 that ensures no loss, with this thinking.

Maximax is the optimistic strategy. In the pay-off table above is indicated a asterisk against the maximum pay-off of each strategy. The maximum of these maximums is 450 under strategy a_6 which, therefore, is the maximax or the optimists' strategy. The decision-maker, with the knowledge of this pay-off table thinks optimistically that the demand would be at best that is 5 and he goes in for a_6 which ensures highest pay-off with this thinking.

A firm in a miserable financial position would tend to adopt maximin strategy to play safe as it ensures the least loss. If, for example, it goes in for a_5 it may, with some probability earn a pay-off of 360 ; but there is also some probability that it ends up with a pay-off—240 which may ruin it. Likewise, a firm in a happy financial position would adopt a_6 with an eye on a handsome pay-off of 450 although it can with some probability suffer a loss of 300 which it is in a position to absorb.

The above table is converted into the opportunity loss or the regret table below :

	a_1	a_2	a_3	a_4	a_5	a_6
0	0	60	120	180	240	300
1	90	0	60	120	180	240
2	180	90	0	60	120	180
3	270	180	0	0	60	120
4	360	270	180	90	0	60
5	450	360	270	180	90	0
Expected Regret	225	160	120	105	115	150

For this case expected regret of strategy a_4 is minimum. This strategy may be adopted. This assumes that each state has the same prob$=\frac{1}{6}$

Example 2. Given the following pay-off function for each act a_1 and a_2

$$Qa_1=-30+50x$$

$$Qa_2=-90+20x$$

(*i*) What is the break even value of x ?

(*ii*) If $x=10$, which is the better act ?

(*iii*) If $x=10$, what is the regret of the poor strategy ?

(*iv*) If $x=-5$ which is the better act ?

(*v*) If $x=-5$ what is the regret of the poorer strategy ?

Solution : Equating Qa_1 and Qa_2

$$-30+50x=-90+20x$$

or $$x=-2$$

$\therefore$ -2 is the break even point.

(*ii*) Substituting $x=10$ in Qa_1 and Qa_2

$Qa_1=470$ $Qa_2=110$

Hence a_1 is the better strategy.

(*iii*) Regret with $x=10$ is $470-110=360$

This happens when strategy a_2 is adopted.

(*iv*) for $x=-5$ $Qa_1=-280$

$Qa_2=-190$

a_2 is the better strategy.

(*v*) Regret in (*iv*)$=90=(+280-190)$

Probabilities in Decision-making : The *rational criteria considered above could be taken as a stepping stone for this chapter. Equal weightages or probabilities were assigned to each of the state to derive expected pay-offs under different strategies. As a matter of fact, it can be generalised that even Maximax and Maximin criteria also involve probabilities insofar as probability of 1 is assigned to one state and zero to the other states.

Obviously decision-making could be improved if the probability distribution of all the state were known.

The associated probability with each state may either be guess-estimated subjectively, derived empirically from past records or on the basis of a sample result. Empirically derived probability distribution is called a *priori* distribution. Subjective

*Expected pay-off or regret is known as the rational criterion and is generally used when probabilities of states of nature are known.

prior distributions guess estimates are not to be discarded as useless and unscientific, Businessmen usually develop sound intuition of the various possibilities of a particular state occurring by taking into account a large number of factors.

Newsboy's Problem :

Example 3. Consider a newsboy's problem who buys newspapers every day in bulk. The surplus, at the end of the day, has to be disposed of at a very cheap rate. Also, in the case of his purchases falling short of the day's demand would result into regret in the sense discussed above newspaper costs him 5 N.P. The newsboy, aided by a statistician, keeps records of actual papers sold for the last 50 days tabulated below :

No. of papers demand	14	15	16	17	18	19	20
No. of Days of frequency	4	11	10	7	7	6	5
Probability	0.8	0.22	0.20	0.14	0.14	0.12	0.10

It is assumed that the 50 days' statistics adequately describe the probabilities. Newsboy's problem is to find the optimum number of papers to buy each day. He sells a newspaper for 10 N.P.

Solution : The pay-off function may be described as follows :—

$Qa_i = 5a_i$ N.P. for $a_i \leqslant \theta_j$

$Qa_i = \{10\theta_j - 5a_i\}$ N.P. for $a_i > \theta_j$

a_i here represents the strategies available to the newsboy, *i.e.*, a_1 might represent purchase of 14, a_2 the purchase of 15 and on.

Similarly θ_j represents the j states *i.e.*, θ_1 might represent the demand for 14 papers, θ_2 for 15 papers and so on.

Pay-off table is constructed below, together with the empirical probability distribution of θ which has been derived from records of actual sales during the last 50 days.

The pay-off table is followed by a Regret Table by the conversion rule stated earlier.

Pay-off Table

States	Prob	a_1	a_2	a_3	a_4	a_5	a_6	a_7
θ	P	14	15	16	17	18	19	20
$\theta_1=14$	0.08	70	65	60	55	50	45	40
$\theta_2=15$	0.22	70	75	70	65	60	55	50
$\theta_3=16$	0.20	70	75	80	75	70	65	60
$\theta_4=17$	0.14	70	75	80	85	80	75	70
$\theta_5=18$	0.14	70	75	80	85	90	85	80
$\theta_6=19$	0.12	70	75	80	85	90	95	90
$\theta_7=20$	0.10	70	75	80	85	90	95	100
		70	74.2	76.2	76.2	74.8	72	68

The bottom row gives the expected pay-off against each strategy :

$$E\left\{(Qa_3)\right\} = 60\times 0.08 + 70\times 0\;22 + 80\times 0.20 + 80\times 0.14 + 80\times 0.12 + 80\times 0.10 = 76.2$$

Thus 16 or 17 is the optimal number with the highest expected pay-off of 76.2 each.

The regret function for the problem would be

$$Ra_i = 5(a_i - \theta_j) \text{ N.P.} \qquad \text{for } a_i > \theta_j$$
$$= 5(\theta_j - a_i) \text{ N.P.} \qquad \text{for } a_i \leqslant \theta_j$$

Regret Table

		a_1	a_2	a_3	a_4	a_5	a_6	a_7
θ	P(θ)	14	15	16	17	18	19	20
14	0.08	0	5	10	15	20	25	30
15	0.22	5	0	5	10	15	20	25
16	0.20	10	5	0	5	10	15	20
17	0.14	15	10	5	0	5	10	5
18	0.14	20	15	10	5	0	5	10
19	0.12	25	20	15	10	5	0	5
20	0.10	30	25	20	15	10	5	0
	E(R)	14	9.8	7.8	7.8	9.2	12.2	16 0

Regret is obviously zero when $\theta_j = a_i$, when $a_i > \theta_j$ the newsboy will regret over the left overs and when $a_i < \theta_j$ he would regret for being out of stock when there was demand for more newspapers.

The expected regret is least for 16 or 17 newspapers ordered and is equal to 7.8. (Also see p. 75 for alternative method).

Example 4. An investor was considering stock purchases in two companies. He determined the following pay-off table contingent upon whether Company A or B wins a competition for share of the market.

	Strategies	
State	a_1 (Invest in A)	a_2 (Invest in B)
θ_1 : A wins out	30	0
θ_2 : B wins out	40	80

(*a*) What is the maximin strategy ? the maximax strategy ?

(*b*) At what probability of A's winning out is the expected value of the two strategies the same ?

Solution :

(a) Maximin Strategy ; Invest in A with pay-off of 30.

Maximax Strategy : Invest in B with a pay-off of 80.

(b) Let p the required probability.

$$30p+40(1-p)=0p+80(1-p)$$

or $70p=40$

i.e., $p=4/7$.

Expected value of Perfect Information (E.V.P I) : Assume for a moment that the newspaper boy in the regret table on page 66 knows with certainty what the demand is going to be. He would naturally choose the strategy that has zero regret against this known demand or state of nature. If he is faced with such a situation over and again he would always choose the strategy having zero regret provided, of course, he always knows beforehand what the demand is going to be each time. With such perfect information his regret in the long run would be zero. Without this perfect information he would always go in for the strategy with optimal regret. In other words, the expected regret of the optimal policy is the E V.P.I. Now, how the newspaper boy can get perfect information ? He can nndertake a sample survey and ascertain the magnitude of demand. The larger the sample size the more the precision. But the maximum he can expend on sampling is the EVPI(=7.8 in this case). More than this should he never spend in buying perfect information *i.e.*, sampling etc.

Example 5. A company has an opportunity to computerize its records department. However, existing personnel have job security under a union agreement. The cost of the three alternative programmes for the changeover depend upon the attitude of the union and are estimated below :

Attitude of Union	General Retraining a_1	Selective Retraining a_2	Hire new employees a_3
θ_1, Antagonist	940*	920*	*900
θ_2, Passive	810	800	820←
θ_3, Enthusiastic→	700	710←	860

(*a*) What is the minimax strategy ? the minimin strategy ?

(*b*) Construct the opportunity loss table.

(*c*) The probabilities of states are expressed at 0.5, 0.3 and 0.2 respectively. Find the expected cost of each act.

(*d*) Given the above probabilities, what is the expected value of perfect information ? (Expected value of Perfect Information—Regret of optimal strategy.

Solution : The table of this problem gives cost and *not* pay-off.

Therefore minimax and minimin criteria are applicable here and they correspond to maximax and maximin criteria in pay-off tables explained earlier.

(a) Minimax Strategy, a_3 pay-off=900

Minimum Strategy, a_1 pay-off=700

(b) Opportunity Loss or Regret Table

		a_1	a_2	a_3
θ_1	P(0)=0.5	40	20	0
θ_2	0.3	10	0	20
θ_3	0 2	10	10	160

(c) Expected cost of a_1

$940 \times 0.5 + 810 \times 0.3 + 700 \times 0\ 2 = 853$

Expected cost of a_2

$920 \times 0.5 + 800 \times 0.3 + 710 \times 0.2 = 842.$

Expected cost of a_3

$900 \times 0.5 + 820 \times 0.3 + 860 \times 0.2$

$= 450 + 246 + 172 = 868.$

Expected opportunity loss of a_1

$= 40 \times 0.5 + 10 \times 0\ 3 + 0 = 23.$

Expected opportunity loss of a_2

$= 20 \times 0.5 + 0 + 10 \times 0.2 = 12$ (is also expected value of perfect information).

Expected opportunity loss of a_3

$= 0 + 20 \times 0.3 + 160 \times 0.2 = 38.$

Example 6. A TV dealer finds that the cost of a TV in stock for a week is Rs. 20 and the cost of a unit shortage is Rs. 50. For one particular model of TV the probability distribution of weekly sales is as follows :

Weekly Sales	0	1	2	3	4	5	6
Probability	0 10	0.10	0 20	0.20	0.20	0.15	0.05

How many units per week should the dealer order ? Also, find EVPI.

Solution : The cost matrix is constructed below. Also derived from it are the expected costs for the various strategies ;

Demand	prob.	Strategies (Buy so many) 0	1	2	3	4	5	6
0	0.10	0	20	40	60	80	100	120
1	0.10	50	20	40	60	80	100	120
2	0.20	100	70	40	60	80	100	120
3	0.20	150	120	90	60	80	100	120
4	0.20	200	170	140	110	80	100	120
5	0.15	250	220	190	160	130	100	120
6	0.05	300	270	240	210	180	150	120
Expected		147.5	122.5	102.5	92.5	92.5	102.5	120

Thus either buy 3 or 4.

ALTERNATIVELY

(Implying profit of Rs. 50 per TV for the shortage cost of Rs. 50)

The pay-off matrix is compiled below.

Demand	Prob.	Strategies (Buy so many) 0	1	2	3	4	5	6
0	0.10	0	−20	−40	−60	−80	−100	−120
1	0.10	0	30	10	−10	−30	−50	−70
2	0.20	0	30	60	40	20	0	−20
3	0.20	0	30	60	90	70	50	30
4	0.20	0	30	60	90	120	100	80
5	0.15	0	30	60	90	120	150	130
6	0.05	0	30	60	90	120	150	180
Ex.		0	25	45	55	55	45	27.5

Thus buy 3 or 4.

EVPI (from the cost matrix).

Demand Col. (1)	Prob. (2)	Min. cost (in cost matrix row) (3)	Cost under certainty (4)=(2)×(3)
0	0.10	0	0
1	0.10	20	2
2	0.20	40	8
3	0.20	60	12
4	0.20	80	16
5	0.15	100	15
6	0.05	120	6
			59

Thus EVPI$=92.5-59=33.5$ **(Answer)**

EVPI (from the Pay-off Table)

Demand	Prob.	Max. Pay-off under certainty (from Pay-off matrix row)	
Col. : (1)	(2)	(3)	(4)=(2)×(3)
0	0.10	0	0
1	0.10	30	3
2	0.20	60	12
3	0.20	90	18
4	0.20	120	24
5	0.15	150	22.5
6	0.05	180	9
			88.5

Thus EVPI$=88.5-55=33.5$ (it tallies).

Example 7. Following are the records of demand of an item for the past 300 days.

θ=Demand in units	No. of days	Prob. (θ)
10,000	18	0.06
11,000	90	0.30
12,000	120	0.40
13,000	60	0.20
14,000	12	0.04
	300	1.00

(*i*) What is the expected demand ?

(*ii*) It costs Rs. 15 to make an item which sells for Rs. 20 normally but at the end of the day any surplus has to be disposed at Rs. 10 per item What is the optimum output ?

Solution : Expected demand $=10\times0.06=0.60$

$=11\times0.30=3.30$

$=12\times0.40=4.80$

$=13\times0.20=2.60$

$=14\times0.04=0.56$

11.86

Expected demand=11,860 (**Answer**)

The daily output profit matrix is constructed below :

P(θ)		10	11	12	13	14
0.06	10	50	45	40	35	30
0 30	11	50	55	50	45	40
0.40	12	50	55	60	55	50
0.20	13	50	55	60	65	60
0.04	14	50	55	60	65	70
Expected		50	54.4	55.8	53.2	48.6

Demand of 12,000 is the optimum with an expected pay-off of $55.8 \times 1000 =$ 55,800 rupees.

Example 8. The sales manager of Beta Co. is highly experienced in the fad market. He is sure that the sales of JUMBO (during the period it has special appeal) will not be less than 25,000 units. Plant capacity limits total production to a maximum of 80,000 units during JUMBO's brief life. According to the sales manager, there are 2 chances in 5 for a sales volume of 50,000 units. The probability that it will be more than 50,000 is four times the probability that it will be less than 50,000. If sales exceed 50,000 units volumes of 60,000 and 80,000 are equally likely. A 70,000 unit volume is 4 times as likely as either. It costs Rs. 30 to produce a unit of JUMBO whereas its selling price is estimated at Rs. 50 per unit. Initial investment is estimated at Rs. 8,00,000. Should the venture of production be undertaken.

Solution :

Prob. $(50{,}000) = \frac{2}{5} = 0\ 40$

Prob. (less than or more than 50,000)=0.60

Prob. (less than 50,000) : Prob. (More than 50,000) : : 1 : 4

Thus Prob (less than 50,000)$=0.60 \times \frac{1}{1+4} = 0.12$

Prob (More than 50,000)$=0.60 \times \frac{4}{1+4} = 0.48$

Prob (60,000) : Prob (70,000) : Prob (80,000) : : 1 : 4 : 1

Thus Prob (60,000)$=0.48 \times \frac{1}{1+4+1} = 0\ 08$

∴ Prob (80,000)=0.08

and Prob 70,000$= \frac{4}{1+4+1} \times 0.48 = 0.32$

Now we have complete probability distribution of sales except that for sales less than 50,000 we have a summarised probability of 0.12. The payoff matrix is compiled below. We used 25,000 in place of less than 50,000 as the worst contingency.

Demand	Prob.	< 50,000 (take it 25,000)	50,000	60,000	70,000	80,000
>50,000 (worst 25,000	0.12	500,000	−25,000	−55,000	−85,000	−115,000
50,000	0.40	500,000	1,000,000	70,000	40,000	10,000
60,000	0.08	500,000	1,000,000	120,000	90,000	60,000
70,000	0.32	500,000	1,000,000	120,000	140,000	110,000
80,000	0.08	500,000	1,000,000	120,000	140,000	160,000
Expected Pay off		500,000	850,000 ↑	79,000		

This exceeds 800,000 the initial investment; therefore, the venture ought to be undertaken.

Exercises

1. Assume a merchant dealing in high fashion apparel must decide how many fur coats to rush order. Since the demand for the coats is caused by a fad, the merchant believes he will only be able to place one order : and if the coats are not sold within several weeks they will have no market value. The coat costs Rs. 400 and sells for Rs. 800, and the estimated demand function is ;

Demand	Probability of demand
3	0.1
4	0.2
5	0.3
6	0.25
7	0.10
8	0 05
	1.00

(*a*) Construct the pay-off matrix ann find the optimal order size for the coats **(Answer 5)**

(*b*) Also construct the opportunity loss table and verify solution to part (*a*). What is the EVPI as read from the opportunity loss table. **(Answer : Rs. 400)**

2. **Indicate which of the following acts are inadmissible and assign reasons for the same.**

Events	*Acts*			
	A_1	A_2	A_3	A_4
E_1	100	101	50	25
E_2	80	30	20	10
E_3	20	19	40	–5

Answer. A_1 is inadmissible because A_2 *has higher maximum as well as the* minimum Pay-off.

A_1 is not inadmissible because of a higher minimum pay-off compared with A_2. Although it has equal minimum pay-off compared with A_3 the former is preferable because of a higher maximum pay-off compared with A_3

3. Decide in following situations :

Events	*Acts*	
	A_1	A_2
E_1	1	–2
E_2	–1	100

(*i*) Which act gives the maximum pay-off ?

(*ii*) What is the expected pay-off if the event probability of Event E_1 is. 99.

(*iii*) Which act has maximum pay-off if the event probabilities are equal.

[**Answer.** (*i*) A_1, (*ii*) A_1=98 and A_2=–98, (*iii*) choose A_2 with 49.]

4. A farmer wants to decide about the crop to be raised on his 100 hectare farm. The following are the yields of crops depending on weather condition :

Yields (quintal hectare)

Weather condition	*Wheat*	*Barley*	*Gram*
Dry	50	60	40
Moderate	70	80	80
Damp	120	40	60
Price (Rs) per quintal	120	110	165

You have to indicate :

(*a*) Pay-off from each crop in terms of the cash receipt and indicate the act with maximum pay-off.

(*b*) The expected pay-off for each crop and identify the crop which has maximum expected pay-off if probability of different weather conditions are :

Dry	.3
Moderate	.5
Damp	.2

5. Indicate minimax opportunity loss act and calculate the lowest expected opportunity loss from the following opportunity loss table :

	Probability	Acts A_1	A_2	A_3
E_1	.4	10	0	20
E_2	.5	0	30	40
E_3	.1	50	5	0

[**Answer.** Minimax opportunity loss act is A_2. The lowest opportunity loss of 9 is in A_1]

6. A milkman buys milk at Rs. 2 per litre and sells for Rs. 2.50 per litre. Unsold milk has to be thrown away. The daily demand has the following probabilities.

Demand (Litres) :	46	48	50	52	54	56	58	60	62	64
Probability :	.01	.03	.06	.10	.20	.25	.15	.10	.05	.06

If each day's demand is independent of previous day's demand, how much litres should be ordered every day.

1. A university association is considering inviting a troupe for dramatic performance. Tickets would be sold only to students for Rs. 3 per ticket. The fee for hiring the troupe is Rs. 3600. In order to estimate attendance the association studied records for similar events and compiled the following rate :

Attendance	Relative frequency
750	0.04
900	0.06
1050	0.20
1200	0.40
1350	0.22
1500	0.08

The total student body is of 3,000, members. Find the expected regret.

(**Answer : 171**)

An alternative appposch to solve the Newsboy's problem.

Let P (n) be the probability for demand for n units or more.

Let MP be the profit per unit sold and ML be the loss from each unit that is unsold.

Expected profit from the sale of the nth unit=E(n)=MP. P(n)—ML (1—P(n) where 1—P(n) is the probability of not selling the n—th unit.

E(n) ought to be greater than zero if the n—th unit is to fetch a profit.

Thus $\quad E(n) \geqslant 0$

$\Rightarrow \quad MP.P(n)-ML(1-P(n)) > 0$

$$\Rightarrow \quad P(n) = \frac{ML}{MP+ML}$$

Let us reconsider example 3 by this approach.

θ	Prob.	Cum. Prob.
14	0.08	0.08
15	0.22	0.30
16	0.20	0.50
17	0.14	0 64
18	0.14	0.78
19	0.12	0.90
20	0.10	1.00

ML=5 NP

MP=10−5=5 NP

$$\text{Thus } P(n) = \frac{5}{5+5} = 0.50$$

(Since P (n) is immediately less or equal to cum. Prob. against θ=16 and 17 answer is 16 or 17.

APPENDIX I

Exponential Smoothing

Exponential smoothing refers to a family of forecasting models. The exponential smoothing model estimates average smoothed demand for the upcoming period F_t by adding or subtracting a fraction of the difference between actual current demand and the last smoothed average F_{t-1}. The new smoothed average F_t is then given by

New smoothed average = old smoothed average+ α (new demand—old smoothed average);

When symbolized, it takes the form

$$F_t = F_{t-1} + \alpha (D_t - F_{t-1}) \quad ...(1)$$

or

$$F_t = \alpha D_t + (1-\alpha) F_{t-1} \quad ...(2)$$

D_t = actual current value of variable being forecast in period t.

and α = smoothing constant.

The value of α lies between 0 and 1. It determines the degree of smoothing that takes place and indicates how responsive the model is to fluctuations in the forecast variable. The setting of α is usually done by trial and error and is not a scientific method. The commonly used values of α are between 0.01 to 0.30.

It is evident from the above discussion that the major advantage is that it is *not necessary* to store all the historical data for the forecasting model and compute average each time. Instead, exponential smoothing requires only the previous forecast and the actual value of the previous period.

To illustrate, Govind Ram & Sons Company uses exponential smoothing to forecast energy demand on a monthly basis. Experience has shown that the appropriate constant to use is $\alpha=0.10$.

If $D_t=110$ and $F_{t-1}=100$, then the new smoothed average is

$$F_t=0.1\times 110+0.9\times 100=11.0+90=101.0$$

Since the new demand figure D_t includes possible random variations, we are discounting 90 per cent of these variations. Obviously then, small values of α will have a stronger smoothing effect than large values. Conversely, large values of α will react more quickly to real changes (as well as random variations) in actual demand. As an example, if $\alpha=0.4$ and the previous data remain the same, the new smoothed average would be

$$F_t = 0.4 \times 110 + 0{\cdot}6 \times 100=44{\cdot}0+60=104{\cdot}0$$

Note that the component of current actual demands and the old average are now weighted quite differently, giving considerably more weight to current actual demand. The equation (2) actually gives weight to all past actual demand data though this is not obvious. This weighting occurs through the chain of the periodic calculations to produce smoothed averages for each period. Thus, the smoothed averages are based on a sequential process representing all previous actual demands.

Generally, exponential smoothing is considered superior to the other forecasting methods. Its cost is equivalent and its accuracy especially in the short term is better. Owing to their relatively small computational cost and computer storage requirement, exponential smoothing models are probably the most widely used of the time series techniques. For longer term forecast, however, exponential smoothing is also considered a poor technique.

Trend Effects

The model, given by (2) above, does not take into account the seasonal or trend component. There exists other exponential smoothing models that compensate

for the various components of a time series such as the seasonal or trend component. To illustrate more complex exponential smoothing models, let us consider a model that has an adjustment for trend built into it If trend exists in either a positive or a negative form, there will be a lag if we make use of the simple exponential smoothing model just described. (see figure 1).

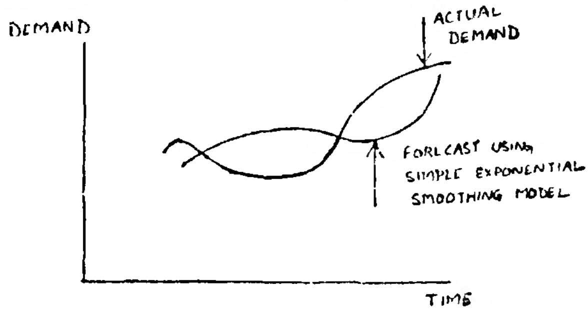


Fig. 1

The basic idea behind the trend-adjusted model is to calculate a simple, exponentially smoothed forecast and adjust the forecast for a trend lag.

Mathematically; the trend-adjusted model can be described as follows :

$$F_t' = F_t + \frac{1-\beta}{\beta} \cdot T_t$$

Where F_t' = trend-adjusied forecast for time period t,

F_t = simple exponential smoothing forecast for time period t,

and β = trend smoothing factor.

T_t is computed using the following formula :

$$T_t = T_{t-1} + \beta(t_t - T_{t-1}), \qquad \text{where } t_t = F_t - F_{t-1}$$

Computation of a trend-adjusted forecast is a four step process.

Step 1. Compute F_t, a simple forecast for time period t.

Step 2. Compute t_t by using $t_t = F_t - F_{t-1}$.

Step 3. Calculate the exponentially smoothed trend by using

$$T_t = T_{t-1} + \beta\,(t_t - T_{t-1})$$

Step 4. Finally, calculate a trend-abjusted forecast by using

$$F_t' = F_t + \frac{1-\beta}{\beta}\, T_t.$$

Tabel 1

Period	*Demand*
1	13
2	18
3	20
4	17
5	23
6	25
7	31
8	30
9	34
10	35
11	38
12	[illegible]

Let us illustrate how to compute a trend adjusted exponentially smoothed forecast using the demand data in Table 1 and smoothing constants α and β equal to .3 and .25 respectively. Let the initial forecast be 12 5. The trend-adjusted forecast for period 2 is computed as follows :

The first step is to compute F_2.

$$F_2 = F_1+\alpha\,(D_1-F_1) = 12.5+.3\,(13-12.5)$$
$$= 12.65.$$

The next step is to calculate t_2

$$t_2 = F_2-F_1 = 12.65-12.50 = .15$$

T_2 can now be calculated assuming the initial trend adjustment is 0 ;

$$T_2=T_1+\beta\,(t_2-T_1) = 0+.25\,(.15) = .0375.$$

Finally, the trend adjusted forecast for time period 2 can be computed.

$$F_2' = F_2+\frac{1-.25}{.25}T_2 = 12.65+3\,(.0375) = 12.7625.$$

Table 2

Time period	*Actual Demand*	F_t	t_t	T_t	F_t
1	13	12.5000	0	0	12.5000
2	17	12 6500	0.15000	0.037500	12.7625
3	20	14.2550	1.60500	0.429575	15.5431
4	17	15.9785	1.72350	0.752906	18.2372
5	23	16.2849	3.06450	0.641292	18.2088
6	25	18.2995	2.01451	0.984598	21.2533
7	31	20.3096	2.01016	1 24099	24 0326
8	30	23 5167	3.20711	1.73262	28 7143
9	34	24.4617	1.94498	1.78563	30,8186
10	35	28-0232	2.56149	1 97960	33.9620
11	38	30.1162	2.09304	2.00796	36 1401
12	39	32.4814	2.36513	2.09725	38 7731

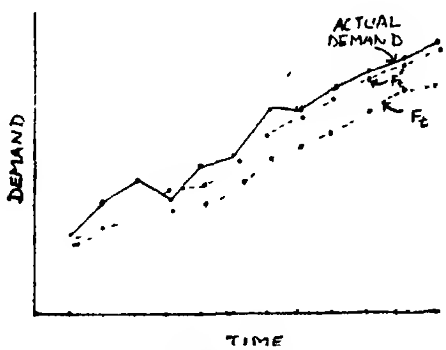


Fig. 2

Doing the same calculations for the remaining time periods for the time series date shown in Table 1 results in trend adjusted forecast are reflected in Table 2. Fig. 2 graphically contrasts the unadjusted and adjusted forecast for the time series in Table 2.

Ex. Consider the set of data values in Fig. 2

x	y	x	y
100	57	60	46
70	43	50	45
30	35	20	26
40	33	10	26
80	56	90	53

Now assume that the variable referred to as x is really time and that we associate the x values 10, 20, 30 and so on with the values 1, 2, 3 and so on.

(*a*) Would you expect the exponential smoothing model

$F_t = \alpha D_t + (1-\alpha) F_t$ to produce a goodfit for these data ? Why ?

(*b*) What relationship would you expect to hold between S_t and Y_t ?

(*c*) Suppose α was originally set equal to 0.5, would you increase or decrease α to get a better fit solution.

Solution : Consider the scatter diagram for the viven data.

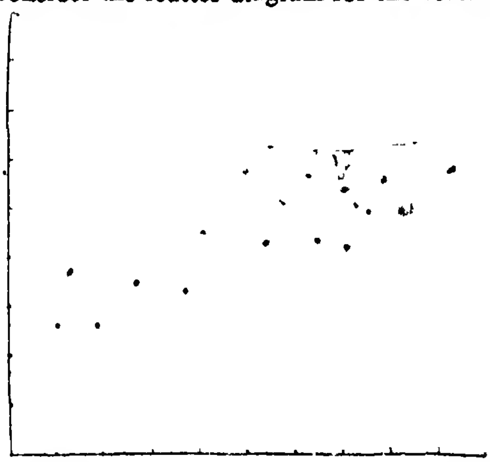


Fig. 3.

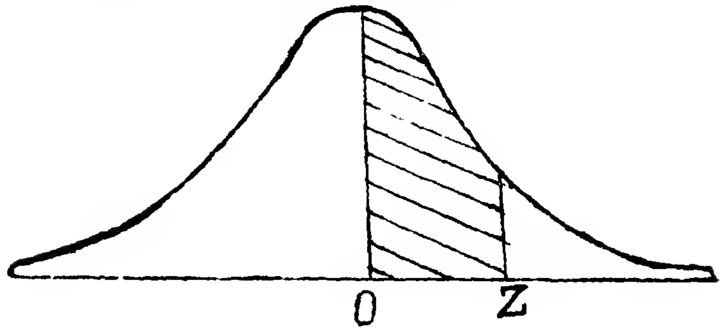
(*a*) The scatter diagram shows that Y appears to increase with t. Since the model $F_t = \alpha D_t + (1-\alpha) F_{t-1}$ is designed for systems in which variations in Y_t occur at random (that is, where there is not a regular patter of change) the model will probably not produce a very good fit.

(*b*) One would expect F_t to be smaller than D_t because F_t is a weighted sum of terms, most of which are less than or equal to Y_t.

(*c*) One would expect a better fit by increasing the value of α since this puts more weight on the most recent, and thus larger, values of Y.

APPENDIX

AREA UNDER STANDARD NORMAL CURVE



↓ Z →	0	1	2	3	4	5	6	7	8	9
0	0000	0040	0080	0120	0160	0199	0239	0279	0319	0359
1	0398	0438	0478	0517	0557	0596	0636	0675	0714	0753
2	0793	0832	0871	0910	0948	0987	1026	1064	1103	1141
3	1179	1217	1255	1293	1331	1368	1406	1443	1480	1517
4	1554	1591	1628	1664	1700	1736	1772	1808	1844	1879
.5	1915	1950	1985	2019	2054	2088	2123	2157	2190	2224
.6	2257	2291	2324	2357	2389	2422	2454	2486	2517	2549
.7	2580	2611	2642	2673	2704	2734	2764	2794	2823	2852
.8	2881	2910	2939	2967	.2995	3023	3051	3078	3106	3133
.9	3159	3186	3212	3238	3264	3289	3315	3340	3365	3389
1.0	.3413	3438	3461	3485	3508	3531	3554	3577	3599	3621
1.1	3643	3665	3686	3708	3729	3749	3770	3790	3810	3830
1.2	3849	3869	3888	3907	3925	3944	3962	3980	3997	4015
1.3	4032	4049	4066	4082	4099	4115	4131	4147	4162	4177
1.4	4192	4207	4222	4236	4251	4265	4279	4292	4306	4319
1.5	4332	4345	4357	4370	4382	4394	4406	4418	4429	4441
1.6	4452	4463	4474	4484	4495	4505	4515	4525	4535	4545
1.7	4554	4564	4573	4582	4591	4599	4608	4616	4625	4633
1.8	4641	4649	4656	4664	4671	4678	4686	4693	4699	4706
1.9	4713	4719	4726	4732	4738	4744	4750	4756	4761	4767
2.0	4772	4778	4783	4788	4793	4798	4803	4808	4812	4817
2.1	4821	4826	4830	4834	4838	4842	4846	4850	4854	4857
2.2	4861	4864	4868	4871	4875	4878	4881	4884	.4887	4890
2.3	4893	4896	4898	4901	4904	4906	4909	4911	4913	4916
2.4	4918	4920	4922	4925	4927	4929	4931	4932	4934	4936
2.5	4938	4940	4941	4943	4945	4946	4948	4949	4951	4952
2.6	4953	4955	4956	4957	4959	4960	4961	4962	4963	4964
2.7	4965	4966	4967	4968	4969	4970	4971	4972	4973	4974
2.8	4974	4975	4976	4977	4977	4978	4979	4979	4980	4981
2.9	4981	4982	4982	4983	4984	4984	4985	4985	4986	4986
3.0	4987	4987	4987	4988	4988	4989	4989	4989	4990	4990
3.1	4990	4991	4991	4991	4992	4992	4992	4992	4993	4993
3.2	4993	4993	4994	4994	4994	4994	4994	4995	4995	4995
3.3	4995	4995	4995	4996	4996	4996	4996	4996	4996	4997
3.4	4997	4997	4997	4997	4997	4997	4997	4997	4997	4998
3.5	4998	4998	4998	4998	4998	4998	4998	4998	4998	4998
3.6	4998	4998	4999	4999	4999	4999	4999	4999	4999	4999
3.7	4999	4999	4999	4999	4999	4999	4999	4999	4999	4999
3.8	.4999	4999	4999	4999	4999	4999	4999	4999	4999	4999
3.9	5000	5000	5000	5000	5000	5000	5000	5000	5000	5000

F. III. B-3

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STUDY—III

LINEAR PROGRAMMING

Contents :

Suggested Reading : Operations Research *by Goel & Mittal, Pragti Prakashan, Meerut.*

Linear Programming

Linear programming is a mathematical technique for determining the optimal allocation of resources and obtaining a particular objective when there are alternative uses of the resoures : money, manpower, materials, machines and other facilities. The objective in resource allocation may be cost minimisation or inversely profit maximisation. The technique of linear programming is applicable to problems in which the total effectiveness can be expressed as a linear function of individual allocations, and the limitations on resources give rise to *linear* equalities or inequalities of the individual allocations. The adjective : linear, is to be particularly noted. It implies that all the limitations or constraints and the objective must afford expression as linear functions

Although the technique of linear programming is applicable to all such problems there are some more easy methods for solving specific categories of problems. Special algorithms have been evolved to ease up computational task. The algorithms have acquired special names of their own. This has led to the following categories of the linear programming problems :—

(*i*) General Linear Programming problems.
(*ii*) Transporation Problems.
(*iii*) Assignment Problems.

General Linear Programming Problem

An examination of the following simple example should illustrate the basic concepts of a linear programming problem (abbreviated as LPP).

Example 1. A small scale industry manufactures two products, X_1 and X_2 which are processed in the machine shop and the assembly shop The times (in hours) required for each product in the shops are given in the matrix below. Profits per unit are also given along.

	Machine	*Assembly*	*Profit unit*
Product X_1	2	4	Rs. 3
Product X_2	3	2	Rs. 4
Total time available (in a day)	16	16	

Assuming that there is unlimited demand for both the products how many units of each should be produced every day to maximse total profit ?

Let x_1 and x_2 be the number of units of X_1 and X_2 respectively that maximise the profit. The profit objective function to be maximised may be expressed, symbolically as

$$Z=3x_1+4x_2$$

which is subject to

$$2x_1+3x_2\leqslant 16 \quad \text{Machnining constraint}$$
$$4x_1+2x_2\leqslant 16 \quad \text{Assembly constraint}$$

Also, $x_1 > 0$, $x_2 > 0$ since non-negative units of any product is meaningless. By analogy the general linear programming problem can be defined as

Maximize (or minimize) $Z = c_1x_1 + c_2x_2 + \ldots\ldots + c_nx_n$ subject to

$$a_{11}x_1 + a_{12}x_2 + \ldots\ldots\ldots + a_{1j}x_j + \ldots\ldots\ldots + a_{1n}x_n (\leqslant = \geqslant) b_1$$
$$a_{21}x_1 + a_{22}x_2 + \ldots\ldots\ldots + a_{2j}x_j + \ldots\ldots\ldots + a_{2n}x_n (\leqslant = >) b_2$$
$$\vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + \ldots\ldots\ldots + a_{mj}x_j + \ldots\ldots\ldots a_{mn}x_n (\leqslant = >) b_m$$

and the non-negativity restrictions

$x_j > 0$ where $j = 1, 2, \ldots\ldots, n$

Also, all c's, b's and a_{ij}'s are constants and x_j's are variables.

We have used $(\leqslant = >)$, which means any one of the signs could be there. The linear function that is to be optimised is known as the objective function. Conditions are called the constraints. Solving a linear programming problem means finding non-negative values of the variables (x_1, $x_2 \ldots \ldots x_n$) which optimise the objective function and satisfy the contraints also.

Methods of Solving LPP's

Graphical method : We shall first study this method and then pass on to the simplex method for solving more complex problems. The graphical method is being discussed to provide necessary grounding in understanding the computational steps of the simplex algorithm, it itself being of little use in handling practical problems. There is also the trial and error method which, again, has limited utility with regard to practical problems but is quite useful for exposition of the simplex algorithm.

Consider the constraints as equalities rather than inequalities and then, draw lines in two-dimensional plane corresponding to each equation and non-negativity restriction. These lines will border the region of permissible values of the variables. The region enclosed will be the desired feasible region of the variables. The constraints are shown as equalities in Figs. 1 & 2. The procedure to be followed is as below.

For Machining : $2x_1 + 3x_2 = 16$

Put	$x_2 = 0$,	and	$x_1 = 0$, in turn.
so that	$x_1 = 8$	and	$x_2 = \frac{16}{3}$

In figure 1, point N is plotted along X_1-axis as (8,0) and M is plotted along X_2 axis as $\left(0, \frac{16}{3}\right)$

Exactly in the same manner, the assembly restriction is plotted in figure 2. It is to be noted that we have shaded the areas in both the figures just in the first quadrant to obey the non-negative constraints. The implication of this being that we must produce 0 or more units of each of the two products. The shaded area in the first figure represents the feasible region for machining *i e.*, area of possible solutions obeying the machining restriction. Any point in this region represents, by its co-ordinates, the amounts of X_1 and X_2 that can be machined. Likewise, the shaded region of figure 2 can be interpreted.

In figure 3, we have superimposed the previous 2 figures. The shrunken shaded region enclosed by the polygon OMSP reprsents the overall feasible region *i e*, any point in it, by its co ordinates, represents units of the two products that can *both* be machined and assembled in a day. It is a different matter that all the points do not furnish us with the optimal combination which can only be ascertained by bringing the objective function into the picture.

Now $$Z=3x_1+4x_2$$

Choose a profit figure arbitrarily, say, $3\times4=12$, the HCF of the coefficients, so that $12=3x_1+4x_2$. Plot this profit line as in figure 3. This is shown as PQ in this figure. All the points on PQ would represent not only feasible combinations of the two products but also the same profit of 12. Such a line is, therefore, called an iso-profit line. We "pull PQ outwards (away from the origin) *i.e*, draw lines paralled to it until we reach the exterme of the feasible region, S by the third iso-profit line. The more an iso-profit line is away from the origin the higher the profit it ensures. Going beyond S would mean infeasible region. We stop here so that S gives the optimal combination. Its co-ordinates can be read as (2,4) meaning that 2 units of X_1 ought to be produced and 4 units of X_2 with an optimal profit of $3\times2+4\times4=22$.

Exercise : With the same constraints find graphically optimal profit for an objective function $Z=6x_1$ in the above example. (Answer : 24)

We summarise below the graphical procedure :

Step 1. Transform constraints into equalities.

Step 2. Graph each equality. Shade the feasible region.

Step 3. Choose a convenient profit and graph the iso-profit line.

Step 4. Pull the profit line until you reach extreme of the feasible region.

Step 5. Read co-ordinates of the extreme point and find the maximum profit. *An alternative graphical method follows.*

Alternative Method

Steps 1 and 2 as above.

Step 3. Read the co-ordinates of the vertices of the convex polygon *i.e.*, shaded feasible region as below for the example on hand.

Co-ordinates of vertex		$Z=3x+4x_2$
O	(0, 0)	$Z_1=3\times0+4\times0=0$
P	(4, 0)	$Z_2=3\times4+4\times0=12$
S	(2, 4)	$Z_3=3\times2+4\times4=22$
M	(0, $5\frac{1}{3}$)	$Z_4=3\times0+4\times5\frac{1}{3}=21\frac{1}{3}$.

Thus Z_3 gives $Z_{max} = 22$ with $x_1 = 2$ and $x_2 = 4$.

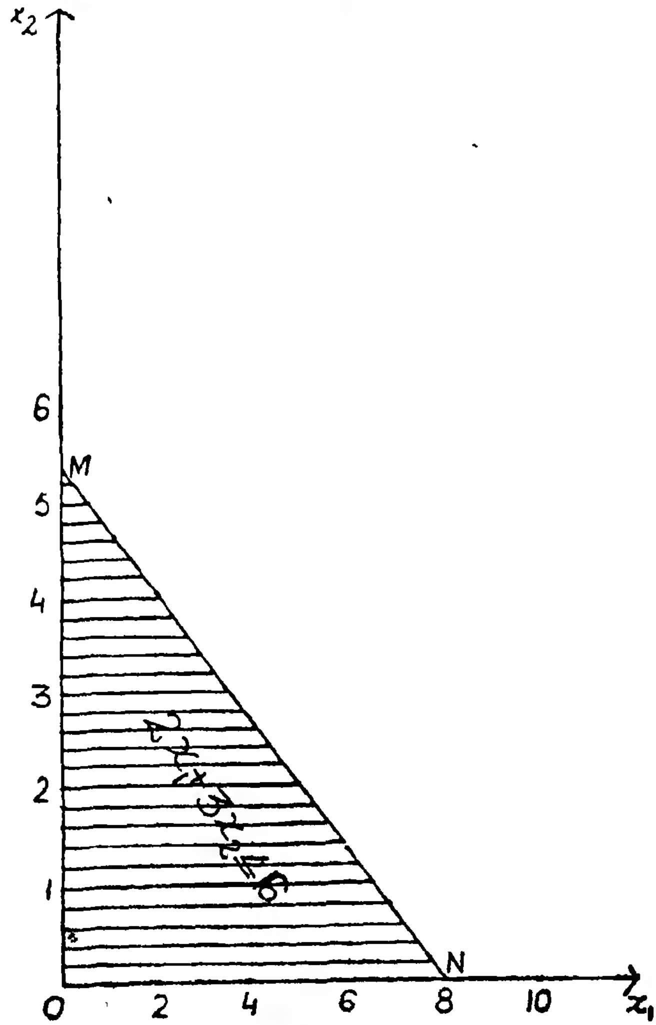


Fig. 1

Minimisation Case (Example 2).

Consider the following minimisation LPP,

Minimise $Z = 50x_1+20x_2$

subject to $2x_1-1x_2\geqslant 0$

$1x_1+4x_2\geqslant 80$

$0.90x_1+0.80x_2\geqslant 40$

$x_1, x_2\geqslant 0$

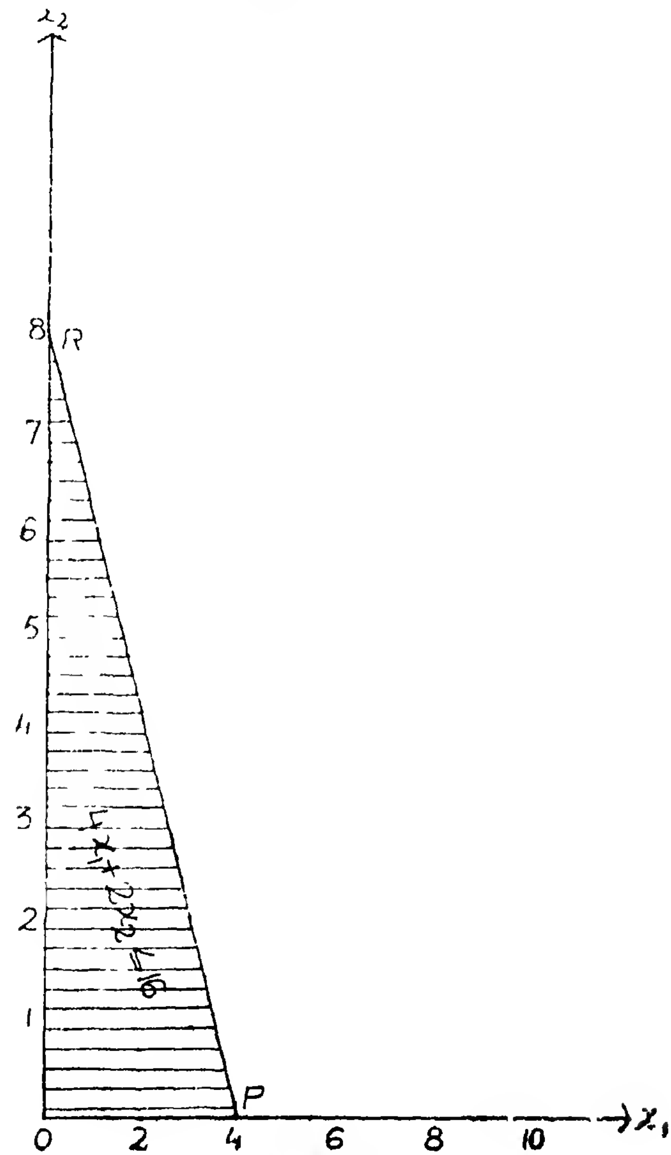


Fig. 2

(A convex POLYGON like OMSP is known as simplex)

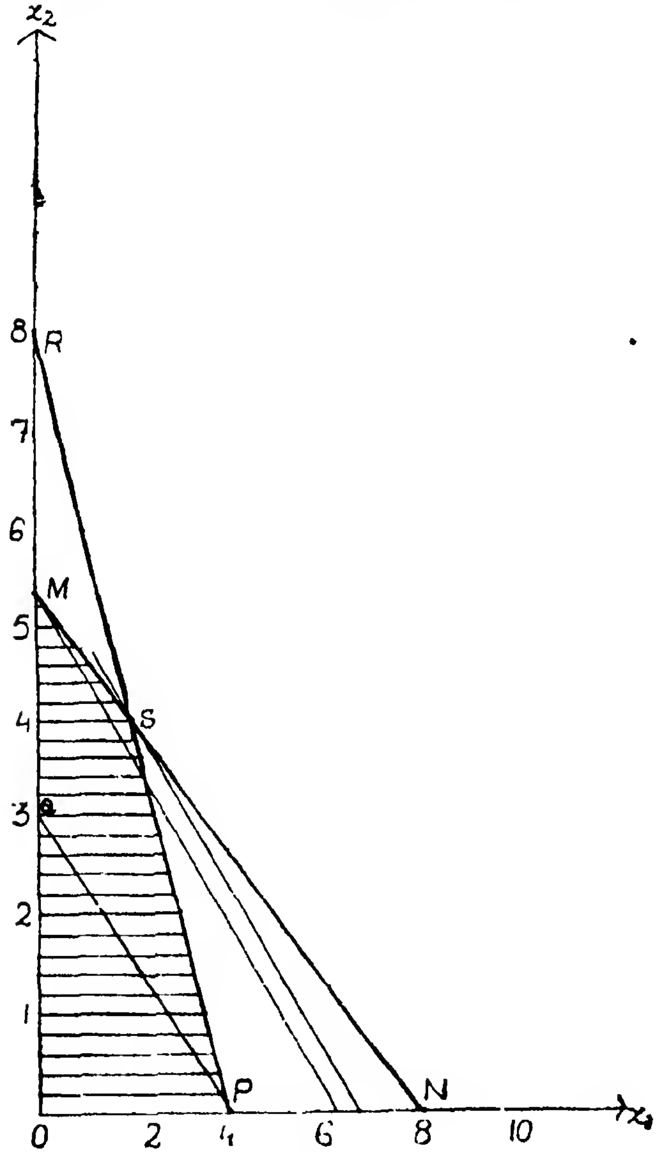


Fig. 3

Steps 1 and 2 above for the maximisiation case are also followed in the minimisation problem. The constraints, in their limiting form, become

$$2x_1 - 1x_2 = 0$$
$$1x_1 + 4x_2 + 80$$
$$0.90x_1 + 0.80x_2 = 40$$
$$x_1 = 0$$
$$x_2 = 0$$

These are plotted below in fig. 4. The feasible region is shaded. It is situated outwards the constraints in their limiting forms.

Step 3. A convenient cost figure is chosen and a straight line corresponding to it is also plotted in the diagram below.

Step 4. The cost straight line is 'pulled' outwards until we reach the *first* vertex. The co-ordinates of this vertex S read (16, 32).

Step 5. Minimum cost $(=5\times16+20\times32=)$ Rs. 1440 is obtained by substitution in the objective function.

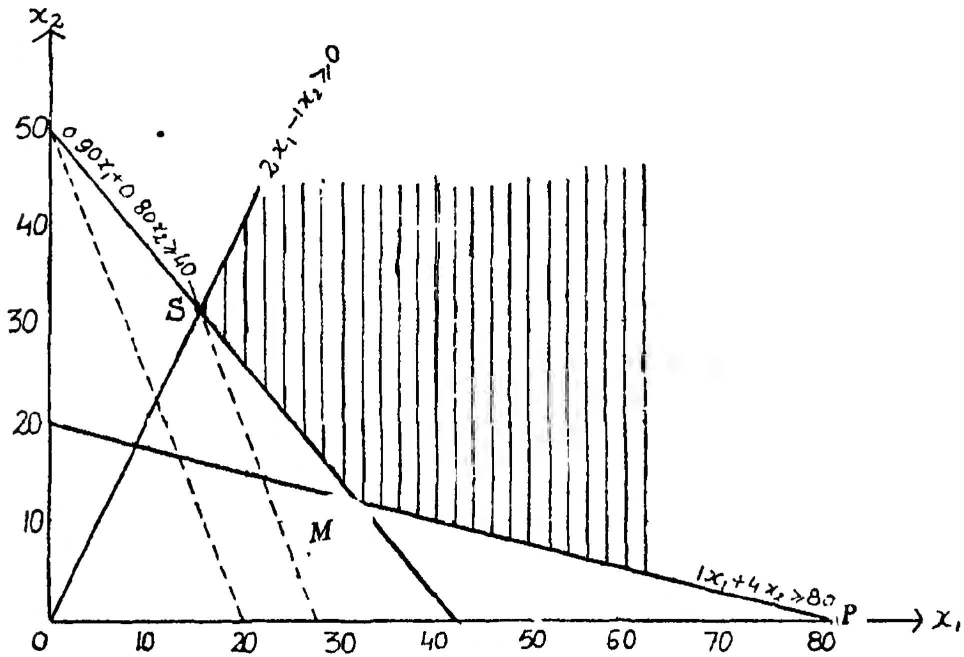


Fig. 4

Alternatively, read co-ordinates of S, M and P, substitute them in turn in the objective function and pick up the least total cost solution as optimal.

Example 3. A company makes 2 kinds of leather belt. Belt A is high quality belt, and belt B is of lower quality. The respective profits are Rs. 0.40 and Rs. 0.30 per belt. Each belt of type A requires twice as much time as a belt of type B and, if all belts were of type B the company could make 1,000 per day. The supply of leather is sufficient for only 800 belts per day (both A and B combined). Belt A requires a fancy buckle, of which only 400 per day are available. These are only 700 buckles a day available for belt B. Set linear programming equations for the problem, obtain its solution by the graphical method.

Solution. Suppose the company makes x_1 belts of type A and x_2 belts of type B each day. From the restrictions on the number of buckles available we have

$$x_1 \leqslant 400$$
$$x_2 \leqslant 700$$

From the availability of leather we have

$$x_1+x_2 \leqslant 800$$

and from the limitation on time

$$2x_1+x_2 \leqslant 1{,}000.$$

$x_1=400$ and $x_2=700$ are plotted straightway as constraints 1 and 2 below

For $x_1+x_2=800$

when $x_1=0$, $x_2=800$; $x_1=400$, $x_2=400$ — Two points are thus gotten and plotted as constraint 3.

For $2x_1+x_2=1000$.

when $x_2=0$, $x_1=500$; $x_1=100$, $x_2=800$ — plotted as constraint 4.

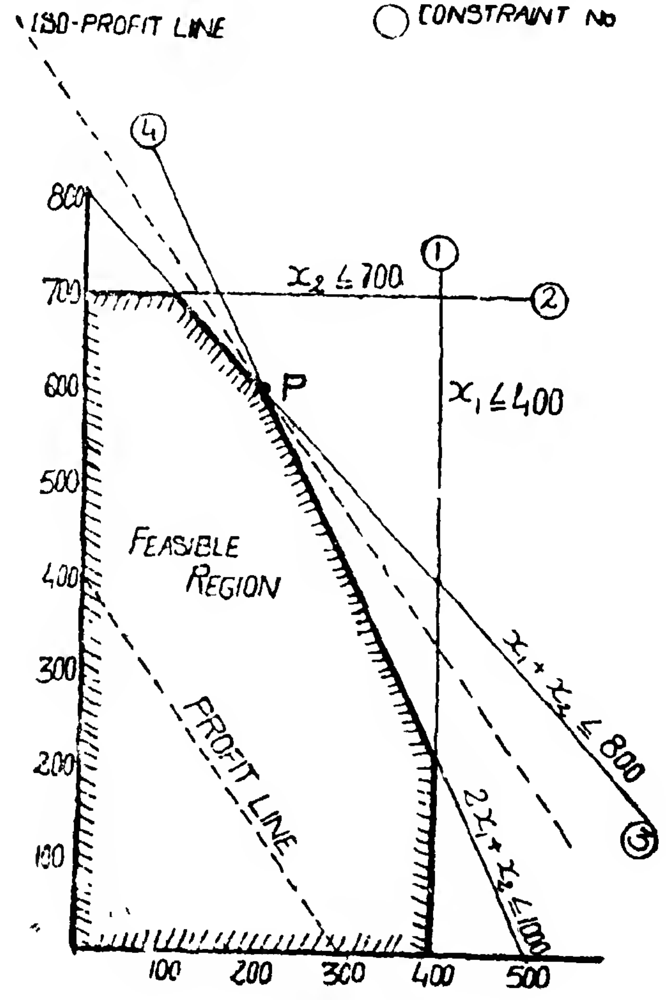


Fig. 5.3

Objective function: $0.4x_1+0.3x_2=120$

when $x_1=0$ $x_2=400$

when $x_2=0$ $x_1=300$

(Plotted as dotted profit line)

The extreme iso-profit line passes through P of the feasible region. The co-ordinates of P can be read as (200, 600).

As an exercise find the optimal solution by the alternative method.

Example 4. Minimise $Z=20x_1+10x_2$

Subject to $x_1+2x_2\leqslant 40$ ...(*i*)

$3x_1+x_2\geqslant 30$ ...(*ii*)

$4x_1+3x_2\geqslant 60$ ...(*iii*)

$x_1, x_2\geqslant 0.$

Constraint (*i*) in the limiting form : $x_1+2x_2=40$

when $x_1=0$ $x_2=20$

$x_2=0$ $x_1=40$

Constraint (*ii*) in the limiting form : $3x_1+x_2=30$

when $x_1=0$ $x_2=30$

$x_2=0$ $x_1=10$

Constraint (*iii*) in the limiting form : $4x_1+3x_2=60$

when $x_1=0$ $x_2=20$

$x_2=0$ $x_1=15$

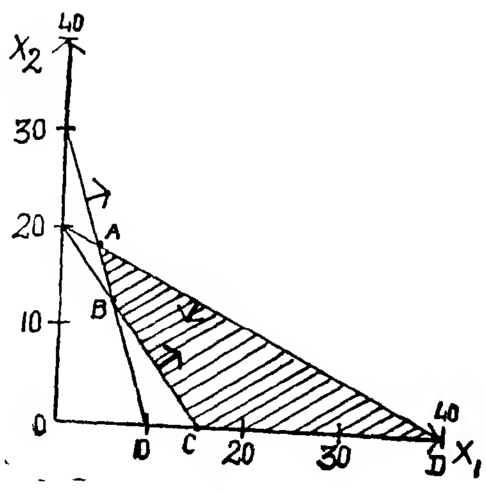


Fig. 6

The three constraints in the limiting form are plotted above. Note that the arrow for constraint (*i*) points towards the origin since it is $\leq$ type ; whereas arrow for the other two constraints point away from the origin since they are of $\geq$ type.

The feasible region has been shaded and ABCD constitutes the feasible polygon. Co ordinates of these 4 vertices are read as,

$$Z=20x_1+10x_2$$

A (4, 18)	$Z_A=20\times4+10\times18=260$
B (6, 12)	$Z_B=20\times6+10\times12=240$
C (15, 0)	$Z_C=20\times15+10\times0=300$
D (40, 0)	$Z_D=20\times40+10\times0=800$

Hence $Z_{min}=Z_B=240$ with $x_1=6$ and $x_2=12$

Exercise 1. Consider two different types of foodstuffs, say F_1 and F_2. Assume that these foodstuffs contain vitamins V_1, V_2 and V_3 respectively. Minimum daily requirements of these are 1 mg of V_1, 50 mg of V_2 and 10 mg V_3. Suppose that the foodstuff F_1 contains 1 mg of V_1, 100 mg of V_2 and 10 mg of V_3. Whereas foodstuff F_2 contains 1 mg of V_1, 10 mg of V_2 and 100 mg of V_3. Cost of 1 unit of foodstuff F_1 is Re 1 and that of F_2 is Re 1.5.

Formulate the symbolic model of the problem and solve it for the least cost.

(Answer : Z=1)

Exercise 2. A pineapple firm produces two products—canned pineapple and canned juice, The specific amounts of material, labour and equipment required to produce each product and the availability of each of these resources are shown below :

	Canned Juice	Canned Pineapple	Available Resources
Labour (man-hours)	3	2.0	12.0
Equipment (machine-hours)	1	2.3	6.9
Material (units)	1	1.4	4.9

Assuming one unit each of canned juice and canned pineapple has profit margins of Rs. 2 and Re. 1 respectively, formulate the mathematical model and then solve it the for the maximum profit.

(Answer : Z=8).

Exercise 3. Find the maximum value of

$$Z=2x+3y$$

subject to
$$x+y\leq30$$
$$y\geq3$$
$$0\leq y\leq12$$

$$x-y \geq 0$$
$$0 \leq x \leq 20$$

Answer : $Z_{max}=72$.

Exercise 4. Maximise $7x_1+3x_2$

subject to
$$x_1+2x_2 \geq 3$$
$$x_1+x_2 \leq 4$$
$$0 \leq x_1 \leq 5/2$$
$$0 \leq x_2 \leq 3/2$$

Answer : $Z_{max}=22$.

Exercise 5. The manager of an oil refinery must decide on the optimal mix of two possible blending processes of which the inputs and outputs per production run are as follows :

	Input (Units)		Output (Units)	
Process	Crude A	Crude B	Gasoline X	Gasoline Y
1	5	3	5	8
2	4	5	4	4

The maximum amounts available of crude A and B are 200 units and 150 units respectively. Market requirements show that at least 100 units of gasoline X and 80 units of gasoline Y must be produced. The profits per production run from process 1 and process 2 are Rs. 300 and Rs. 400 respectively. Solve the LPP by the graphical method.

Trial & Error Method

Graphical method cannot be used when there are more than 2 variables in an LPP. In that case, we use the simplex method which is highly efficient and versatile as also amenable to further mathematical treatment and offers interesting economic interperations. However, its underlying concepts are rather lengthy to discuss and the student should patiently go through the following material on the trial and error method to gain a good grasp over the simplex technique.

Slack Variables

Example 1 is rewritten below :

Maximise $Z=3x_1+4x_2$

Subject to
$$2x_1+3x_2 \leq 16$$
$$4x_1+2x_2 \leq 16$$
$$x_1, x_2 \geq 0.$$

The $\leq$ type inequalities can be transformed into equalities by the addition of non-negative variables, say x_3 and x_4 (known as slack variables) as below. These variables represent imaginary products with zero profit per unit.

$$\left.\begin{array}{l}2x_1+3x_2+1x_3=16\\4x_1+2x_2+1x_4=16\end{array}\right\}-A$$

And the objective function may be rewritten as below.

Maximise $\quad Z=3x_1+4x_2+0x_3+0x_4$

The linear programming Theorems

The trial and error and simple methods are based on the concept of slack variables and theorms described below :

Extreme point theorem states that an optimal solution to an LPP occurs at one of the vertices of the feasible region. This should be obvious from the discussion on the graphical method.

Now the vertices are defined by the intersection of equations. The first step of the method is, therefore, to convert the inequalities into equalities by the addition (or subtraction) of the slack (or surplus variables) depending on the direction of the inequality. In $\geqslant$ type inequality we subtract a variable (called the surplus variable) to make it an equality.

It is to be noted that the system of equations (A) above has more variables than the number of equations. Such a system of equations has an infinite number of solutions, yet it has a finite and few vertices the co-ordinates of which can be determined by applying the basis theorem.

Basis theorem states that for a system of m equations in n variables (where $n>m$) a solution in which at least $(n-m)$ of the variables have value of zero is a vertex. This solution is called a basic solution.

Extreme point theorem can be extended to state that the objective function is optimal at least at one of the basic solution. Some of the vertices may be infeasible in that they have $-$ve co-ordinates and have to be dropped in view of the non-negativity condition on all variables including the slack and surplus variables.

Consider the LPP of example 1 towards elucidation of the basis theorem.

Maximise $Z=3x_1+4x_2$

Subject to $2x_1+3x_2\leqslant 16$

$4x_1+2x_2\leqslant 16$

$x_1,x_2\geqslant 0.$

Introducing slack variables x_3, and x_4

Maximise $Z=3x_1+4x_2+0x_3+0x_4$

Subject to

$$\left.\begin{array}{l}2x_1+3x_2+1x_3+0x_4=16\\4x_1+2x_2+0x_3+1x_4=16\end{array}\right]-B$$

$x_1,x_2,x_3,x_4\geqslant 0$

Here n (number of variables)$=4$ and m (number of equations)$=2$. Thus $n-m=2$. According to the basis theorem we set $2(=n-m)$ variables in (B) equal to zero at a time, solve the resulting system of equations and obtain a basic solution. Thus if we zeroise x_1 and x_2 the resulting system of equations would be

$$\left.\begin{array}{l}1x_3+0x_4=16\\0x_3+1x_4=16\end{array}\right]-(C) \qquad \text{Set 1 } (x_1=x_2=0)$$

These equations directly yield $x_3=16$ and $x_4=16$ as the basic solution *i.e.*, the co-ordinates of a vertex.

The other sets of equations, upon zeroising two variables at a time in (B), would be as follows :

$$\left.\begin{array}{l} 2x_1+3x_2=16 \\ 4x_1+2x_2=16 \end{array}\right] \qquad \text{Set 2 } (x_3=x_4=0)$$

$$\left.\begin{array}{l} 2x_1+0x_4=16 \\ 4x_1+1x_4=16 \end{array}\right] \qquad \text{Set 3 } (x_2=x_3=0)$$

$$\left.\begin{array}{l} 2x_1+1x_3=16 \\ 4x_1+0x_3=16 \end{array}\right] \qquad \text{Set 4 } (x_2=x_4=0)$$

$$\left.\begin{array}{l} 3x_2+1x_3=16 \\ 2x_2+0x_4=16 \end{array}\right] \qquad \text{Set 5 } (x_1=x_4=0)$$

$$\left.\begin{array}{l} 3x_2+0x_4=16 \\ 2x_2+1x_4=16 \end{array}\right] \qquad \text{Set 6 } (x_1=x_3=0)$$

It is a simple matter to solve these six sets of simultaneous equations and obtain the six basic solutions *i e*, co-ordinates of the six vertices of the feasible region. The solutions are given below. The student may verify these by solving each of the sets of simultaneous equations.

Set	Solution
1	$x_3=16$, $x_4=16$
2	$x_1=2$, $x_2=4$
3	$x_1=8$, $x_4=-16$
4	$x_1=4$, $x_3=8$
5	$x_2=8$, $x_3=-8$
6	$x_2=16/3$, $x_4=16/3$

Since solutions 3 and 5 yield a negative co-ordinate each, contradicting thereby the non-negativity constraints, these are infeasible and have to be dropped from consideration.

Now according to the basis theorem the optimal solution lies at one of these vertices. By substituting these co-ordinates the values of objective function are derived below :

Set	Solution	Z (Profit)
1	$x_3=16$, $x_4=16$	0
2	$x_1=2$, $x_2=4$	22
3	Infeasible	
4	$x_1=4$, $x_3=8$	12
5	Infeasible	
6	$x_2=16/3$, $x_4=16/3$	$21\frac{1}{3}$

Thus soluion 2 is optimal with a profit of 22. This is how we can solve an LPP simply by employing the theorems stated above ; but the simplex method is a further improvement over the trial and error method. There are three inefficiencies in the trial and error approach as follows.

Inefficiencies of Trial and Error Method

I. Not, perhaps, in the above example but in other LPP's where m and n are larger solving numerous sets of simultaneous equations would be extremely cumbersome and time-consuming.

II. Scanning the profit table we notice that we jump from profit 0 to 22 to $21\frac{1}{3}$ *i.e.*, there are ups and downs. The simplex method ensure that successive solutions yield progressively higher profits, culminating into the optimal one.

III· Some of the sets yield infeasible solutions. There should be means to detect such sets and not to solve them at all.

Simplex method

Inefficiency I is obviated by row operations which is a systematic way of solving sets of simultaneous equations It is quicker than the ordinary methods. Other inefficiencies are taken up later.

Row Operations

The fact that $x_3=16$ and $x_4=16$ could be directly read in eqns (C) no p 13 upon zeroising x_1 and x_2 is utilised in row operations. It is to be noted that the coefficients of x_3 and x_4 form an identity matrix in (C) and it is this matrix that helps in directly reading these values.

In row operations, demonstrated below for the example on hand, we do not abruptly jump from x_1 and x_2 for the first extreme point to zeroising x_3 and x_4 for the 2nd extreme point. Zeroisation is done systematically. Having found an extereme point by zeroising x_1 and x_2 we take up either of

$$x_1,\ x_3$$
$$x_1,\ x_4$$
$$x_2,\ x_3$$
$$x_2,\ x_4$$

for zeroisation for the next extreme point (syn ; vertex) *i e*, we retain one of the alread zeroised variable and take another one alongwith it, Say, we choose x_2 and x_3 for zeroisation. By so doing we have retained the already zerosid x_2 and taken x_3 for zeroisation alongwith it. We proceed on this way. Row operations are carried out below for the example on hand. Also, further explanation on these follows.

1	Non basic *i.e.*, Zeroised Variables 2	Basic *i.e.*, non zeroised variables 3	4
Solution A	$x_1=0$ $x_2=0$	$x_3=16$ $x_4=16$	↓ $2x_1+3x_2+1x_3+0x_4=16$...(*i*) $4x_1+2x_2+0x_3+1x_4+16$...(*ii*)
B	$x_2=0$ $x_3=0$	(x_3 outgoing from A) $x_1=8$ $x_4=-16$ (x_1 incoming)	Multiply eq (*i*) by $\frac{1}{2}$ to make coefficient of x_1 unity. This gives equation (*iii*) below. Multiply eq. (*iii*) by 4 and substract if from equation (*ii*) to get equation (*iv*) $1x_1+\frac{3}{2}x_2+\frac{1}{2}x_3\times 0x_4=8$. (*iii*) $0x_1-4x_2-2x_3+1x_4=-16$. (*iv*)
C	$x_3=0$ $x_4=0$	$x_1=2$ $x_2=4$	Multiply eq. (*iv*) by $-\frac{1}{4}$ to make coefficient of x_2 unity. This gives equation (*vi*) Multiply eq. (*vi*) by $\frac{3}{2}$ and subtract it from eq. (*iii*) to give eq, (*v*) $1x_1+0x_2-\frac{1}{4}x_3+\frac{3}{8}x_4=2$. (*v*) $0x_1+1x_2+\frac{1}{2}x_3-\frac{1}{4}x_4=4$;..... (*vi*) Proceed on this way
D	$x_2=0$ $x_4=0$	$x_1=4$ $x_3=8$	$1x_1+\frac{1}{2}x_2+0x_3+\frac{1}{4}x_4=4$ $0x_1+2x_2+1x_3-\frac{1}{2}x_4=8$
E	$x_1=0$ $x_4=0$	$x_2=8$ $x_3=-8$	$2x_1+1x_2+0x_3+\frac{1}{2}x_4=8$ $-4x_1+0x_2+1x_3+0x_4=-8$
F	$x_1=4$ $x_3=0$	$x_2=\frac{16}{3}$ $x_4=\frac{16}{3}$	$\frac{2}{3}x_1+1x_2+\frac{1}{3}x_3+0x_4=\frac{16}{3}$ $\frac{8}{3}x_1+0x_2+\frac{2}{3}x_3+1x_4=\frac{16}{3}$

Col 2 gives zeroised variables.

Col. 3 gives same solutions as of p. 14.

It can be seen that there is an outgoing variable and incoming variable in the basis *i.e.*, the basic variable. The choice of the incoming and outgoing variables is however arbitary.

Net profits

Now let us take up inefficiency II. It concerns proceeding progressively to higher and higher values of the objective function. The original objective function is $Z=3x_1+4x_2+0x_3+0x_4$. The initial solution A of the row operations above is $x_3=16$, $x_4=16$ and of course, $x_1=0$, $x_2=0$ so that initial Z is 0. Now the question

arises if x_1 should be the incoming variable or x_2 should be the incoming variable. Interpreting example I as product-mix problem, initially we start with imaginary products X_3 and X_4 with a profit of 0 and now, in an endeavour to improve profit we wonder whether we should bring in X_1 or X_2. Obviously X_2 has a higher contribution or net profit (coefficient of 4 in the objective): therefore, it should be brought into the next soultion. The next solution must have x_2. Whether x_3 should go or x_4 should go is a matter of inefficiency III. Anyhow, the original set of equations is

$$2x_1+3x_2+1x_3+0x_4=16$$
$$4x_1+2x_2+0x_3+1x_4=16$$

We have decided to bring x_2, keep $x_1=0$ and whether to keep x_3 or x_4 zeroised is a still a question mark. By retaining x_2 and keeping x_1 zeroised the set reduces to

$$3x_2+1x_3=16$$
$$2x_2+1x_1=16$$

Our problem is whether to zeroise x_3 or x_4

Replacement Ratios

Interpreting the problem again as a product-mix problem the two equations are designated as machining and assembly restrictions meaning that only 16 hours of each at most are available to us.

$$3x_2+1x_3=16 \qquad \text{Machining restriction}$$
$$2x_2+1x_4=16 \qquad \text{Assembly restriction}$$

Case (i) If we zeroise x_3 we can machine maximum of $\frac{16}{3}$ units of x_2 according to the first equation.

Now $\frac{16}{3}$ units of x_2 require $\frac{16}{3}\times 2=\frac{32}{3}$ hours of assembly which are less than 16. Thus we can at most produce $\frac{16}{3}$ units of X_2 because of the machining restriction. Thus if we set $x_3=0, x_2=\frac{16}{3}$.

Case (ii) If, on the other hand, we zeorise x_4 we can assemble a maximum of units 8 of x_2 according to the 2nd equation. Now 8 units of x_2 require $8\times 3=24$ hours of machining which exceed the available 16. In other words setting $x_4=0$ in the 2nd equation leads to $x_2=8$. Putting now $x_2=8$ in the first equation leads to $x_3=-8$ which means an infeasible solution.

Hence we should zeroise x_3 and not x_4. The conclusion, then, is divide the R.H.S. of each equation by the coefficient of the incoming variable (x_2 here) and call these *replacement ratios*. In this case, these are $\frac{16}{3}$ and $\frac{16}{2}$. The incoming variable should be set at the value of the least+ve ratio *i.e.*, $x_2=\frac{16}{3}$. And the other variable in the equation of the least ratio should be zeroised *i.e.*, x_3 should be zeroised here.

[*Negative and infinite replacement ratios would simply be ignored*].

To summarise, in simplex method (*i*) we perform row operations so that the simultaneous equations are solved systematically and easily as required by the basis theorem, (*ii*) we jump to as high a profit as possible in the successive solutions and (*iii*) we skip infeasible solutions.

Example 1 is solved by the simplex method below. The explanation on the composition of the simplex tables and computations involved is also given.

Maximise $Z=3x_1+4x_2$

subject to $2x_1+3x_2\leqslant 16$

$4x_1+2x_2\leqslant 16$

$x_1\geqslant 0,\ x_2\geqslant 0$

Introducing slack, variables,

Maximise $Z=3x_1+4x_2+0x_3+0x_4$

subject to $2x_1+3x_2+1x_3+0x_4=16$

$4x_1+2x_2+0x_3+1x_4=16$

x_1 to $x_4\geqslant 0$

The simplex solution follows.

	1	2	3	4	5				6
					3	4	0	0	
	FIXED RATIO	PROGRAMME	PROFIT	QTY	x_1	x_2	x_3	x_4	RATIO
I		x_3	0	16	2	3	1	0	16/3 ←
	2/3	x_4	0	16	4	2	0	1	8
		NER			3	4 ↑	0	0	
II	1/4	x_2	4	16/3	2/3	1	1/3	0	8
		x_4	0	16/3	8/3	0	−2/3	1	2 ←
		NER			1/3 ↑	0	−4/3	0	
III		x_2	4	4	0	1	1/2	−1/4	
		x_1	3	2	1	0	−1/4	3/8	
		NER			0	0	−5/4	−1/8	

The main simplex table, a side from headings, is seen divided into 3 horizontal sub-tables, I to III. There are 6 main headings 1 to 6 as explained below for sub-table I.

Col. 1. Fixed ratio—explanation deferred.

Col. 2. Programme—it contains the basic variables. Initially, in sub-table 1. they are slack variables, so that we start with production of imaginary products with the minimum profit of zero.

Col. 3. Profit/unit as read from the objective function.

Col. 4. This heading is divided in as many sub-headings as the number of real and slack variables. It indicates each of the variable with profit/unit as read from the objective function at its top. The body of the subtable carries the coefficients of the variables as read from the constraints. That the coefficients of the slack variables constitute a unit matrix in this initial sub-table is to be noted.

Col. 5. Replacement ratio as discussed earlier.

Computational aspects for the derivation of improved solutions are explained step by step below.

Step I. Select the incoming variable in sub-table I in the NER (net evaluation row) to be explained shortly. The NER entries in the first sub-table are simply the profit/unit figures as read from the objective function or copied from heading 5. *However there is a more rigorous method of making these entries which alone has to be followed in making entries in the subsequent NER's and explanation on which is defferred.* Selection of the incoming variable is simple indeed. We select the one with highest entry in NER and this happens to be x_2 with a value of 4. We note this selection by putting an arrow below 4 in the NER of sub-table I. The column with the arrow is known as the key column.

Step II. Select the outgoing variable. For this, we compute the replacement ratios of col. 6 This is done by dividing col. 4 by the key column and we get $\frac{16}{3}$ and 8 as the ratios. The variable x_3 against the least ratio of $\frac{16}{3}$ is selected as the outgoing variable and the fact is noted by putting an arrow against the least ratio of $\frac{16}{3}$. This row with the arrow is called the key row. The element at the interection of the key row and the key column is known as the pivot or key element and it is encircled.

Step III. Having noted the incoming variable (x_2) and the outgoing variable (x_3) we are ready to perform the row operations on sub-table I and fill in column 4 and 5 of sub-table II.

Before that, however, we fill in cols. 2 and 3 of sub-table II which is straightforward; x_3 is replaced by x_2 in col. 2, x_4 stays in it and in col. 3 the profit/unit figures are copied from heading 5.

Now let us take up the row operations. The key row of sub-table I (under cols. 4 and 5) is divided by the pivot element and this becomes the corresponding row of the sub-table II. It reads $\frac{16}{3}$, $\frac{2}{3}$, 1, $\frac{1}{3}$, 0.

The fixed ratio of the non-key (row) is (are) derived by dividing the element (s) in the key column by the pivot element. It (they) is (are) entered in the non-key row (s) under column I in the sub-table I itself. The key row (s) in sub table I is multiplied by its fixed ratio. This leads to

$$16\times\frac{2}{3}=\frac{32}{3},\ 2\times\frac{2}{3}=\frac{4}{3},\ 3\times\frac{2}{3}=2,\qquad 1\times\frac{2}{3}=\frac{2}{3},\ 0\times\frac{2}{3}=0.$$

This, however, is a rough work. Entries are not made any where. The result of this multiplication are subtracted from the non-key row of sub-table I to yield the non-key row of the sub-table II as below

$$16-\frac{32}{3}=\frac{16}{3},\ 4-\frac{4}{3}=\frac{8}{3},\ 2-2=0,\ 0-\frac{2}{3}=\frac{-2}{3},\ 1-0=1.$$

These entries are made for the non key row of sub-table II.

Step IV. This consists of deriving the NER of sub-table II. Each of its elements is derived by multiplying col. 3 of sub-table II with col. 5, summing these up and subtracting the sum from the profit/unit in heading 5—computations for each element in the 2nd NER are shown below.

$$3-\left(4\times\frac{2}{3}+0\times\frac{8}{3}\right)=\frac{1}{3}$$

$$4-(4\times1+0\times0)=0$$

$$0-\left(4\times\frac{1}{3}+0\times\frac{2}{3}\right)=-\frac{4}{3}$$

$$0-(4\times0+0\times1)=0.$$

But what is the underlying logic ? The constraints in the sub-table II have become.

$$\frac{2}{3}x_1+1x_2+\frac{1}{3}x_3+0x_4=\frac{16}{3}$$

$$\frac{8}{3}x_1+0x_2-\frac{2}{3}x_3+1x_4=\frac{16}{3}$$

i.e.,

$$x_2=\frac{16}{3}-\frac{2}{3}x_1-\frac{1}{3}x_3 \qquad \ldots(i)$$

$$x_4=\frac{16}{3}-\frac{8}{3}x_1+\frac{2}{3}x_3 \qquad \ldots(ii)$$

Substituting these values of x_2 and x_4 in the original objective function

$$Z=3x_1+4x_2+0x_3+0x_4$$

we get the following modified function,

$$Z=3x_1+4\left\{\frac{16}{3}-\frac{2}{3}x_1-\frac{1}{3}x_3\right\}$$

$$+0x_3+0\left\{\frac{16}{3}-\frac{8}{3}x_1+\frac{2}{3}x_3\right\}$$

$$= x_1\left\{3 - \frac{2}{3}\times 4 - \frac{8}{3}\times 0\right\} + 0x_2 + x_3\left\{0 - 4 \times \frac{1}{3} + 0\times\frac{2}{3}\right\} + 0x_4$$

i.e., $$Z = \frac{1}{3}x_1 + 0x_2 - \frac{4}{3}x_3 + 0x_4 + \frac{64}{3}$$

These coefficient have been entered in the 2nd NER.

[This might appear to be a mathematical jugglery but there is a sound interpretation to this. The reason for this becomes clear if we keep in mind that $\frac{1}{3}x_1 + 0x_2 - \frac{4}{3}x_3 + 0x_4 + \frac{64}{3}$ becomes the modified objective function corresponding to the solution stage given by the programme 2 *i e.*, sub table II, that has fully utilised the machining capacity. Thus the production of 1 unit of X_1 at this stage will mean (according to the 1st original constraint $2x_1+3x_2\leqslant 16$) that 2 units of machining capacity (required for producing 1 unit of X_1) must be reallocated from X_2. Since each unit of X_2 requires 3 units of machining capacity current level of production will thus be reduced by $\frac{2}{3}$ units which will mean reduction of Rs. $\frac{8}{3}$ $\left(= 4\times\frac{2}{3}\right)$ in the contribution. Thus, introduction of 1 unit of X_1 at this stage results in a decrease of Rs. $\frac{8}{3}$ in the profit contribution of X_2. Hence the introduction of 1 unit of X_1 at this stage will give a net profit *i e.*, of $3 - \frac{8}{3}$ = Rs. $\frac{1}{3}$. Similar interpretation can be made for the other coefficients in the modified objective function].

By applying these 4 steps on sub-table II we derive sub-table III. Since its NER does not have any positive element *i.e.* opportunity cost we stop here. This is an indication of the optimal solution with $x_1 = 2$ and $x_2 = 4$ (and $Z = 22$), for if we derive another sub-table by bringing in x_3 or x_4 into the solution we shall decrease the profit. To sum up, when, in a maximization problem, all the elements in an NER are negative or zero we stop further iterations. The student may himself derive sub-table III as an exercise.

It is obvious that the student needs a plenty of practice to be thorough with the computationally tedious, though otherwise very elegant and versatile, simplex method. *As a tip, the fractions in the simplex tables must be retained and by trying to decimalise them the student may land himself in trouble.* Numerous examples and exercises follow to give understanding and practice to the students.

Example 5 : Maximise $Z = 2x_1 + 5x_2$

subject to $x_1 + 4x_2 \leqslant 24$

$3x_1 + x_2 \leqslant 21$

$x_1 + x_2 \leqslant 9$

$x_1, x_2 \geqslant 0$

Solution. By introducing slack variables S_1, S_2, and S_3 the problem becomes

Maximise $2x_1+5x_2+0S_1+0S_2+0S_3$

Subject to $x_1+4x_2+S_1=24$

$3x_1+x_2+S_2=21$

$x_1+x_2+S_3=9$

$x_1, x_2, S_1, S_2, S_3 > 0$.

The simplex tables in this case are :

FR $=\frac{1}{4}$, $\frac{1}{4}$

Programme	Profit/Unit	Qty.	2	5	0	0	0	Ratio
			x_1	x_2	S_1	S_2	S_3	
S_1	0	24	1	4	1	0	0	6←
S_2	0	21	3	1	0	1	0	21
S_3	0	9	1	1	0	0	1	9
			2	5	0	0	0	
				↑				

F.R. $=\frac{1}{3}$, $\frac{11}{3}$

x_2	5	6	$\frac{1}{4}$	1	$\frac{1}{4}$	0	1	24
S_2	0	15	$\frac{11}{4}$	0	$\frac{-1}{4}$	1	0	$\frac{60}{11}$
S_3	0	3	$\frac{3}{4}$	0	$\frac{-1}{4}$	0	1	4←
			↑ $\frac{3}{4}$	0	$\frac{-5}{4}$	0	0	

x_2	5	5	0	1	$\frac{1}{3}$	0	$-\frac{1}{3}$	NER elements are all non-positive
S_2	0	4	0	0	$\frac{2}{3}$	1	$-\frac{11}{3}$	
x_1	2	4	1	0	$-\frac{1}{3}$	0	$\frac{4}{3}$	

Hence $\left.\begin{array}{l} Z_{max}=5\times5+4\times2=33 \\ x_1=4\ ;\ x_2=5 \end{array}\right\}$ **Answer**

Example 6. An electronics firm is undecided as to the most profitable mix for its products, The products now manufactured are transistors, resistors and carbon tubes with a profit (per 100 units) of Rs. 10, Rs. 6 and Rs. 4 respectively. To produce a shipment of transistors containing 100 units requires 1 hour of engineering, 10 hours of direct labours and 2 hours of administration service. To produce 100 resistors are required 1 hour, 4 hours and 2 hours of engineering, direct labour and administrative time respectively. To produce one shipment of the tubes (100 units) requires 1 hour of engineering, 5 hours of direct labour and 6 hours of administration. There are 100 hours of engineering services available, 600 hours of direct labour and 300 hours of administration. What is the most profitable mix ?

Solution. Let us tabulate data in convenient manner.

	Products			
	Transistors	Resistors	Carbon Tubes	
Engineering	1	1	1	100
Labour	10	4	5	600
Administration	2	2	6	300
	10	6	4	

Symbolically, maximise $10x+6y+4z$

Subject to

$$1x+1y+1z<100 \text{ and } x\geq 0$$
$$10x+4y+5z<600 \qquad y\geq 0$$
$$2x+2y+6z<300 \qquad z\geq 0$$

Introducing slak variables

Maximise $10x+6y+4z$

Subject to

$$x+y+z+S_1=100$$
$$10x+4y+5z+S_2=600$$
$$2x+2y+6z+S_3=300$$
$$x\geq 0,\ y\geq 0,\ z\geq 0,\ S_1\geq 0,\ S_2\geq 0,\ S_3\geq 0$$

	Programme	Profit/Unit	Qty:	10	6	4	0	0	0	
				x	y	z	S_1	S_2	S_3	
F.R.=1/10	S_1	0	100	1	1	1	1	0	0	100
	S_2	0	600	10	4	5	0	1	0	60←
1/5	S_3	0	300	2	2	6	0	0	1	150
	Net Evalution Row			10	6	4	0	0	0	

↑

	S_1	0	40	0	6/10	5/10	1	−1/10	0	400/6=67←
F.R.=2/3	x	10	60	1	4/10	5/10	0	1/10	0	600/4=150
F.R.=2	S_3	0	180	0	12/10	5	0	−2/10	1	150
	Net Evalution Row			0	2	−1	0	−1	0	

↑

y	6	400/6	0	1	5/6	10/6	−1/6	0
x	10	100/3	1	0	1/6	−2/3	1/6	0
S_3	0	100	0	0	4	−2	0	1
			0	0	−ve	−ve	−ve	0

Hence the most profitable mix is $\frac{400}{6}$ resistors and $\frac{100}{3}$ transistors. The maximum profit is $400+\frac{1000}{3}=733\ \frac{1}{3}$.

Minimisation (Revised Simplex Method).

The simplex algorithm applies to both maximisation and minimisation problems. The only difference in the algorithm involves the selection of the incoming variable. In the maximisation problem it is the one highest +ve NER element. Conversely, it is the most −ve variable that is selected as the incoming variable in a minimisation problem. And if all elements in the NER are either positive or zero it is the indication for the optimal solution and we stop there.

Let us take an example straightway to explain the solution procedure.

Example 7. A small township of 15,000 people requires, on the average, 300,000 gallons of water daily. The city is supplied from a central water-works *where the water is purified by such conventional methods as filteration and chlorination. In addition, two different chemical compounds : (i) softening chemical and (ii) health chemical, are needed for softening the water and for health* purposes. The waterworks plans to purchase two popular brands that contain these chemicals. One unit of Chemico Corporation's product gives 8 pounds of softening chemical and 3 pounds of health chemical. One unit of Indian Chemical's product contains 4 pounds and 9 pounds per unit, respectively, for the same purposes.

To maintain the water at a minimum level of softness and to meet a minimum in health protection, experts have decided that 150 and 100 pounds of the two chemicals that make up each product must be added to water daily. At a cost of Rs. 8 and Rs. 10 per unit respectively for Chemico's and Indian Chemical's products what is the optimal quantity of each product that should be used to meet the minimum level of softness and a minimum health standard ?

Solution. The relevant data may be tabulated as below :

Chemical	Brand Chemico	 Indian	Daily Requirement
(*i*) Softening	8	4	150
(*ii*) Health	3	9	100
Cost/unit of each brand	8	10	

Let us formulate the LPP as below

Minimise (cost), $Z=8x+10y$

Subject to

$$8x+4y \geqslant 150$$
$$3x+9y \geqslant 100$$

and $\quad x \geqslant 0, y \geqslant 0.$

Subtracting surplus variables to convert inequalities into equalities

$$8x+4y-S_1=150$$
$$3x+9y-S_2=100$$

Physical Interpretation of Surplus Variables

S_1 and S_2 represent the extra units (if any) of the two chemicals over 150 and 100 units respectively. Their magnitude being such as to just convert

the inqualities into equalities, thereby restricting them to non-negative values The problem may then be restated as below :

Minimise (cost), $Z=8x+10y+0S_1+0S_2$

Subject to $8x+4y-S_1=150$

$3x+9y-S_2=100$

and $S_1>0,\ S_2\geq 0,\ x\geq 0,\ y>0.$

Artificial Slack Variables

If x and y are set equal to zero S_1 and S_2 turn out to be negative, violating the non-negativity restriction. Therefore, to circumvent this we introduce another similar device of artifical slack variables. Let us represent these by A_1 and A_2 respectively. The problem then becomes.

Minimise (cost), $Z=8x+10y+0S_1+0S_2+MA_1+MA_2$

Subject to $\left.\begin{array}{l}8x+4y-S_1+A_1=150\\3x+9y-S_2+A_2=100\end{array}\right\}$ All variables ≥ 0

Physical Interpretation of the Artificial Variables

These are imaginary brands, each unit containing 1 unit of the pertinent chemical. Both are restricted to non-negatives. Whereas surplus variables have zeros as their cost coefficient, each artificial variable is assigned an infinitely large cost coefficient (usually denoted be M) This renders inclusion of the artificial variable in the solution prohibitively. These will therefore be out in the final table.

Designing the Initial Programme

It is obtained by letting each of the variables x, y, S_1 and S_2 assume value of zero, *i.e*, retaining artificial variables in the initial solution.

Programme	Cost per unit	Quantity	8	10	0	0	M	M	
			x	y	S_1	S_2	A_1	A_2	
A_1	M	150	8	4	−1	0	1	0	150/4
A_2	M	100	3	9	0	−1	0	1	100/9←
Net Evaluation Row			8−11M	10−13M	M	M	0	0	
				↑					

Initial Solution

It is to be noted that in the NER we do not copy the coefficients from the cost objective but derive these be step IV on page 20 as below :

$$\left.\begin{array}{ll} x & 8-8M-3M=8-11M \\ y & 10-4M-9M=10-13M \\ S_1 & 0+M-0=M \\ S_2 & 0-0+M=M \\ A_1 & M-M-0=0 \\ A_2 & M-0-M=0 \end{array}\right\} \quad A$$

The reason for this is explained below The problem is rewritten below :

Minimise $\quad Z=8x+10y+0S_1+0S_2+MA_1+MA_2$

Subject to $\quad 8x+4y-S_1+A_1=150 \quad \ldots(i)$

$\quad 3x+9y-S_2+A_2=100 \quad \ldots(ii)$

all variables≥ 0.

From (i) $\quad A_1=150-8x-4y+S_1$

From (ii) $\quad A_2=100-3x-9y+S_2$

Substituting these in the objective function,

$$Z=8x+10y+0S_1+0S_2+M(150-8x-4y+S_1)+M(100-3x-9y+S_2)$$
$$(8-11M)x+(10-13M)y+MS_1+MS_2+0A_1+0A_2.$$

These coefficients correspond to set A above.

Now consider the inital solution with an initial cost of (150+100M) for further iterations.

10−13M is the most−ve and, therefore, the column under 'y' is the key column. Also, the ratio 100/9 is less than 150/4 (on extereme right). The pivot element, then, is 9. The outgoing variable is A_2 being replaced by y. The cost of the existing solution, 250 M is forbiddingly high. The stage is set for revising the initial programme.

The key row is revised by dividing it through by 9, the pivot element and the results are as follows :

$$\frac{100}{9},\ \frac{3}{9},\ 1, 0,\ \frac{-1}{9},\ 0,\ \frac{1}{9}.$$

The fixed ratio for revising the non-key row is $\frac{4}{9}$. The revised figures for this row are computed below :

$$150-100\times\frac{4}{9}=\frac{1350-400}{9}=\frac{950}{9}$$

$$8-3\times\frac{4}{9}=\frac{75-12}{9}=\frac{60}{9}$$

$$4-9\times\frac{4}{9}=0$$

$$-1-0=-1$$

$$0+\frac{4}{9}=\frac{4}{9}$$

$$1-0=1$$

$$1-0\times\frac{4}{9}=\frac{-4}{9}$$

The new table is shown below :

Programme	Cost/unit	Quantity	8	10	0	0	M	M	
			x	y	S_1	S_2	A_1	A_2	
A_1	M	950/9	60/9	0	−1	4/9	1	−4/9	$\frac{950}{60}$ ←
y	10	100/9	3/9	1	0	−1/9	0	1/9	100/3
Net Evaluation Row			42/9− 60M/9	0	M	10/9− 4M/9	0	$\frac{13M-10}{9}$	

Revising the key row by dividing it through by $\frac{60}{9}$, *i.e.*, multiply by $\frac{9}{60}$ we get

$$\frac{950}{60},\ 1,\ 0,\ \frac{-9}{60},\ \frac{4}{60},\ \frac{9}{60},\ \frac{-4}{60}$$

For the non-key row the fixed ratio is $\frac{3}{9}\div\frac{60}{9}=\frac{3}{60}$ Revising the non-key row.

$$\frac{100}{9}-\frac{950}{9}\times\frac{3}{60}=\frac{35}{6}$$

$$\frac{3}{9}-\frac{60}{9}\times\frac{3}{60}=0$$

$$1-\frac{3}{60}\times0=1$$

$$0+1\times\frac{3}{60}=\frac{1}{20}$$

$$-\frac{1}{9}-\frac{4}{9}\times\frac{3}{60}=\frac{-2}{15}$$

$$0-\frac{3}{60}=\frac{-1}{20}$$

$$\frac{1}{9}+\frac{4}{9}\times\frac{3}{60}=\frac{2}{15}$$

The results are tabulated below :

Programme	Cost per unit	Quantity	8	10	0	0	M	M
			x	y	S_1	S_2	A_1	A_2
x	8	95/6	1	0	−3/20	1/5	3/20	−1/15
y	10	35/6	0	1	1/20	−2/15	−1/20	2/15
NER (all non-negative)			0	0	$\frac{7}{10}$	$\frac{4}{5}$	$M-\frac{7}{10}$	$M-\frac{4}{5}$

Hence the optimal cost is $\frac{95}{6}\times 8+\frac{35}{6}\times 10$

$$=\frac{760+350}{6}$$

$$=\frac{1110}{6}=185$$ **Answer.**

Some Remarks

1. It may be desired to convert a maximisation problem into a minimisation one and *vice versa*. Mathematically, this can be accomplished by reversing signs through of *just* the objective funntion.
2 *Inequalities in the wrong direction* : Consider the problem :

$$\text{Maximise } Z=x_1+5x_2$$

$$\text{Subject to } 3x_1+4x_2\leqslant 6 \qquad \ldots(i)$$

$$x_1+3x_2\geqslant 2 \qquad (ii)$$

$$x_1,\ x_2\geqslant 0$$

[Whether to introduce slack or surplus artificial variables depends on the type of inequality and has nothing to do with the type of the problem *i e*, maximisation or minimisation].

The 2nd inequality is in the *wrong* direction. Upon introducing the "surplus" variable.

$$x_1+3x_2-S_2=2.$$

If S_2 is taken in the initial solution it would be −ve when x_1 and x_2 are zero. To circumvent this an artificial variable is also introduced in this inequality. The problem becomes :

$$\text{Maximise } Z=x_1+5x_2-MA_2$$

$$\text{Subject to } 3x_1+4x_2+S_1=6$$

$$x_1+3x_2-S_2+A_2=2$$

$$x_1,\ x_2,\ S_1,\ S_2,\ A_2\geqslant 0.$$

The initial solution consists of S_1 and A_2. Several examples on inequalities in the wrong direction follow. Surplus variable can never come in initial solution.

(Note that in maximisation problems M always has — ve sign and in minimisation problems M always has a + ve sign)

3. For minimisation problems containing < inequalities and/or equalities example 12 should suffice

4 It is to be carefully noted that except the artificial variables. others, once driven out in an iteration, may re-enter in a subsequent iteration. But, an artificial variable, once driven, can never re-enter, because of the big M coefficient associated with it in the objective function. Advantage can be taken of this fact by not computing its column in iteration subsequent to the one from which it was driven out. See example 8 in this connection

5. In some problems, if there is an equality, it pays to eliminate one variable (say, x_i) from all the inequalities and treat it is a slack variable for equality. Big M method may then by employed. Example 13 should clarify this point further. Prior to this, however, example 9 is intended to show the possibility of treating the very existing variable as a slack variable

6 Any linear programming problem can be re-formulated into what is known as its dual Any of the primal (the original) or the dual may be selected for iterating by the simplex method The selection is made on the basis af computational burden. Also the dual provides interesting insights into the methodology of the LPP solution. This matter is discussed in a following section at a greater length

7. If two or more variables share the maximum positive coefficient in the net evaluation row any one may be chosen for introduction for the new solution arbitrarily, *viz.*, in $Z=2x_1+2x_2+x_3$ it matters little if x_1 or x_2 is chosen.

8. Lower bounds may be specified in an LPP. For example, over and above to the three usual constraints, it may be stipulated that x_1 cannot be less than 25 or 40 or l_1, *i e.* $x_1 \geq l_1$. This can be handled quite easily by introducing a variable y_1 such that $x_1=l_1+y_1$ Substitute $x_1=l_1+y_1$ wherever it occurs and solve the LPP. Computations would be greatly reduced. Please see example 14 in this connection.

9. In all the simplex tables there is bound to be a unit matrix of size $p \times p$ where p is the no of rows (excluding net elevation row). The columns that constitute such a unit matrix need not be adjacent.

10. In view of the tediousness of computational aspects it is useful to make a check at each iteration. This can be done by deriving the net evaluation row in two ways : (*i*) just like any other row in the simplex tableau by deriving its fixed ratio (*ii*) by summing the product of the quantities column with the profit/cost column, and subtracting this sum from the original profit contribution or cost coefficient of the variable. These should tally. Also, having obtained the optimal solution it is desirable to verify it if it obeys the given constraints. See example 9 where NER is calculated is both the ways.

11. The simplex method, the revised simplex method, the graphical and trial and error merhods, the dual approach provide sevreral ways of doing an LPP. The student may want to do each LPP in more than one way for the sake of verification of the answer and practice.

12. For degnerate problems (tie in replacement ratios) see example 19.

Example 8 : Minimise $Z=4x_1+2x_2$

Subject to $3x_1+x_2 \geqslant 27$

$-x_1-x_2 \leqslant -21$

$x_1+2x_2 \geqslant 30$

$x_1, x_2 \geqslant 0$

Solution : The second 'equation' is the less than inquality which can be converted to more than inequality by reversing all the sings on both the sides. Introducing surplus and artifical variables the system of Inequalities reduces to

Minimise $Z=4x_1+2x_2+0S_1+0S_2+0S_3+MA_1+MA_2+MA_3$

Subject to $3x_1+x_2-S_1+A_1=27$

$x_1+x_2-S_2+A_2=21$

$x_1+2x_2-S_3+A_3=30$

$(x_1 \geqslant 0 \; ; x_2 \geqslant 0 \, , S_1 \geqslant 0 \, , S_2 \geqslant 0 \, , S_3 \geqslant 0 \, , A_1 \geqslant 0 \, , A_2 > 0 \, , A_3 \geqslant 0)$

	Programme	Cost per Unit	Qty.	4	2	0	0	0	M	M	M	
				x_1	x_2	S_1	S_2	S_3	A_1	A_2	A_3	
F.R. $=\frac{1}{3}$	A_1	M	27	3	1	−1	0	0	1	0	0	9←
	A_2	M	21	1	1	0	−1	0	0	1	0	21
F.R. $=\frac{1}{3}$	A_3	M	30	1	2	0	0	−1	0	0	1	30
Net evaluation Row				↑ 4−5M	2−4M	M	M	M	0	0	0	

	Programme	Cost per Unit	Qty.	4	2	0	0	0	M	M	M	
				x_1	x_2	S_1	S_2	S_3	A_1	A_2	A_3	
F.R. $=\frac{1}{5}$	x_1	4	9	1	1/3	−1/3	0	0	left blank*	0	0	27
F.R. $=\frac{2}{5}$	A_2	M	12	0	2/3	1/3	−1	0		1	0	18
	A_3	M	21	0	5/3	1/3	0	−1		0	1	63/5←
	Net Evaluation Row			0	$\frac{2}{3} - \frac{7}{8}M$	$\frac{4}{3} - 2/3M$	M	M		0	0	

*See point 4 on page 30

				x_1	x_2	S_1	S_2	S_3	A_1	A_2	A_3	
	x_1	4	24/5	1	0	−2/5	0	1/5		0		24
	A_2	M	18/5	0	0	1/5	−1	2/5		1		9←
−3/5	x_2	2	63/5	0	1	1/5	0	−3/5		0		−21
	Net Evaluation Row			0	0	$6/5 - M/5$	M	$\frac{2}{5} - \frac{2M}{5}$ ↑		0		

x_1	4	3	1	0	$\frac{1}{2}$	$\frac{1}{2}$	0				
S_3	0	9	0	0	$\frac{1}{2}$	$\frac{-5}{2}$	1				
x_2	2	18	0	1	$\frac{1}{2}$	$\frac{-3}{2}$	0				

Net Evaluation Row : All non-negative

This provides the optimal solution with

$$x_1 = 3 \qquad \text{and} \qquad x_2 = 18$$

$$\text{Minimum cost} = 4 \times 3 + 2 \times 18 = 48 \text{ (Answer)}$$

Example 9. Minimise $\quad x_2 - 3x_3 + 2x_5$

Subject to
$$x_1 + 3x_2 - x_3 + 2x_5 = 7$$
$$-2x_2 + 4x_3 + x_4 = 12$$
$$-4x_2 + 3x_3 + 8x_5 + x_6 = 10$$
$$x_i \geq 0.$$

Solution : Equivalently, maximize $Z = -x_2 + 3x_3 - 2x_5$. It is to be seen that x_1 x_4 and x_6 can be treated as slack variables and constraints may be rearranged as

$$3x_2 - x_3 + 2x_5 + x_1 = 7$$ (See remark 1 on p. 29).
$$-2x_2 + 4x_3 + x_4 = 12$$
$$-4x_2 + 3x_3 + 8x_5 + x_6 = 10$$

Programme		Profit	Qty.	−1	3	−2	0	0	0	
				x_2	x_3	x_5	x_1	x_4	x_6	
$-\frac{1}{4}$	x_1	0	7	3	−1	2	1	0	0	−7
	x_4	0	12	−2	4	0	0	1	0	3←
3/4	x_6	0	10	−4	3	8	0	0	1	10/3
3/4				−1	+3 ↑	−2	0	0	0	
	x_1	0	10	5/2	0	2	1	$\frac{1}{4}$	0	4←
−1/5	x_3	3	3	−1/2	1	0	0	$\frac{1}{4}$	0	−6
−1	x_6	0	1	−5/2	0	8	0	−3/4	1	−2/5
1/5				$\frac{1}{2}$ ↑	0	−2	0	−3/4	0	
	x_2	−1	4	1	0	4/5	2/5	1/10	0	
	x_3	3	5	0	1	2/5	1/5	3/10	0	
	x_6	0	11	0	0	10	1	−1/2	1	
				0	0	−12/5	−1/5	−4/5	0	

Hence $x_2=4$: $x_3=5$

and Z min. $=4-15=-11$

Example 10 ; Maximise $Z=x_1+2x_2$

subject to $x_1-x_2\geqslant 3$

$2x_1+x_2\leqslant 10$

$x_1\geqslant 0;\ x_2\geqslant 0$

Solution : Introducing slack, surplus and artificial variables the problem reduces to

Maximise $Z=x_1+2x_2-Mx_4$

subject to $x_1-x_2-x_3+x_4=3$

$2x_1+x_2+x_5=10$

Programme	Profit	Qty.	1	2	0	$-M$	0	
			x_1	x_2	x_3	x_4	x_5	
x_4	$-M$	3	1	-1	-1	1	0	3←
x_5	0	10	2	1	0	0	1	5
			$M+1$ ↑	$2-M$	$-M$	0	0	
x_1	1	3	1	-1	-1	1	0	3
x_5	0	4	0	3	2	2	1	4/3←
			0	3 ↑	1	$-M-1$	0	
x_1	1	13/3	1	0	$-\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	
x_2	2	4/3	0	1	$\frac{2}{3}$	$-\frac{2}{3}$	$\frac{1}{3}$	
			0	0	-1	$-M+1$	-1	

Hence $x_1=\frac{13}{3}$, $x_2=\frac{4}{3}$; $Z=\frac{13}{3}+\frac{8}{3}=7$ (Answer).

Example 11. Minimise $3x_1-x_2$

$$2x_1+x_2\geqslant 2$$
$$x_1+3x_2\leqslant 3$$
$$x_2\leqslant 4$$
$$x_1,\ x_2\geqslant 0$$

Introducing slack, surplus and artificial variables,

Minimise $\quad 3x_1-x_2+Mx_4$

subject to $\quad 2x_1+x_2-x_3+x_4=2$

$$x_1+3x_2+x_5=3$$
$$x_2+x_6=4$$

All variables $\geqslant 0$.

Solution

Fixed Ratio	Prog-ramme	Cost	Qty.	3	−1	0	M	0	0	Replacement Ratio
				x_1	x_2	x_3	x_4	x_5	x_6	
	x_4	M	2	2	1	−1	1	0	0	1←
1/2	x_5	0	3	1	3	0	0	1	0	3
0	x_6	0	4	0	1	0	0	0	1	∞
				3−2M ↑	−1−M	M	0	0	0	
1/5	x_1	3	1	1	1/2	−1/2	1/2	0	0	2
	x_5	0	2	0	5/2	1/2	−1/2	1	0	4/5→
2/5	x_6	0	4	0	1	0	0	0	1	4
				0	−5/2 ↑	3/2	M−3/2	0	0	
	x_1	3	3/5	1	0	−3/5	3/5	−1/5	0	
	x_2	−1	4/5	0	1	1/5	−1/5	2/5	0	
	x_6	0	16/5	0	0	−1/5	1/5	−2/5	1	
				0	0	2	M−2	1	0	

Hence $x_1=3/5$

$x_2=4/5$

$Z_{min}=1$ **(Answer)**

Note : One or more of the constraints could be given as an equality rather than an inequality in right or wrong direction. There are 2 alternative methods for attacking such problems as demonstrated in the following couple of examples. In the first method, we used an artificial variable in the equality with confidence that it will be driven out ultimately because of M in the objective function, thereby reatining intact the implication of the equality. This method is followed in the next example.

Example 13 follows the remark 5 on page 30. If, however, the student so wishes he can skip over the 2nd method and example 13.

Example 12. *Minimise* $Z=30x_1+20x_2$, *subject to*

$$x_1+x_2\leqslant 8,\ 6x_1+4x_2\geqslant 12,\ 5x_1+8x_2=20, \text{ and } x_i\geqslant 0$$

Solution : Introduction slack, surplus and artifical variables the restrictions become

$$x_1+x_2+x_3=8$$

$$6x_1+4x_2-x_4+1A_1=12$$

$$5x_1+8x_2+1A_2=20$$

	Programme	Cost	Qty.	30 x_1	20 x_2	0 x_3	0 x_4	M A_1	M A_2	
1/8	x_3	0	8	1	1	1	0	0	0	8
1/2	A_1	M	12	6	4	0	−1	1	1	2
	A_2	M	20	5	8	0	0	0	0	5/2←
				30−11 M	↑ 20−12 M	0	M	0	0	
3/28	x_1	0	11/2	3/8	0	1	0	0	−1/8	44/3
	A_1	M	2	7/2	0	0	−1	1	−1/2	4/7←
5/28	x_2	20	5/2	5/8	1	0	0	0	1/8	16/25
				$\frac{-7}{2}M - \frac{140}{8}$ ↑	0	0	M	0	$\frac{3}{2}M - \frac{20}{8}$	
	x_3	0	74/14	0	0	1	3/28	−3/28	−4/56	
	x_1	30	4/7	1	0	0	−2/7	2/7	−1/7	
	x_2	20	15/7	0	1	0	5/28	−5/28	3/15	
				0	0	0	5	M−5	M	

Example 13 : Maximise $Z=4x_1+5x_2-3x_3$

Subject to $x_1+x_2+x_3=10$ (*i*)

$x_1-x_2\geqslant 1$ (*ii*) (See point 5 on p. 30)

$2x_1+3x_2+x_3\leqslant 40$ (*iii*)

$x_1, x_2, x_3\geqslant 0$

Solution : Subtracting (*i*) from (*iii*) with a view to eliminate x_3 in the latter and retaining x_3 as a slack variable for the former, the restrictions are modified as follows. Also, introduced are slack, surplus and artificial variables.

$$x_1+x_2+x_3=10$$
$$x_1-x_2-x_4+x_5=1$$
$$x_1+2x_2+x_6=30$$
$$Z=4x_1+5x_2-3x_3-Mx_5$$

Programme		Profit	Quantity	4	5	−3	0	−M	0	Ratio
				x_1	x_2	x_3	x_4	x_5	x_6	
1	x_3	−3	10	1	1	1	0	0	0	10
	x_5	−M	1	1	−1	0	−1	1	0	1←
1	x_6	0	20	1	2	0	0	0	1	30
				7+M ↑	8−M	0	−M	0	0	
	x_3	−3	9	0	2	1	1	−1	0	9/2←
−½	x_1	4	1	1	−1	0	−1	1	0	−ve
3/2	x_6	0	19	0	3	0	1	−1	1	29/3
				0	15 ↑	0	4	-M-7	0	
	x_2	5	9/2	0	1	½	½	−½	0	
	x_1	4	11/2	1	0	½	−½	½	0	
	x_6	0	11/2	0	0	−3/2	−½	½	1	
				0	0	−15/2	−½	-M+½	0	

Hence $x_1=\frac{11}{2}$, $x_2=\frac{9}{2}$.

Example 14 : A manufacturer of three products tries to follow a policy of producing those which contribute most to fixed cost and profit. However, there is also a policy of recognising certain minimum sales requirements. Currently these are ;

Product	*Units per week*
X	*20*
Y	*30*
Z	*60*

There are three producing departments. The product times in hour per unit in each department and the total times available for each week in each department are :

	Time required per product, in hours			*Total hours available*
Product :	X	Y	Z	
Department 1	0.25	0.20	0.15	420
2	0.30	0.40	0.50	1,048
3	0.25	0.30	0.25	529

The contribution per unit of product X, Y, Z is Rs. 10.50, Rs. 9.00 and Rs. 8.00 respectively. The company has scheduled 20 units of X, 30 units of Y and 60 units of Z for production in the following week.

Your are required to state :

(*a*) whether the present schedule is an optimum one from a profit point of view and if it is not, what it should be ;

(*b*) The recommendations that should be made to the firm about their production facilities [following the answer to (*a*) above].

Solution : Let us formulate the L.P.P.

Objective : Maximise $10.50X+9.00Y+8.00Z$

Subject to : $0.25X+0.20Y+0.15Z\leqslant 420$

$0.30X+0.40Y+0.50Z\leqslant 1048$

$0.25X+0.30Y+0.25Z\leqslant 529$

$0\leqslant X\geqslant 20$ $\quad$ $0\leqslant X$ means

$0\leqslant Y>30$ $\quad$ $X>0$, etc.

$0\leqslant Z>60$

Since the company is already producing minimum of Y and Z it should, at least, produce maximum of X limited by the first constraint. It can be arithmetically verified that there is a scope for improvement.

Substituting (Please refer to point 8 on P 30) ;

$$X=x_1+20$$
$$Y=x_2+30$$
$$Z=x_3+60$$

This problem becomes

Maximise $10.50x_1+9.00x_2+8.00x_3+$a constant

Subject to $0.25x_1+0.20x_2+0.15x_3 \leqslant 400$

$0.30x_1+0.40x_2+0.50x_3 \leqslant 1000$

$0.25x_1+0.30x_2+0.25x_3 \leqslant 500$

$x_1 \geqslant 0$; $x_2 \geqslant 0$; $x_3 \geqslant 0$

The student may now derive the optimal soultion, as an exercise.

Some special cases

1. No Feasible Solution

Example 15. Maximise $z=x_1+2x_2$ (See figure 7 on p. 40)

Subject to $x_1+x_2 \leqslant 4$

$x_1+x_2 \geqslant 6$

$x_1, x_2 \geqslant 0$

It is obvious the two constraints are mutually exclusive and therefore there cannot be any feasible solution to this problem. In more practical problems, however, the constraints may be far too numerous and it would be difficult indeed to sort out this matter at a cursory glance. Consider the simplex approach.

Maximise $z=x_1+2x_2$

Subject to $x_1+x_2 \leqslant 4$, *i.e.*, $x_1+x_2+x_3=4$

$x_1+x_2 \geqslant 6$, *i.e.*, $x_1+x_2-x_4+A_1 \geqslant 6$

$x_1, x_2, x_3, x_4, A_1 \geqslant 0$

Programme	Profit	Quantity	1	2	0	0	$-M$
			x_1	x_2	x_3	x_4	A_1
x_3	0	4	1	1	1	0	0 4/6=4←
A_1	$-M$	6	1	1↓	0	-1	1 6/1=6
			$1+M$	$2+M$	0	$-M$	0
x_2	2	4	1	1	0	0	0
A_1	$-M$	2	0	0	-1	-1	1
			-1	0	$-M$	$-M$	0

Since A_1 persists in the "solution", *i.e.*, even though the net evaluation row contains all negatives A_1 has not been removed, there, therefore, is no feasible solution to this problem.

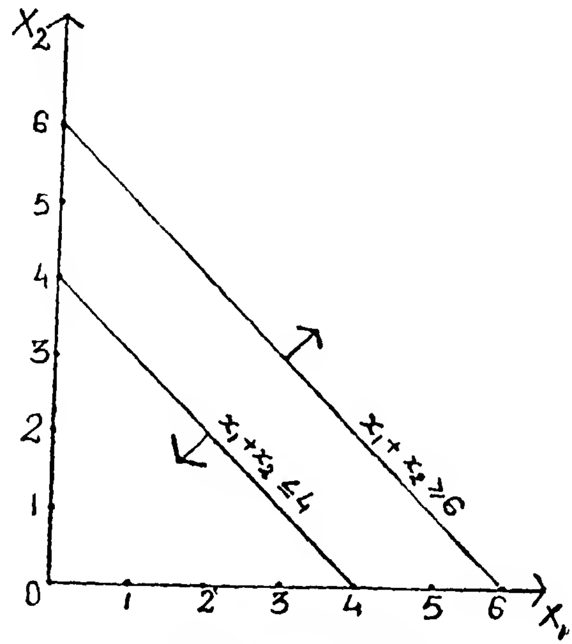


Fig. 7 No Feasible Region

2. **Multiple Optimal Solutions** are obtained, if in the graphic solution (see Fig. 8 on p. 41) the profit line is parallel to one of the straight lines representing the constraints. Thus CD is the extreme line in this case and not just an extreme point. Any point on CD give optimal solution.

In this simplex final table this situation is indicated by a non-basic variable having zero in the net evaluation row. Consider the following problem.

Example 16. Maximise $Z=3x_1+5x_2+5x_3$

Subject to $0.1x_1+0.25x_2+0x_3 \leq 120$

$0.20x_1+0.30x_2+0.40x_3 \leq 260$

$x_1, x_2, x_3 > 0$

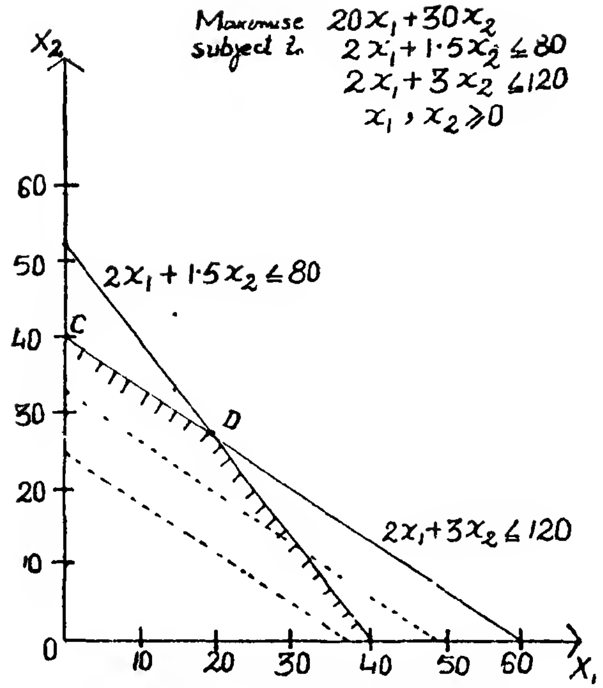


Fig. 8. Multiple Optimal Solution,
(Profit lines is parallel to 2nd constraint).

			3	5	5	0	0	
Programme	profit	Quantity	x_1	x_2	x_3	x_4	x_5	
x_4	0	120	0.10	0.25	0	1	0	
x_5	0	260	0.20	0.30	0.40 ↑	0	1←	
			3	5	5	0	0	
x_4	0	120	0.10	0 25	0	1	0←	
x_3	5	650	0 50	0.75 ↑	1	9	2.50	
			0.50	1.25	0	0	—12.50	
x_2	5	480	0.40	1	0	4	0	1200
x_3	5	290	0.20	0	1	—3	2.50	1450

Zero under a non-basic variable x_1 in the final net evaluation row indicates that the problem has multiple (infinite) solutions. If x_1 were to be brought into the solution in place of x_3 the following table would result.

x_1	1200	1	2.50	0	10	0
x_2	500	0	—0.50	1	—5	2.50
		0	0	0	—5	12.50

Thus we have infinite optimal sulutions as below :

	1	2	General Solution
x_1	0	1200	$p\times 0+1200(1-p)$
x_2	480	0	$480p+0(1-p)$
x_3	290	500	$290p+500(1-p)$
x_4	0	0	$0p+0(1-p)$
x_5	0	0	$0p+0(1-p)$

Any arbitray value of p would provide the same optimal profit.

3 **Unrestricted Variables** can arise in a few situations. For example, a negative value of a variable may imply increase in inventory. Consider the following problem :

Example 17 : Maximise $z=3x_1+5x_2$

Subject to
$$2x_1+5x_2\leqslant 132$$
$$3x_1+2x_2\leqslant 100$$
$x_2\geqslant 0$, x_1 *unrestricted in sign*

Replace x_1 by $x_1=y_1-z_1$, yielding thereby

Maximise $Z=3y_1-3z_1+5x_2$

Subject to
$$2y_1-2z_1+5x_2\leqslant 132$$
$$3y_1-3z_1+2x_2\leqslant 100$$
$$x_2,\ y_1\ z_1\geqslant 0.$$

The *initial* table is given below :

Programme	Profit	Quantity	3	—3	5	0	0
			y_1	z_1	x_2	x_3	x_4
x_3	0	132	2	−2	5	1	0
x_4	0	100	3	—3	2	0	1
			3	—3	5	0	0

(By referring to examples 14 and 17 do excrise 6 at the end of the study).

Unbounded Solution

Example 18. Consider the LPP : Maximise $z=x_1+2x_2$

Subject to $-x_1+x_2 \leqslant 2$

$x_1+x_2 \geqslant 4.$

$x_1, x_2 \geqslant 0.$

Refer to figure 9 below. There are two extreme points on the feasible region. But the objective function can be increased indefinitly. There is a fundamental simplex theorem that states the optimal value of the objective function occurs at an extreme point of the feasible region, provided that a unique finite optimal solution exists.

Such a situation may be diagnosed in the *final* simplex table as below :

			1	2	0	0	−M	
Programme	Profit	Quantity	x_1	x_2	S_1	S_2	A_1	
x_2	2	3	0	1	½	−½	½	−6
x_1	1	1	1	0	−½	−½	½	−6
	NER		0	0	−½	3/2	−M−3/2	

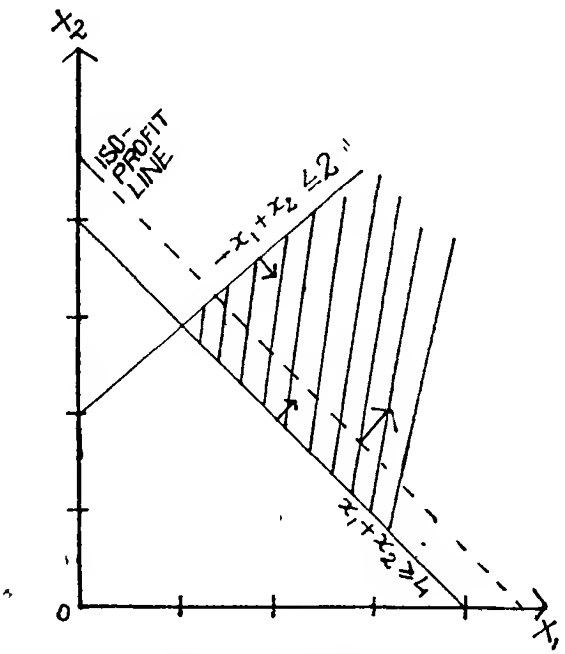


Fig. 9. Unbounded Solution.
(Feasible Region extends indefinitely)

S_2 should enter the basis. But it has negative elements in this table. The ratio on R H.S. are therefore also negative. Such a problem has unbounded solution.

Example 19. (Degeneracy)

Maximise

$$z=3x_1+9x_2$$

subject to the constraints :

$$x_1+4x_2\leqslant 8$$
$$x_1+2x_2\leqslant 4$$
$$x_1,\quad x_2\geqslant 0.$$

Solution :

Introudcing slack variables,

Maximise $z=3x_1+9x_2+0x_3+0x_4$

Subject to $x_1+4x_2+x_3=8$

$x_1+2x_2+x_4=4$

x_1 to $x_4\geqslant 0$.

Fixed Ratio	Programme	Profit	Qty.	3	9	0	0	Replacement Ratio
				x_1	x_2	x_3	x_4	
2	x_3	0	8	1	4	1	0	2
	x_4	0	4	1	2	0	1	2←
$\frac{9}{2}$		0		3	9 ↑	0	0	
	x_3	0	0*	−1	0	1	−2	
	x_2	9	2	$\frac{1}{2}$	1	0	$\frac{1}{2}$	
		18		$-\frac{1}{2}$	0	0	$\frac{-9}{2}$	optimal

Degeneracy occurs in an LPP when there is a tie in the lowest two+ve replacement ratios in the initial or any subsequent tables. In case of such a tie we select any one of them arbitrarily. Here we selected x_4 as the outgoing variable arbitrarily and we find that the next table provides the optimal solution. (*Please note a zero in the quantity column in the 2nd table which is bound to come in a degenerate problem solution). A zero in the initial also means degeneracy which is resolved as discussed below.

We see the case when we select x_3 as the outgoing variable.

Fixed Ratio	Programme	Profit	Qty.	3	9	0	0	Replacement Ratio
				x_1	x_2	x_3	x_4	
$\frac{1}{2}$	x_3	0	8	1	4	1	0	2←
	x_4	0	4	1	2	0	1	2
$\frac{9}{4}$				1	9 ↑	0	0	
$\frac{1}{2}$	x_2	9	2	$\frac{1}{4}$	1	$\frac{1}{4}$	0	8
	x_4	0	0	$\frac{1}{2}$	0	$-\frac{1}{2}$	1	0←
$\frac{3}{2}$				$\frac{3}{4}$ ↑	0	$-\frac{9}{4}$	0	
	x_2	9	2	0	1	$\frac{1}{2}$	$-\frac{1}{2}$	←
	x_1	3	0	1	0	-1	2	0 persists
				0	0	$-\frac{3}{2}$	$-\frac{3}{2}$	

In the first table, we took x_3 as the outgoing variable. Since the two replacement ratios were equal in this table there occurs a zero replacement ratio in the 2nd

table. We always select this zero replacement ratio as the least and hence x_4 is the outgoing variable in the 2nd table. *Zero replacement ratio* persists in the 3rd table (*which incidentally provides the optimal solution*. Had it not been optimal **we would not have accepted now the persisting zero ratio but the non-zero, least+ve one of the next iteration**).

Marginal value of a resource.

Let us pick up Example 1 to explain this concept. In that example, there are two resources machining and assembly The optimal tableau of that example is reproduced below. We seen in its NER $-5/4$ and $-1/8$ below x_3 and x_4. The marginal value of the machining and assembly resources are then respectively 5/4 and 1/8. The implication of this is that if now (at the optimal stage) we wish to bring in x_3 in the solution the total profit will be reduced from 22 (the optimal) by 5/4 times the no of units of X_3 brought in the programme. This is demonstrated below where we bring x_3 in the programme.

Table of page 18 is reproduced below.

Fixed Ratio	Pro-gramme	Profit	Qty.	3	4	0	0	Replace ments Ratio
				x_1	x_2	x_3	x_4	

Optimal table follows (rest omitted) x_3 is brought in the next to optimal table.

	x_2	4	4	0	1	1/2	−1/4	8←
−1/2	x_1	2	2	1	0	−1/4	3/8	−8
Optimal table	NER	22*		0	0	↑ −5/4	−1/8	Bring x_3 in solution
	x_3	0	8	0	2	1	−1/2	
	x_4	3	4	1	1/2	0	1/2	
		12*		−1	5/2	0	−3/2	

*Profit has been reduced from 22 to 12 because 8 units of x_2 have been brought in the new solution, thereby the net reduction of profit, 10 naturally equals $8 \times 5/4$.

PRIMAL AND DUAL

Example 1 is restated below.

$$\begin{aligned} &\text{Maximise } Z = 3x_1 + 4x_2 \\ &\text{Subject} \quad 2x_1 + 3x_2 \leqslant 16 \\ &\qquad\qquad 4x_1 + 2x_2 \leqslant 16 \\ &\qquad\qquad x_1, x_2 \geqslant 0 \end{aligned}$$

Call this as the primal problem the meaning of which should be clear from the fact that it has a unique counterpart problem known as the dual formulated below. The rules for dual formulation also follow.

DUAL

$$\begin{aligned} &\text{Minimise } Z' = 16y_1 + 16y_2 \\ &\text{subject to} \quad 2y_1 + 4y_2 \geqslant 3 \\ &\qquad\qquad 3y_1 + 2y_2 \geqslant 4 \\ &\qquad\qquad y_1, y_2 \geqslant 0 \end{aligned}$$

Rules for obtaining the dual.

1. Select as many variables as the number of constraints in the primal problem. We select y_1 and y_2 (both of them being non-negative).

2. Objective function of the dual, Z' is obtained from R.H.S. coefficients of the primal constraints as below.

Minimise $Z' = 16y_1 + 16y_2$.

Please note that maximisation changes to minimisation (and *vice versa*) in the dual formulation.

3. Constraints of the dual would be as many in number as the number of variables in the primal. Thus each constraint of the dual is derived from the coefficients of one variable in the primal. The profit coefficient of the primal variables in the objective function consitute the R H.S. of the dual problem. As such tne constraints of the dual are gotten below.

$$\begin{aligned} 2y_1 + 4y_2 &\geqslant 3 \\ 3y_1 + 2y_2 &\geqslant 4 \\ y_1, y_2 &\geqslant 0. \end{aligned}$$

Note that the direction of the inequalities also changes during dual formulation.

In short, the dual is obtained by transposing the rows and columns of the constraint coefficients, transposing the coefficients of the objective function and

R.H.S. of the constraints, reversing the inequalities and finally minimising instead of maximising (and vice-versa).

In fact, it does not matter which of the two problem we call primal.

[The student may take the DUAL above as the primal and derive its dual. He can verify that it turns out to be Example 1 itself.].

General Statement of the Primal and the Dual

PRIMAL : Find $x_j \geqslant 0$ $(j=1, 2, \ldots\ldots\ldots n)$ so that

$$Z_x = c_1x_1 + c_2x_2 + \ldots\ldots\ldots + c_nx_n \text{ is minimised.}$$

Subject to $a_{11}x_1 + a_{12}x_2 + \ldots\ldots\ldots + a_{1n}x_n \leqslant b_1$

$$a_{21}x_1 + a_{22}x_2 + \ldots\ldots\ldots + a_{1n}x_n \leqslant b_2$$

$$\vdots$$

$$a_{m1}x_1 + a_{m2}x_2 + \ldots\ldots\ldots + a_{mn}x_n < b_m$$

DUAL

Find $y_i \geqslant 0 (i=1, 2, \ldots\ldots\ldots)$ so that

$$Z_y = b_1y_1 + b_2y_2 \ldots\ldots\ldots + b_my_m \text{ is maximised.}$$

Subject to

$$a_{11}y_1 + a_{21}y_2 + \ldots\ldots\ldots + a_{m1}y_m \geqslant c_1$$

$$a_{12}y_1 + a_{22}y_2 + \ldots\ldots\ldots + a_{m2}y_m \geqslant c_2$$

$$\vdots$$

$$a_{1n}y_1 + a_{2n}y_n + \ldots\ldots + a_{mn}y_n > c_n$$

Correspondence between Primal and Dual Optimal Solutions.

We solve the dual (**Example 19**) of page 47

Example 20 : Minimise $Z' = 16y_1 + 16y_2$

Subject to $2y_1 + 4y_2 > 3$

$$3y_1 + 2y_2 > 4; \quad y_1, y_2 > 0$$

Introducing surplus and atificial variables,

Minimise $Z' = 16y_1 + 16y_2$

Subject to $2y_1 + 4y_2 - S_1 + A_1 = 3$

$$3y_1 + 2y_2 - S_2 + A_2 = 4$$

$$y_1, y_2, S_1, S_2, A_1, A_2 \geqslant 0$$

simplex method, we want to replace a basic variable with a non-basic variable *i.e.*, *want to zeroise an existing allocation cell and instead make some allocation in an empty cell. The problem then boils down to determination of an* "outgoing allocation" and an "incoming allocation" that brings us closest to the optimal, if at all possible.

4 (4)	6 (✓)	9 (3)	2 (3)	7	8	10
3	5	4	8	10	0 (12)	12
2 (4)	6	9	8	4	13	4
4	4 (8)	5 (1)	9	3	6 (9)	18
9	8	7 (12)	9	2 (8)	14	20
8	8	16	3	8	21	

Towards this, we arbitrarily select the empty cell (1,2), put a tick in it meaning that we want to make some allocation in it and zeroise one (or more) existing allocations. For reason mentioned above one, and only one loop, can be formed in the allocation pattern. It is shown is the table above. If we bring one unit to the ticked cell other allocations on the corners of the loop would be adjusted as below.

(2) These allocations are independent *i.e.*, a loop cannot be formed in them.

Without giving the proof, we may also mention that whenever there are $m+n-1$ allocations and a tick is placed in an empty cell *one and only one* loop can be passed through the ticked cell and some or all of the $m+n-1$ allocations.

$\boxed{4}$ 4	6	$\boxed{3}$ 9	$\boxed{3}$ 2	7	8	10
3	5	4	8	10	$\boxed{12}$ 0	12
$\boxed{4}$ 2	6	9	8	4	13	4
4	$\boxed{8}$ 4	$\boxed{1}$ 5	9	3	$\boxed{9}$ 6	18
9	8	$\boxed{12}$ 7	3	$\boxed{8}$ 2	14	20
8	8	16	3	8	21	

Cell Evaluations

Consider the initial solution to the transporation problem on hand as obtained by VAM on page 56 and reproduced above.

The allocations are $m+n-1$ in number and independent. It may be stated in passing that initial allocations obtained by Northwest Corner rule or VAM are always in independent positions though they may be $m+n-1$ or less than $m+n-1$ in number. The motivation in the optimality test is to see if it is possible to improve upon the existing solution. In the simplex language, are any of the net evaluations negative ? Before we attempt this for the problem on hand let us see what net evaluations mean in the context of transporation problems. As in the

The dotted lines constitute, in these allocation patterns what is known as loops. A loop *may or may not involve all the allocations. It* consists of (at least 4) horizontal and vertical lines with an allocation at each corner which, in turn is a join of a horizontal and vertical line. At this stage the last loop above is to be particularly noted. Here, two lines intersect each other at cell (2,4) and do not simply join, therefore, this is not be regarded as a corner. Such allocations in which a loop can be formed are known as non-independent. Whereas those (of page 61) in which a loop cannot be formed are regarded as independent. Towards explanation of this nomenclature consider the following allocation pattern with a loop in it.

4	6		10
8	2		10
		5	5
12	8	5	

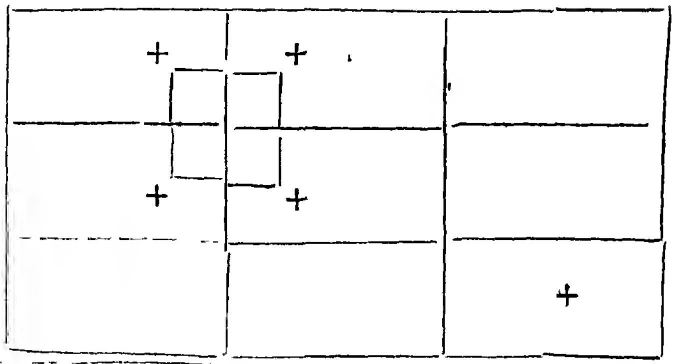
It is possible to progressively adjust the allocations along the corners of the loop without violating the row and column totals. One adjusted allocations pattern for the above table is shown below as an example.

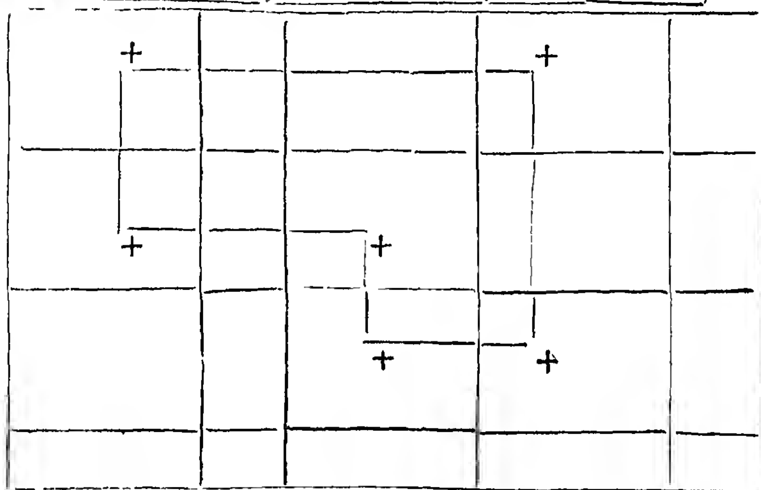
4−2=2	6+2=8	
8+2=10	2−2=0	
		5

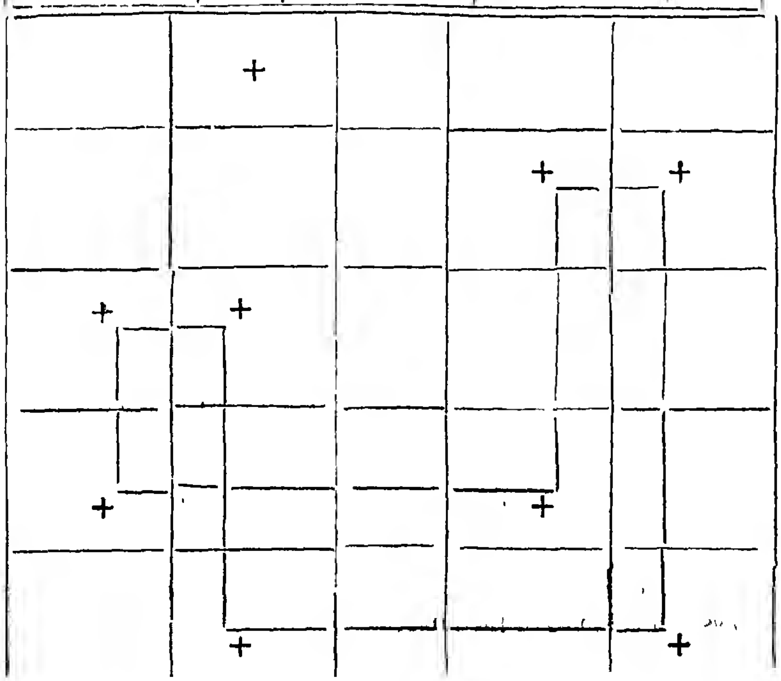
But such a possibility of reallocation does not exist where a loop cannot be formed *i.e.*, allocations are independent.

Coming back now to the optimality test to be described shortly it can be applied to a transportation table if it satisfies the following conditions.

(1) It contains exactly $m+n-1$ allocations where m and n represent the number of rows and columns of the table.







Since only one column is left in the table above there is no question of finding the differences and 16 units are allocated to the various cells of this column straightway, as below.

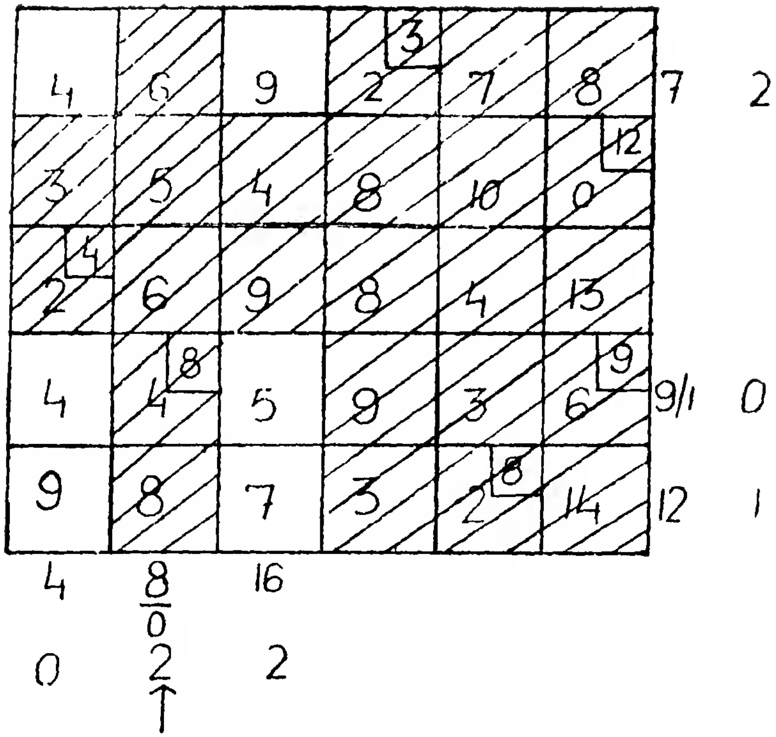
$\boxed{4}$ 4	6	$\boxed{3}$ 9	$\boxed{3}$ 2	7	8	10
3	5	4	8	10	$\boxed{12}$ 0	12
$\boxed{4}$ 2	6	9	8	4	13	4
4	$\boxed{8}$ 4	$\boxed{1}$ 5	9	3	$\boxed{9}$ 6	18
9	8	$\boxed{12}$ 7	3	$\boxed{8}$ 2	14	20
8	8	16	3	8	21	

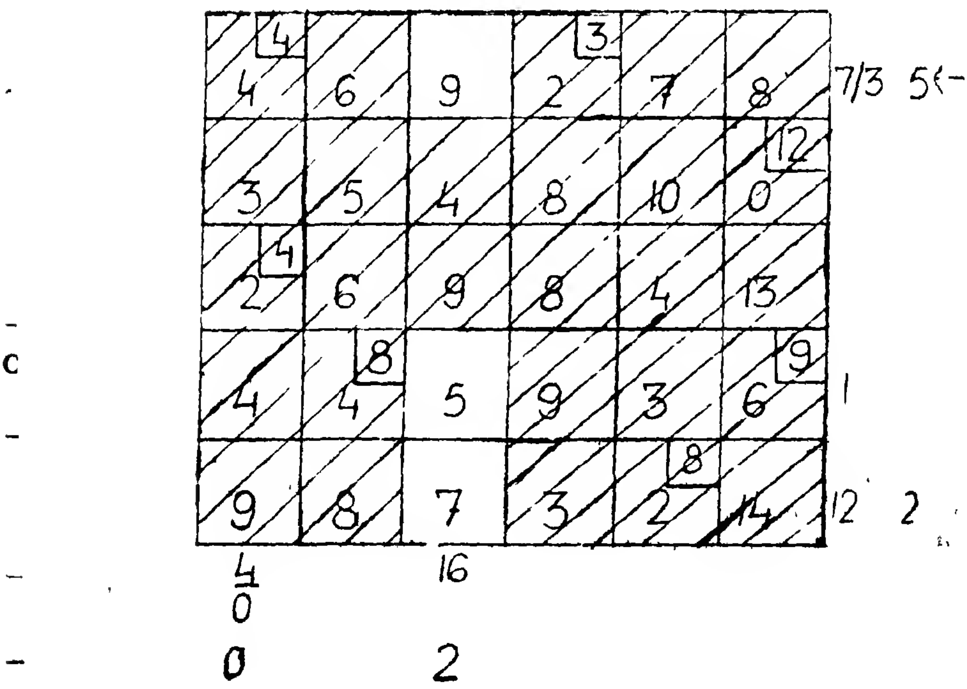
So many tables above have been drawn merely for exposition. As we shall see with the following examples the initial allocation by VAM can as well be obtained in just one table.

The superiority of VAM lies in the fact that, unlike the Northwest Corner rule, not only the availabilities and requirements are taken into account but also due regard is paid to the unit costs. A row and column difference actually indicates the minimum unit penalty incurred by failing to make an allocation to the smallest cost cell in that row or column.

Optimality Test

Now, we know how to obtain the initial basic feasible solution but it remains to be tested if it is optimal and if it is not so how do we go about deriving the optimal solution ? Before we take up this matter it is necessary to explain the concept of **independence** and **non-independence** amongst allocations. Towards this consider the following transportation tables of different problems. The allocations in these are shown by plus signs.


3
4 6 9 2 7 8 7 2
12
3 5 4 8 10 0
4
2 6 9 8 4 13
8 9
4 4 5 9 3 6 9/1 0
8
9 8 7 3 2 14 12 1
4 8/0 16
0 2 2


4 3
4 6 9 2 7 8 7/3 5←
12
3 5 4 8 10 0
4
2 6 9 8 4 13
8 9
c
4 4 5 9 3 6 1
8
9 8 7 3 2 14 12 2
4/0 16
0 2

4	6	9	2 $\boxed{3}$	7	8	7	2
3	5	4	8	10	0 $\boxed{12}$		
2	6	9	8	4	13	4	2
4	4	5	9	3	6 $\boxed{9}$	9	
9	8	7	3	2 $\boxed{8}$	14	20/12	5 ←
8	8	16		8/0			
2	2	2		1			

4	6	9	2 $\boxed{3}$	7	8	7	2
3	5	4	8	10	0 $\boxed{12}$		
2 $\boxed{4}$	6	9	8	4	13	4/0	4 ←
4	4	5	9	3	6 $\boxed{9}$	9	0
9	8	7	3	2 $\boxed{8}$	14	12	1
8/4	8	16					
2	2	2					

4	6	9	2	7	8	10	2
3	5	4	8	10	0 $\boxed{12}$		
2	6	9	8	4	13	4	2
4	4	5	9	3	6 $\boxed{9}$	18/9	1
9	8	7	3	2	14	20	1
8	8	16	3	8	$\frac{9}{0}$		
2	2	2	1	1	2		

4	6	9	2 $\boxed{3}$	7	8	10/7	2 ←
3	5	4	8	10	0 $\boxed{12}$		
2	6	9	8	4	13	4	2
4	4	5	9	3	6 $\boxed{8}$	9/0	1
9	8	7	3	2	14	20	1
8	8	16	$\frac{3}{0}$	8			
2	2	2	1	1			

							AV.	
	4	6	9	2	7	8	10	2
	3	5	4	8	10	0 (12)	12/0	3
	2	6	9	8	4	13	4	2
	4	4	5	9	3	6	18	1
	9	8	7	3	2	14	20	1
REQ	8	8	16	3	8	21/9		
	1	1	1	1	1	6 ↑		

Step 2. From amongst these row and column differences, select the one with the largest difference. If there occurs a tie amongst the largest differences the choice may be made arbitrarily. Allocate the maximum possible to the *least cost cell* in the selected column or row. Hatch that column or containing this cell whose totals have been exhausted so that this column or row is ignored in further consideration.

Step 3. Recompute the coumn and row differences for the reduced transportation table and go to step 2. Repeat the procedure until all the column and row totals are exhausted.

VAM is applied to the previous problem on the table above as an illustration. Entered in the table are the given unit shipping costs.

The arrow indicates that the last column having the largest difference from amongst all rows and columns is selected for allocation. Cell (2,6) with the least cost picked up and allocated 12 units. Row 2 is hatched since its total is exhausted.

This procedure is continued in the following tables until all the columns and rows are hatched

Row-wise allocation, as above, are made below. The maximum that

8	2					10/2/0
	6	6				12/6/0
		4				4/0
		6	3	8	1	18/12/9/1/0
					20	20/0
8/0/0	8/6	16/10/6/0	3/0	8/0	21/20/0	

can be allocated in cell (1,1) is 8. This satisfies completely the requirements of column 1, but availabilities of row 1 are not completely exhausted; therefore, we proceed to cell (1,2) in row 1 still and allocate the remaining 2 units. This exhausts completely the availabilities of row 1 so that we can take up row 2. We can start at cell (2,2) only since column's requirements have been completely satisfied, there is nothing that we can allocate in its first cell (2,1). By the aforesaid procedure, we allocate 6 units to cell (2,2) and another 6 units to cell (2,3) exhausting completely the availabilities of row 2. This process is continued until we reach the cell (5,6).

Now, there exist quite a few alternative methods for obtaining an initial basic feasible solution. However we shall consider only the Vogel's Approximation Method (VAM) which is considered to be superior to all other alternatives including the Northwest Corner rule in that it usually provides an initial solution that is optimal or nearly so. Therefore, we shall also stick to it for the discussion ahead. However, the reader may want to try his hand on the following solved examples by the Northwest corner rule for sake of practice.

Vogel's Approximation Method (VAM)

VAM entails the following steps:—

Step 1. For each row of the transportation table identify the smallest and second smallest costs. Find the difference between the two and display it to the right of that row. Likewise, find such a difference for each column and display it below that column.

(*ii*) Any x_{ij} appears only once in the first m equations and once in the last n equations.

Our problem is to determine x_{ij}, the quantity that is to be shipped from the i—th origin to the j—th destination in such a way that the total transportation cost is minimum. The quantities of interest can be tabulated as below.

Destinations

Origins	D_1	$D_2 \ldots\ldots\ldots D_j \ldots\ldots D_n$	Available
O_1	c_{11}	$c_{12} \ldots\ldots c_{1j} \ldots \ldots c_{1n}$	a_1
O_2 $\vdots$	c_{21}	$c_{22} \ldots\ldots c_{2j} \ldots\ldots c_{2n}$	a_2
O_i $\vdots$	c_{i1}	$c_{i2} \ldots\ldots c_{ij} \ldots\ldots c_{in}$	a_i
O_m	c_{m1}	$c_{m2} \ldots \ldots c_{mj} \ldots\ldots c_{mn}$	a_m
Required	b_1	$b_2 \ldots\ldots b_j \ldots\ldots b_n$	Total

The method for solving this class of problems consists of finding a basic feasible solution. If it is not optimal an iterative procedure is used to improve it.

Initial Solution by the Northwest Corner Rule

The idea is to find an initial basic feasible solution *i e.*, a set of allocations that satisfies the row and column totals. This method simply consists of making allocations to each row in turn, apportioning as much as possible to its first cell and proceeding in this manner to its following cells until the row total is exhausted. Consider, for example, the following sample problem. This method does not use transportation costs which we shall bring in later in the other method.

							Available
							10
							12
							4
							18
							20
Required	8	8	16	3	8	21	

$$\sum_{j=1}^{n} x_{ij}=a_i,\ i=1\ldots\ldots..m \qquad (1)$$

$$\sum_{i=1}^{m} x_{ij}=b_j,\ j=1,\ldots\ldots..n \qquad (2)$$

Minimise $$Z=\sum_{i=1,}^{m}\sum_{j=1}^{n} c_{ij}\, x_{ij} \qquad (3)$$

$$x_{ij}\geqslant 0 \text{ for all } i,j \qquad (4)$$

Equation 1 is interpreted as the sum of what leaves each factory for the various destinations is equal to what is produced at the factory, (2) implies that the sum of what arrives at each warehouse from the various origins is equal to the demand at the warehouse. The double sum of equation 3 represents the total transproatation cost. The non-negative conditions (4) arise because negative values for any x_{ij} have no physical meaning.

Solution to the problem described by equations 1 to 4 is given under the condition that

$$\sum_{i=1}^{m} a_i=\sum_{j=1}^{n} b_j$$

From the physical point of view this condition means that the system of equations is in balance *i.e.*, the total production is equal to the total requirements. Equations (1) and (2) may be expanded as below.

$$\begin{array}{llllll}
x_{11}+x_{12}\ldots\ldots+x_{1n}=a_1 & & & & & \\
 & & x_{21}+x_{22}+\ldots\ldots\ldots+x_{2n}=a_2 & & & \\
 & & \vdots & & & \\
 & & & x_{m1}+x_{m2}+\ldots\ldots\ldots+x_{mn}=a_m & & \\
x_{11} & +x_{21}+ & \ldots\ldots\ldots+x_{m1} & & & =b_1 \\
\quad x_{12} & +x_{22} & \qquad +x_{m2} & & & =b_2 \\
\quad\vdots & & & & & \\
\qquad x_{1n} & & +x_{2n}+\ldots\ldots\ldots\ldots+x_{mn}=b_n & & &
\end{array}$$

This is a system of $(m+n)$ equations in mn unknowns ; but the equations are not independent. Two important observations about the system of equations are worth noting.

(*i*) The coefficient of x_{ij}'s are either 1 or 0

Example 22. Find the dual of the following problem.

Minimise $Z=30x_1+20x_2$

Subject to
$$x_1+x_2 \leqslant 8$$
$$6x_1+4x_2 \geqslant 12$$
$$5x_1+8x_2=20 \qquad \ldots(iii)$$
$$x_1,\ x_2 \geqslant 0.$$

Solution. The equality (*iii*) can be restated as two inequalities as below.

$$\left.\begin{array}{r} 5x_1+8x_2 \geqslant 20 \\ 5x_1+8x_2 \leqslant 20 \end{array}\right\}$$

or
$$\left.\begin{array}{r} 5x_1+8x_2 \geqslant 20 \\ -5x_1-8x_2 \geqslant 20 \end{array}\right\}$$

The entire problem is now restated as below.

Minimise $Z=30x_1+20x_2$

Subject to
$$-x_1-x_2 \geqslant -8$$
$$6x_1+4x_2 \geqslant 12$$
$$5x_1+8x_2 \geqslant 20$$
$$-5x_1-8x_2 \geqslant -20.$$

The dual is now formulated below.

Maximise $-8y_1+12y_2+20y_3-20y_4$

Subject to
$$-y_1+6y_2+5y_3-5y_4 \leqslant 30$$
$$-y_1+4y_2+8y_3-8y_4 \leqslant 20$$
$$y_1,\ y_2,\ y_3,\ y_4 \geqslant 0.$$

The Transportation Problem

This chapter is devoted to special problems that belong to the so called transportation class. These special problems are quite important from the practical point of view. Their practical importance arises because several real situations can be described by systems of equations that fall into the transportation class. A sizeable application of linear programming problems have been made in this field.

A transportation problem can be paraphrased by considering m factories which supply to n warehouses or distribution centers. The factories produce goods at level $a_1,\ a_2 \ldots\ldots\ldots a_m$ and the demand or requirements of the distribution centres for these goods are $b_1,\ b_1 \ldots\ldots b_n$ respectively. If the unit cost of shipping from i-th factory to warehouse j is c_{ij} what shipping pattern minimises the transportation cost ?

Example 20. Find the dual of the following problem.

Maximise $Z=x_1+2x_2$

Subject to $x_1-x_2 \geqslant 3$

$2x_1+x_2 \leqslant 10$

$x_1 \geqslant 0 \; ; \; x_2 > 0$

Solution The 1st constraint must be brought to "$\leqslant$"type by changing signs through before we can derive the dual. This is done below.

Maximise $Z=x_1+2x_2$

Subject to $-x_1+x_2 \leqslant -3$

$2x_1+x_1 \leqslant 10$

$x_1, x_2 \geqslant 0$

Dual is now formulated below.

Minimise $Z^1=-3y_1+10y_2$

Subject to $-y_1+2y_2 \geqslant 1$

$y_1+y_2 \geqslant 10$

$y_1, y_2 \geqslant 0.$

Example 21. Formulate the dual for the following problem.

Minimise $3x_1-x_2$

Subject to $2x_1+x_2 \geqslant 2$

$x_1+3x_2 \leqslant 3$

$x_2 \leqslant 4$

$x_1, x_2 \geqslant 0$

Solution. Since this is a minimisation problem first of all we make the "$\leqslant$" type inequalities to "$\geqslant$" type as below.

Minimse $3x_1-x_2$

Subject to $2x_1+x_2 > 2$

$-x_1-3x_2 \geqslant -3$

$-x_2 \geqslant -4$

$x_1, x_1 \geqslant 0.$

The dual is now derived below.

Maximise $2y_1-3y_2-4y_3$

Subject to $2y_1-y_2 \leqslant 3$

$y_1-3y_2-y_3 \leqslant -1$

$y_1, y_2, y_3 \geqslant 0$

Primal Optimal Table

Prog.	Profit	Qty.	x_1	x_2	x_3	x_4
x_2	4	**4	0	1	$\frac{1}{2}$	$-\frac{1}{4}$
x_1	3	**2	1	0	$-\frac{1}{4}$	$\frac{3}{8}$
NER			0	0	$*-\frac{5}{4}$	$*-\frac{1}{8}$

Dual Optimal Table

Prog.	Cost	Qty.	y_1	y_2	S_1	S_2	A_1	A_2
y_2	16	$*\frac{1}{8}$	0	1	$-\frac{3}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$-\frac{1}{4}$
y_1	16	$*\frac{5}{4}$	1	0	$\frac{1}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{2}$
			0	0	2**	4**	M$-$2	M$-$4

Please note that marginal value of a resource is synonymous with *opportunity cost or shadow price.*

Economic Interpretation

We restate the primal and the dual below.

Primal	Dual
Maximise $Z=3x_1+4x_2$	Minimise $Z'=16y_1+16y_2$
Subject to $2x_1+3x_2 \leqslant 16$	Subject to $2y_1+4y_2 \geqslant 3$
$4x_1+2x_2 \leqslant 16$	$3y_1+2y_2 \geqslant 4$
$x_1, x_2 \geqslant 0$	$y_1, y_2 \geqslant 0$

In the dual problem, the management is interested in knowing the price it should assign to each resource, say for example, there is an alternative opportunity to the management to subcontract its machining and assembly capacities. For this alternative, $2y_1+4y \geqslant 3$ means that the total price of the two resources on product x_1 must exceed its profit contribution of Rs. 3. If it does not exceed the subcontract alternative is inferior to own production. The objective function $16y_1+16y_2$ to be minimised means that the management is interested in minimising the total price to be charged from the sub-contractors so that it is indifferent to own production and subcontract alternatives. Thus the dual, here, seeks to find for the management the optimal (shadow) prices to be *imputed* to the two resources.

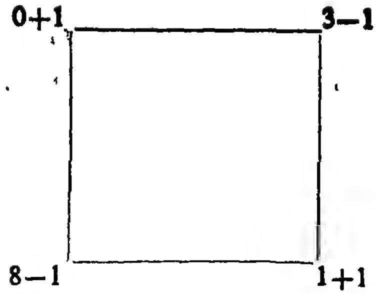
Fixed Ratio	Prog	Cost	Qty.	16	16	0	0	M	M	Replacement Ratio
				y_1	y_2	S_1	S_2	A_1	A_2	
	A_1	M	3	2	4	-1	0	1	0	$\frac{3}{4}$ ←
$\frac{1}{2}$	A_2	M	4	3	2	0	-1	0	1	2
				$16-5M$	$16-6M$ ↑	M	M	0	0	
$\frac{1}{4}$	y_2	16	$\frac{3}{4}$	$\frac{1}{2}$	1	$-\frac{1}{4}$	0	$\frac{1}{4}$	0	$\frac{3}{2}$
	A_2	M	$\frac{5}{2}$	2	0	$\frac{1}{2}$	-1	$-\frac{1}{2}$	1	$\frac{5}{4}$
				$8-2M$ ↑	0	$4-\frac{1}{2}M$	M	$-4+\frac{1}{2}M$	0	
	y_2	16	$\frac{1}{8}$	0	1	$-\frac{3}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$-\frac{1}{4}$	
	y_1	16	$\frac{5}{4}$	1	0	$\frac{1}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{1}{2}$	
			.	0	0	2	4	$M-2$	$M-2$	

We put the optimal tables of the primal and the dual below to bring out the correspondence between them,

*Except for sign reversal the value in the primal and the dual are the same. In other words, the dual problems gives the soluton in terms of marginal values of resource for the primal Problem.

**There is exact correspondence between the primal and the dual.

Thus, we can extract the primal optimal solution from the dual optimal table and vice-versa.



This would mean lowering of the cost by

$$6\times1-9\times1+5\times1-4\times1=-2$$

This then is the net evaluation of the ticked cell and obviously it would pay to bring this cell into solution since the cost would be reduced by 2 per unit transferred to this along the loop. Likewise, we can compute cell evaluations for all the empty cells and select that as the incoming one which has the most−ve cell evaluation.

The procedure would consist of ticking each empty cell, forming a loop involving the tick and computing the cell evaluation for a unit transfer to the ticked cell along the loop. Obviously this would be a rather lengthy procedure. Fortunately there exists a much easier way of computing cell evaluations for all empty cells at one stroke. This is what we described as the optimality test above and it is applicable to transportation tables containing exactly $m+n-1$ independent allocations. It consists of the following steps :

1. Determine a set of $m+n$ numbers

$$u_i \qquad i=1, 2......m$$
$$v_j \qquad j=1, 2......n$$

such that, for each occupied cell

$$C_{pq}=u_p+v_q$$

9. Compute the cell evaluations Δpq for each empty cell (p, q), by the relationship $\Delta pq=c_{pq}-(u_p+v_q)$

Let us apply this optimality test to the problem on the hand. Below is the table indicating just the unit costs in allocated cells in the problem on hand. We set $u_i=0$ for row, 1. It could be assigned any other value but zero has been assigned for ease in subsequent computations.

Also, it is desirable to select that row or column that has maximum number of allocations. In this case row 1 has a maximum of 3 allocations. Now, we wish to find the value of v_1, v_3 and v_4 for the columus of these 3 allocations which can be accomplished as below.

$$u_1+v_1=4 \quad \text{Since,} \quad u_1=0 \quad \Rightarrow \quad v_1=4$$
$$u_1+v_3=9 \quad \Rightarrow \quad v_3=9$$
$$u_1+v_4=2 \quad \Rightarrow \quad v_4=2$$

						u_i
	4		9	2			0 (u_1)
						0	-10 (u_2)
	2						-2 (u_3)
		4	5			6	-4 (u_4)
					2		-2 (u_5)
v_j	4 (v_1)	8 (v_2)	9 (v_3)	2 (v_4)	4 (v_5)	10 (v_6)	

Values of $v_1=4$, $v_3=9$, $v_4=2$ have been entered at the foot of each column.

Now let us proceed with v_1, v_3 and v_4 in turn

v_1 : $u_3+v_1=2$ Since $v_1=4$ $u_3=-2$ is entered

v_3 : $u_4+v_3=5$ Since $v_3=9$ $u_4=-4$ is entered

v_4 : No more allocation in its column and we ignore it.

Having established and entered values of u_3 and u_4 let us take up these in turn.

u_3 ; No more allocation in its row and we ignore it.

u_4 : $u_4+v_2=4$ Since $u_4=-4$ $v_2=8$ is entered

$u_4+v_6=6$ Since $u_5=-4$ $v_6=10$ is entered

Proceeding in this manner we fill up all u_i's and v_j's

In the following table, we derive u_i+v_j for each empty cell.

	8			4	10
-6	-2	-1	-8	-6	
	6	7	0	2	8
0			-2	0	
2	6		0		8

u_i+v_j

The following table gives the cell evaluations derived by subtracting the above figures from the original unit costs.

	—2			3	—2
9	7	5	16	16	
	0	2	8	2	5
4			11	3	
7	2		3		6

$\Delta_{ij}=(u_i+v_j)$

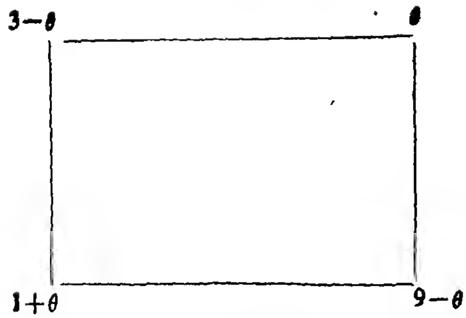
Since some of the Δ_{ij}'s are —ve the initial solution is not optimal. It would pay the most to zeroise an existing allocation and allocate as much as possible to the most —ve Δ_{ij} cell (6, 1). There are two cells having the same most —ve Δ_{ij}. We pick up one of them arbitrarily for ticking.

This is done below in the allocation table by ticking cell, 1, 6 and involving it in the only loop possible.

4		3	3		√
					12
4					
	8	1			9
		12		8	

Transferring the maximum amount to the ticked Cell

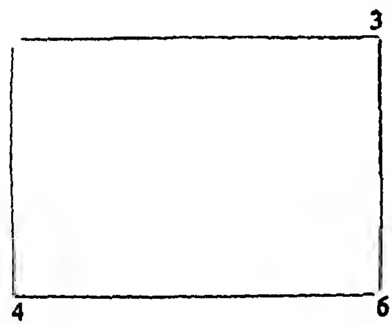
Reallocation is done by transferring the maximum amount to the ticked cell. The rule for obtaining the maximum amount (say, Q max.) that can be transferred



Starting with the ticked cell θ is added to and subtracted from the corner allocations alternately.

$$\theta \max = \min \text{ of} \begin{Bmatrix} 3-\theta=0 \\ 9-\theta=0 \end{Bmatrix} = 3 \qquad \text{Thus} \quad \theta \max = 3$$

Reallocation along this loop is shown below for clarity :



Complete reallocations follows :

4			3		
					12
4					
	8	4			6
		12		8	

Reallocation matrix (optimal as tested below).

The optimality test is performed below.

							u_i
	4			2		8	0
						0	−8
	2						−2
		4	5			6	−2
			7		2		0
v_j	4	6	7	2	2	8	

	6	7		2	
−4	−2	−1	−6	−6	
	4	5	0	0	6
2			0	0	
4	6		2		8

(u_i+v_j) matrix.

	0	2		5	
7	7	5	14	18	
	2	4	8	4	7
2			9	3	
5	6		1		2

Δ_{ij} matrix

Since none of the Δ_{ij} is $-$ve this solution is optimal.

Least cost

$4\times4=16$
$3\times2=6$
$3\times8=24$
$0\times12=0$
$4\times2=8$
$8\times4=32$
$4\times5=20$
$6\times6=36$
$12\times7=84$
$8\times2=16$

242 **(Answer)**

Unbalanced Transportation Problem

Example 2 : A company has factories at A, B and C which supply warehouses at D, E, F and G. Monthly factory capacities are 160, 150 and 190 units respectively. Monthly warehouse requirements are 80, 90, 110 and 160 units respectively Unit shipping costs (in rupees) are as follow :

To

Form		D	E	F	G
	A	42	48	38	37
Form	B	40	49	52	51
	C	33	38	40	43

Determine the optimum distribution for this company to minimise shipping costs

Solution. Availabilities$=160+150+190=500$

Requirements$=80+90+110+160=440$

Availabilities$-$Requirements$=500-440=60$.

Therefore, a dummy warehouse H is introduced, and initial solution obtained below by VAM in just one table.

To

	D	E	F	G	H		
A	42	48	38	160 37	e 0	160/0	37/1/1/1
B	80 40	49	10 52	51	60 0	150/90/10/0	40*/9/11*/1
C	33	90 38	100 40	43	0	190/100/0	38/1/1/3
	80/0	90/0	110/0	160/0	60/0		
	1	10*	2	6*	0		

Since there are only 6 (one less that $m+n-1$) allocations an infinitesimally small alloction e is placed in the *least cost* and *independent cell* (1, 5). This solution is tested for optimality below. (N.B. : If allocations were $m+n-3$ we would place two e's, e_1, e_2 which are virtually zero in the 2 least cost independent cells). This device enables us to apply the optimality test on $(m+n-1)$ alloctions

						u_i
				37	0	0
	40		52		0	0
		38	40			−12
v_j	40	50	52	37	0	

40	50	52			
	50		37		(u_i+v_j)
28			25	−12	

2	−2	−14			
	−1		14		Δij
11			18	12	

Since there are −ve Δ_{ij}'s the initial solution is not optimal Reallocation is done below by ticking the most−ve Δ_{ij} cell, (1, 3) and involving it in the loop.

$$\theta max = \min\begin{pmatrix} e-\theta=0 \\ 10\text{-}\theta=0 \end{pmatrix}$$

		√	160	e
80		10		64
		90	100	

Note that the maximum that can be transferred to the ticked cell is *e*. Since *e* is infinitestimally small it leaves other corner allocations unaffected. (*Intermediate i.e., non-corner allocations are never altered in the process of reallocation*).

		e	160	
80		10		60
	90	100		

Reallocation

This solution is tested for optimality below.

						u_i
			38	37		38
	40		52		0	52
		38	40			40
v_j	−12	−2	0	−1	−52	

26	36			−14	
	50		51		(u_i+v_j)
28			39	−12	

16	12			14	
	−1		0		Δ_{ij}
11			4	12	

Since there is a —ve Δ_{ij} this solution too is not optimal. Reallocation is done below.

$$\theta \max = \min \begin{Bmatrix} 10-\theta=0 \\ 90-\theta=0 \end{Bmatrix} = 10$$

		e	160	
80	√	10		60
	90	100		

		e	160		
80	10			60	**Reallocation**
	80	110			

This solution is tested for optimality below.

						u_i
			38	37		−13
	40	49			0	0
		38	40			−11
v_j	40	49	51	50	0	

27	26			−13	
		51	50		(u_i+v_j)
29				−11	

15	12			13	
		1	1		Δ_{ij}
10			4	11	

Since all Δ_{ij}'s are +ve this solution is optimal. The student may compute the minimum total cost.

Example 3 : Solve the following transporation problem

	D_1	D_2	D_3	D_4	
O_1	5	3	6	2	19
O_2	4	7	9	1	37
O_3	3	4	7	5	34
	16	18	31	25	

Solution : The initial solution is obtained below by Vogel's method

				Available	Cost Difference
5	18 / 3	1 / 6	2	19/1	1/2/3
12 / 4	7	9	25 / 1	37/12/0	3*/3*
4 / 3	4	30 / 7	5	34/30	1/1/3
Requd.					
16/4/0	18/0	31	25/0		
Cost Diff.					
1/2*	1/1	1/1	1		

Since there are 6 $(=m+n-1)$ allocations the optimality test is performed below straightway :

					u_i
		3	6		0
	4			1	2
	3		7		1
v_j	2	3	6	−1	

2			−1
	5	8	
	4		0

(u_i+v_j)

3			3
	2	1	
	0		5

Δ_{ij}

Since all Δ_{ij}'s are positive the initial solution itself is optimal. **Also, since one Δ_{ij} is zero there exists other optimal solution, too.**

Example 4 : A company has 3 factories manufacturing the same product and 5 sole agencies in different parts of the country. Production costs differ from factory to factory, and the sales prices for agency to agency. The shipping cost per unit product from each factory to any agency is known. Given the following data, find the production and distribution schedules most profitable to the company.

Production cost/unit	Max capacity No. of units	Factory i
20	150	1
22	200	2
18	125	3

Shipping Costs

1	1	5	9	4	
9	7	8	3	6	
4	5	3	2	7	
1	2	3	4	5	Agency j
80	100	75	45	125	Demand to be met
30	32	31	34	29	sales price

Solution : The profit matrix is derived below from the equation, profit =sales—production cost—shipping cost.

9	11	6	5	5	
—1	3	1	9	1	Profit matrix
8	9	10	14	4	

It is converted to loss matrix below by changing the signs of all the figures in the above matrix so that we can apply our minimisation algorithm.

—9	—11	—6	—5	—5
1	—3	—1	—9	—1
—8	—9	—10	—14	—4

We can now proceed with this matrix for minimisation of loss. However it would be slightly cumbersome to work with negative figures. They can be made positive by adding, say 14 the highest —ve number above, to each figure. This leads to the following relative loss table,

5	3	8	9	9
15	11	13	5	13
6	5	4	0	10

Since the problem is unbalanced a dummy warehouse is added and the initial solution obtained by VAM below.

(50) 5	(100) 3	8	9	9	0	150/50/0	3/2/2/2*/4
15	11	13	(45) 4	(105) 13	(50) 0	200/150/105	5*/6*/2/2/2
(30) 6	5	(75) 4	0	(20) 10	0	125/50/20	0/4/1/1/4
80/30/0	100/0	75/0	45/0	125	50/0		
1/9*	2	4	5*	1/3	0		

Since there are 8 ($=m+n-1$) allocations the optimality test is straightway performed below :

							u_i
	5	3					—4
				5	13	0	0
	6		4		10		—3
v_j	9	7	7	5	13	0	

		3	1	9	—4	
9	7	7				(u_i+v_j)
	4		2		—3	

		5	1	0	4	
6	4	6				Δ_{ij}
	1		—2		3	

The most —ve Δij cell is ticked and reallocation done below.

$$\theta\ \max = \min \left\{ \begin{array}{l} 20-\theta = 0 \\ 40-\theta = 0 \end{array} \right) = 20$$

50	100				
			45	105	50
30		75		20	

50	100					
			25	125	50	Reallocation Matrix
30		75	20			

5						—6
			5	13	0	0
6		4	0			—5
11	9	9	5	13	0	

		3	—1	7	—6
11	9	9			
	4			8	—5

$(ui+vj)$ matrix

		5	10	2	6
4	2	4			
	1			2	5

Δij matrix

Since all Δij's are positive this solution is optimal.

$50 \times 9 = 450$

$100 \times 11 = 1100$

$25 \times 9 = 225$

$125 \times 1 = 125$

$30 \times 8 = 240$

$75 \times 10 = 750$

$20 \times 14 = 280$

Rs. 3,170 **Answer.**

Example 5 : Solve the following transportation problem where cell entries are unit costs.

						Availability
	73	40	9	79	20	8
	62	93	96	8	13	7
	96	65	80	50	65	9
	57	58	29	12	87	3
	56	23	87	18	12	5
Requirements	6	8	10	4	4	

Solution : The initial solution by vogel's method is obtained below :

73	40	9 (8)	79	20	8/0	11
62	93	96	8 (3)	13 (4)	7/3/0	5/5/54*
96 (6)	65 (3)	80	50	65	9	15/15/15
57	58	29 (2)	12 (1)	87	3/1/0	17/45*
56	23 (5)	87	18	12 (e)	5/0	6/6
6	8/3	10/2/0	4/3/0	4/0		
1/1/6/34	17/35/42*/28	20*/51*	4/4/10/42	1/1/1/52		

Since there are 8 (one less than $m+n-1$) allocations an infinitesimally small allocation e is placed in the least cost, independent cell (5, 5) and the optimality test performed below :

					u_i
		9			−16
			8	13	0
96	65				41
		29	12		4
	23			12	−1
v_j 55	24	25	8	13	

39	8		−8	−3
55	24	25		
		66	49	54
59	28			17
55		24	7	

(u_i+v_j)

34	32		87	23
7	69	71		
		14	1	11
—2	30			70
2		63	11	

Δij

Since there is a negative Δij the initial allocation is not optimal. Reallocation is done below by ticking the —ve Δij cell and involving the tick in the loop.

$$\theta_{max} = min \left\{ \begin{array}{l} 1-\theta=0 \\ 4-\theta=0 \\ 5-\theta=0 \\ 6-\theta=0 \end{array} \right\} = 1$$

		8		
			3	4
6	3			
√		2	1	
	5			θ

		8		
			4	3
5	4			
1		2		
	4			1

Reallocation

The optimality test is performed below :

					u_i
			9			37
				8	13	55
	96	65				96
	57		29			57
		23		12		54
v_j	0	−31	−28	−47	−42	

37	6		−10	−5
55	24	27		
		68	49	54
	26		10	15
54	26		7	

(u_i+v_j)

36	34		89	25
7	69	69		
		12	1	11
	32		2	72
2		61	11	

Δ_{ij}

Since all Δ_{ij}'s are +ve this solution is optimal. The student may compute the minimum total cost.

Example 6. The Link Manufacturing Company has several plants, three of which manufacture two principal products, standard card table and deluxe card table. A new deluxe card table will be introduced which must be considered in term of selling price and costs. The selling prices are : Standard, Rs. 14.95 deluxe Rs. 18.95 and new deluxe, Rs. 21.95

Requirements (Units)		Variable Costs			Available Plant Capacity	
Model	Qty.	Plant A	Plant B	Plant C	Plant	Capacity
Standard	450	8.00	7 95	7.10	A	800
De-luxe	1050	8.50	8 60	8 45	B	600
New Deluxe	600	9.25	9.20	9.30	C	700

Solve this problem by the transportation technique for the greatest contribution.

Solution : (This student should be in a position to follow the computational steps below without explanation).

6.95	7.00	7.85
10 45	10.35	10.50
12.70	10.75	12.65

Note : 6.95=14 95−8.00
10.45=18.95−8.50

Contribution matrix.

The following relative loss matrix has been obtained by subtracting each figure in the above contribution matrix from the highest figure of 12.75 and multiplying by 100 to get whole numbers for ease in subsequent arithmetic.

450 580	575	590	450	5/10
350 230	240	700 225	1050/350	5/5
e 5	600 0	10	600/0	
800	600	700		
	0	0		
225	240	215		
350		365		

The above allocation has been obtained by Vogel's rule.

Since they are 4 *i.e.*, 1 less than $m+n-1$ a small allocation, e is placed in he least cost, independent cell 3, 1. The optimality test is performed below :

			u_i
580			580
230		225	230
5	0		5
v_j 0	−5	−5	

u_i+v_j matrix

	575	575
	225	
		0

Δ_{ij} matrix

	0	15
	15	
		10

Since no Δ_{ij} is $-$ve this initial solution itself is normal. Also since there is one zero Δ_{ij} there exists optimal solution too. The largest contribution

$6\ 95 \times 450 + 10\ 45 \times 350 + 12.75 \times 600 + 10\ 50 \times 700 = 12785$ **Answer.**

Note . In the examples solved herein we see that sometimes we get allocations less than $m+n-1$ in the initial solution We place e's then in the least cost and independent cells It is also possible that we get less than $m+n-1$ allocations in an intermediate reallocation in which case we would place e's in the most$-$ve Δ_{ij} *independent* cell An example follows .

Example 7 Solve the following transporation problem for the least total cost using the N W. Corner rule for the initial solution. Also, formulate the mathematical (syn symbolic model) for the problem.

(a) The initial solution by the North-West corner rule is obtained below.

					Available
	$\boxed{4}$ 1	$\boxed{2}$ 2	4	4	6
	4	$\boxed{4}$ 3	$\boxed{4}$ 2	0	8
	0	2	$\boxed{4}$ 2	$\boxed{6}$ 1	10
Required	4	6	8	6	

This solution (with $6 = m+n-1$ allocations) is tested below.

					u_i
	1	2			0
		3	2		1
			2	1	1
v_j	1	2	1	0	

		1	0
2			1
2	3		

(u_i+v_j)

		3	4
2			—1
—2	—1		

Δ_{ij}

Since there are $-\Delta_{ij}$'s the initial solution is not optimal. The reallocation is therefore done below by ticking cells (3, 1) and involving in the only possible loop.

4	2		
	4	4	
θ		4	6

θ_{max}=min. of $\left.\begin{array}{l} 4-\theta=0 \\ 4-\theta=0 \\ 4-\theta=0 \end{array}\right\} =4$

	6		
		8	e_2
4	e_1		6

Reallocation which is tested for optimality below by putting 2 hypothetical (infinitesimally small) allocations e_1 and e_2 in the most −ve Δ_{ij} and independent cells.

				u_i
	2			0
		2	0	-1
0	2		1	0
v_j 0	2	3	1	

0		3	1
-1	1		
		3	

(u_i+v_j) matrix

1		1	3
5	2		
		-1	

Δ_{ij} matrix

Since there is a $-$ve Δ_{ij} this solution too is not optimal. Reallocation is done below by ticking $-$ve Δ_{ij} cell and involving it in the only possible loop.

	6		
		8	e_2
4	e_1	e_2	√

$$\theta_{max} = \min \left.\begin{array}{r} 6-\theta=0 \\ 8-\theta=0 \\ =6 \end{array}\right\}$$

	6		
		2	6
4	e_1	6	

Reallocation which is tested for optimality below.

				u_i
	2			0
		2	0	0
0	2	2		0
v_j 0	2	2	0	

0		2	0
0	2		
			0

(u_i+v_j)

1		2	4
4	1		
			1

Δ_{ij}

Since none of Δ_{ij}'s is negative, this solution is optimal.

$$\text{Least cost} = 1\times2+2\times2+0\times6+0\times4+2\times6$$
$$= 26 \text{ (Answer).}$$

Formulation of the mathematical model for the transportation problem.

x_{11} 1	x_{12} 2	x_{13} 4	x_{14} 4	6
x_{21} 4	x_{22} 3	x_{23} 2	x_{24} 0	8
x_{31} 0	x_{32} 2	x_{33} 2	x_{34} 1	10
4	6	8	6	

x_{ij}'s are allocations.

$$\text{Minimise} \quad 1x_{11}+2x_{12}+4x_{13}+4x_{14}+$$
$$4x_{21}+3x_{22}+2x_{23}+0x_{24}+$$
$$0x_{31}+2x_{32}+2x_{33}+1x_{34}$$

$$\text{Subject to} \quad x_{11}+x_{12}+x_{13}+x_{14} = 6$$
$$x_{21}+x_{22}+x_{23}+x_{24} = 8$$
$$x_{31}+x_{32}+x_{33}+x_{34} = 10$$
$$x_{11}+x_{21}+x_{31} = 4$$
$$x_{12}+x_{22}+x_{32} = 6$$
$$x_{13}+x_{23}+x_{33} = 8 : x_{ij} \geqslant 0$$

One of the column constraints is left out.

The Assignment Problem

It is another special case of LPP It occurs when n jobs must be assigned to n facilities on a one-to-one basis with a view to minimising an effectiveness objective. An example follows.

Example 1 : An accountant has 4 subordinates and 4 tasks. The subordinates differ in efficiency. The tasks also differ in their intrinsic difficulty. His estimates of the time each would take to perform each task is given in the matrix below. How should the tasks be allocated, one to one man, so that the total manhours are minimised.

	I	II	III	IV
1	8	26	17	11
2	13	28	4	26
3	38	19	18	15
4	19	26	24	10

The assignment problem can be solved by applying the following :—

Step 1. Subtract the minimum element of each row from all the elements in that row. From each column of the matrix so obtained, subtract its minimum element. The resulting matrix is the starting matrix for the following procedure.

Step 2. Draw the minimum number of horizontal and vertical lines that cover all the zeros. If this number of lines is n, order of the matrix, optimal assignment can be made by skipping steps 3 and 4 and proceeding with step 5. If, however, this number is less than n go to the next step.

Step 3. Here, we try to increase the number of zeros in the matrix We select the smallest element out of these which do not lie on any line. Subtract this element from all such (uncovered) elements and add it to the elements which are

placed at the instersections of the horizontal and vertical lines Do not alter the elements through which only one line passes.

Step 4. Repeat steps 1, 2 and 3 until we get the minimum number of lines equal to *n*.

Step 5A. Starting with first row, examine all rows of matrix in step 2 or 4 in turn until a row containing exactly one zero is found Suround this zero by, □ indication of an assignment there. Draw a vertical line through the column containing this zero. This eliminates any confusion of making any further assignments in that column. Process all the rows in this way.

(B) Apply the same treatment to columns also. Starting with the first column, examine all columns until a column containing exactly one zero is found Mark □ around this zero and draw a horizontal line though the row containing this marked zero

Repeat steps 5A and B, until one of the following situations arises :

(*i*) no unmarked (by □) or uncovered (by a line) zero is left.

(*ii*) there be more than one unmarked zero in one column or row. In this case, put□around one of the unmarked zero arbitrarily and pass 2 lines in the cells of remaining zeros in its row and column. Repeat the process until no unmarked zero is left in the matrix.

Let us take the above example for solution by applying the aforesaid steps.

Step 1

By substracting the minimum element of each row from all its elements in turn the given matrix reduces to

0	18	9	3
9	24	0	22
23	4	3	0
9	16	14	0

Next we subtract the minimum element of each column from all elements in turn, obtaining.

0	14	9	3
9	20	0	22
23	0	3	0
9	12	14	0

Step 2

We draw the minimum number of lines to cover all zeros in the last matrix above as follows. To do so the first line is row 3 that contains the hightest

$$\begin{matrix} 0 & 14 & 9 & 3 \\ 9 & 20 & 0 & 22 \\ 23 & 0 & 3 & 0 \\ 9 & 12 & 14 & 0 \end{matrix}$$

number of zeros It can be seen that 4 $(=n)$ lines cover all the xeros ; hence optimal assignment is possible and it is obtained by the application of step 5 straightway below.

$$\begin{matrix} \boxed{0} & 14 & 9 & 3 \\ 9 & 20 & \boxed{0} & 22 \\ 23 & \boxed{0} & 3 & 0 \\ 9 & 12 & 14 & \boxed{0} \end{matrix}$$

The optimal assignment, then is

1→1, 2→III, 3→II, 4→IV.

Minimum time taken$=8+4+19+10=41$ hours.

Example 2 : A manager has 5 jobs to be done. The following matrix shows the time taken by the j-th job ($j=1, 2...5$) on the i-th machine ($i=1, 2, 3...5$). Assign 5 jobs to the 5 machines so that the total time taken is minimised.

Jobs

	1	2	3	4	5
I	9	3	4	2	10
II	12	10	8	11	9
III	11	2	9	0	8
IV	8	0	10	2	1
V	7	5	6	2	1

Solution : Subtracting the minimum element of each row from all its elements in turn the given matrix reduces to

7	1	2	0	8
4	2	0	3	1
11	2	9	0	8
8	0	10	2	1
5	3	4	0	7

Now subtracting the minimum element of each column from its column, the matrix reduces to

3	1	2	0	7
0	2	0	3	0
7	2	9	0	7
4	0	10	2	0
1	3	4	0	6

Next, we draw the minimum number of lines as follows :

3	1	2	0	7
0	2	0	3	0
7	2	9	0	7
4	0	10	2	0
1	3	4	0	6

MATRIX A

Since there are only 3 lines (less than 5) optimal assignment cannot be made as yet. We, therefore, perform step 3 on the last matrix above. The minimum uncovered element is 1. It is subtracted from all the uncovered elements and adding it to those at intersections, giving thereby the following matrix.

2	0	1	0	6
0	2	0	4	0
6	1	8	0	6
4	0	10	3	0
0	2	3	0	5

As to be seen below minimum number of lines that cover all zeroes is 5.

$$\begin{array}{ccccc} 2 & 0 & 1 & 0 & 6 \\ 0 & 2 & 0 & 4 & 0 \\ 6 & 1 & 8 & 0 & 6 \\ 4 & 0 & 10 & 3 & 0 \\ 0 & 2 & 3 & 0 & 5 \end{array}$$

Optimal assignment is, therefore, possible and is made as per step 5 below.

$$\begin{array}{ccccc} 2 & \boxed{0} & 1 & 0 & 6 \\ 0 & 2 & \boxed{0} & 4 & 0 \\ 6 & 1 & 8 & \boxed{0} & 6 \\ 4 & 0 & 10 & 3 & \boxed{0} \\ \boxed{0} & 2 & 3 & 0 & 5 \end{array}$$

Optimal assignment, then is

I→2, II→3, III—4, IV—5, V—1.

Minimum time=3+8+0+1+7=19.

Example 3 5 salesmen are to be assigned to 5 districts. Estimates of sales revenue in thousands of rupees for each salesman are given below.

	A	B	C	D	E
1	32	38	40	28	40
2	40	24	28	21	36
3	41	27	33	30	37
4	22	38	41	36	36
5	29	33	40	35	39

Find the assignment pattern that maximises the sales revenue.

Solution. In order to convert this maximisation problem into a minimisation

problem to be able to apply the assignment algorithm we subtract each element from the highest, 41 and obtain the following loss matrix.

9	3	1	13	1
1	17	13	20	5
0	14	8	11	4
19	3	0	5	5
12	8	1	6	2

Applying step 1 to the loss matrix we derive the following matrix, in which 4 lines are drawn to cover all zeros.

$$\begin{matrix} 8 & 0 & 0 & 7 & 0 \\ 0 & 14 & 12 & 14 & 4 \\ 0 & 12 & 8 & 6 & 4 \\ 19 & 1 & 0 & 0 & 5 \\ 11 & 5 & 0 & 0 & 1 \end{matrix}$$

The minimum uncovered element is 4 that is subtract from all elements and added to all elements at intersections. This yields the following matrix in which 5 lines are needed to cover all zeros.

$$\begin{matrix} 12 & 0 & 0 & 7 & 0 \\ 0 & 10 & 8 & 10 & 0 \\ 0 & 8 & 4 & 2 & 0 \\ 23 & 1 & 0 & 0 & 5 \\ 15 & 5 & 0 & 0 & 1 \end{matrix}$$

Step 5A and B is applied below to obtain the optimal assignment.

$$\begin{matrix} 12 & \boxed{0} & 0 & 7 & 0 \\ 0 & 10 & 8 & 10 & 0 \\ 0. & 8 & 4 & 2 & 0 \\ 23 & 1 & 0 & 0 & 5 \\ 15 & 5 & 0 & 0 & 1 \end{matrix}$$

Condition (*ii*) of Step B arises above ; therefore cell 2, 1 is arbitrarily chosen and □ put around it and a line is also drawn in the second row.

12	$\boxed{0}$	0	7	0
$\boxed{0}$	10	8	10	0
0	8	4	2	0
23	1	0	0	5
15	5	0	0	1

This process is repeated below by putting □ around cell 3, 4 arbitrarily chosen. Therefore, 2 lines are drawn to cover 4th raw and 3rd column.

12	$\boxed{0}$	0	7	0
$\boxed{0}$	10	8	10	0
0	8	4	2	0
23	1	$\boxed{0}$	0	5
15	5	0	0	1

The same process is continued to yield the optimal pattern as below.

12	$\boxed{0}$	0	7	0
$\boxed{0}$	10	8	10	0
0	8	4	2	$\boxed{0}$
23	1	$\boxed{0}$	0	5
15	5	0	$\boxed{0}$	1

Optimal assignment, then is

1—B, 2→A, 3→E, 4→C and 5→D.

The maximum assignment profit is given by Z=38+40+37+41+35=191 thousand rupees.

Unbalanced Assignment Problems

Like the unbalanced transportation problems there could arise unbalanced assignment problems too. They are to be handled exactly in the same manner *i.e.*, by introducing dummy jobs or dummy men, etc. The student may solve the

following unbalanced problem as an exercise.

Exercise. A management consulting firm has a backlog of 4 contracts. Work on these contracts must be started immediately. Three project leaders are available for assignment to the contracts. Because of the varying work experience of the project leaders, the profit to consulting firm will vary based on the assignment as shown below. The unassigned contract can be completed by subcontracting the work to an outside consultant. The profit on the subcontract is zero.

		Contract			
		1	2	3	4
	A	13	10	9	11
Project	B	15	17	13	20
Leader	C	6	8	11	7
	Dummy	0	0	0	0

Find the optimal assignment. Note that the problem is basically not only unbalanced (though now balanced by inclusion of dummy) but also a maximisation one

Answer : A→1, B→4, C→3, Dummy→2

Before we explain the rationale of the assignment alogorithm the student is urged to solve the following exercise to familiarise himself with its mechanics.

Exercises. Solve the following assignment problems :

(A)

Men	Jobs				
	1	2	3	4	5
1	2	9	2	7	1
2	6	8	7	6	1
3	4	6	5	3	1
4	4	2	7	3	1
5	5	3	9	5	1

(B)

8	4	2	7	1
0	9	5	5	4
3	8	9	2	6
4	3	1	0	3
9	5	8	9	5

(C)

5	0	6	8	7	4
5	2	3	0	6	7
3	4	4	3	5	2
3	9	7	2	7	6
9	8	7	8	4	5
1	8	7	4	2	3

Rationale of the Assignment Alogarithm

Step 1. The relative cost of assigning facility i to job j is not changed b the subtraction of a constant from either a column or a row of the original co matrix.

Step 2. An optimal assignment exists if total reduced cost of the assignme is zero This is the case when the minimum number of lines necessary to cover a zeros is equal to the order of the matrix If, however, it is less than n a furth reduction of the cost matrix has to be undertaken.

Step 3 The underlying logic can be explained by means of matrix A page 93 in which only 3 ($=n-2$) lines can be drawn

An optimal assignment is not possible. Further reduction is necessary. Th reduction is made by subtracting the smallest non-zero element from all elements the matrix which is 1,

This yields the following matrix,

2	0	1	−1	6
−1	1	−1	2	−1
6	1	8	−1	6
3	−1	9	1	−1
0	2	3	−1	5

This matrix contains—ve values. Since the objective is to obtain an assig ment with the reduced cost of zero the —ve numbers must be eliminated. This c be done by adding 1 to each of the rows and columns crossed by the 3 lines MATRIX A of p. 93 Doing so yields the following matrix :

2	0	1	2	6
0	2	0	4	0
6	1	8	0	6
4	0	10	2	0
0	2	3	0	5

All this, in fact, amounts to step 3 *i.e.,* add the least non zero uncover element to elements at intersections, subtract it from all the uncovered elements a leave other elements unaltered.

Self-examination Questions

Exercise 1. A media specialist has decided on the allocation of adverti ment in three media vehicles. Let x_i be the number of messages carried on the i— media, $i=1, 2, 3$. The unit costs of a message in the 3 media are Rs. 1000, Rs. 7 and Rs. 500. The total budget available is Rs. 20,C00 for the campaign period a year. The first medium is a monthly magazine and it is desired to advertise r more than one insertion in one issue. At least six messages should appear in t 2nd medium. The number of messages in the third medium should strictly

between 4 and 8. The expected effective audience for unit message in the media vehicles is shown below :

Vehicle	Expected effective audience
1	80,000
2	60,000
3	45,000

It is desired to build the linear programming model to maximise the total effective audience.

(Answer on p 103)

Exercise 2. A fertiliser company produces two types of fertilisers called grade I and grade II Each of these types is processed through two critical chemical plants Plant A has maximum of 120 hours available in a week and plant B has a maximum of 180 hours available Manufacturing 1,000 Kg of grade I fertiliser requires approximately 6 hours in Plant A and 4 hours in plant B : manufacturing 1,000 kg of grade II fertiliser requires 3 hours in plant A and 10 hours in plant B. The maximum that the company can sell of grade I fertiliser is 18,000 kg. If profit is Rs. 450 per 1,000 kg of grade I fertiliser and Rs. 550 per 1,000 kg of grade II fertiliser, it is required to build a mathematical model for amounts of the fertilisers that maximise the profit.

(Answer on p 103)

Also solve the problem graphically.

(**Answer** : I=10.75 units
II=12.50 units
Z=Rs. 13062.50)

Exercise 3. The Manager of Sohan Oil Co. wishes to find the optimal mix of two possible blending processes. For process 1, an input of 1 unit of crude oil A and three units of crude oil B produces an out put of 5 units of gasoline X and two units of gasoline Y. For process 2, an input of 4 units of crude oil A and 2 units of crude oil B produces an output of 3 units of gasoline X and 8 units of gasoline Y. Let x_1 and x_2 be the number of units the company decides to use of process 1 and process 2, respectively. The maximum amount of crude oil A available is 100 units and that of crude oil B is 150 units. Sales commitments require that at least 200 units of gasoline X and 75 units of gasoline Y are produced. The unit profits of process 1 and process 2 are p_1 and p_2 respectively. Formulate the blending problem as a linear programming model.

(Answer on p. 104)

4. Use the simplex method to maximise.

$$z=6x+5y-3z-4w$$

Subject to the following restrictions :

$$\begin{aligned} 2x+3y+2z-4w &= 24 \\ x+2y &\leqslant 10 \\ x+y+2z+3w &\leqslant 15 \\ y+z+w &\leqslant 8 \\ x, y, z, w &\geqslant 0 \end{aligned}$$

5. *Find the values of x and y that maximise*

$$z=2x+5y$$

Subject to the conditions

$$0 \leqslant x \leqslant 400$$
$$0 \leqslant y \leqslant 300$$
$$x \leqslant y \leqslant 600$$

Solve both graphically and by the simplex method (**Answer** : $x=y=300$)

6. *Find* x_1, x_2, x_3 *such that*

$$x_1+x_2+x_3=10{,}000$$
$$x_1 \leqslant 3000,\ x_2 \geqslant 1500,\ x_3 \geqslant 2000$$

$Z=8x_1+10x_2+11x_3$ is minimum.

7. Find the non-negative values of x, y and z that maximise $3x+5y+4z$.
Subject to the restrictions

$$2x+3y \leqslant 8$$
$$2y+5z \leqslant 10$$
$$3x+2y+4z=15$$

Also formulate the dual of this primal.
(**Answer** : $x=89/41$, $y=50/41$, $z=62/41$).

8. Minimise $z=x_2-3x_3+2x_5$

$$3x_2-x_3+2x_5 \leqslant 7$$
$$-2x_2+4x_3 \leqslant 12$$
$$-4x_2+3x_3+8x_5 \leqslant 10$$
$$x_2, x_3, x_5 \geqslant 0.$$

9. Use duality to obtain solution to the following LPP :

Maximise $z=6x_1+4x_2+6x_3+x_4$

Subject to
$$4x_1+5x_2+4x_3+8x_4=21$$
$$3x_1+7x_2+8x_3+2x_4 \leqslant 48$$
$$x_1 \text{ to } x_4 \geqslant 0.$$

$\left(\textbf{Answer} : x_1=\frac{21}{4} ; \text{max. } z=\frac{63}{2}\right)$

10. Maximise $z=2x_1+3x_2$

Subject to
$$-x_1+2x_2 \leqslant 4$$
$$x_1+x_2 \leqslant 6$$
$$x_1+3x_2 \leqslant 9$$
x_1, x_2 unrestricted.

(**Answer** : $Z_{max}=27/2$).

11. A company makes three products, X, Y, Z out of three materials P_1, P_2 and P_3. The three products use units of three materials according to the following table.

	P_1	P_2	P_3
X	1	2	3
Y	2	1	1
Z	5	2	1

The unit profit contribution of the three products are Rs. 3, Rs. 4 and Rs. 5 respectively Availabilities of the materials are 10, 12 and 15 units respectively. The problem is to determine the product mix that will maximise the total profit. Solve the primal problem, write its dual and give the economic interpretations.

Q 12. Find the maximum and minimum values of $z=5x_1+3x_2$

Subject to constraints

$$x_1+x_2\leqslant 6$$
$$2x_1+3x_2\leqslant 3$$
$$0\leqslant x_1\leqslant 3$$
$$0\leqslant x_2\leqslant 3$$

Solve the problem both graphically and by the simplex approach.

(**Answer** Opt. $z=24$).

Q 13. Find the optimal solution of the following transportation problem :

	D_1	D_2	D_3	D_4	Available
O_1	23	27	16	18	30
O_2	12	17	20	51	40
O_3	22	28	12	32	53
Required	22	35	25	41	123

The cell entries are unit transportation costs.

(**Answer** : Opt. $z=2221$).

Q. 14. Solve the following transportation problem :

	D_1	D_2	D_3	D_4	D_5	Av.
O_1	9	12	9	6	9	5
O_2	7	3	7	7	5	4
O_3	6	5	9	11	3	2
O_4	6	8	11	2	2	9
Required	4	4	6	2	4	20

(**Answer :** Opt. $z=102$).

Q. 15. Solve the following transportation problem :

				Available
	8	7	3	60
	3	8	9	70
	11	3	5	80
Required	50	80	80	

(**Answer** : opt. $Z=75$).

Q. 16. A company has 4 machines on which to do 3 jobs. Each job can be assigned to one and only one machine. The cost of each job on each machine is given in the following table.

		Machine			
		W	X	Y	Z
	A	18	24	28	32
Job	B	8	13	17	19
	C	10	15	19	22

What are the job assignments which will minimise the cost

Q 17. Solve the following transportion problem.

		Stores 1	2	3	4	5	6	Requirement
Factories	1	9	12	9	6	9	10	5
	2	7	3	7	7	5	5	6
	3	6	5	9	11	3	11	2
	4	6	8	11	2	2	10	9
Av.		4	4	6	2	4	2	

(**Hint** · The initial solution by VAM gives $m+n-2$ allocations ; apparently requiring e to be put in the least cost empty cell (3, 5) but it is not independent ; therefore, the student would have to place e in cell (2, 3) that is next higher in cost and also independent)

Also formulate the mathematical model

How would you go about the problems solution if it were prohibited to ship anything from factory 3 to store 5 ?

Hint (Put a very high cost M in cell, 3, 5).

Answer :

Exercise 1. Minimise $80{,}000\, x_1+60{,}000\, x_2+45{,}000\, x_3$

Subject to $1{,}000\, x_1+750\, x_2+500\, x_3 \leqslant 20{,}000$ (budget)

$x_1 \leqslant 12$

$x_2 \leqslant 6$

$x_3 \leqslant 4$

$x_3 \leqslant 8$

x_1 to $x_3 \geqslant 0$

Exercise 2. Let x_1 be the amount in 1,000 kg. of grade 1 fertiliser produced and x_2 the amount in 1,000 kg. of grade II produced. The total profit in $Z=450\, x_1+550\, x_2$.

Constraints $6x_1+3x_2 \leqslant 120$ Plant A Capacity

$4x_1+10x_2 \leqslant 180$ Plant B capacity

$x_1 \leqslant 18$

$x_1, x_2 \geqslant 0.$

Exercise 3. Minimise $p_1x_1+p_2x_2$

subject to

$$\left.\begin{array}{r} x_1+4x_2 \leqslant 100 \\ 3x_1+2x_2 \leqslant 150 \end{array}\right\} \text{Availability}$$

$$\left.\begin{array}{r} 5x_1+3x_2 \geqslant 200 \\ 2x_1+8x_2 \geqslant 75 \end{array}\right\} \text{Demand}$$

$x_1, x_2 \geqslant 0$

PRACTICAL APPLICATIONS OF LINEAR PROGRAMMING

In addition to its wide use in industrial and administrative applications, linear programming has had extensive application to agricultural, aircraft and several military problems. These are briefly discussed below. The student need not memorise the numerical examples. They are given merely to enhance understanding of

applications. However, he is advised to *keenly study the formulation of various LPP's given as examples.*

INDUSTRIAL APPLICATIONS

These are basically product-mix problems in which the general objective is to derive the optimal production and procurement plan for the time period under consideration. The measure of effectiveness is either a defined return that is sought to be maximised or a defined cost that is to be minimised. Some examples follow:

Production Planning—the product-mix problem

An industrial concern has available a certain production capacity on various manufacturing processes and has the opportunity to utilise this capacity to manufacture various products. Typically, different products will have different selling prices, will require different amounts of production capacity at the several processes, and therefore will have different unit profits, there may also be stipulations on maximum and/or minimum production levels The problem is to determine the optimal mix so that the total profit is maximised Please see example 1.

Production Planning—the production smoothing problem

An industrial concern has the problem of scheduling its production (or procurement) over a number of future time periods with the total time span being considered the "planning horizon".

Example. The Voltex Company produces an air conditioner/heating unit, The company currently has firm orders for 6 months into the future. The company can schedule its production over the next 6 months to meet order on either a regular or on overtime basis. Consider order and the associated production costs for the next 6 months are as follows:

Month	Jan.	F.	M.	A.	M.	June
Orders	590	610	650	700	500	700
Cost/unit (Rs.)						
Regular Production	50	52	51	55	47	50
Cost/unit (Rs.)						
Overtime Production	62	58	63	60	55	52

With 75 airconditioners in stock at the beginning of January the company wishes to have at least 100 air conditioners in stock at the end of June. The inventory-carrying cost for air-conditioners is Rs. 10 per unit per month.

Let x_{ij} = number of units produced in month i ($i=1, 2......, 6$), on a regular or overtime basis ($j=1, 2$)

y_i—number of units of ending inventory in month i ($i=1, 2......, 6$)

Minimise (cost) $Z = 50x_{11}+62x_{12}+52x_{21}+58x_{22}+$
$51x_{31}+62x_{32}+55x_{41}+60x_{42}+$
$47x_{51}+55x_{52}+50x_{61}+52x_{62}+$
$10(y_1+y_2+y_3+y_4+y_5+y_6)$

subject to (January)

$$\left.\begin{aligned} 75+x_{11}+x_{12}-590&=y_1\\ y_1+x_{21}+x_{22}+610&=y_2\\ y_2+x_{31}+x_{32}-650&=y_3\\ y_3+x_{41}+x_{42}-700&=y_4\\ y_4+x_{51}+x_{52}-500&=y_5\\ y_5+x_{61}+x_{62}-700&=y_6 \end{aligned}\right\}\text{ Monthly Inventory Constraints}$$

$y_6 \geqslant 7100$ (Ending Inventory Constraints)

$$\left.\begin{aligned} x_{11} \leqslant 500 \quad x_{12} \leqslant 300\\ x_{21} \leqslant 500 \quad x_{22} \leqslant 300\\ x_{31} \leqslant 500 \quad x_{32} \leqslant 300\\ x_{41} \leqslant 500 \quad x_{42} \leqslant 300\\ x_{51} \leqslant 500 \quad x_{52} \leqslant 300\\ x_{61} \leqslant 500 \quad x_{62} \leqslant 300 \end{aligned}\right\} \qquad \begin{aligned} x_{ij} &\geqslant 0, \text{ all } i \text{ and } j\\ y_j &\geqslant 0, \text{ all } j \end{aligned}$$

Blending Problems. These problems are likely to arise when a product can be made from a variety of available raw materials of various composition and prices. The manufacturing process involves blending (mixing) some of these materials in varying quantities to make a product conforming to given specifications. The supply of raw materials and specifications serve as constraints in obtaining the minimum cost material blend. The solution would state the number of unit of each raw material which are to be blended to make one unit of product.

Example. A refinery makes 3 grades of petrol (A, B, C) from 3 crude oils (*d, e, f*). Crude can be used in any grade but the others must satisfy the following specifications

Grade	*Specification*	*Selling Price per litre*
A	Not less than 50% crude *d*	8 0
	Not more than 25% crude *e*	
B	Not less than 25% crude *d*	6 5
	Not more than 50% crude *e*	
C	No specifications	5.5

There are capacity limitations on the amounts of the three crude elements that can be used ,

Crude	*Capacity*	*Price/litre*
d	500	9.5
e	500	9.5
f	300	6.5

It is required to produce the maximum profit.

Formulation. Let there be x_1 litres of *d* in A
x_2 litres of *e* in A
x_3 litres of *f* in A
y_1 litres of *d* in B
y_2 litres of *e* in B
y_3 litres of *f* in B
z_1 litres of *d* in C
z_2 litres of *e* in C
z_3 litres of *f* in C

Then $\dfrac{x_1}{x_1+x_2+x_3} \geqslant \frac{1}{2}$ *i e.* $-x_1+x_2+x_3 \leqslant 0$

$\dfrac{x_2}{x_1+x_2+x_3} \leqslant \frac{1}{4}$ *i e.* $-x_1+3x_2-x_3 \leqslant 0$

$\dfrac{y_1}{y_1+y_2+y_3} \geqslant \frac{1}{4}$ *i e* $-3y_1+y_2+y_3 \leqslant 0$

$\dfrac{y_2}{y_1+y_2+y_3} \leqslant \frac{1}{2}$ *i e.* $-y_1+y_2+y_3 \leqslant 0$

Also,

$$x_1+y_1+z_1 \leqslant 500{,}000$$
$$x_2+y_2+z_2 \leqslant 500{,}000$$
$$x_3+y_3+z_3 \leqslant 300{,}000$$

Profit $= -1\ 5\,x_1 + 2\ 5\,x_2 + 1\ 5\,x_3 - 3\ 0\,y_1 + 1\ 0\,y_2 - 4\ 0\,z_1 - 1\ 0\,z_3$

Diet Problems are not much different from the blending problems as brought out by the following example.

Example. The vitamins V and W are found in two different foods, F_1 and F_2. The amount of vitamin in each of the two foods, respective prices per unit of each food, and the daily vitamin requirements are given in the following table. The data indicate that one unit of F_1 contains 2 units of vitamin V and 3 units of vitamin W. Similarly one unit of F_2 contains 4 units of vitamin V and 2 units of vitamin W. Daily requirements of vitamin V is at least 40 units and Vitamin W of least 50 units.

The problem is to determine optimal quantities of foods F_1 and F_2 to be bought so that the daily vitamin requirements are met and, simultaneously the cost of buying the foods is minimised.

	Food		*Daily*
Vitamin	F_1	F_2	*Requirements*
V	2	4	40
W	3	2	50
Cost/Unit of food Formulation	3	2.5	

Let x_1 and x_2 be the quantities of F_1 and F_2 respectively.

$$\begin{aligned} \text{Minimise} \quad & 3x_1+2.5x_2 \\ \text{subject to} \quad & 2x_1+4x_2 > 40 \\ & 3x_1+2x_2 > 50 \\ & x_1 > 0 \\ & x_2 \geqslant 0 \end{aligned}$$

Trim Problems are applicable to paper industry where paper of a standard width has to be cut into smaller width as per customer requirements with the objective of minimising the waste produced.

Example. The Fine Paper Company produces rolls of paper used in cash registers Each roll of paper is 500 ft. in length and can be produced in widths of 1, 2, 3 and 5 inch. The company's production process results in 500′ rolls that are

12 inches in width. Thus the company must cut its 12 inch roll to the desired width. It has six basic cutting alternatives as follows:

Cutting Alternative	No. of Rolls 1″	2″	3″	5″	Waste (inches)
1	6	3	0	0	0
2	0	3	2	0	0
3	1	1	1	1	1
4	0	0	2	1	1
5	0	4	1	0	1
6	4	2	1	0	1

The maximum demand requirements for the four rolls are as follows:

Roll Width (inches)	*Demand Requirements (Rolls)*
1	3000
2	20C0
3	1500
5	1000

The Company wishes to minimise the waste generated by its production meeting its demand requirements. Formulate the LP model.

Formulation. Let x_j be the number of times cutting alternative i $(j=1, 2, \ldots, 6)$ is employed.

Minimise (waste produced) $Z = 1\,x_3 + 1\,x_4 + 1\,x_5 + 1\,x_6$ subject to

$$6x_1 + 1x_3 + 4x_6 \geqslant 3000$$
$$3x_1 + 3x_2 + 1x_3 + 4x_5 + 2x_6 \geqslant 2000$$
$$2x_2 + 1x_3 + 2x_4 + 1x_5 + 1x_6 \geqslant 1500$$
$$1x_3 + 1x_4 \geqslant 1000$$
$$x_j \geqslant 0, \text{ for all } j$$

Distribution Problems

Example. At three factories company has a total of 2000 bags of a particular food that be shipped to five warehouses. The demands of the warehouses, the supplies avilable at the different factories, and the transportation costs per bag are given in the following matrix.

		Factory 1	2	3	Demand
	1	.50	.65	.54	320
	2	.48	.55	.60	470
Warehouse	3	.72	.60	.58	440
	4	.53	.51	.57	350
	5	.45	.49	.59	420
Supply		900	500	600	

Formulate the LP model.

Production Distribution Problems. These problems occur when the products needed by the various destinations in a transportation problem do not exist in finished form but rather must be manufactured at the sources before shipment.

Example : A decision is to be made how many units of a product to manufacture at each of the 4 factories to be transported to five markets. The following table lists the availabilities and demands and profit per unit for the distribution from each market. These profits take into account the sale price in each market, the variable cost in each factory and the transportation costs from each factory to each market.

		Factory 1	2	3	4	*Requirements*
Market	1	5	6	3	8	420
	2	7	8	5	4	490
	3	3	6	7	5	510
	4	4	9	2	7	530
	5	4	7	5	3	550
Availability		700	500	800	1000	

Formulate the LP Model.

Marketing Applications

Advertising-mix problem is analogous to the product-mix problem as brought out by exercise 1 in the earlier section.

Financial Applications : The investment protfolio selection prob'ems can be satisfactorily handled by linear programming as brought out by the following example although integer programming is more generally used in this function :

Example. *The Agro Promotion Bank is trying to select investment portfolio for a cotton farmer. The bank has chosen a set of five investment alternatives. with subjective estimates of rates of raturn and risk, as follows :*

Investment	*Annual Rate of Return*	*Risk*
Tax-free municipal bonds	6.0	1.3
Corporate bonds	8.0	1.5
High grade common stock	5.0	1.9
Mutual fund	7.0	1.7
Real estate	15.0	2.7

The bank officer in charge of the protfolio would like to maximise the average annual rate of return on the portfolio. However, the wealthy investor has specified that the average risk of the portfolio should not exceed 2.0; and does not want more than 20% of the investment to be put into real estate. Formulate an LP model for the problem.

Formulation : Let x_{ij} be the percent of portfolio allocated to investment j (j=1, 2, 3, 4, 5).

Maximise (average annual rate of return on the portfolio) $Z = 6\%x_1 + 8\%x_2 + 5\%x_3 + 7\%x_4 + 15\%x_5$; subject to (Average risk constraint)

$$1.3x_1 + 1.5x_2 + 1.9x_3 + 1.7x_4 + 1.7x_5 \leqslant 2.0$$

(Real estate constraint) $x_5 \leqslant 0.2$

(Total investment constraint) $x_1 + x_2 + x_3 + x_4 + x_5 = 1.0$

$$x_1, x_2, x_3, x_4, x_5 \geqslant 0.$$

Administrative Applications

The use of linear programming as a tool of administrative problem-solving is well-documented in both academic circles and the area of business operations. Included in these areas of application are the following.

Personnel Assignment Problems Q (3)(a) serves as an example.

Bidding Problems as illustrated by the following example.

Example. A company is to subcontract work on four assemblies. The five subcontractors have agreed to submit a bid price on each assembly type and a limit on the total number of assemblies (in any combination) for which they are willing to contract These bids, the contract times, and the requirements for assemblies are gives in the following matrix.

		Subcontractor					
		A	B	C	D	E	Assemblies Required
	1	3 45	3 80	3 00	3 10	3 70	500
	2	3 40	3 13	3.50	3 10	3 20	300
Assembly	3	3.35	3 50	3.50	3.20	3 40	300
	4	3.60	3 00	3.00	3.30	3.90	400
Contract Limits		250	280	330	360	380	

Formulate the LP model. (Hint Treat the problem as a transportation problem).

Balancing Production, Inventories and Workforce

The sales programme cannot usually be accepted as the production programme since it is likely to impose fluctuating load on the work facilities and equipment. Anyhow, in theory at least, the sales programme is a possible production plan on one extreme out of infinite possible plans. The sales programme as the production plan would keep the inventories at a minimum level (zero) but the fluctuations in the rates of production and workforce would be at their highest cost. The other extreme production plan would be to have a constant production rate (at the average sales requirements) so the cost of changing the production rate and work-force are minimal, but the inventory-carrying costs are at their highest. In between, these two extremes infinite number of plans involving varying amounts of inventory-carrying costs on the one hand and costs associated with changing the production rate and work-force level on the other hand.

Example on the production smoothing problem would serve as an illustration of their problem too. Overtime production premiums constitute the "costs associated with the change of the work-force level".

NON-INDUSTRIAL APPLICATIONS

Agriculture Applications

These deal with somewhat different resources, but the objectives are nevertheless same—to maximise the return from the allocation activity or to minimise some defined cost. Two typical problems are illustrated by the following couple of

Example 1. A former has 1,000 acres of land on which he can grow corn, wheat or soyabeams. *Each acre of corn costs Rs 100 for preparation, requires 7 man-days of work and yields* a profit of Rs. 40. An acre of soyabeams costs Rs. 70 to prepare, requires 8 man days of work and yields a profit of Rs 20 If the farmer has Rs. 100,000 for preparation and can count on 8000 man-days of work, how many acres should be allocated to each crop to maximise profits ? Formulate an LP model.

Formulation. Let x_1, x_2 and x_3 designate the acrage of corn, wheat and soyabeams respectively.

Maximise $Z=30x_1+40x_2+20_3$

subject to (Money) $100x_1+120x_2+70x_3 \leqslant 100{,}000$

(Man-day) $7x_1+10x_2+8x_3 \leqslant 8{,}000$

(Acrage) $x_1+x_2+x_3 \leqslant 1000$

$x_1, x_2\ x_3 \geqslant 0$

Example 2. The managers of several cattle feed lots are interested in determining how many of each of several types of livestock feeds to purchase in order to satisfy the nutritional requirements for their livestock. They wish to purchase these foods in a manner that minimises the cost of feeding their livestock Relevant costs and nutritional data is as below.

Required Nutrient	*Alfa*	*Corn*	*Soyabeams*	*Sorghum*	*Minimum Nutrient Requirement*
	Units of Nutritional Elements				
Nutrient A	40	50	30	60	500
Nutrient B	30	60	35	40	750
Nutrient C	25	30	25	50	600
Cost per unit	Rs. 1.00	Rs. 1.25	Rs. 95	Rs. 1.35	

Formu'ation. Let x_j to be the number of units of food type $j=(1, 2, 3, 4)$ used

Minimise (Cost) $z = 1.00x_1+1.25x_2+0.95x_3+1.35x_4$

Subject to (Nutrient A) $40x_1+50x_2+30x_3+60x_4 \geqslant 500$

(Nutrient) B) $30x_1+60x_2+35x_3+40x_4 > 750$

(Nutrient C) $25x_1+30x_2+25x_3+50x_4 > 600$

$x_1, x_2, x_3, x_4 > 0.$

Flight Scheduling Applications

Linear programming has been effectively applied to a variety of operational scheduling problems.

The most common example of this type is the flight scheduling problem.

Example. North-East Aircraft company, which operaters out of a central terminal, has 8 aircrafts of Type I, 15 aircrafts of Type II, and 12 aircrafts of Type III available for to-day's flights. The tonnage capacities (in thousands of tons) are 4.5 for Type, 1.7 for Type II and 4 for Type III.

The company dispatches its planes to cities A and B. Tonnge requirements (in thousands of tons) are 20 at city A and 30 at city B ; excess tonnage capacity supplied to a city has no value. A plane can fly once only during the day.

The cost of sending a plane from the terminal to each city is given by the following table :

	Type I	Type II	Type III
City A	23	5	1.4
City B	58	10	3.8

Formulate the LPP model to minimise the air-transportation cost.

Formulation

Aircraft Type

	I	II	III	
	8	15	12	(Nos.)
	4 5	7	4	(capacity in tons)
Tons	x_{11}	x_{12}	x_{13}	Cost Matrix
A (20)	23	5	1·4	
Requirements	x_{21}	x_{22}	x_{12}	(Cost of sending a plane)
B (30)	58	10	3 8	

Let x_{11} be the I planes coming to A, x_{12} the II planes coming to A ; and so on.

$$\text{Min.}\quad Z=23x_{11}+5x_{12}+1\ 4x_{13}+58x_{21}+10x_{12}+3\ 8x_{23}$$
$$4.5x_{11}+7x_{12}+4x_{13} \geqslant 20$$
$$4.5x_{21}+7x_{22}+4x_{23} \geqslant 30$$
$$x_{21}+x_{11} \leqslant 8$$
$$x_{22}+x_{12} \leqslant 15$$
$$x_{23}+x_{13} \leqslant 12$$
$$x_{ij} \geqslant 0.$$

Other applications of linear programming include structural design, scheduling military tanker fleet, minimising the number of carriers to meet a fixed schedule, the least ballast shipping required to meet a specific shipping program, cost cutting in business, fabrication scheduling computations of maximum flows in network, steel production scheduling, stocks and flows, the balancing of assembly lines, etc.

As such linear programming has contributed greatly to the enhancement of productivity at the firm, industry and national levels and continues to be promising in this regard.

Limitations of Linear Programming

1. **All relationships are linear.** This may not strictly hold good in many real-life situations. The following are few examples.

(*a*) Profit co-efficients in the objective function would spoil the linearity assumption if there are significant semi-variable costs, in which case the objective function would, in fact, be stepped.

(The learning curve phenomenon would render the constraints non-linear in a product-mix problem. Consider, the example, two models of a new helicopter as the products in such a problem. The first helicopter (of, say, the first model) would take more time than the 2nd helicopter (of the same model) because the workers would have gained experience or learnt. In linear constraints it is assumed —wrongly—that time per helicopter stays the same.

(c) Often the production cost may not be a linear function of the output. It may, for example, vary in direct proportion to the square root, square, etc. of the quantity to be made Also price and demand are often not independent. Consequently, the profit may be a non linear function of the quantity which renders the objective function non-linear.

In fairness, however, many non-linear relationships can be well approximated by linear curves.

2. **All constraints and coefficients are stated with certainty.** This may not be so, for example, in the case of an objective function that is in the form of a payoff table discussed in Study II. Nevertheless, the probabilistic LPP can be solved with advanced and complicated/lengthy arithmatic under such circumstances.

3. **Solutions in fractional units are permissible.** This would be valid it the variables are continuous (viz., gasolines) but if they are discrete (viz. helicopters) the optinal solution $\left(\text{say } x_1=2\frac{1}{4}\text{ and } x_2=3\frac{7}{8}\right)$ may or may not provide near-optimal solution upon rounding off. Sometimes the rounded values may yield the profit (or cost) that is widely off the optimal value. Under such situations the technique of integer programming is more appropriate.

4. In real-life problems the constraints and variables are far too numerous and the computer becomes a necessary adjunct.

Sensitivity Analysis

Sensitivity Analysis is concerned with the extent of sensitivity of the optimal solution to an LPP for change in one or more of (*i*) the profit or cost coefficients of the objective function. (*ii*) L.H.S. coefficients of the variables in the constraints and (*iii*) the R.H.S. quantities of the constraints. The need for this analysis arises from two reasons : (1) the aforesaid coefficients are after all estimates only and are liable to be inaccurate to some extent. If the optimal solution is rather insensitive

not much effort need be expended in estimating them. If, however it is sensitive to changes in these the coefficients would have to be estimated after painstaking research in the relevants data (2) The management can manipulate these coefficients and relax the constraints in several situations. For example, selling prices can be increased in a product-mix problem to increase the unit profit or time requirements of a product at a workcentre may be diminished by improved methods or extra equipment may be installed to increase the capacity availability *i.e.* the R H.S. quantity. Management would then naturally be interested in knowing what effects these manipulations have on the optimal solution.

Now the optimal solution to an LPP consists of two parts. For example, in a product-mix problem, it would comprise (*i*) the optimal profit and (*ii*) the decision variables or products in the basis and their quantities to be manufactured. Sensitivity Analysis may be carried out graphically for a two product problem, but for larger real-life problems use has to be made of advanced simplex theory on the final simplex tableau.

For the profit (or cost) coefficients, sensitivity analysis determines the range within which each of these can be varied without altering the basic variables. Likewise, it indicates to what extent the R.H.S. quantities can be increased without altering the existing basic variables. (The determination of effects on the optimal product-mix for addition or deletion of a constraint or a variable also falls under sensitivity analysis).

It is easy to discern that from the management's point of view the information provided by sensitivity analysis may be far more useful than the single optimal result In a way, sensitivity analysis a makes the static LP solution a dynamic tool that evaluates changing conditions.

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STUDY MATERIAL

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FINAL COURSE (N)

OPERATIONS RESEARCH & STATISTICAL ANALYSIS

STUDY—IV

CPM/RESOURCE ALLOCATION

Contents :

Suggested Reading : A Guide to CPM/PERT *by Levy and Wiest, Prentice-Hall.*

1 **Introduction**

Several management planning and control situations involve one time projects rather than continuing processes Examples from fairly diverse fields could be cited at once Some of these are as follows

1 Building a new bridge over the Ganges

2 Introducing a new product.

3 Manufacturing 20 helicopters of a special design for the IAF

4 Shifting a manufacturing unit to another site.

5 Overhaul of a blast furnace.

6 Executing a large and complex order on jobbing production.

7 Sending a spacecraft to the Mars

8. Construction of a 15-storey building

All these projects are characterised by the following set of common implications although they pertain to widely different fields,

(A) Discrete beginning *e g*, 31st Jan '79

(B) Discrete end, *e g*, 1st Jan '82

(C) They are more or less unique in nature Neither in the past nor in the future they are likely to be undertaken substantially in the same form

(D) They involve co-ordination of activities of numerous agencies, departments, vendors, research institutions, contractors, consultants, etc

(E) Durations of the various activities involved are rather uncertain At this stage, however, the projects can indeed be dichotomised. Some projects, such as construction of a 15-storey building, are really not fraught with much uncertainty. Over the years the contractors do acquire empirical timings of such activities as pouring concrete, laying bricks fitting window panels and so on. As we shall see a little later, such projects are amenable to the Critical Path Method (CPM) for planning and control Now take the case of the projects of introduction of a new product and sending a space-craft to the Mars. Not to speak of durations of activities, it is extremely difficult even to list all the activities comprehensively. For the latter problem, there is no solution but to review the project periodically and add to or drop activities from the list of activities foreseen at the commencement of the project, depending upon the turns the research takes. But this is no excuse for starting with an ill-conceived list. All possible activities should be visualised and listed Such projects are handled by PERT.

Let us define an activity formally to make the discussion more meaningful. This is in the context of CPM and PERT, both of which entail network of activities and events represented by arrows and circles respectively. An activity is a part of a plan to which a known resource will be applied. It is preferable for one activity to only refer to a single resource, responsibility, or homogeneous combination. For example, "lay pipe" is an activity requiring mainly effort and "mix concrete" is an activity requiring mainly equipment.

Having sorted out the activities a network model depicting the activity dependency relationships is constructed, *i.e.*, drawn. Three kinds of activity dependencies can be distinguished. First, an activity cannot be started until one or

more other activities have been completed. You cannot place the roof until you laid the foundation and built the walls Therefore, for each activity such preceding activities, upon completion of which only the former can be undertaken, have to be sorted out. Obviously, the very first activity is exceptional in that it is preceded by none other Second, when you have completed an activity you are in a position to start one or more other activities. Obviously, again, if the activity is the last one there is no question of starting any activity after finishing it. After having put the roof you are in a position to put a false ceiling or erect the terrace. It is necessary, therefore, to sort out all the activities that can follow at once after completion of each activity Third, there are activities that can be undertaken concurrently with/without some off-setting in time. You do not need to wait to lay pipe until you have dug the whole trench You may do a bit of trenching and then lay the pipe. Then on, the two activities can proceed concurrently—dig, lay , dig, lay and so on, last bit being lay.

In a more relevant situation, you may find several departments in a jobbing manufacturing unit doing design, printing drawings, writing operating sheets, estimating time and costs, designing special tools, jigs and fixtures, actual manufacture—all being done concurrently with offsetting in the above sense Once you have sorted out these dependency relationships you can draw a network of arrows representing activities and portraying dependency relationships. There are simple arithmetic/analytical means by which the critical path, that is the sequence of activities with the longest duration, can be singled out. (Occasionally, there may be more than one critical path, *i e*, two or more sequences equalling in duration). Such a critical path, may be the one encompassing activities of floating enquiries for imported components, choosing the right bid, placement of orders for imports after clearance from appropriate authorities, receiving materials, inspecting these and finally issuing them. This sequence is, say, 18 months, whereas that of designing, writing operation sheets, manufacturing special tools, jigs and fixtures may be, say, 6½ months. There could be several other sequences of durations less than 18 months Thus you can be slack on the non-critical paths and concentrate your effort on the critical path activities. The non-critical path activities would have, then, some slack (i.e , float or leeway) associated with them. Whereas any delay on the critical sequence would surely lead to the corresponding delay in the project completion this may not be so with the non critical sequences. You can delay those by utilising their float without pushing the completion date forward.

Now another interesting and extremely useful point arises at this stage. We can indeed arrange the activities in order of their floats. Roughly, the activity with the least float, *i.e.*, the one on the critical path should receive the high priority and the priorities of the non-critical activities may be established in order of the floats. The more the float the lower is its priority. This then, provides us a criterion for apportioning the resources that are usually limited. This process of apportioning the resources may also be called resource allocation.

Like the activities, the resources may also be critical and non critical ones. You may be having a surplus of cranes, but a scarcity of bulldozers. This we say

and emphasise for it is practical to consider only those resources that are limited ; because the number of activities in the sort of projects under discussion would easily run into hundreds, if not thousands If, therefore, numerous resources are sought to be permuted with myriads of activities, the problem of resource allocation would be virtually intractable even with a computer Several computer packages have been offered by the computer manufacturers and other agencies for resource allocation none of which is technically and practically perfectly sound. It is highly desirable therefore to screen the resources patiently and 'retain' only those that are really limited ; it would not be surprising to end up with few such resources thereby making the resource allocation problem manageable.

Regarding the other set of projects which have a great deal of uncertainty associated with the activity durations. *PERT is more handly Three time estimates of each activity are made—optimistic, normal and pessimistic. We shall define these inherently subjective estimate but it is easy to get the essence Beta distribution is assumed for the these "guess estimates" and the expected time and its standard deviation, S_t are derived as shown below :

$$t_e = \frac{t_o + 4t_n + t_p}{6}; \quad St = \frac{t_p - t_o}{6}.$$

Where t_0=optimistic estimate
t_n=normal estimate
t_p=pessimistic estimate

There are other minor differences between CPM and PERT. Many of them have submerged with time. And often the two names are used interchangeably.

The student is already familiar with jobbing production. Gantt chart was applied in the past for planning and progressing of such production It is still useful for minor jobbing work, but for complex projects, it had to give way to CPM and PERT. It is neither capable of portraying the activity technological dependencies nor slippages or shortfalls against the plans which themselves are changing to suit the circumstances. Several States in the U.S.A. have accorded recognition to this fact and made it incumbent upon the contractors to supplement their bids with a network plan.

Projects of great size and complexity would call for substantial investment and it is obvious that a great deal of judgment is needed on the part of the management before granting their financial approval and the signal for go ahead. Also it would normally not be just one project they would be concerned with. There may be numerous other projects under execution and several others awaiting approval. Management has to apportion capital appropriately. There exist techniques such as discounted cash flow to analyse the feasibility of such projects as regards investment ; the student is referred to study No. V in Financial Management.

In view of technical complexity and large investment, it may take several months for the management at various levels up the hierarchy to review the proposal once conceived and put in writing by someone in the organisation until it is

*Abbreviation for Program Evaluation & Review Technique

approved or discarded. Another interesting and heartening aspect with regard to these projects is that the customer is unusually co-operative. This is of necessity since only few firms would be capable of undertaking such projects, occasionally even on global basis Also, the project management would not be in a position to conceive the project technically in detail at the outset. It is inherent in view of the oft-repeated complexity. The customer would, therefore, usually be asking for design modifications even when the items on which such changes are sought are in mid stream, *i e* under production He would be willing to provide assistance in design and even supply some especial raw materials difficult to procure Negotiations regarding price and item-wise delivery schedules may be un-ending but only when the project is completed.

Activities

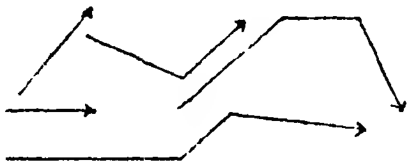


FIG 1

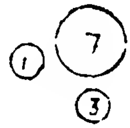


FIG 2

FIG 3

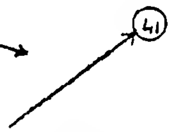


FIG 4

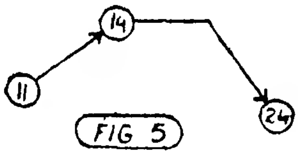


FIG 5

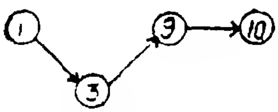


FIG 6

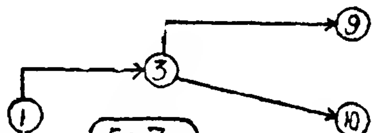


FIG 7

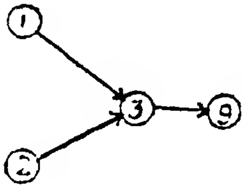


FIG 8

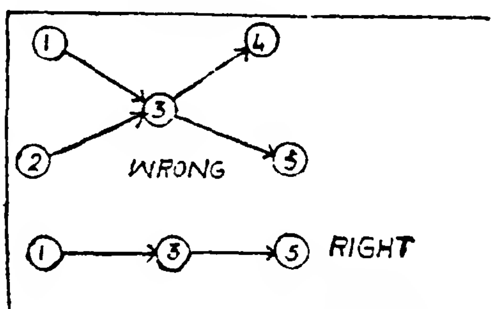


FIG-9

CPM portrays a real-life project by a Network Diagram. The basic component of this diagrammatical model is the arrow that represents an activity. One

activity is represented by one arrow The tail of the arrow represents the starting point of the activity and the head represents the completion The arrow is not a vector and need not be drawn to scale It may be straight, slanting, or bent, but is not broken. In Fig 1, five arrows representing activities (e g., obtain materials, lay bricks, grind tool etc) are shown A project entails several activities. The arrows are arranged to show the plan of logical sequence in which the activities of the project are to be accomplished. The sequence is ascertained for each activity by the following three queries :

1 What activities must precede this one ?

2 What activities can be concurrent with this one ?

3 What activities must follow this one ?

2 1 Events

An event is represented, usually, by a circle and the event number is enclosed in it, e g , events in Fig 2. Examples of events would be.

1. Material procured
2. Design completed
3. Project completed
4. Bricks laid, etc

All activities must have commencement marked by an event Such events are called the tail events because they are connected to the tail of an activity as shown in Fig 3 Similarly, all activities have terminal point called the head event, because it is at the head of an activity These are shown in Fig 4 Fig. 5 depicts the tail and head events connected by arrows representing activities, i e., depicts the dual role of an event. Event (14) is head event for one activity and tail event for another activity

In the CPM tabulations to follow shortly, i is used as a short hand for the tail event and j for the head event of an activity. The activity, then, being $i—j$. Thus in Fig. 6 activity 1-3 is followed by activity 3-9 or activity 3-9 is preceded by activity 1-3

It is not unusual to designate the events by symbols other than a circle, e g , square, oval, rectangle, etc. We shall, however, stick to the most common convention, the circle.

2 2. Logical Connection of Activities

There are a number of abstract logical rules which are useful in portraying the activity inter-dependencies in an appealing manner.

If activities are in series representation is to be as per Fig. 6.

In a project of laying pipe the three activities : 1-3, 3-9 and 9-10 may designate trenching, laying pipe and welding pipe. The three activities have to be carried out in series the reasoning being that the pipe cannot be laid until trench-

ing has been done and welding cannot be undertaken until the pipe has been laid.

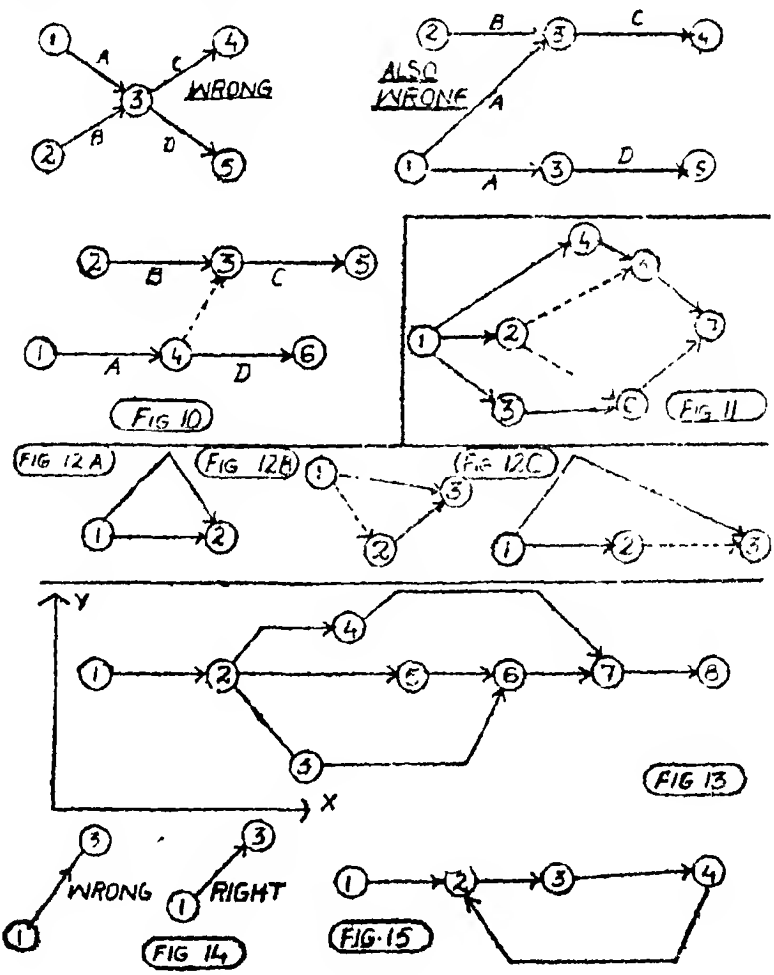
If, however, two activities say, 3—9 and 3-10 are both dependent upon activity 1-3 the representation would be as in Fig 7 Fig 8 shows a case where activity 3-9 is dependent upon two activities 1-3 and 2-3

Fig. 9 shows an incorrect portrayal and the correct one for the following activity dependency relationship :

3—5 dependent upon 1-3

3—4 dependent upon 2-3

2.3. The Dummy Activity. It is represented by a dot [illegible] It is a hypothetical activity that is supposed to consume no resource and ti[illegible] [illegible]ver. It is put to the following couple of uses



Setting the logic right : Fig 10 shows three cases for the following set of dependency relationships.

Activity C dependent upon both A and B.

Activity D dependent upon A alone.

The first portrayal (on top left of Fig 10) is clearly wrong since it shows D as dependent upon not only A but also B which is not wanted. The other portrayal is wrong too since A is being shown twice and this contravenes the fundamental axiom of network that there must be one arrow for each activity. The way out to this dilemma is the representation by means of the dotted arrow called a dummy activity. It has no duration and consumes no resources : but is used to set right the logic In this example, C is dependent upon both A and B (via dummy) whereas D is dependent upon just A The dummy activity may be likened to zero in Mathematics

Exercise 1 : Interpret the dependency relationship is portrayed by the network in Fig 11.

The dummy activity, in addition to setting the logic right, serves another purpose also It is customary to identify an activity as i-j or 1-2, 2-6, 17-31 etc. In Fig 12A, two separate activities are inadvertently designated by 1-2. This is circumvented by introducing a dummy in either of the diagrammatic ways of Fig 12B or 12C. Thus, a dummy activity does not consume any resources ; but is merely used to set the logic right or for proper identification of more than one activities having the same tail and head events.

2.4 Caution about loops : In all the above diagrams the arrow points from left to right. This is a convention to be strictly adhered to as this would avoid the illogical looping as shown wrongly in Fig. 15.

Activities 2-3, 3-4 and 4-2 constitute an illogical loop.

If, as an instance, they represent trenching, laying pipe and welding pipe respectively this would constitute a loop and illogically imply a never-ending cycle of trenching, laying pipe and welding pipe. Furthermore conventionally, coding of an activity by 4-2 is incorrect. j should alwrys be greater than i. This is more fully discussed below :

2.5. Numbering the Events : Numbering could be random. It does not matter, for example, if an activity is identified by 1-12 or 12-1 etc, but keeping i smaller than j has justifiable grounds. When the numbers are in the $i \angle j$ sequence it facilitates rapid processing of the network, because it becomes possible to move in an orderly way towards the end event. Such sequencing also minimises the possibility of dropping an activity in the process of numbering when the network has

been drawn With a computer the sequence is not so useful and random numbering is sometimes employed. It is, however, a good practice to make $i<j$ a universal rule. The numbering scheme of Fig. 13 is to be recommended when the network has been finally drawn.

Proceed from left to right. The event with the least x-coordinate is assigned the smallest integer, say, 1. Other events are assigned progressively higher integers with regard to x-coordinate. If two or more events (4 and 5 in the above example) have the same x-coordinate the one towards arrow should have higher number.

Furthermore, it is not necessary, and in fact also not desirable to number the events consecutively It would be a better scheme to number the events as 10, 20, 30, 40, 50, 60, 70, in the above diagram instead of 1, 2, 3, 4, 5, 6, 7. This affords insertion of more activities and events omitted by oversight or having become necessary in view of certain logic revisions

It was mentioned earlier that it is desirable that all the activity arrows point from left to right. If the arrow is vertical it may point downwards or upwards.

For the sake of presentability, it is to be recommended that activities emanating from one event or converging to another make as great angles between themselves as possible.

A few more conventions are given below ;

(i) Keep the arrow to the extreme right. (ii) As far as possible avoid drawing arrows that cross each other. Usually by suitable 'stretching' the network diagram it is possible to avoid this. (iii) Where, however, crossing is unavoidable, bridging may be done. This applies to dummies as well. Draw boldly a big network. Smaller ones are confusing. Use of pencil and rubber is recommended.

Example 2 · Depict the following dependency relationships by means of network diagrams. The alphabets stand for activities.

1. A and B control F ; B and C control G.
2. A and B control F ; B controls G with C controlling G and H.
3. A controls F and G ; B controls G ; whilst C controls G and H.
4. A controls F and G ; B and C control G with H depending upon C.
5. F and G are controlled by A, G and H are controlled by B with H controlled by B and C.
6. A controls F, G and H ; B controls G and H with H controlled by C. (Answers are to be found at the end of the Study.) Trial and Error is the way out ; therefore, use a pencil and a rubber.

Example 3 : *Find out the superfluous (unnecessary) dummy* ***activities in*** *the networks below.*

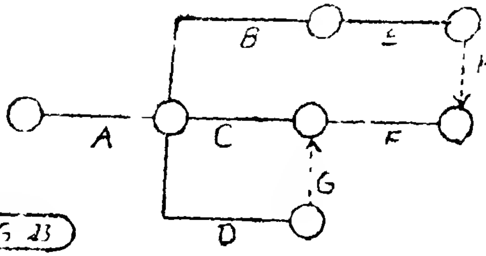


FIG 23

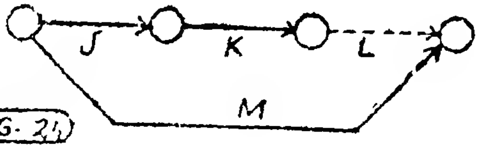


FIG. 24

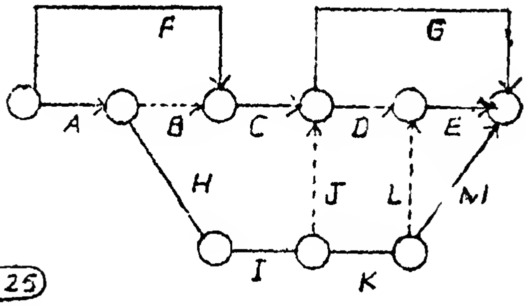


FIG 25

Network Construction

Example . The activities involved in the computer installation process are detailed below You are required to draw the network.

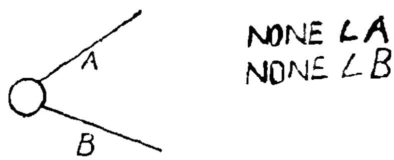
	Activity	*Predecessor Activities*
A.	Physical preparation	None
B.	Organisational planning	None
C.	Personnel Selection	B
D.	Equipment Installation	A
E.	Personnel Training	C
F.	Detailed Systems Design	C
G.	File Conversion	F
H.	Establish Standards and Controls	F
I.	Program preparation	H
J.	Program Testing	I
K	Parallel operations	D, E, G, J
L.	Finalise systems documentation	I
M.	Follow up	K, L

Solution :

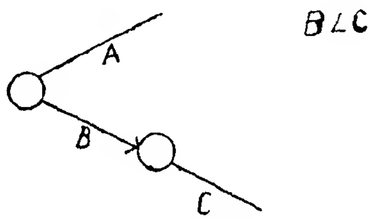
The network is drawn below. The construction has been carried out step by step which is explained below. The symbol '∠' is to be read as "precedes" in these figures.

(i) Since A and B are preceded by no activity they are shown emanating from the start event. We do not put arrows and the head events for A and B because they have been drawn as such only tentatively In the following step (s) their position *may* have to be changed

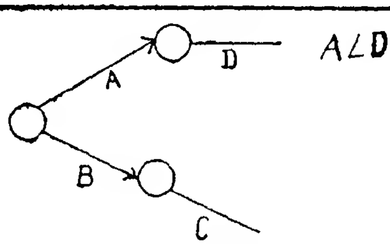
(ii) Here C is shown as dependent upon B. Again we neither show arrow and nor the head event of C because its position is tentative.



(i)

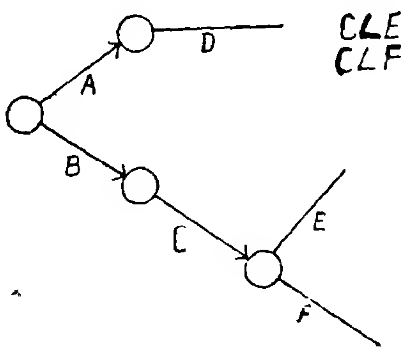


(ii)



(iii)

(IV)



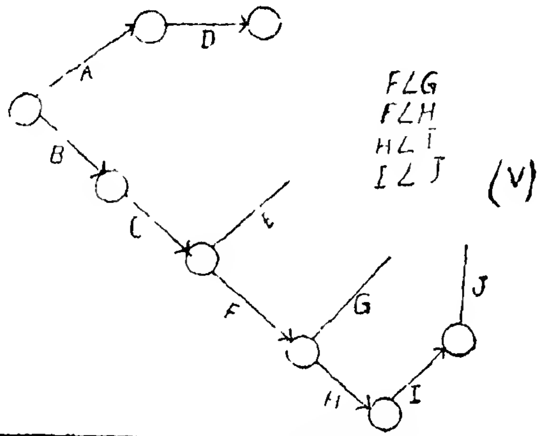
(iii) D is shown as dependent upon *A*

(iv) *We get bold now and in the same step we show C preceding E and F.*

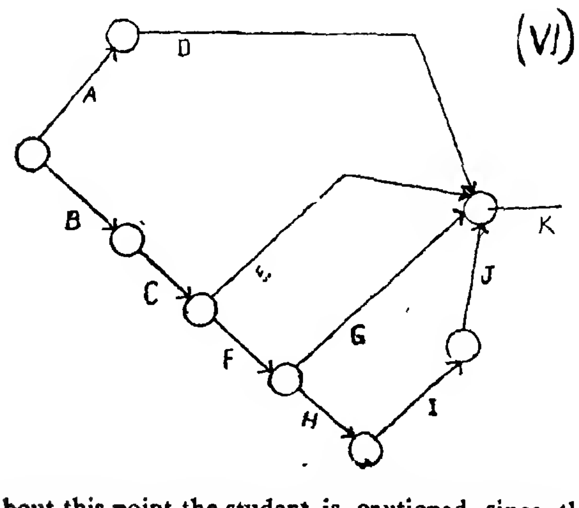
(v) *F is shown preceding G and H, H precedes I and I precedes J.*

(vi) K is shown as dependent upon D E, G and J, *the latter four, therefore,* are made to converge on the same head event.

(vii) L is shown as dependent upon I, and M as dependent on K and L. Note carefully that K and L are the terminal activities and, therefore, converge on

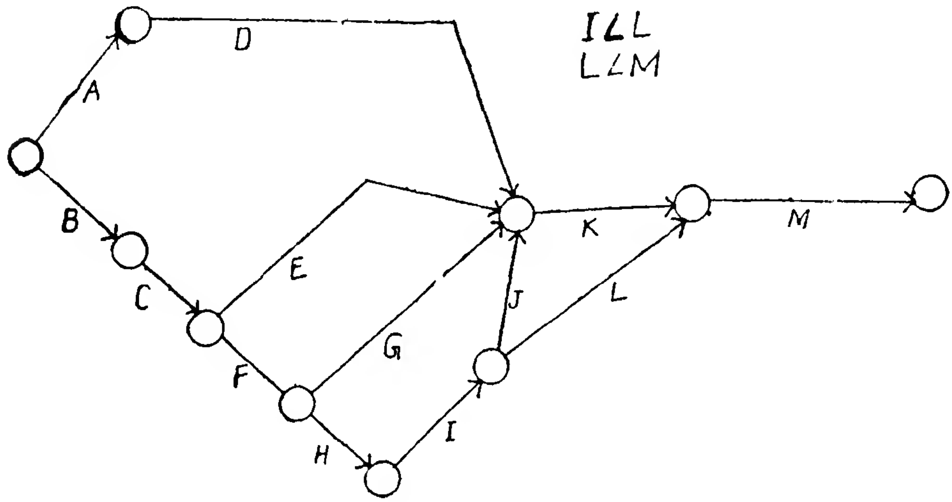


D, E, G, J ∠K



the last event. About this point the student is cautioned since the beginners do tend to show the last activities dangling and do not converge them to the common last event.

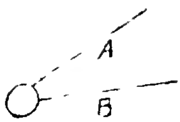
Of course, the network would be constructed in just one diagram. We drew so many for explanation only. However, for a stylish presentation it is desirable that this diagram is suitably stretched *i.e.*, faired up in another diagram. The student may number the events as an exercise.



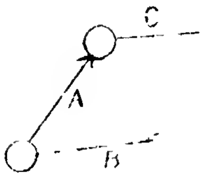
Example : Construct the network for the following activity data.

Activity	*Preceded by*
A	None
B	None
C	A
D	B,C
E	D
F	E
G	B,C
H	F
I	F,G
J	H,I
K	B
L	F,G,K

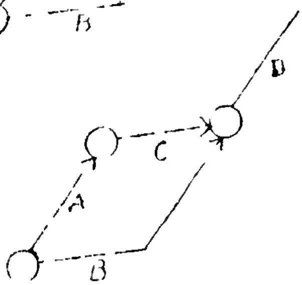
The network is drawn below step by step with dependency relationships that are incorporated at each step which the student should, by now, be able to follow. Please carefully review the steps where dummies are incorporated.



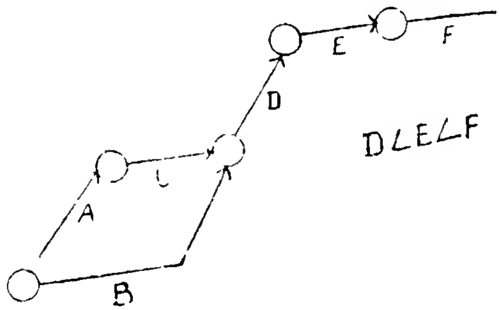
NONE ∠A
NONE ∠B



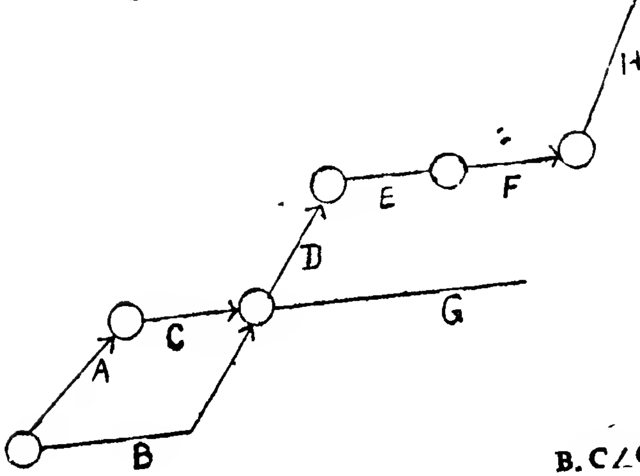
A∠C



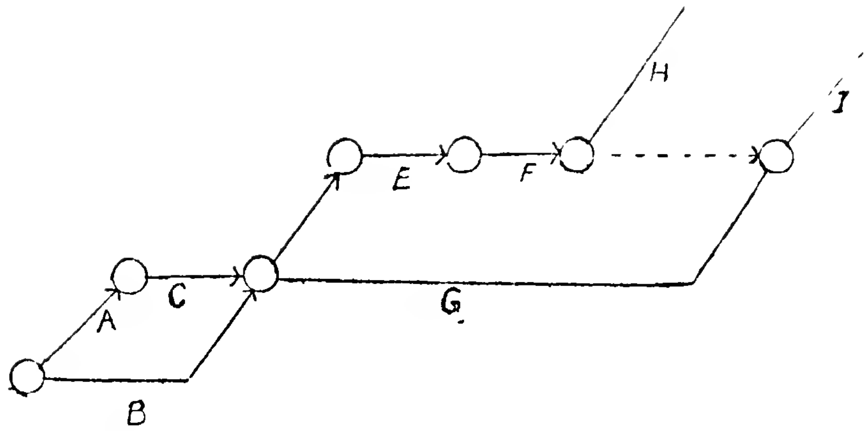
B∠D
C∠D



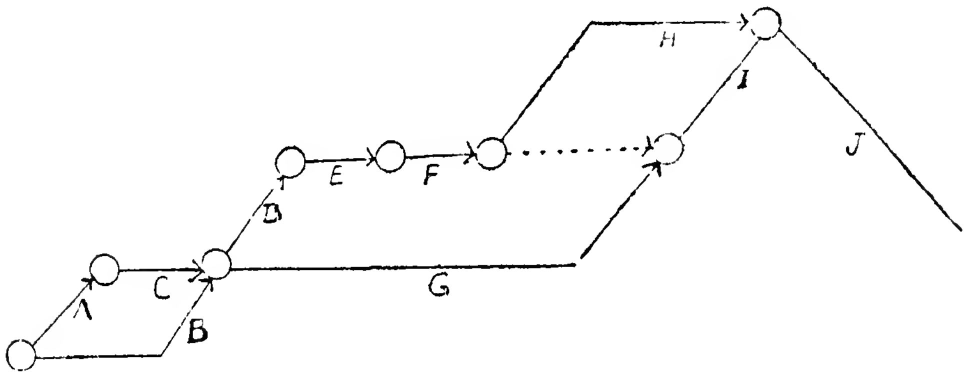
D∠E∠F



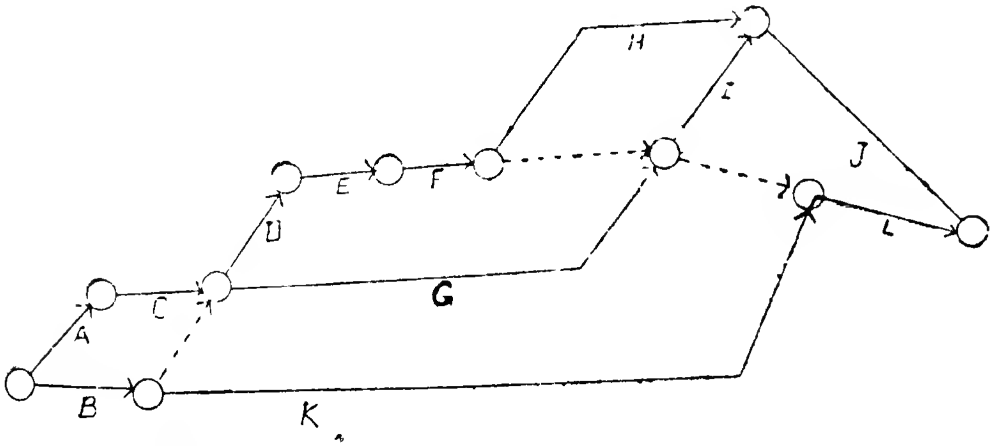
B.C∠G



F∠H, I
G∠I



I, H∠J



B∠K
F,G, K∠L

The student may number the events as an exercise.

2 6. Concurrent activities

Activities may not always be discrete, i e , they may be done in part allowing the subsequent activities to commence before the preceding activity is fully completed. Activities of this kind are to be frequently encountered in batch production. If, for example, a batch of 50 spindles is to be processed on two machines obviously it is not necessary to process all the items of the batch on the first machine and then transfer these to the next machine A few items processed on the first machine may be transferred to the second machine before completion of the entire batch on the first machine. Since this is a matter of great practical importance we shall dwell upon it at a greater length Such simultaneous or concurrent activities are to be encountered in sewarage work e g , trenching, laying pipe, welding pipe and back filling all going, on simultaneously with suitable lags on construction work

Example: A batch of 4 axles is to be processed on the following three machines in this sequence :

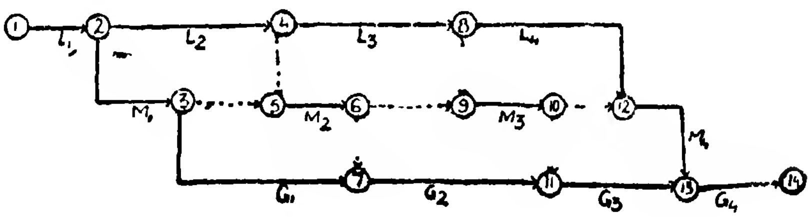
Lath (L), Milling (M) and Grinding (G)

Instead of first working on these 4 axles on lathe and then on milling and finally on grinding in sequence, it is desired to process the first axle on the lathe and as and when it is processed, it is taken up on milling and the 2nd axle on the lathe, and so on In other words, each of the three activities L, M and G have been quartered for the sake of concurrent operations. You are required to draw the network

Solution : The dependency relationships are sorted out hereunder :

Quartered Activity	*Preceded by*
L_1	None
L_2	L_1
L_3	L_2
L_4	L_3
M_1	L_1
M_2	L_2, M_1
M_3	L_3, M_3
M_4	L_4, M_3
G_1	M_1
G_2	M_2, G_1
G_3	M_3, G_2
G_4	M_4, G_3

The network is now constructed below (The student is urged to draw his own network. Mere understanding of our solution is not enough).



(N.B. : The concurrent activities so drawn are known as ladders' in the network jargon.)

4. Critical Path Analysis

You have already been familarised with the logic of the network analysis by way of introduction of this study paper. It is as simple as sorting out for each activity : (*i*) activities that should immediately precede it, (*ii*) the activities that are concurrent with it and (*iii*) the activities that can be undertaken immediately after its completion. For projects that are worth treating by the network analysis, a great deal of practice is, however, needed in listing the activities comprehensively and then establishing these technological dependency relationship. This is because the real life projects are so complex that activities tend to run into several hundreds in number. Obviously the analyst should be an expert in the field. If, for example, it is a construction project, he may be an architect or a civil engineer. More than one analyst may be needed for such projects as the space-crafts. The analyst would be constructing the network. The analysis of the network may either be done by the computer or himself, *i.e.*, finding critical path and floats.

The purpose of the analysis is two-fold : (*i*) to find the critical path, *i.e.*, the sequence of activities with the longest duration. Once it is found it is marked in bold sequence of arrows on the network. For a simple network as of Fig. 26 the various sequences can be enumerated and the durations of activities encompassed by them simply added, to find the critical sequence. As stated earlier, one could indeed end up with more than one critical sequence ; and (*ii*) to find the float associated with each non-critical activity.

Systematic analysis : The enumeration method would be too cumbersome computationally for any real-life project. Even the computer would be hard pressed to proceed this way. There is a systematic way that cuts short the analysis time to manageable proportions. Let us say that event 1 starts at zero time. Zero time is called the base—the base could well be calendarised *e.g.*, Jan. 15, 1976. The activities are normally listed as below and shown in fig. 26 as a network

Activity	*Description*	*Duration in weeks*
1—2	Market research	15
1—3	Make drawings	15
2—3	Decide production policy	3
2—5	Prepare sales programme	5
3—4	Prepare operation sheets	8
3—6	Buy materials	12
4—5	Plan labour force	1
4—6	Make tools	14
5—6	Schedule production	3
6—7	Produce product	14

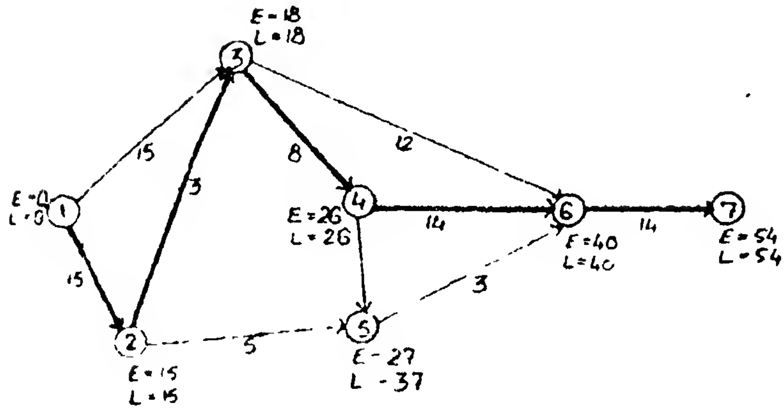


Fig 26

Analysing the Network

Consider Event 3. It can be reached directly from event 1 or via event 2. The times for the two sequences being 15 and 15+3=18 Activities emanating from event 3 can only be undertaken when 1—3 and 1—2 & 2—3 are finished It should be obvious that 1—3 can be delayed by 18−15=3 days which is called the float associated with 1—3. It can easily be noticed that the other 2 activities on the critical path can have no float They must be completed within the duration specified ; and delay on these would delay the completion of the entire project Also, it can be seen that the earliest the activities emanating from event 3 can commence is the 18th day This is designated by putting E=18 around its node in the network diagram. Likewise, we can compute the earliest start, E, for each event by proceeding progressively from left to right by the following rule .

If only one activity converges on an event its earliest start E is given by E of the tail event of the activity plus activity duration. If more than one activity converge on it E's via all the paths would be computed and the highest value chosen and put around the node.

The E's worked for the example at hand are shown in the network diagram.

Now work backwards 'proceeding' progressively from the end event to the start event by equating L, the latest time of the completion of the last event, to its E, *i e.*, L=54 for the end event. The L's for the remaining events would be computed by the following rule.

If only one activity emanates from an event compute L by subtracting activity duration from L of its head event. If more than one activity emanates from an event compute L's via all the paths and choose the smallest. Put it around the node of event at hand.

The L's are worked for the example at hand and are shown in the network diagram. The following analysis table is then compiled.

Network for Jobbing Production

Activity	*Duration*	*Start* *Earliest*	*Start* "*Latest*"	*Finish* "*Earliest*"	*Finish* '*Latest*'	*Float* *Total*
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1—2	15	0	0	15	15	0
1—3	15	0	3	15	18	3
2—3	3	15	15	18	18	0
2—5	5	15	32	20	37	17
3—4	8	18	18	26	26	0
3—6	12	18	28	30	40	10
4—5	1	26	36	27	37	10
4—6	14	26	26	40	40	0
5—6	3	27	37	30	40	10
6—7	14	40	40	54	54	0

Explanation of the table : Under col. 3 are noted the E's for the tail events. The following two columns are left blank for the time being. Under col. 6 are noted the L's of the head events.

col (4)=col (6)−col (2) col (7)=col (6)−col (5)
col (5)=col (4)+col (2) col (7)=col (4)−col (3)

Example : Consider activity 1—2. Its tail event has E=0.

Put 0 against this activity under column 3. Likewise, its head event has L=15. Put 15 against it under col. 6 Under col. 4 note the value in col. 6 minus the activity duration. Under col. 5 note the value in column 3+activity duration. Compute total float by subtracting col. 5 from col. 6. Float is the duration by which an activity can be delayed without delaying the project. Critical path is to be sought against the activities with 0 float and it is 123467. It is shown by bold arrows.

Note the above analysis is valid if resources are liberally available.

5. Program Evaluation Review Technique

Hitherto our emphasis has been on CPM which is applicable where activity durations can be estimated from experience, past historical records and work study techniques fairly precisely. CPM is incapable of handling uncertainty in timing which is a rule rather than an exception with innvoational projects such as introducing a new product or oil exploration project. We have already stated that the Beta distribution is useful here. Those experienced and possessing expertise in the various technical aspects of the projects are required to make three time estimates from which is computed te, the expected time and S_t, the standard deviation of the activity duration The expected time may then be used as the activity duration and the critical path may be found by the analytical method explained earlier. But to what use is S_t put ? Well this provides quite a useful information at the expense of a bit of computational effort. We can find the probability of completing a project by a given date. Here is the procedure.

Compute variance, Vt ($=S^2_t$) of all the activity durations of the critical path. Sum these up and take the square root. This yields the S.D. of the total duration. Let the critical path duration be designated by T_{cp}. Assuming normal distribution for the total duration, you should be in a position to find the confidence interval for T_{cp}. 3 SD give the limits of the total possible duration with 99% confidence. put it in another way. Suppose you wish that the project be completed within a duration of T. Find $\frac{T-T_{cp}}{SD}=Z$, the standard normal variate. Look to the standard normal probability distribution tables (Appendix Study II) for the probability of completing the project within the given duration of T.

Example 1. If the critical path of a project is 20 months alongwith a standard deviation of 4 months what is the probability that the project will be completed within :

(*a*) 20 months (*b*) 18 months (*c*) 24 months ?

Solution : (a) $Z=\frac{20-20}{4}=0$ Probability $=$ 0.50

(b) $Z=\frac{18-20}{4}=-0.50$; Probability $=$ 0.31

(c) $Z=\frac{24-20}{4}=1.00$; Probability $=$ 0.84

around the node.

Example 2. PERT calculations yield a project length of 60 weeks with variance of 9. Within how many weeks would you expect the project to be completed with probability of 0.99 ? (That is the project length that you would expect to be exceeded only by 1% of the time if the project were repeated many times in an identical manner).

Solution : $T_{cp} = 60$, S D. $= \sqrt{9} = 3$.

$60+3\times 3 = 69$ weeks. **(Answer)**

Example 3. A small project is composed of 7 activities whose time estimates are listed in the table below. Activities are identified by their beginning (*i*) and ending (*j*) node numbers.

(*a*) Draw the project network and identify all the paths through it.

(*b*) Find the expected duration and variance for each activity.

What is the expected project length ?

(*c*) Calculate the variance and standard deviation of project length. What is the probability that the project will be completed ;

(*i*) at least 3 weeks earlier than expected ?

(*ii*) no more than 3 weeks later than expected ?

(*d*) If the project due date is 18 weeks, what is probability of *not* meeting the due date ?

(*e*) What due date has about 90% chance of being met ?

Activity	Estimated Duration in Weeks		
i—j	Optimistic	Most Likely	Pessimistic
1—2	1	1	7
1—3	1	4	7
1—4	2	2	8
2—5	1	1	1
3—5	2	5	14
4—6	2	5	8
5—6	3	6	15

Solution :—(*a*) The network is drawn in Fig. 26C. The various paths are as follows ;

1—2—5—6, 1—3—5—6, 1—4—6.

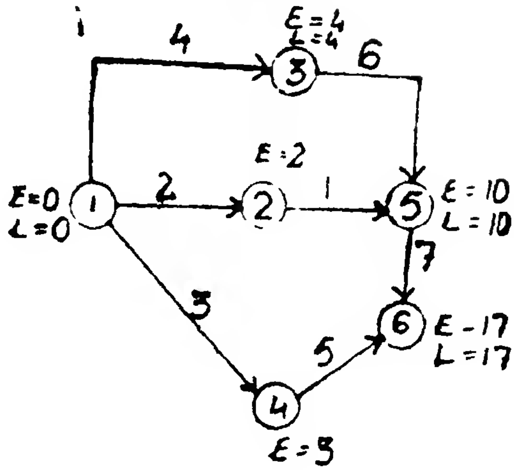


Fig. 26C

(*b*) Expected duration and variances for various activities are computed below :

	te	v_t	
1—2;	$\dfrac{1+1\times4+7}{6}=2;$	$\left[\dfrac{7-1}{6}\right]^2=1$	
1—3;	$\dfrac{1+16+7}{6}=4,$	$\left[\dfrac{7-1}{6}\right]^2=1$	
1—4;	$\dfrac{2+8+8}{6}=3;$	$\left[\dfrac{8-2}{6}\right]^2=1$	
2—5;	$\dfrac{1+4+1}{6}=1;$	$\left[\dfrac{1-1}{6}\right]^2=0$	(See p. 4 for formulas of t_e, and V_t ($=S_t^2$))
3—5;	$\dfrac{2+20+14}{6}=6;$	$\left[\dfrac{14-2}{6}\right]^2=4$	
4—6;	$\dfrac{2+20+8}{6}=5;$	$\left[\dfrac{8-2}{6}\right]^2=1$	
5—6;	$\dfrac{3+24+15}{6}=7;$	$\left[\dfrac{15-3}{6}\right]^2=4$	

(*c*) **Expected project Length = 17 weeks (Answer)**

Variance of the critical path 1—3—5—6 = 1 + 4 + 4 = 9 (Answer.)

$\therefore$ **Standard Deviation = $\sqrt{9}$ = 3 (Answer)**

(*i*) $Z = -3/3 = -1$; Probability = 0 159 **(Answer)**

(*ii*) $Z = 3/3 = 1$; Probability = 0.841 **(Answer)**

(*d*) $T_{ep} = 17$; $T = 18$, $Z = \frac{18-17}{3} = 0.333$

Therefore, probability of meeting the due date = 0.63

And probability of not meeting the date = 0.37 (= 1—0.63)

Example 4. Find the event variances in the network of Example 3.

Solution : Event variances for both the TE and TL. of each event are derived below. The computational procedure should be self evident. We shall put to use the variances of the various activities derived in part (b) of solution to example 3 above.

Earliest Time, TE (D=Duration)

Event	*Longest path leading to it*	**Variance*
1	Nil (D=0)	0
2	1—2 (D=2)	1
3	1—3 (D=4)	1
4	1—4 (D=3)	1
5	1—2, 3—5 (D=4+6=10)	1+4=5
6	1—3, 3—5, 5—6 (D=17)	1+4+4=9

Suppose we wish to find the probability of reaching event 5 in 9 days. This can be computed as below.

$$Z = \frac{19-10 \mid}{\sqrt{5}} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5} = \frac{2\ 236}{5} = 0.4472$$

Probability (from Appendix Study II). 0 5—.1723=·3275 Likewise we can determine probabilities of reaching other events)

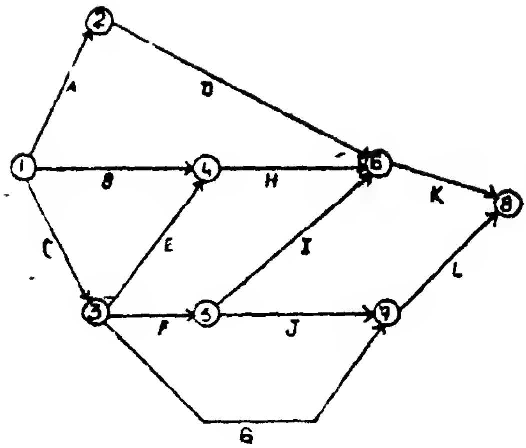
LATEST TIME, TL

Event	*Longest path from it to Last event 6*	**Variance*
1	1—3, 3—5, 5—6	9
2	2—5, 5—6	4
3	3—5, 5—6	8
4	4—6	1
5	5—6	4
6	Nil	0

Example 5. Shown below is a PERT network and a related set of activity times :

i—j	A	B	C	D	E	F	G	H	I	J	K	L
t_o	10	12	8	4	0	12	6	9	4	0	5	9
t_m	13	15	11	7	0	18	12	12	6	0	8	12
t_p	22	18	20	16	0	36	18	27	8	0	11	33

*(N.B. If there are two equal longest paths higher of the two variances would be picked up).



Required

(a) Determine the expected completion time of each activity.

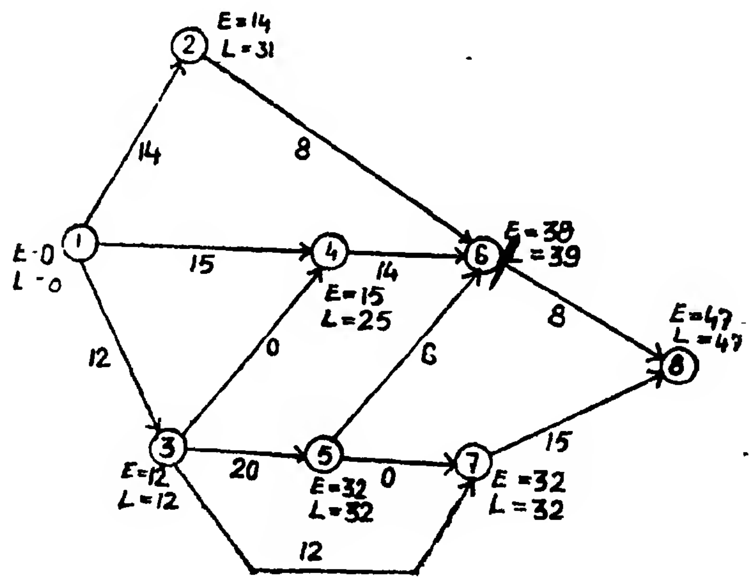
(b) Determine the earliest expected completion time, the latest expec completion time and float of each activity.

(c) What is the total project completion time, and what are activities the critical path ?

(d) Determine S.D. of expected completion time for only those activi on the critical path.

(e) Determine the probability that the project will be completed within weeks.

Solution ;



$$t_e = \frac{t_p+4t_m+t_0}{6};\ V_t = \left(\frac{t_p-t_0}{6}\right)^2$$

					Start		Finish				
i—j	to	$4t_m$	t_p	t_e	E	L	E	L	float	V_t	S.D.
1—2	10	52	22	14	0	17	14	31	17		
1—3	8	44	20	12	0	0	12	12	0	4	2
1—4	12	60	18	15	0	10	15	25	10		
2—6	8	28	16	8	14	31	22	39	17		
3—4	0	0	0	0	12	25	12	25	13		
3—5	12	72	36	20	12	12	32	32	0	16	4
3—7	6	48	18	12	12	20	24	32	8		
4—6	9	48	27	14	15	25	29	39	10		
5—6	4	24	8	6	32	33	38	39	1		
5—7	0	0	0	0	32	32	32	32	0	0	0
6—8	5	32	11	8	38	39	46	47	1		
7—8	9	48	33	15	32	32	47	47	0	16	4

$V_t = 36$

Hence S.D. of the critical path$=\sqrt{36}=6$.

Probability of completing critical path is 41 weeks :

$$Z = \frac{41-47}{6} = -1.$$

∴ probability = 0.159 **(Answer)**.

Exercise : 1 The following table lists the jobs of a network alongwith their time estimates.

i—j	Optimistic Duration	Most Likely Duration	Pessimistic Duration
1—2	3	6	15
1—6	2	5	14
2—3	6	12	30
2—4	2	5	8
3—5	5	11	17
4—5	3	6	15
5—8	1	4	7
6—7	3	9	26
7—8	4	19	28

(*a*) Draw the project network.

(*b*) Calculate the length and variance of critical path.

(*c*) What is the approximate probability that jobs on the critical path will be completed by the due date of 41 days ?

(*d*) What is the approximate probability that jobs on the next most critical path will be completed by the due date ?

Answer · (*b*) Length=36 days , Variance 25(*c*) 0 84 (*d*) 0 84

Exercise 2. Below are given the three time estimates of each activity of a project network. Construct the network Find the variance of each activity and variances of TE and TL of each event

$i-j$	1—2	1—3	1—4	2—5	3—5	4—6	5—6
t_o	7	16	7	9	20	14	2
t_m	10	18	8	12	24	18	3
t_p	12	20	9	17	26	20	7.

Answer

$i-j$	Variance	TE Event No	Variance	TL Event No	Variance
1—2	0 69	1	0	1	2.14
1—3	0 45	2	0 69	2	2 46
1—4	0 11	3	0 45	3	1 69
2—5	1 77	4	0 11	4	1.00
3—5	1 00	5	1 45	5	0 69
4—6	1 00	6	2 14	6	0
5—6	0 69				

6 A few comments on assumptions of PERT & CPM

1 Beta distribution may not always be applicable

2. The formulae for the expected duration and S D are simplifications. Maccrimnon and Ryavec reached the conclusion that in certain cases the errors, because of these assumptions, may even be to the tune of 33%.

3. The errors owing to the aforesaid simplification and assumption may be compounded or may cancel each other to an extent.

4. In computing the S D of the critical path independence of activities is implied Limitations of resources may invalidate the independence which exists by the very definition of an activity.

5. It may not always be possible to sort out completely identifiable activities and to state where they begin and where they end.

6. In projects fraught with uncertainty, it is natural that there exist alternatives with differing outcomes. For example, if a particular hardness is not obtained in a metal, an alloy might have to be used that is more expensive and also inferior on certain technical considerations. There have been theoretical developments in this regard,

and it may be worthwhile to incorporate the concept of decision tree analysis depending upon the situation.

7. Time estimates have an element of subjectiveness and, to that extent, the techniques could be weak. The contractors react to this weakness shrewdly whilst bidding. If these are cost plus contracts they would deliberately "under estimate" the times for chances of being awarded with the contract. Incentive type contracts might lead to an opposite bias.

8. Cost-time tradeoffs, for deriving the cost curve slopes, to be discussed soon, are subjective again and call for a great deal of expertise of the technology as well as genuine effort to estimate. Often the engineers tend to be lax here , occasionally with honest deliberation even, the guesses may be wide off the mark

7. Free and Independent Floats

Free Float · It is computed by subtracting the head event slack from the total float. The head event slack is its (L—E) Free float is that portion of the total float within which an activity can be manipulated without affecting the float of subsequent activities.

Independent Float : It is computed by subtracting the tail event slack from the free float. *If the obtains in—ve it is to be taken as zero.* It is that portion of the

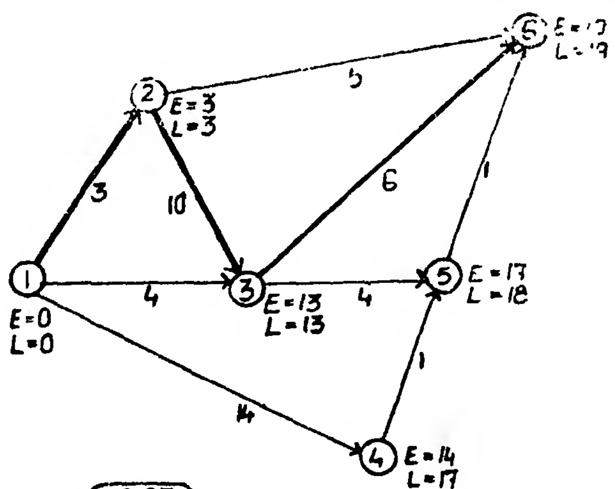


FIG 27

total float within which an activity can be delayed for start without affecting floats of the preceding activities.

The following example is intended to explain the computational aspect of the three floats. Please refer to Fig. 27.

Network Analysis Table

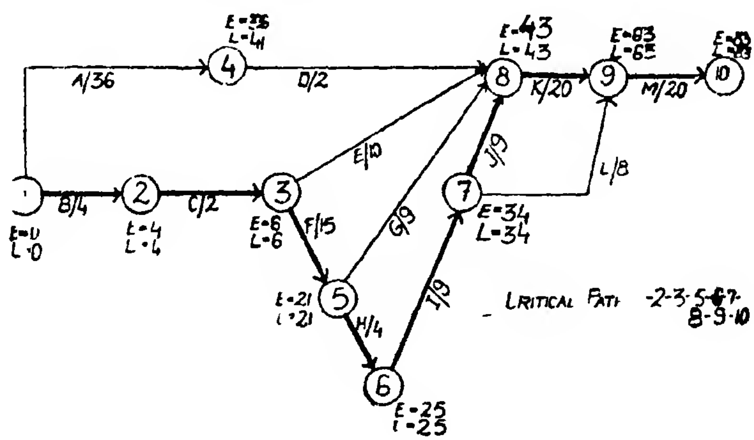
Activity	Duration	Start E	Start L	Finish E	Finish L	Float T	Float F	Float I
1—2	3	0	0	3	3	0	0	0
1—3	3	0	9	4	13	9	9	9
1—4	14	0	3	14	17	3	3	0
2—3	10	3	3	13	13	0	0	0
2—6	5	3	14	8	19	11	11	11
3—5	4	13	14	17	18	1	1	0
3—6	6	13	13	19	19	0	0	0
4—5	1	14	17	15	18	3	3	0
5—6	1	17	18	18	19	1	1	0

(N B. : Activities must always be arranged in the *i—j* sequence)

Explanotion : Consider activity 1—3. The slack of head event 3 is 13—13 =0. Therefore, free float=9—0=9. Likewise slack of tail event 1 is 0—0=0. Therefore independent float=9.

Example : Analyse the network below for the critical path and the three floats for each activity.

Solution :



Network Analysis Table

i—j	D	Start		Finish		Floats		
		E	L	E	L	T	F	I
1—2	4	0	0	4	4	0	0	0
1—4	36	0	5	36	41	5	0	0
2—3	2	4	4	6	6	0	0	0
3—5	15	6	6	21	21	0	0	0
3—8	10	6	33	16	43	27	27	27
4—8	2	36	41	38	43	5	5	5
5—6	4	21	21	25	25	0	0	0
5—8	9	21	34	30	43	13	13	13
6—7	9	25	25	34	34	0	0	0
7—8	9	34	34	43	43	0	0	0
7—9	8	34	55	42	63	21	21	21
8—9	20	43	43	63	63	0	0	0
9—10	20	63	63	83	83	0	0	0

Critical Path : BCFHIJKM=83 weeks.

8. The Project Plan

In the discussion on PERT, we saw how the probability of completion of a project can be computed for a specified duration. There are usually compelling reasons to complete the project earlier than would be the case with the duration of the critical path computed on the basis of normal activity times by employing extra resources. An example would be introduction of a new product The motive in hastening the project might be to ensure that the competitors do not steal a march. It is necessary to introduce the concept of activity cost slope at this stage. A few definitions are in order. Please refer to Fig. 28.

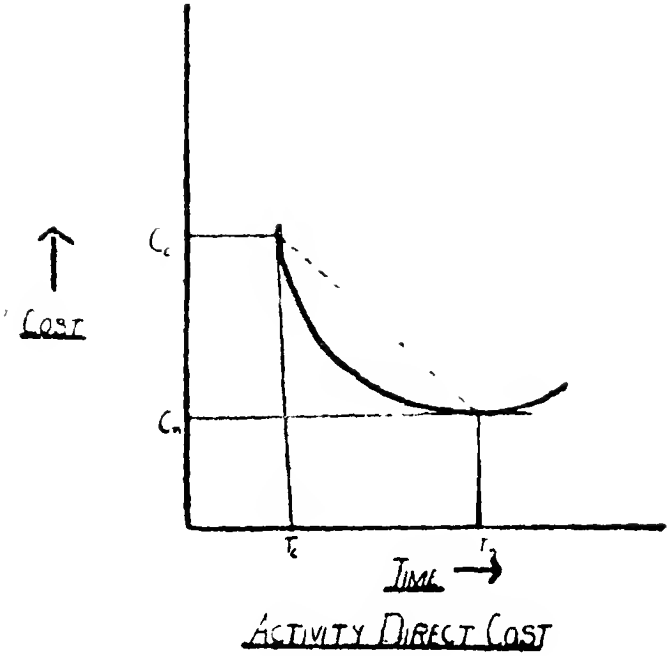


Fig. 28

Definitions

Activity Cost is the cost of performing (completing) a particular activity or task As is to be seen in figure 28 both shortening and protracting the activity duration is more expensive than would be the case with normal time.

Crash Cost Cc · This is the direct cost that is anticipated in completing an activity within the crash time

Crash time, Tc : This is the minimum time required to complete an activity.

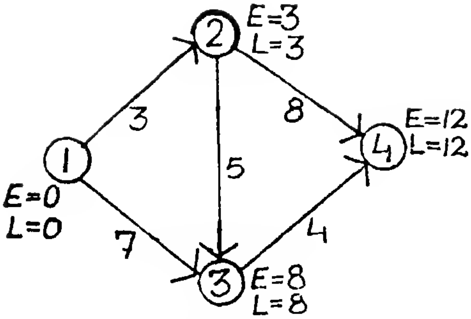
Normal Cost, Cn : This is the lowest possible direct cost required to complete an activity.

Nnrmal Time, Tn : This is the minimum time required to complete an activity at normal cost.

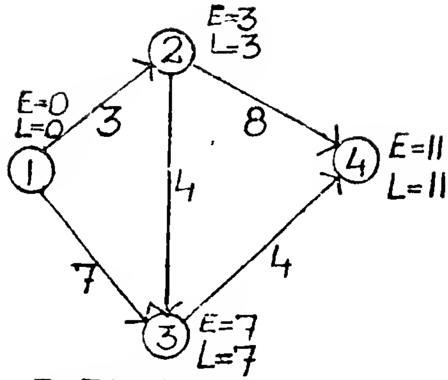
Activity Cost Slope : The cost slope indicates the additional cost incurred per unit of time saved in reducing the duration of an activity

It can be seen from the figure that by a straight line as an approximation to the cost curve the activity cost slope may be expressed as :

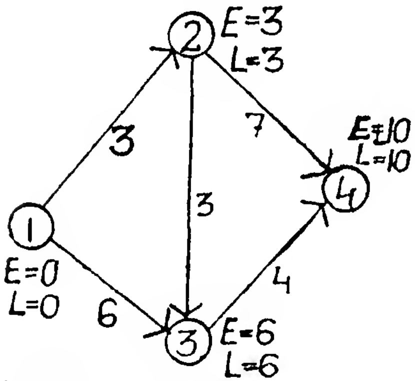
$$\text{Activity Cost Slope} = \frac{C_c - C_n}{T_n - T_c}$$



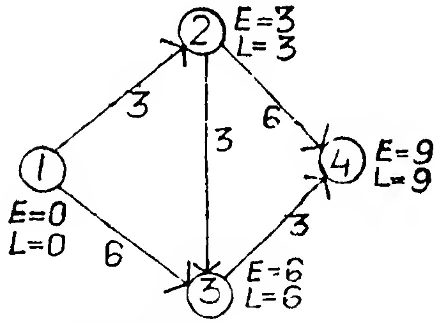
DURATION= 12
REDUCE 2-3 BY 1 DAY.
FIG·A



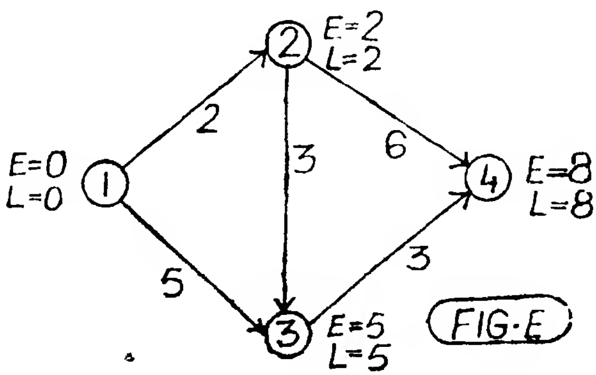
DURATION =11.
REDUCE 2-4, 1-3 & 2-3
BY 1 DAY EACH. FIG·B



DURATION=10.
REDUCE 2-4 & 3-4 BY
1 DAY EACH FIG·C



DURATION = 9.
REDUCE 1-2 & 1-3 BY 1 DAY
EACH. FIG D



FIG·E

The approximation by the straight line is fairly reasonable The estimate of times and costs would be made by those who possess expertise in the field for which these are required.

The example below which the student should go through carefully is intended to explain cost implications of hastening a project

Example : The following data pertains to the network drawn in figure A of p. 31. It is desired to compress the project to the least possible duration day by day and estimate the extra cost.

i—j	T_n	T_c	Cost slope
1—2	3	2	700
1—3	7	4	200
2—3	5	3	100
2—4	8	6	200
3—4	4	2	400

Solution : The critical path is 1—2—3—4 in figure A. It is to be seen that the critical path, 1—2—3—4 is longer than either of the paths ; 1—3—4 and 1—2—4 by one day. Therefore, the project can be compressed by one day along the critical path, 2—3 having the least cost slope is therefore crashed by a day.

Extra Cost = Rs. 100

The revised network is depicted in figure B where all activities have become critical. The following choices of compression exist now. Each set of activities is so chosen that it *reduces all the paths by* a day.

Crash each activity in one of the following sets by a day.

1—2	1—2	2—4	1—3
1—3	3—4	3—4	2—3
			2—4
Cost=900	Cost=1100	Cost=600	Cost=500

The last set of crashing 1—3, 2—3 and 2—4 is the least expensive and these activities are crashed accordingly.

Extra cost =200
100
200
Rs. 500

This is shown in figure C. 2—3 having been crashed to the limit is dropped out from further consideration. In C the following choices of crashing each activity by a day exist :

1—2	1—3	2—4
3—4	1—2	3—4

The last is selected **Extra Cost** =600

The revised network is shown in figure D where 2—4 joins 2—3 in that it is also crashed to limit. The only possibility of compressing the network in figure D is to crash 1—2 and 1—3 by a day each This is done and the final network is shown in figure E.

Extra Cost =900

Although 1—3 and 3—4 have not reached their crashing limits in figure E there is no use also to crash these since this would not compress the project which can be compressed to 8 days only.

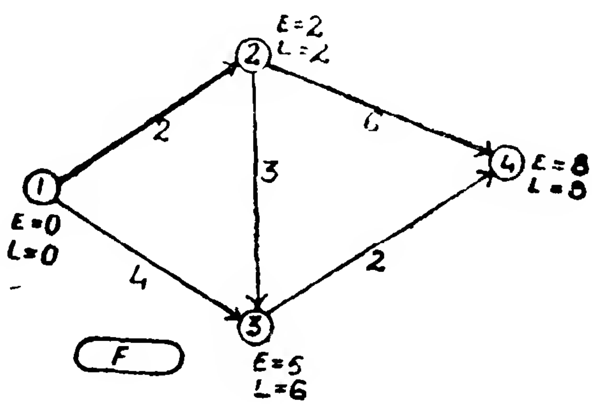
Total extra cost = 100+500+600+900 = Rs 2,100.

[If, however, just the least duration plan was required one could go about the problem in a much simpler way as follows Draw the network with T_e's. This is done in the figure below The network is analysed with 1—2—4 as the critical path of 8 days' duration. The other paths have to be protracted to 8 days' duration This can be done in one of the following ways :

(i) Increase 1—3 and 3—4 by a day each with cost reduction of Rs. 200+400 = 600.

(ii) Increase 2—3 and 1—3 by a day and 2 days respectively with cost reduction of Rs. 100+400 = 500

Obviously the 1st course is to be preferred and the network, if now revised, would be identical with the one of figure E].



Often, however, management would be interested in the least total cost duration rather than the least possible duration. The components of the total cost are depicted below :

Total Cost

Direct Cost	Indirect Cost

Indirect Cost

Opportunity loss (i.e loss of profits owing to delay in the project)	Overheads

Fig. 29 depicts these costs with respect to the duration As the duration decreases both the components of the indirect cost fall But the direct costs, as brought out in the above example, rise with compression or crashing of the duration.

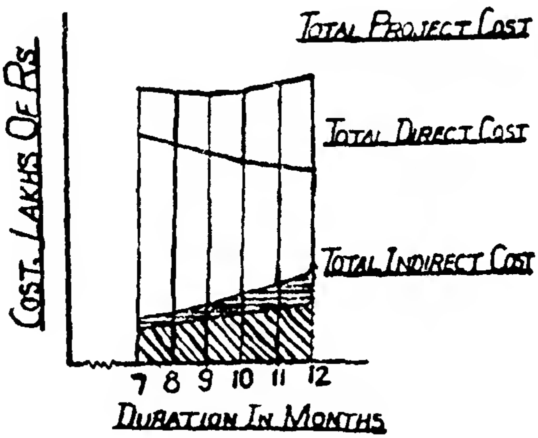


Fig. 29

The topmost curve for the total project cost has a minima that the management should really opt for. The corresponding network is the project plan.

Now, suppose there is an indirect cost of Rs. 800 per day. What would then be the least cost project duration for the example on hand ? The various cost data are tabulated below :

Duration	Direct (crashing) cost Rs.	Indirect cost Rs.	Total cost Rs.
12	0	9600	9600
11	100	8800	8900
10	600	8000	8600
9	1200	7200	8400
8	2100	6400	8500

Thus 9 days is the least total cost duration and rationally the management should go in for this unless high opportunity losses compel them to select a lower duration project plan.

If the management were merely interested in compressing the project to the east possible duration or least total cost solution the problem is solved in the nanner of the above example. The practical problems are, however, much more omplex and entail scores if not, hundreds of activities. The above hit and trial nethod would be cumbersome to perform manually and even on the computer. There exist several mathematical alogarithms which require far less calculations hough the solution they yield is usually *near* optimal and not exactly optimal

Exercise. A small maintenance project consists of jobs in the table below. Vith each job is listed its normal time and a minimum, or crash time, in days. The ost (in Rs per day) of each job is also given.

Job $i-j$	Normal days duration	Minimum (crash) days duration	Cost/day
1—2	9	6	20
1—3	8	5	25
1—4	15	10	30
2—4	5	3	10
3—4	10	6	15
4—5	2	1	40

(a) What is the normal project length and the minimum project length ?

(b) Determine the minimum crashing cost of schedules ranging from nornal length down to, and including, the minimum length schedule That is, if L= ength of the schedule, find the costs of schedules which are L, L—I, L—2 and so in days long.

(c) Overhead costs total Rs. 60 per day What is the optimum length schelule in terms of both crashing and overhead cost ? List the scheduled duration of ach job for your solution :

Answer : (a) 20 days : 12 days

(b) Length (days)	Crashing Cost Rs.	Total Cost Rs.
20	0	1200
19	15	1155
18	30	1110
17	45	1065
16	85	1045
15	130	1030
14	195	1035
13	260	1041
12	335	1055

(c) Optimum length = 15 days.

9. Resources Allocation

Before we take up this topic in a formal manner we shall give below a couple of numerical examples that can be handled merely by commonsense.

Example.

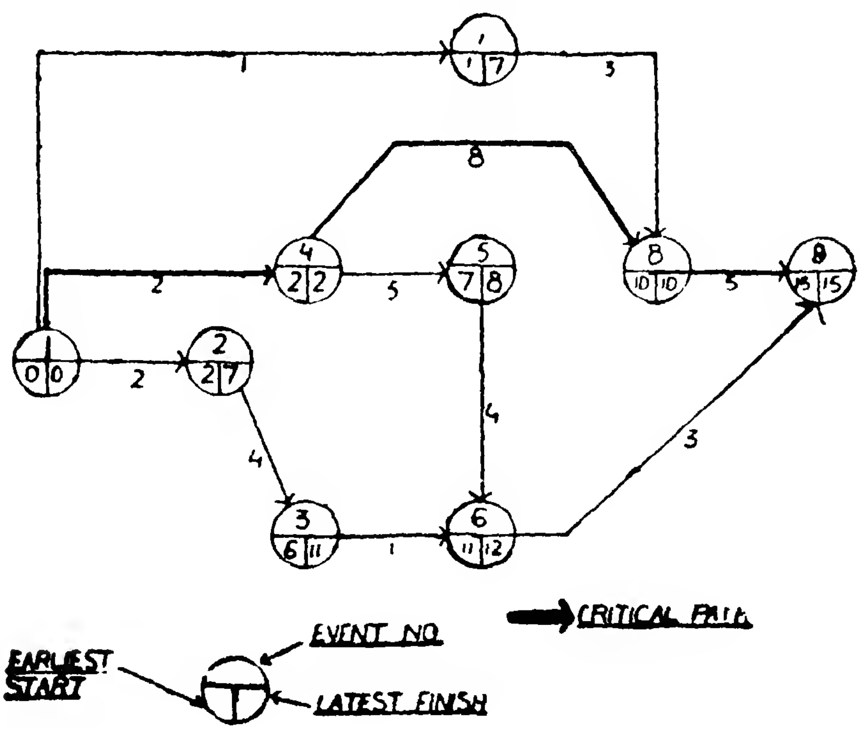
A project schedule has the following characteristics—

Activity	*Time*	*Activity*	*Time*
1—2	*2*	*4—5*	*5*
1—4	*2*	*4—8*	*8*
1—7	*1*	*5—6*	*4*
2—3	*4*	*6—9*	*3*
3—6	*1*	*7—8*	*3*
		8—9	*5*

(a) Construct PERT network and find critical path and time duration of the project

(b) Activities 2—3, 4—5, 6—9 each requires one unit of the same key equipment to complete it Do you think availability of one unit of the equipment in the organisation is sufficient for completing the project without delaying it, if so, what is the schedule of these activities ?

Answer : The network is drawn below. Also derived are E's and L's.



Network Analysis Table

i—j	D	Start E	Start L	Finish E	Finish L	Total float
1—2	2	0	5	2	7	5
1—4	2	0	0	2	2	0
1—7	1	0	6	1	7	6
2—3	4	2	7	6	11	5
3—6	1	6	11	7	12	5
4—5	5	2	3	7	8	1
4—8	8	2	2	10	10	0
5—6	4	7	8	11	12	1
6—9	3	11	12	14	15	1
7—8	3	1	7	4	10	6
8—9	5	10	10	15	15	0

Hence critical path (along 0 floats) is 1—2—8—9 with duration of 15.

(b) Activity 6—9 can only be undertaken when both 2—3 and 4—5 and their following activities are over. Thus 2—3 and 4—5 contend for the equipment. The two alternative schedules for these are :

	Start			Start
2—3	2		4—5	2
4—5	6		2—3	7
6—9	15		6—9	12

The 2nd alternative does not delay the project and is, therefore, recommended.

Example. Electronics Production Company is nearly through with a project to produce a small warning signal generator. Owing to a reduction in required performance characteristics, the remainder of the work is simpler to do than was originally planned. A new estimate of time to completion must be made. The following is known :

Activity	*Time (days)*	*Immediate predecessors*	*Description*
A	1	—	Check total weight and approve
B	2	—	Check power consumption
C	2	—	Check temperature requirements
D	2	A, B	Choose connecting plug
E	4	B, C	Fix resistor final values
F	1	C	Choose encapsulating foam
G	4	D	Ensure hermetic seal
H	8	G, E, F	Perform final test

(*a*) Draw a critical path network, and indicate the critical path.

(*b*) What is the minimum time to completion.

(*c*) During the second day of work [day 1,0 being the start day], it is discovered that activity F (choose encapsulating foam) will take 4 days instead of 1 Will this delay the project ? If the activity takes 6 days, will the project be delayed ?

(*d*) The company has limited number of men available to work on the project. Only two activities can be under way at the same time. Will this delay the project over what the time would have been with unlimited resources ? (Activity F takes six days to complete)

Solution (*a*) The network is constructed below with the critical path (1—2—4—6 7—8 or BDGH) shown in bold lines.

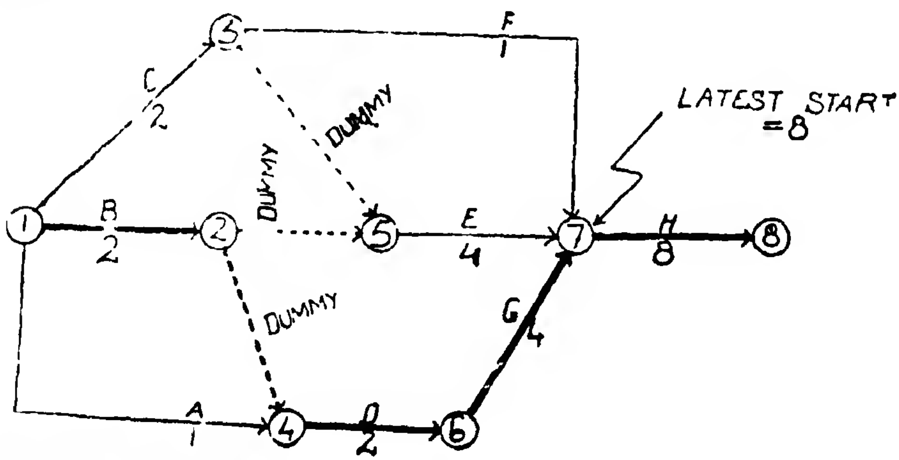
(*b*) The minimum time to completion is 2+2+4+8=16.

(*c*) Whether F takes 4 days or 6 days it would not delay the project activity provided C was undertaken on the first (*i e.*, 0-th) day If, however, C, in view of plenty of float available with path CFH, was not worked in the 0-th day F would delay the project if F takes now 6 days.

(*d*) Since the critical path takes 16 days it can be assigned to one resource. The other resource would have to do the following non-critical activities :—

	Duration	
A	1	Total time=13 days.
C	2	
E	4	
F	6	

But the 2nd resource would have to do all these activities by time 8 which is the latest start of event 7, shown by the crooked arrow in the network. Obviously the 2nd resource cannot do 13 days work in 8 days , therefore, the project is bound to be delayed.

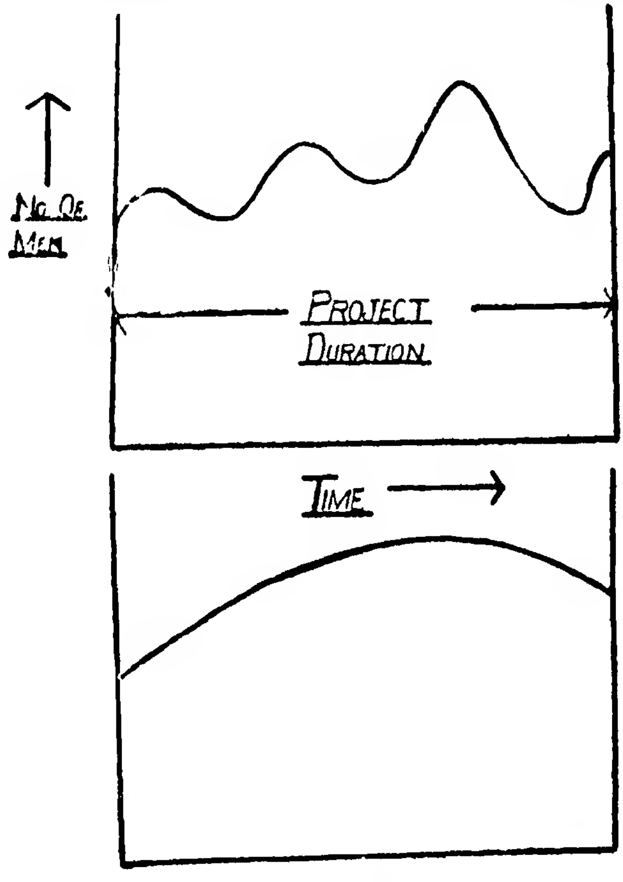


Formal Resource Allocation

A resource may be defined as a variable quantity of men, materials, machines or space, i.e., money that is required to carry out a project. The critical path is valid insofar the availability of resources is liberal i.e., activities need not

be delayed beyond their estimated duration for reasons of resource limitations. But this is seldom so in practice Resources are often sorely resticted.

Earlier, we discussed the planning aspects of the project. In case of projects with uncertain durations the PERT model is capable of providing estimates of probability for the desired completion date. We saw how the project could be compressed in less duration if there are compelling reasons to do so. The management can be provided with a table offering trade-offs between total duration times and costs. Now except for the critical path there is some leeway with all the non-critical activities Their earliest start dates can be delayed within allowable float so that the resources are diverted to more urgent jobs. The fixing of the actual start dates for the various activities is a scheduling problem to be solved by resource



Top: Actual Manpower Requmt
Bottom: Practical Way Of
Application Of Manpower
Resources

Figs. 30 and 31

allocation and it is an extremely complex one at that on mathematical grounds. Considering myriads of activities and dozens of resources the problem is too complex to be pursued for an optimal solution. A near-optimal solution is therefore welcome. The complexity is overmuch for the computer too Several heuristic programs (i.e., those based on rules of thumb) have been written and successfully executed In this section, our endeavour is to explain concepts of the resource allocation problem Several program packages are offered for resource allocation It is futile to explain all these *in toto*. The student, after having got the essence, should be in a position to make out the salient features of these packages and choose the one appropriately for the given situation.

Let us consider the resource of man-power Throughout the discussion we shall use this resource for illustration Other resources may be treated exactly in the same manner Some of the manpower *e g*, welders, brick-layers, pipe-fitters, etc may be computed fairly accurately for a particular situation. This is because standard times can be derived by time and motion study In the case of such manpower as engineers, lawyers, architects etc , it would be hard to make time estimates and compute exact requirements Some of them may look very busy, but may be performing less work than those who know better how to get work done from others For such professionals, therefore, the following analytical techniques which incidentally are amenable to soluation with a computer may not be strictly applicable. Ours, as noted above, is a limited purpose to provide an insight into the techniques

9 1 Variables Resource Levelling Management just cannot hire and fire its personnel as and when desired The management is restrained by considerations of morale and security of the personnel as well as supply and demand position of the labour. Build-up of labour and termination of their services has, therefore, to be gradual. Amidst fluctuating requirements (Fig. 30) a more uniform manpower utilisation pattern may be desired Under such circumstances, the objective would be to establish a gradual build up in personnel, preferably reaching a peak near the end of the project One of the resource allocation procedure discussed below is based on the practice to increase the labour force, say pipefitters, only when additional manpower is needed. Having employed such labour force they are not to be transferred to another project until their services are no longer needed on the current one. This is to be seen in Fig 31. Once the maximum is reached there is a gradual reduction of the work-force. This is the reason for calling it variable resource levelling.

9.2 Fixed Levelling Consider diagram 32A & B of welders for a project. Here, fixed number of welders is supposed to be at our disposal. As is quite conspicuous from the diagram this results both in excessive overtime as well as idle or standby labour force. The aim would be to select the optimum number of welders to minimise the consequences of both overtime and unproductive standby welders

as shown in Fig. 32B. It can be noticed that the [illegible] is eliminated altogether. This can apply to all trades of workers, equipment and facilities If, for example,

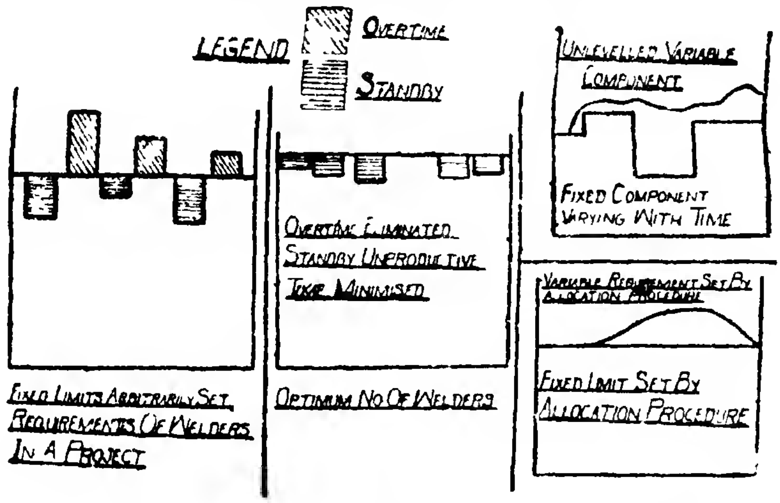


Fig. 32A Fig. 32B Fig 32C,D

an organisation is handling several projects simultaneously and its design department has engineers of different specialisations under fixed levelling, the problem would be to find the best limit of engineers of each specialisation and assign the projects appropriately.

9.3. Combined Levelling. The third and the final possibility is the combination of fixed and variable levelling Here a fixed complement of a talent is assigned to each project In order to satisfy its increasing needs a variable component is also assigned. This is shown in Fig 32 C & D

9.4. Queuing Interpretation of the Resource Allocation Problem. The resource allocation problem may be likened to a queuing problem for exposition. The resource *e g.*, men, machines etc , constitute what is service station in the queuing theory terminology. Activities "arrive" in queues with certain discipline The discipline is determined by the logical activity-dependency relationship upon the basis of which the network is constructed The float provides the play on the basis of which priorities are assigned for serving the activities. There may be time when the service stations, i.e , resources are idle and there is no activity in the queue. At other times there may be too many activities in the queue to be serviced simultaneously. As a consequence some of activities might have to be delayed. If the delay is within the free float this would not affect the following activities. If, however, free float is exceeded it would cause repercussions on the following activities.

At each time period, therefore, it is to be ascertained if there are activities in the queue and resources idle to service these. If so, a particular service

station can be booked for the duration of the activity. If either no activity is queuing or all the stations are booked already we can simply skip to the next time period. We shall develop a mechanical alogarithm on the basis of this discussion.

· Consider the network shown in Fig. 33 on p 45. Each activity requires 1 man, say, a welder. This is noted under the M column of table below. Man-days for the activity are noted under the M—D column. The usual network analysis table follows The total man-days requirements are 30 whereas project duration is 15. Therefore, roughly 30/15=2 men would be needed to complete the project in time in approximately 15 days. At time 0, the queue of activities and the available men are listed below :

1—2	R1
1—3	R2
1—5	

Network Analysis Table

				Start		Finish		Total
i—j	D	M	M—D	E	L	E	L	Float
1—2	5	1	5	0	0	5	5	0
1—3	3	1	3	0	2	3	5	2
1—5	2	1	2	0	7	2	9	7
2—3	0	0	0	5	5	5	5	0
2—6	4	1	4	5	8	9	12	3
3—4	5	1	5	5	5	10	10	0
4—7	5	1	5	10	10	15	15	0
5—6	3	1	3	2	9	5	12	7
6—7	3	1	3	9	12	12	15	3

Obviously, two activities can be token up at 0 time. The 3rd one would have to deferred until there are resources available again. Activity floats are as below :

1—2	0
1—3	2
1—5	7

The priorities are assigned on the basis of the float so that 1—2 and 1—3 with first and second priority are loaded on R1 and R2. It is presumed all men are equally and sufficient versatile to undertake any activity.

Activity 1—5 is delayed because there is no more man available at zero time. We should get R2 free from 1—3 at time 3 only. Then only it may be possible to take up 1—5. This would call for changing or updating the earliest start and finish and float as below :

Floating out of Activity 1—5

	Original	Updated
Earliest Start	0	3
Earliest Finish	2	5
Total Float	7	4

Also, earliest start of event 5 is modified from 2 to 5. This may affect

activities emanating from 5 and even the following activities and events. The consequences of floating out 1—5 can be inferred from the free float of 1—5. If it is less than this delay there are bound to be repercussions. If, however, it is more than this delay there would be no effect on the following paths.

It will be noted that the consequences of floating out of 1—5 eneded at event 6 because its E is still determined via the path containing activity 2—6. In another network, the repercussions may be traced down to several following activities and exents, even the last event.

There is neither any more free resource nor a queuing activity at time period 3. Same is the situation at 4. At 5, however, we do have the pending 1—5 and a few other activities as well as free resources, R 1 and R 2. The activities alongwith their pertinent data are listed below :

i—j	D	Float	Priority	
2—3	0	0	Top	(Dummy)
2—6	4	3	2	
3—4	5	0	1	
5—6	3	4	3	

Dummy activity is always assigned the top priority or it may even be ignored. 2—6 and 3—4 are loaded on R1 and R2 respectively. 5—6 would have to be deferred i e., floated out and would call for updating for the following activities and events in the manner described above.

We can proceed in this manner right up to the end of the network.

The *following points are worth noting* for developing algorithmic way of resource allocation.

A. Halt where both activities and resources are available.

B. Prior to allocation at a halt update the E S , EF and total float of activities affected by the floating out of an earlier activity.

C. The priorities to activities are assigned on the following basis :

1. Assign 1st priority to the activity with least float, 2nd to the activity with the next higher float, and so on
2. In case of tie in floats, assign 1st priority to the activity with higher M—D.
3. In case of tie in M—D's even again 1st priority to the activity with higher M.
4. In case of tie in M's even assign 1st priority to an activity with lower *i* where *i* is the tail event number of an activity.

N.B. : **These rules may, however, be overruled in a special case noted in p. 49 in bold letters.**

We need the following data for Resource Allocation, Activity List, D's, M's, M—D's, earliest start and finish and total float of each activity. The output is a schedule which fixes the start day of each activity on the manpower loading chart. This is arranged in 2 tables and a chart to scale as below :

(A) *Activity Table* (Activities are listed as in the analysis table earlier)

Activity i—j	Duration D	No of men M	Man days M—D	Earliest Start	Earliest Finish	Total float
1	2	3	4	5	6	7

(B) *Resource Allocation Table* (Specimen).

Halting Time	Available Resource	Activities in the queue				Allocated to Resoure
		i—j	float	M×D	Priority	
1	2	3	4	5	6	7
0	R1	1 - 2	0	5	1	R1
	R2	1 —3	2	3	2	R2
		1—5	7	2	3	

A brief description of the Resource Allocation Table is in order.

Col. 1. **Hatling Time.** Halt at a time period when there are both available resources and queuing activities. At time 0, it goes without saying that these two conditions are met and halting at 0 for allocations is a must. When allocations of activities to resources (to be explained shortly) is complete advance to further time periods and halt only when both resources and queuing activities are there. This is to be ascertained more easily and quickly from the Activity Table and the Loading Chart, e g., in the Loading Chart of Fig e g if 1—2, 1—3 are the two activities alloacted at halt at 0 the next possible halt could be 3. Then, look to Activity Table if there are queuing activities at their earliest start—Col. 5 of this Table.

Col. 2 *Available Resource* : At 0 all resource, say men, are available obviously. Later, see from the Loading Chart what are the resource available e.g., at 3 only R2 is there. Enter these resource under this col.

Col. 3. i.j. : Activities due to be taken up at halt time are listed from the Activity Table.

Col. 4 5 Enter float and M×D in col. 4 and 5 respectively against each activity.

Col. 6. Establish and enter priority by the criteria mentioned in p. 43. For dummy activities write 'top'. For others : 1, the highest, 2 the next, and so on.

Col. 7. Enter the resources against the activity which is allocated to it.

Consider network of figure 33 again. Let us do the resource allocations for 2(30/15=2) welders. The procedure is given below step-wise :

(A) Determine the halting time.

(B) Fill columns 1 to 7 of resource allocation table, by the priority rules.

(C) Load the activities on the chart

(D) Tick the loaded activities on the activity table meaning that they are not to be considered any more Revise col. 5 to 7 of Activity Table for the delayed activities. Revision is to be carried out by the diffi-erence between the next possible halting time and this halting time. Revise affected E's.

(E) Repeat D until the repercussions of the delayed activities have been traced through to the end of the network

(F) Repeat steps A to E until complete loading.

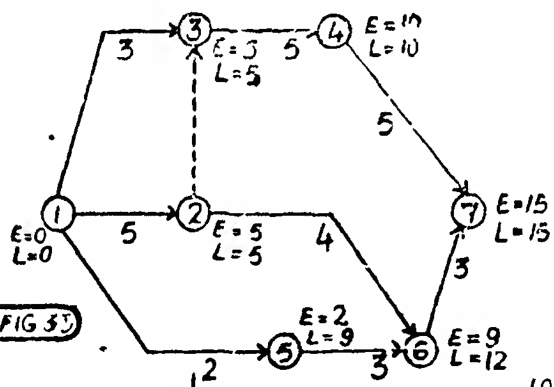


FIG 33

LOADING CHARTS

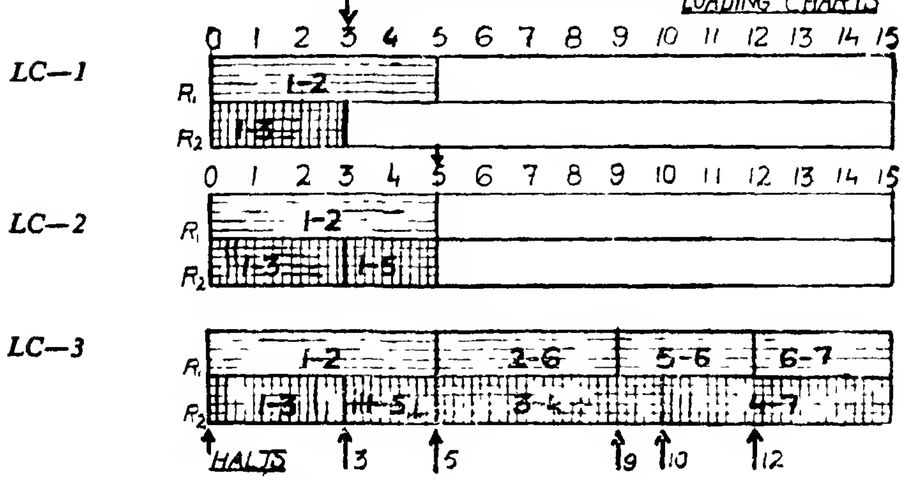


FIG 34

Let us do resource allocation for the network in Fig 33. Various tables are shown in p 47.

Halt at 0 (NB , RAT/1 means col 1 of Resource allocation table, etc)

Enter 0 in RAT/1 At 0 both the men R1 and R2 are obviously available. Enter R1 and R2 in RAT/2

For AT ascertain the activities commencing at 0 from AT/5. Enter i.j's of these activities in RAT/3 These activities are 1-2, 1-3 and 1-5. Enter their pertinent data in RAT/4 and 5 Establish their priorities In this case there is no tie in floats

We can only take up 1-2 and 1-3 and allocate these on LCI I (Loading chart 1).

No more resource left · therefore advance with 1-5 pending still. From LCI, R2 is free at time 3. It is the halting time as there are both 1-5 and R2. Prior to this, however, updating has to be done for floating out of 1-5 This is done in AT/5 to 7. This does have repercussions on the following events/activities. Event 5 has been revised from 2 to 5. Therefore 5-6 is updated in RAT, At. Event 6 is however not affected. Tick AT/1 for activities which have been allocated.

Halt at 3 Enter 3 in RAT/1 and R2 in RAT/2. From AT, 1-5 is the only activity. Fill RAT/3 to 6 Priority is undisputably 1. Allocate it to R2 in RAT/7 and in the Loading Chart. This is depicted in LC 2. Neither any activity nor any resource is left at 3. We advance—up to 5 from LC 2, having ticked AT/1 for 1-5. From AT/5 we find there activities at time 5.

Halt at 5 : Enter R1 and R2 in RAT/2 as both are idle. Also list activities and their data in RAT/3 to 6. 2-3 is a dummy and tick it in AT/1. No more consideration need to be given to it. 2-6 and 3 4 are allocated to R1 and R2 in RAT/7 and is shown in LC3 5-6 is pending for resources and from LC 3 we can halt at 9. Updating for 5-6 is done in AT/5 to 7 Event 6 is changed from 9 to 12 and this calls for updating of 6-7 as well.

Halt at 9 : Enter R1 in RAT/2. Enter the only activity 5-6 that can commence as ascretained from AT/6. It goes to R1 and is shown in RAT/7 and CL-3. Neither any activity nor any resource is left No updating in view of this. From LC-3, 10 is the prospective period for halting. This indeed is so as from AT/5 and we see 4-7 is due to commence at 10.

Halt at 10 : Enter R2 in RAT/2. Fill RAT|3 to 7 for 4-7 and R2. Allocate 4-7 in LC3. No more activity and resource is left From LC 3 we could possibly halt at 12. From AT/5 the last activity 6-7 is due to commence at 12 which is, therefore, the next halting time.

Halt at 12. Enter R1 in RAT 2. Fill RAT/3 to 7 for 6-7 and R1. Allocate 6-7 in LC-3. This completes the resource allocation within the period of 15.

Activity Table

i-j	D	M	M-D	Earliest Start	Earliest Finish	Total Float
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1—2	5	1	5	0	5	0
1—3	3	1	3	0	3	2
1—5	2	1	2	0/3	2/5	7/4
2—3	0	0	0	5	5	3
2—6	4	1	4	5	9	0
3—4	5	1	5	5	10	0
4—7	5	1	5	10	15	0
5—6	3	1	3	2/5/9	5/8/12	7/4/0
6—7	3	1	3	9/12	12/15	3/0

N B. : With practice, the student may want to dispense with RAT and work only with AT and L.C.

Resource Allocation Table

Halting Time	Av Resource	Activity Queue i-j	Float	M×D	Priority	Allocated to Resource
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0	R1, R2	1—2	0	1×5	1	R1
		1—3	2	1×3	2	R2
		1—5	7	1×2	3	
3	R2	1—5	4	1×2	1	R2
5	R1	2—3			Dummy	
	R2	2—6	3	1×4	2	R1
		3—4	0	1×5	1	R2
		5—6	4	1×3	3	
9	R1	5—6	0	1×3	1	R1
10	R2	4—7	0	1×5	1	R2
12	R1	6—7	3	1×3	1	R1

Example : The previous example was quite simple as it was chosen for illustrative purposes only We assumed that each activity required one man only. We try the project of fig 35 with the manpower requirement for the various activities below :

Activity	Men required
1—2	1
1—3	2
1—5	3
2—3	0
2—6	1
3—4	2
4—7	3
5—6	1
6—7	2

Although the rough requirements are $\frac{112}{30}$, we solve the problem for 4, 3 and 5 men. Since some activities require 3 men the case of trying the possibility of 2 men for the entire project does not arise. The procedure is much the same as for single man for each activity. The student may like to proceed with the resource allocation himself and compare the steps and results with that of ours as an exercise.

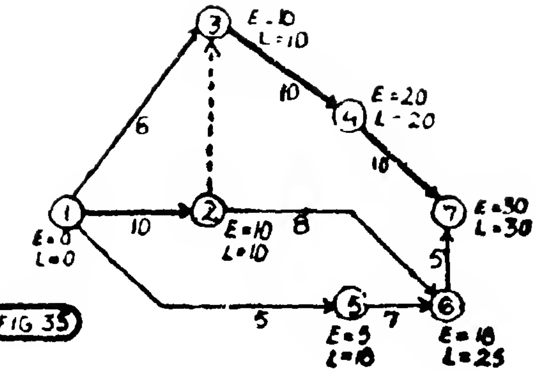
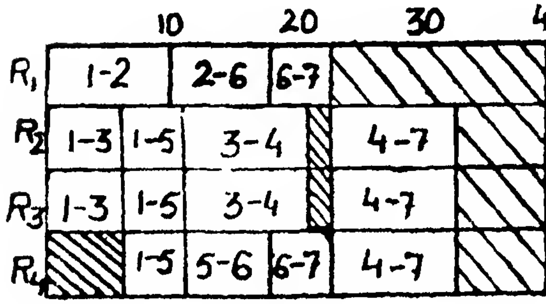
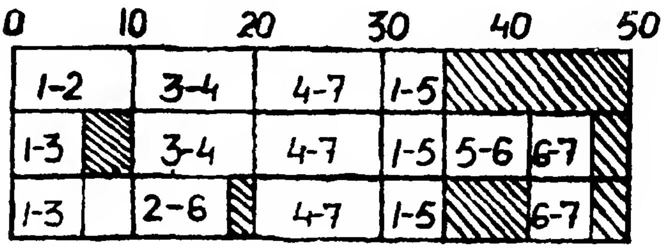


FIG 35



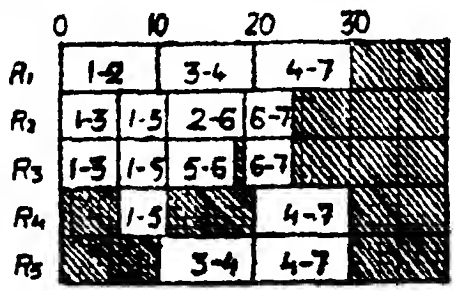
CASE-1
4 WELDERS

FIG 36



CASE 2
3 WELDERS

FIG 37



CASE 3
5 WELDERS

Two things are to be noted particularly here

(i) During the updating procedure the float of an activity may go negative which implies that the project duration is going to be extended beyond the critical path. Once a negative float occurs, therefore, the float criteria for fixing priorities is invalidated. The priorities should then be fixed on the basis of M D, gang size and sequence criteria respectively.

Network Analysis Table (Figure 35 on Page 48)

Activity	D	M	MD	Start		Finish		Total Float
				E	L	E	L	
1—2	10	1	10	0	0	10	10	0
1—3	6	2	12	0	4	6	10	4
1—5	5	3	15	0	13	5	18	18
2—3	0	0	0	10	10	10	10	0
2—6	8	1	8	10	17	18	25	7
3—4	10	2	20	10	10	20	20	0
4—7	10	3	30	20	20	30	30	0
5—6	7	1	7	5	18	12	25	13
6—7	5	2	10	18	25	23	30	7

(ii) When more than one man are required for an activity it may so happen during allocation that in spite of an activity queuing up and a resource available we may have to skip because the resources are not sufficient e g., 3 men are required whereas only 2 are available at a halt. The above procedure is not very sound theoretically for such a situation but, nevertheless, it provides fairly close results to the one expected under a more rigorous procedure.

(iii) The priority rules of p. 43 may be superseded by another consideration.

At each halt, resources are allocated according to priority. If the resources available are sufficient for the job priority of I, then that job is scheduled If, resources are insufficient, they may be made available to the job with the next priority and so on.

Activity Table (case 1 No. of men = 4)

Activity	D	M-D	Earliest Start	Earliest Finish	Total Float
1—2	10	1×10	0	10	0
1—3	6	2×6	0	6	4
1—5	5	3×5	0/6	5/11	13/7
2—3	0	0×0	10	10	0
2—6	8	1×8	10	18	7
3—4	10	2×10	10/11	20/21	0/—1
4—7	10	3×10	20/23	30/33	0
5—6	7	1×7	5/11	12/18	13/7
6—7	5	2×5	18	23	7

Resource Allocation Table

Halting Time	Av. Resources	Activity in the queue i−j	float	M−D	Priority	Allocated to Resource
0	R1 to R4	1—2	0	1×10	1	R1
		1—3	4	2×6	2	R2, R3
		1—5	13	3×5	3	
6	R2 to R4	1—5	7	3×5	1	R2 to R4
10	R1	2—3	Dummy	Top		
		2—6	7	1×8	2	R1 (see rule of p. 51)
		3—4	0	2×10	1	
	Netative Float, no longer the criteria					
11	R2 to R4	3—4		2×10	1	R2, R3
		5—6		1×7	2	R4
18	R1, R4	6—7		2×5	1	R1, R4
23	R1 to R4	4—7		3×10	1	R2 to R4

Case 2 No. of men 3 & Case 3 No. of men=5

The student may do these himself as an exercise and compare his results with the loading chart of Figure 38.

Discussion of the results : The project requires at least 3 men if we keep the matter simple by not introducing overtime. Let the wage rate be Rs 15/day and the indirect cost of Rs. 10/day. The following total costs may be computed :

Case No	*No of Men*	*Completion time*	*Direct*	*Indirect*	*Total*
1	4	33	4×15×33=1980	33×10=330	2310
2	3	47	3×15×47=2115	47×10=470	2580
3	5	30	5×15×30=2250	30×10=300	2550

Obviously employing 4 men on the cost basis alone is the best course of action, but if more objectives are introduced, e g , completion time and the associated opportunity loss it becomes a problem of weighing the objective the student has had in Study Material—1. Yet another objective may be utilization of welder. The idle time can be ascertained from the loading charts and the % utilization may be calculated.

The course of action that provides the best effectiveness upon the 3 objectives.

1. total cost,
2. completion Time (converted to opportunity loss),
3. % labour utilisation

Exercise. Do resource allocation for the network below where each activity needs one man. No. of men available=3.

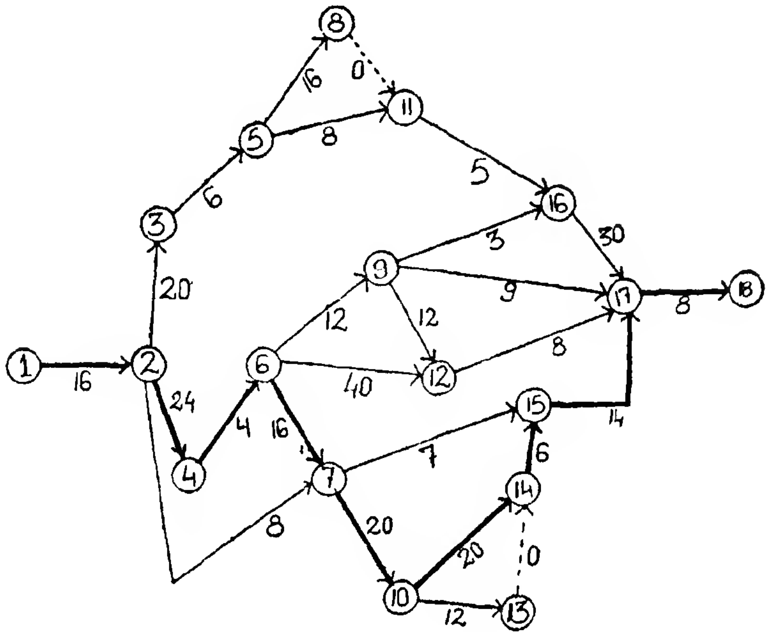


Fig. 39

Answer :

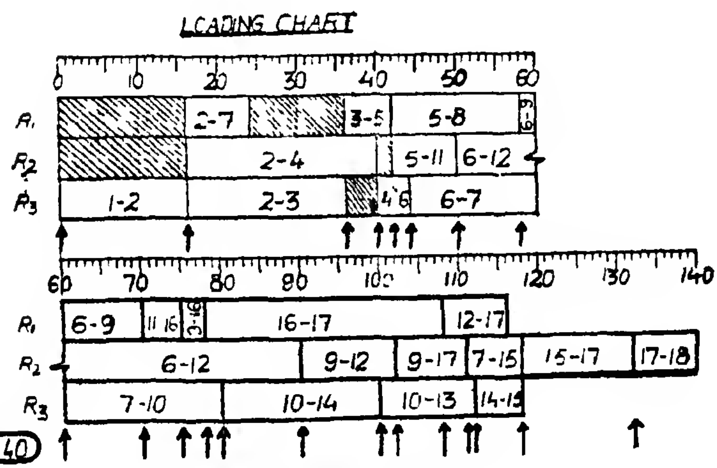
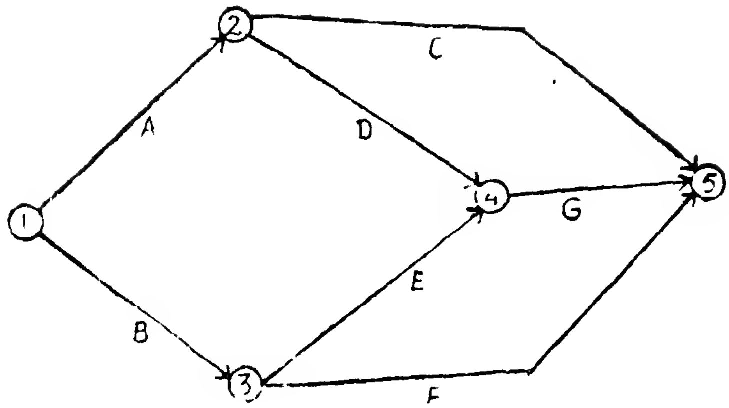


FIG 40

No. of men=33/128≈3

Self-Examination Questions

1. Below we give a PERT network with the related time data. You are required to find the probability of completing the project in 27 days.



Activity	A	B	C	D	E	F	G
t_o	6	5	4	4	4	4	2
t_m	9	8	7	7	7	10	5
t_p	18	17	22	16	10	22	8

Answer Critical path = 24 days, prob = 0 903)

2. The following table gives the activities in a construction project and other relevant information :

Activity	*Normal time days*	*Crash time days*	*Normal cost Rs.*	*Total Crash cost Rs.*
1—2	20	17	600	720
1—3	25	25	200	200
2—3	10	8	300	440
2—4	12	6	400	700
3—4	5	2	300	420
4—5	10	5	300	600

(a) Draw the network for the project.

(b) Find the total, free and independent floats for each activity.

(c) Using all information "crash" or "shorten" the project step by step until the shortest duration is reached.

3. Projects India Limited undertakes special contracts. The following table gives estimates of time and cost for activities involved in completing one contract which has been just offered to them :

Activity	*Previous Activities*	*Normal time days*	*Normal Cost Rs.*	*Minimum time*	*Cost of minimum time*
A	—	12	10,000	8	14,000
B	—	10	5,000	10	5,000
C	A	0	0	0	0
D	A	6	4,000	4	5,000
E	B,C	16	9,000	14	12,000
F	D	16	3,200	8	8,000
		60	31,200	44	44,000

"Previous activities" must be completed before the activity in question can be started The minimum time represents the shortest time in which the activity can be completed given the use of especial costly methods of operation Assume that it is possible to reduce the normal time to the minimum time in small steps and the extra cost incurred will be proportional to the time saved.

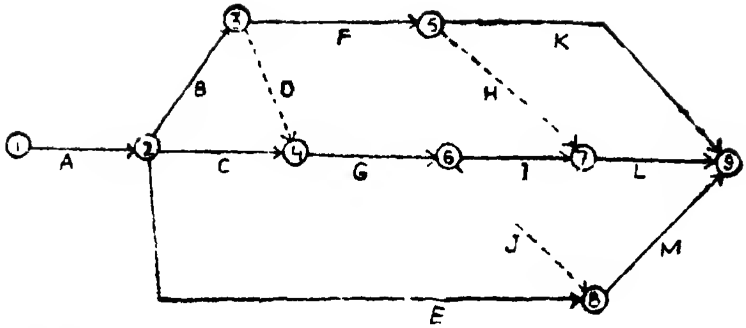
(a) draw a network diagram and identify the critical path for normal procedures

(b) Recommend what programme be followed if the job must be completed in 30 days and calculate the total cost for the programme

(c) Explain how would you modify your analysis if the estimates were subject to uncertainty. Illustrate your answer by assuming that estimates of the time required for E are uncertain Normal time is expected to be in the range of 12 days to 20 days, but 2 days could still be saved by spending an extra Rs 3,000 You remain confident about estimates for other activities Target time for the contract is 30 days and there would be a penalty of Rs 5,000 for late completion

4. Alpha construction company has an opportunity to submit a bid for the construction of a new apartment building From specifications provided by the developer, a PERT network for the project has been developed, and is shown below. Also shown for each activity are estimates of most optimistic, most likely, and most pessimistic completion times in weeks (a, m and b, repectively).

Activity	*a*	*m*	*b*	*Activity Description*
A	5	8	17	Excavate basement
B	5	8	11	Build concrete elevator tower
C	3	5	7	Pour concrete foundation
D		0	0	
E	6	9	18	Excavate parking ramp area
F	5	8	17	Install temporary manual elevator
G	5	7	12	Erect main building
H	0	0	0	
I	0	0	0	
J	0	0	0	
K	4	7	10	Install automatic elevator
L	7	10	31	Complete interior work
M	4	6	11	Erect parking ramp.



Required :

(1) Determine the critical path and its S D.

(2) Alpha's management policy with respect to submitting bids is to bid the minimum amount that will provide a 92% probability of at best breaking even Materials for this project will cost Rs. 900,000 and all other costs will vary at a rate of Rs. 10,000 for every week spent working on the project. What amount should be bid under this policy ?

(3) Assume that Alpha's bid was accepted and that the project has been in progress for 20 weeks. Activities A, B and C have been completed, Activities E, F and G are in progress, with the following estimates made of time required to complete them :

Activity	a	m	b
E	1	2	3
F	3	5	10
G	3	6	9

No change has been made in the time estimates for activites K, L and M. Draw a revised PERT network representing the remainder of the project (excluding completed activities). Determine the critical path for the remainder of the project and the remaining project complection time.

5. Table below shows jobs, normal and crash time and cost for a project :

Job	*Normal*		*Crash*	
i j	Time Days	Cost (Rs)	Time (Days)	Cost Rs.)
1—2	6	1,400	4	1,900
1—3	8	2,000	5	2,800
2—3	4	1,100	2	1,500
2—4	3	800	2	1,400
3—4	Dummy	—	—	—
2—5	6	900	3	1,600
4—6	10	2,500	6	3,500
5—6	3	500	2	800

Indirect cost for the project is Rs. 300 per day.

(*i*) Draw the network of the project.

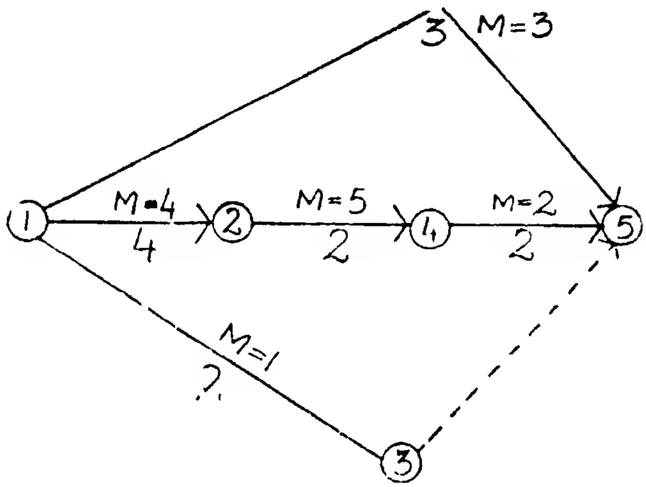
(*ii*) What is the normal duration and cost of project ?

(*iii*) Find the optimum duration and minimum project cost.

(*iv*) If all activities are crashed, what will be the project duration and corresponding cost ?

Ans. (*ii*) 20 days, Rs. 15,200 (*iv*) 12 days, Rs. 13,500

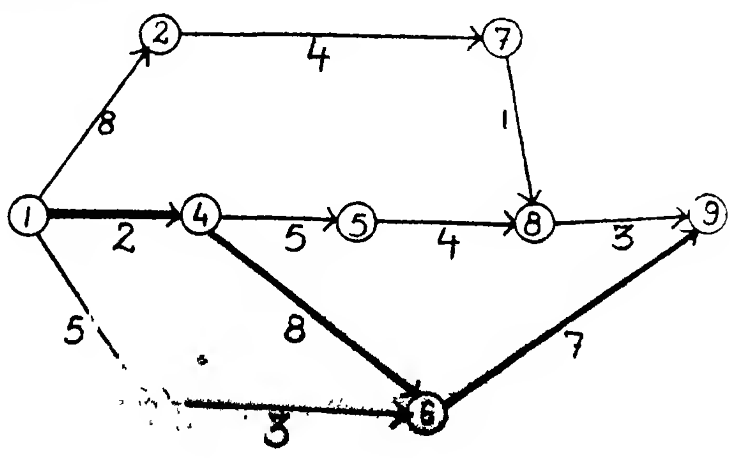
Q. 6.



In the network above are indicated the duration in days, and the number of men required for each activity. You are required to find the number of days in which the project can be completed with (i) unlimited resources and (ii) with a total of only 5 men available.

(**Ans.** (i) 8 days, (ii) 11 days.)

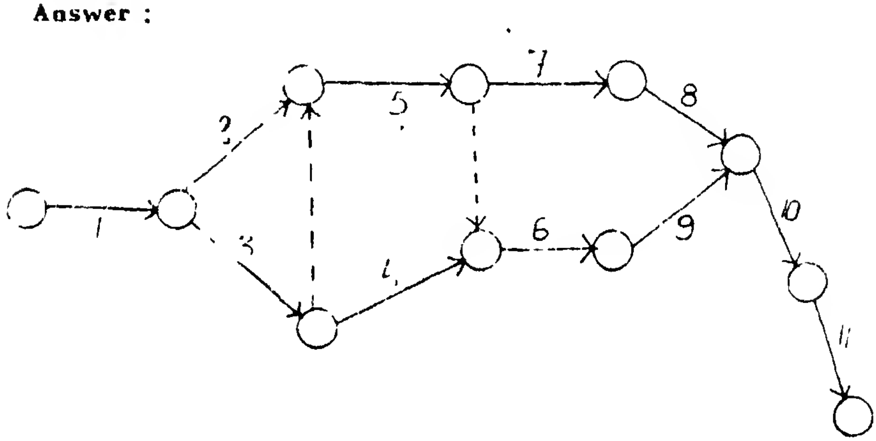
Q. 7. In the network below, suppose we are required to employ a special piece of equipment on activities 2—7, 4—5, and 8—9, one at a time. Will it affect the Project duration ?



Q. 8. A new type of conveyor is to be designed. The follwing list represents the major activities together with the precedence relationships sorted out for you. You are required to construct the network diagram.

Activity No.	Activity Description	Preceding Activity
1	Prepare drawings	—
2	Carry out cost analysis	1
3	Carry out economic feasibility	1
4	Manufacture tools	3
5	Prepare Bill of Materials	2, 3
6	Procure raw materials	4, 5
7	Order sub assemblies	5
8	Receive sub assemblies	7
9	Manufacture components	6
10	Final assembly	9, 8
11	Testing and shipment	10

Answer :



Q. 9. Construct PERT network for the following project, and determine the critical path

Activity	*Time*	*Immediate Predecessors*
A	4	None
B	1	None
C	1	A
D	1	B
E	6	C
F	8	E
G	2	F
H	4	F
I	1	H
J	5	G, I
K	5	D, G
L	7	K.

Q. 10. Assnme that you are letting a contract for widening a street-

tions until contractor go-ahead. Determine the critical path and the shortest time to complete the set of tasks :

Task	*Description*	*Expected time (working days)*	*Immediate Predecessor*
A	Preparations for job	5	—
B	Notify property owners	2	—
C	Select prospective bidders	4	—
D	Contact prospective bidders	4	C
E	Send out requests for bids	3	A,D
F	Hold a bid conference	2	E
G	Secure property owner's approval	14	B
H	Wait for bidders to prepere bids	10	F
I	Receive bids	2	H
J	Evaluate bids	5	I
K	Prepare final job schedule	2	G,J
L	Arrange details with water authorities	4	G,J
M	Get approval of schedule	1	K
N	Negotiate and sign contract	2	L,M

Q. 11. (a) Construct a PERT Diagram from the following information, and determine the critical path.

Activity	t_o	t_m	t_p	*Immediate Predecessor*
A	1	2	4	—
B	2	4	6	A
C	2	6	10	A
D	6	8	10	B
E	4	6	8	C
F	6	10	14	C
G	8	10	12	E
H	12	14	16	F
I	4	8	12	G,H
J	10	12	16	G,H
K	2	4	6	I
L	6	10	14	J

(b) Assuming that the schedule allows 40 days to complete the whole project, calculate the probability of completion by the scheduled date.

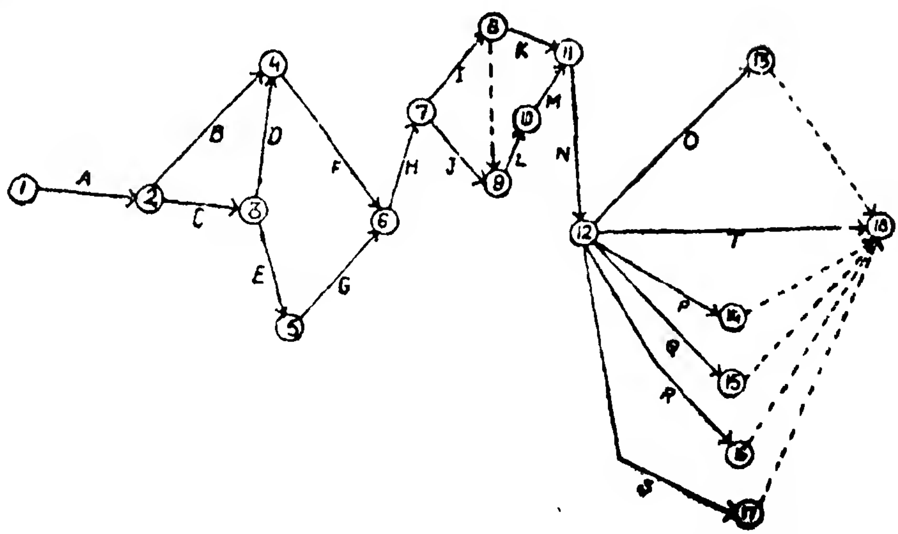
(c) The contractor wants a scheduled completion date that will give him 98% chance of attaining how many days should be allowed in his schedule ?

Q. 12. The activities involved in the project of the introduction of a new product are listed below. Also indicated against each activity is its predecessor. You are required to draw the network diagram making use of dummies where needed.

	Description of Activity	*Preceded By*
A	Perform initial screening	None
B	Check that the product conforms to	

D	Build prototype	C
E	Perform market enalysis	C
F	Estimate cost/price	B, D
G	Review manufacturing facilities	E
H	Make test decision	F, G
I	Manufacture model	H
J	Prepare market survey	H
K	Investigate packaging, label, etc.	I
L	Train interviewers	I, J
M	Conduct market survey	L
N	Make go-ahead decision	K, M
O	Manufacture packaging	N
P	Select name	N
Q	Select sales personnel	N
R	Prepare advertising	N
S	Make patent application	N
T	Manufacturing product	N

Auswer :

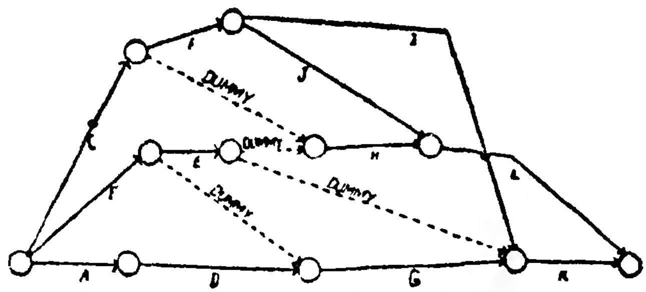


Q. 13. **Construct the network for the following set of activities.**

Activity	*Predecessors*
A	—
B	—
C	—
D	**A**
E	**B**
F	**C**
G	**B, D**
H	**C, E**
I	**F**

Answer :

The network depicting these dependency relationships is shown below.

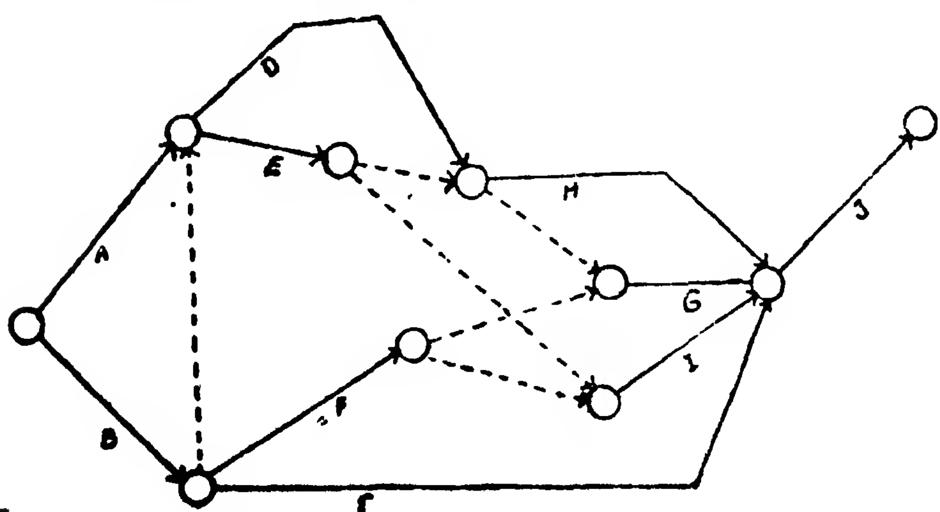


Q. 14. Draw the following network using dummies wherever necessary.

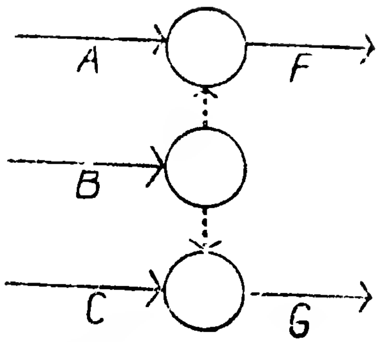
Activity	*Immediate Predecessor*
A	None
B	None
C	B
D	A, B
E	A, B
F	B
G	E,D,F
H	D,E
I	E,F
J	H,G,I,C

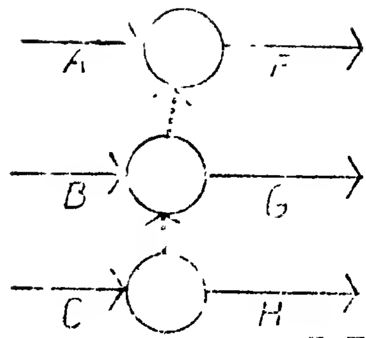
Answer:

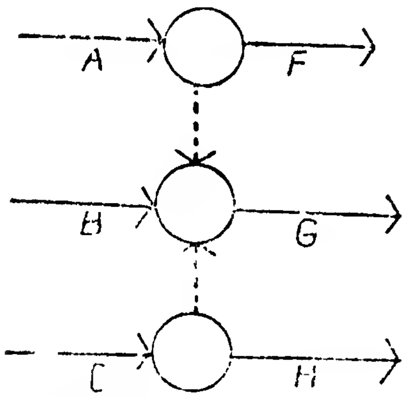
The network is drawn below,

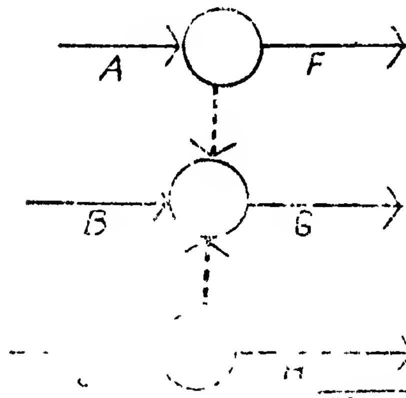


Answers to Exercise 2.









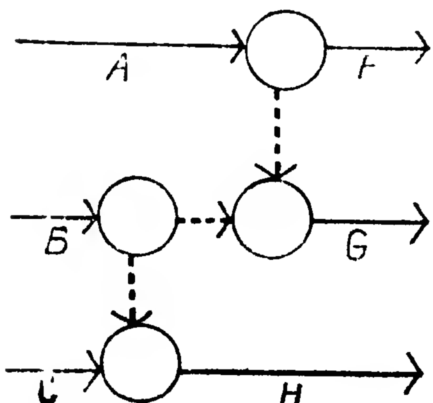


FIG 45

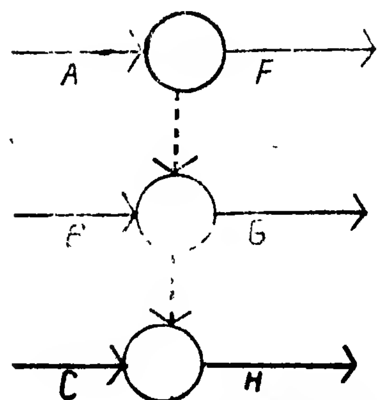


FIG 46

THE INSTITUTE OF CHARTERED ACCOUNTANTS OF INDIA

STUDY MATERIAL F S P. (N) O R —5
Combination—'B'

FINAL COURSE (N)

OPERATIONS RESEARCH

STUDY—V

Contents :

Suggested Readings : Fundamentals of Financial Management by Van—Horne (Chapter on Risk Analysis)

Operations Research by Goel & Mittal—Pragati Prakashan, Meerut

1. Decision Trees

The topic has been covered in Study VIII, Intermediate Statistics. Here, we shall take up a few relatively complex problems. The criteria of evaluation is mainly EMV i.e. Expected Monetary Value, which, the student may recall, is similar to expected regret or expected payoff or expected opportunity loss.

Example 1. Mr. X is trying to decide whether to travel to Sri Lanka from Delhi to negotiate the sale of a shipment of china novelties. He holds the novelties in stock and is fairly confident, but by no means sure that if he makes the trip he will sell the novelties at a price that will give him a profit of Rs 30,000 He puts the probability of obtaining the order at 0.6. If he does not make the trip he will certainly not get the order.

If the novelties are not sold in Sri Lanka there is an Indian customer who will certainly buy them at a price that leaves him a profit of Rs. 15,000 and his offer will be open at least till Mr. X returns from Sri Lanka. Mr X estimates the expenses of trip to Sri Lanka at Rs 2,500 He is, however, concerned that his absence, even for only three days, may lead to production inefficiencies in the factory These could cause him to miss the deadline on another contract, with the consequence that a late penalty of Rs 10,000 will be invoked Mr X assesses the probability of missing the deadline under these circumstances at 0 4 Further, he believes that in his absence there will be a lower standard of house-keeping in the factory, and the raw material and labour costs on the other contract will rise by about Rs 2,000 above the budgeted figure.

Draw an appropriate decision tree for Mr X's problem and using EMV as the appropriate criterion for decision, find the appropriate initial decision

Solution:

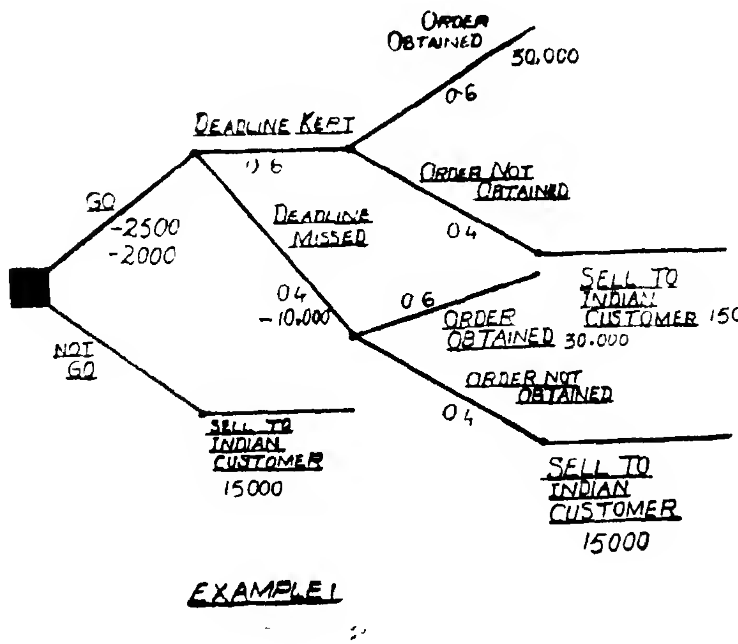


Fig. 1

The decision tree is drawn in the figure above. *Calculations of EMV proceeds from right to left and for this reason the decision tree analysis is also known as the Roll Back Technique.*

Go	*Not Go*
$30,000 \times 0.6 = 18,000$	$15,000 \times 1 = 15,000$
$15,000 \times 0.4 = 6,000$	(where 1 is the probability)

24,000

$24,000 \times 0.6 = 14,400$
plus
$24,000 \times 0.4 = 9,600$
minus
$10,000 \times 0.4 = 4,000$

20,000 Hence Mr. X should proceed to Sri Lanka
Minus 4,500

15 500

Example 2 M/s J Bloggs & Co is currently working with a process, which, after paying for materials, labour etc brings a profit of Rs 10,000 The following alternatives are made available to the company.

(i) The company can conduct research (R_1) which is expected to cost Rs. 10,000 and having 90% probability of success, the company gets a gross income of Rs 25,000.

(ii) The company can conduct research (R_2) expected to cost Rs. 5,000 and having a probability of 60% success. If successful the gross income will be Rs. 25,000.

(iii) The company can pay Rs 6,000 as royalty of a new process which will bring a gross income of Rs 20,000

(iv) The company continues the current process

Because of limited resources, it is assumed, that only one of the two types of research can be carried out a time.

Solution : The decision tree which represents the possible courses of action is depicted in figure 2. Point 1 is a 'decision box' located 'now' on the time scale. The four possibilities arising here are shown Upon failure of a particular research, say, R_1, there are again 3 original alternatives to be sorted out, that of R_1 being excluded If R_2 fails after failure of R_1 the company is left with only two choices, i e., either to pay royalty or continue the existing process.

Branch Current
Net Return = Rs. 10,000.

Branch Licence
Net Return = 20,000 − 6,000 = Rs. 14,000.
Branch R_1 First

Value at Point 3

Current	Rs. 10,000
Licence	Rs. 14,000
Value at point C,	Expected gross profit
	$= 25,000 \times 0.6 + 14,000 \times 0.4$
	$= 20,600.$

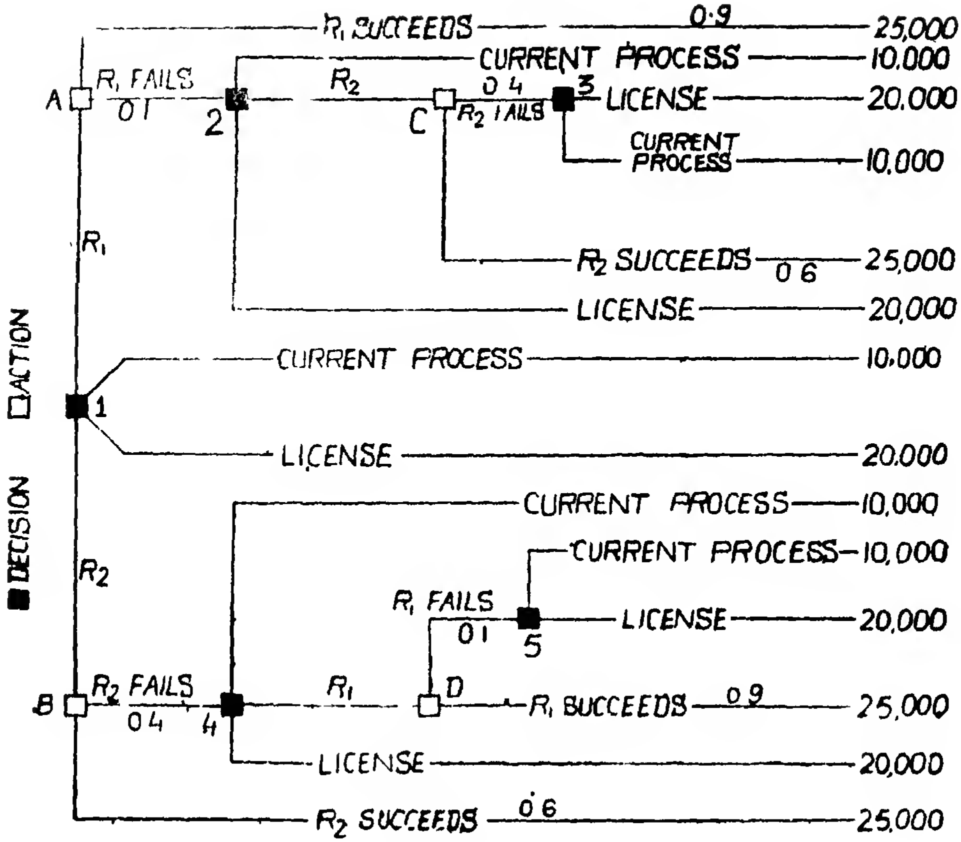


Fig. 2

Value at Point 2	
Licence	Rs 14,000
Current	Rs 10,000
R_2	20,600—5,000=15,600
Value at Point A	25,000×0 9+0.1×15,600=24,060
Value at Branch R_1	24,060—10,000—14,060
Branch R_2 First	
Value at point 5	
Current	Rs. 10,000
Licence	Rs. 14,000
Value at Point at D .	Expected gross profit =0.9×25,000+0.1×14,000 =23,900
Value at point 4	
Current	Rs. 10,000
Licence	Rs. 14,000
R_1	=Rs. 23,900—10,000 =13,900

Value at Point B $=0.4\times14{,}000+0.6\times25{,}000$
$=20{,}600$

Value of Branch R_2 $=20{,}600-5{,}000=15{,}600$

Thus R_2 followed by licence upon former's failure is the best course of action.

Exercises

1. A complex airborne navigating system incorporates a sub-assembly which unrolls a map of the flight plan synchronously with the movement of the aeroplane. This sub-assembly is bought on very good terms from a sub-contractor but is not always in perfect adjustment on delivery. The sub-assemblies can be readjusted on delivery to guarantee accuracy at a cost of Rs. 50 per sub-assmbly. It is not, however, possible to distinguish visually those assemblies that need adjustment.

Alrtenatively the sub-assemblies can each be tested electroincally at a cost of Rs. 12 per sub-assembly tested. Past experience shows that about 40% of those supplied are defective, the probability of the test indicating a bad adjustment when the sub-assembly is faulty is 0 7, while the probability that the test indicates a good adjustment when the sub-assembly is properly adjusted is 0 8. If the adjustment is not made and the sub-assembly is found to be faulty when the system has its final check, the cost of subsequent rectification will be Rs. 150

Draw up an appropriate decision tree to show the alternatives open to the purchaser and use it to determine his appropriate courses of action.

Answer : For either strategy. E M V =Rs 50

2. A client asks an estate agent to sell three properties (A, B and C) for him and agrees to pay him a 5% commission on each sale. He specifies certain conditions. The estate agent must sell A first, and this he must do within 60 days. If and when A is sold the agent receives his 5% commission on that sale. He can then either back out at this stage or nominate and try to sell one of the two remaining properties within 60 days. If he does not succeed in selling the nominated property in that period, he is not given the opportunity to sell the other If he does sell it in the period, he is given the opportunity to sell the third property on the same conditions The following table summarises the prices, selling costs (incurred whenever a sale is attempted) and the estate agent's estimated probability of making a sale.

Data for Property sales

Property	*Price of Property* Rs.	*Selling Costs* Rs	*Probability of Sale*
A	12,000	400	0 7
B	25,000	225	0 6
C	50,000	450	0 5

(i) Draw up an appropriate decision tree for the estate agent.

(ii) What is the estate agent's best strategy under EMV approach ?

Answer : Accept offer to try to sell A—If sell A then try to sell C—If sell C then try to sell B

2. Utility function

In study 11, we paid increasing attention to the expected pay-off criteria after heaving discussed the maximax and maximin criteria briefly The reason being that the expected payoff criteria lends itself to mathematical treatment to a great extent. We saw how it could be utilized in deriving the opportunity loss of the optimal policy which is the cost of perfect information With this cost, the decision maker can be aided to undertake a sampling enquiry *i e.,* the *a-priori* distribution (or probabilities) can be augmented to derive *a posteri* distribution. Thus one is closer to cetainty though at the expense of sampling enquiry. But the expected pay-off or return criteria has shortcomings This was demonstrated by Von Neauman-Morgenstern who developed the concept of utility which is far more realistic than the expected value.

It is a common experience that the worth of an amount of money, say Rs. 100, may mean different not only to different people but even to the same person at different occasions So is also the case in business situations. A company in a sound financial position may opt for a risky venture since it is capable of absorbing the loss, if at all it occurs, without much disruption

Consider the following set of alternative opportunities :

B_1—An award of Rs 1,000 with a chance of 100 in 100

B_2—An award of Rs 5,000 with a chance of 50 in 100 and penalty of Rs 1000 with a chance of the rest of 50 in 100

It is interesting to see that normally one may choose B_1 although the cool mathematical fact by expected value is that B_2 is superior. This to be seen from the computations below :

Expected value of (Rs) *Expected value of.. (Rs.)*

$B_1 = 1000$

$$B_2 = 5000 \times \frac{50}{100} - 1{,}000 \times \frac{50}{100}$$
$$= 2000$$

This simple example, then, points out the inadequancy of the expected value criteria Let us now turn to Neuman—Morgenstren's *'standard'* gambling technique. It is based on betting odds which a rational individual would insist on before accepting some risk

2.1. The Von Neumann Morgestern Method : This method of measuring utility is known as the standard gambling technique. It involves conversion of betting odds, viz , Rs 5000 or zero with a 50/50 chance, which a rational decision-maker would insist upon before accepting some risk into a curve (see Fig. 3 on p. 7) of the utility of money to the individual Utility is measured in the units *utils.* Let us take an example to explain how a point on this curve is obtained. Consider an individual who is offered 2 choices.

(i) Winning 5,000 or nothing with 50/50 chance.

(ii) A sure gift of Rs. 2,000.

Assume, as is expected to be the case with most, he opts for the sure Rs. 2,000. Decrease this gift alternative to Rs. 1,800 and ask him his choice. He

ill dreads the 50/50 gamble and opts for the sure Rs. 1,800. Go on this way until the individual is indifferent. This point is called the *certainty monetary equivalent or* C M E of the gamble. Let us assume that this individual's certainty monetary equivalent to the gamble offered is Rs. 1,000 The first step in plotting the utility curve is to assign utility numbers to two values in the gamble. It is also axiomatic that the utility of Rs. 5,000 for example, is greater than of 0 Suppose we assign a utility of 5 to 5,000 and 0 to Rs. 0. Now the individual is indifferent to

(i) Winning 5,000 or nothing with 50/50 chance.

(ii) A sure gift of Rs. 1,000.

Obviously, then, the utility of Rs. 5,000 with a probability of 0.5 equals utility of Rs. 1,000 with probability of 1.0. Symbolically,

$$\tfrac{1}{2}U_{5000}+\tfrac{1}{2}U_0=1.U_{1000}$$

Now . $U_{5000}=5$ and $U_0=0$.

Thefore, $U_{1000}=\frac{1}{2}\cdot5+\frac{1}{2}\cdot0=5/2$ utils. Thus we have the following three points on the utility curve.

Rupees	*Utils*
5,000	5
1,000	5/2
0	0

By the same procedure, offering some other betting odds against a gift is repeated and more points on the utility curve obtained. For example. the indiuivual may be indifferent to (i) winning Rs. 5,000 with 2/5 and nothing with 3/5 chances and (ii) a gift of Rs. 800.

Thus $$\frac{2}{5}U_{5000}+\frac{3}{5}U_0=1\,U_{800}.$$

or $$U_{800}=\frac{2}{5},\ 5=2.$$

The curve may then be plotted as in Fig. 3. The student might have already recognised the similarity between plotting the utility curve and weighting of

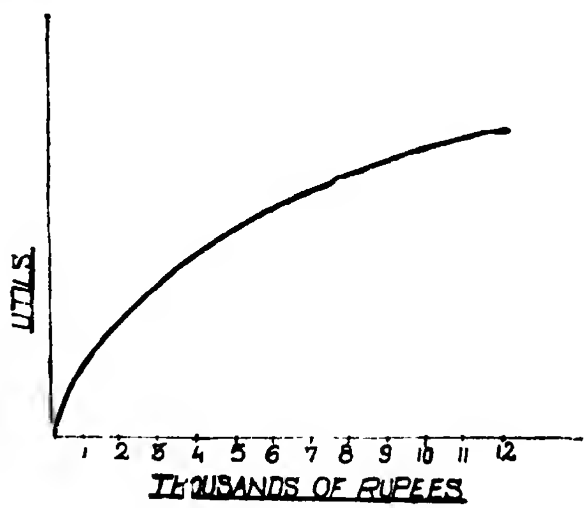


Fig. 3 Utility Function

the objectives discussed in Study 1. In business situations the gifts etc. are outcomes of certain strategies and the businessman is required to assign probabilities to the outcomes or express his indifference in the above sense. Let us, however, restate the above mode of questioning the individual in a slightly different manner

We may offer the same individual a choice between (i) gamble having a probability p of winning Rs 5,000 and a probability of $(1-p)$ of winning Rs. 0 and (ii) a certain gift of Rs. 800 Ask him to assign the value of p where he would be indifferent to the choices Assume that he puts is as $\frac{1}{2}$. Then

$$\tfrac{1}{2}U_{5000}+\tfrac{1}{2}U_0=U_{800}$$

$$U_{800}=5/2 \text{ Utils.}$$

Example 4 · Suppose a businessman arbitrarily assigns a utility of 25 utils to the amount of Rs 100,000 and zero units to zero rupees. Further, suppose by questioning it has been determined that his certainty monetary equivalent to a gamble in which he can win Rs 100,000 with a probability of 0 40 or win nothing with a probability of 0 60 is Rs 30,000 What is the utility of Rs 30,000 ?

Solution .

$$0{\cdot}4\,U_{100,000}+0\,6U_0=U_{80,000}$$

or $$U_{80,000}=0\,4\times25=10 \textbf{ (Ans)}$$

Example 5 : Suppose that in the previous problem when the utility of Rs. 100,000 was 25 utils and the utility of 0 rupee was 0 utils, the businessman was asked to specify the probability of winning a prize of Rs 100,000 or nothing such that he would be indifferent between the gamble and a certain gift of Rs 25,000 Suppose he specified that the probability of winning Rs. 100,000 would have to be 0 50 What is his utility for Rs 25,000 ?

Solution · $$0\,50\times25+0\,50\times0=U_{25,000}$$

i e. $$U_{25,000}=12\tfrac{1}{2}$$

2 5 A Logarithmic Utility Function So far we have been discussing the utility function for a particular individual and it goes without saying that these would differ from one individual to another There are, however, certain common characteristics We mentioned one of those It is axiomatic that the larger the amount of money the larger is its utility But possibly the increase is asymptotic i e each extra rupee has diminishing returns in terms of utility These requirements are easily met by logarithmic functions Symbolically,

$$\log X=U\ (\text{Rs. } X)$$

The interpretation of this being that the logarithm of X rupees is the utility of X rupees Whether it is logarithmic function to the base 10 or e matters little Since logarithm of 0 is ∞ this situation can be circumvented by letting

$$\log (X+1)=U\ (\text{Rs. } X)$$

The interpretation of this being that the logarithm of X+1 equals the utility of X rupees The logarithmic function is obviously a better approximation for an individual's personal utility function for money than a linear function

Example 6 : Assume a utility equation for a rupee was

$$\log \text{Rs. } (X+1)=U\ (\text{Rs } X)$$

Use logarithms to the base 10 and find

(*a*) the utility of Rs. 15.

(*b*) the utility of Rs. 500.

(*c*) the utility of Rs. 2,000

Solution :

(*a*) log 16=2.0412

(*b*) log 501=2 69984

(*c*) log 2,001=3 30124.

Example 7 : An opportunity is offered at a cost of Rs. 22 to participate in a venture with 0 10 probability of gaining Rs 100 and a probability of 0 90 of gaining Rs. 10. Bose has a utility function for money amounts given by $U(M)=M+0.01M^2$ where M is the money amount

(A) Would you expect Bose to participate in the venture ?

(B) Aiyer has a utility function, U(M)=log M. How much would he be willing to pay to buy into this venture ?

Solution · (A) $U(22)=22+0.01\times22^2=26\ 84$

$U(100)=100+0.1\times100^2=200$

$U(10)=10+0.01\times10^2=11.$

$(0\ 10)U(100)+(0\ 90)U(10)=20+9\ 9=29.9.$

Since the utility of Rs 22, 26.84 <29 9, Bose should participate in the *venture*

(B) U(M)=log (M)

Let C be the cost that Aiyer should be willing to pay.

$U\ (100)=\log 100=2$

$U\ (10)\ =\log\ \ 10=1$

$U\ (C)\ =2\times0\ 10+1\times0.90=1.1$

$(C)\ =\text{Antilog } 1.1=\text{Rs. } 12.88.$

Example 8. Corporation XYZ is considering an investment which would utilise all its funds available for investment. The researchers expect either of the two instantaneous events to occur, with the results shown in table below ·

Event	*Conditional Net Monetary Value* Rs	*Probability of Event*
A	1,000,000	0 70
B	−50,000	0.30

Assuming the firm has the utility function which may be approximated as follows :

$U = 0.001\,Y^2$ if $Y < -1,500$

$= Y$ if $Y \geqslant -1,500$

(A) Is it desirable to undertake the venture on the basis of the assumed utility function ?

(B) What would be the decision if the expected payoff criteria is used ?

Solution (A)

Event	*Conditional Net Monetary Value*	*Conditional Utility*	*Probability*	*Expected Utility*
A	1,000,000	1 000,000	0 70	700,000
B	-50,000	2,500,000	0 30	—750,000
				—50,000

It is not desirable to undertake the venture, therefore.

(B)

Event	*Conditional Net Monetary Value*	*Probability of Event*	*Expected payoff*
A	1,000,000	0 70	700,000
B	50,000	0 30	—15,000
			685,000

The venture should be undertaken therefore.

(The contradictory results are to be noted—hence the inadequacy of the expected payoff criteria)

Exercises

1 A decision-maker has the following utilities over the relevant portion of his overall assets scale :

Assets (Rs. 10,000)	5	6	7	8	9
Utility	0.32	0 46	0.59	0 67	0.72

(i) The decision-maker currently possesses total assets of Rs. 70,000. He is offered a place in the lottery where he has a chance of 0 6 of winning Rs. 10,000 together with complementary chance of 0 4 of losing Rs. 10,000. Should the decision-maker accept the offer ?

(ii) If the decision-maker were offered participation in a series of two independent plays of the same lottery with same prizes and probabilities in each play

would this affect his decision to enter (or not to enter) the lottery ? (Note that enter implies participation in both plays of the lottery).

Answer :

(i) No; he should not accept; (ii) Yes, it will affect his decision and he should accept.

2 Mr. X currently has assets of Rs. 100,000 and is presented with a deal which in his opinion, has probability $\frac{1}{3}$ of resulting in a loss of Rs 100,000, but probability 2/3 of resulting in a profit of Rs 100,000, Mr. X's utility function is as follows :

Assets (Rs 0000's)	0	2.5	5	10	15	20
Utility	0	0.45	0.65	0 85	0 95	1 00

(i) Show that Mr. X should refuse the deal.

(ii) Suppose that five people with utility functions exactly like Mr. X's and each with assets of Rs. 100,000 all assign the same probabilities to the possible consequences of this deal. If they all agree to share the profit or loss equally, does the deal become attractive for all of them as a syndicate ?

Answer

(i) Mr. X should refuse the deal as expected end utility is 0.67 (compare with initial utility of 0 85).

(ii) The deal becomes attractive to the syndicate; expected and utility for each individual is 0.86.

(Hint : Plot utility against assets and interpolate).

3. Investment and Uncertainty

3 1. The student is referred to Financial Management Study Note V for introduction to the **Hillier's models** It may be recalled that the following formulae are used for finding the mean and the variance of the net present value by Hillier's model.

$$M = \sum_{i=0}^{n} (1+r)^{-i} M_i$$

$$\sigma^2 = \sum_{i=0}^{n} (1+r)^{-2i} \sigma_i^2$$

where M_i is the mean and σ_i, the standard deviation of the cash flow distribution (assumed normal) of the i-th period, and r is the discounting factor. These formulas apply, if periodic cash flows are independently distributed If, however, the

periodic cash flows are not independently distributed but are perfectly correlated the following formulas may be used

$$M = \sum_{t=0}^{n} (1+r)^{-t} M_t$$

$$\sigma^2 = \left[\sum (1+r)^{-t}\sigma_t\right]^2 \text{ i.e. } \sigma = \sum (1+r)^{-t}\sigma_t$$

The economic interpretation of perfect correlation between two *normally* distributed random cash flow is as follows If X_t deviates randomly from its mean by n standard deviations X_{t+1} will also deviate in the same direction by n standard deviations. *If not stated otherwise in a problem cash flows may be assumed as independently distributed*

Example 9 Assume a discount rate of 0 10 and an investment with the following project cash flows .

Year	*Cash-Flows*
0	−1600
1	1500
2	1000

Assuming the firm has a linear utility function for money, should the investment be accepted '?'

Solution :

This is a capital budgeting problem under certainty. Also, the utility function being linear, the student should be in a position to solve it readily by the traditional methods Any how, the solution is given below.

Period, t	*Cash Flows*	$(1+r)^{-t}$	*Present Value of Cash Flows*
0	−1,600	1.000	−1,600
1	1,500	0.909	1,364
2	1,000	0 826	826
			+590

The NPV being +ve the investment should, therefore, be undertaken.

Example 10. Assume the cash flow of example 9 are independently and normally distributed with the following parametric values :

Period, t	*Mean, M_t*	*Standard deviation, σ_t*
0	Rs. −1,600	Rs. 400
1	1,500	500
2	1,000	600

(A) Compute the mean of the present value distribution.

(B) Compute the standard deviation of the present value distribution.

(C) Compute the probability of the investment having a net present value of zero or less.

(D) Compute the expected value of loss (or the expected value of perfect information).

Solution :

t	$(1+0.10)^{-t}$	M_t	$(1+0.10)^{-t}M_t$	$(1+0.10)^{-2t}$	σ_t^2	$(1+0.10)^{-2t}\sigma_t^2$
0	1.000	−1,600	−1,600	1.000	16×10^4	16.0×10^4
1	0.909	1,500	1,364	0.826	25×10^4	20.65×10^4
2	0.826	1,000	826	0.682	36×10^4	24.55×10^4
			M=590			$61.2\times10^4=\sigma^2$

(A) M=590

(B) $\sigma=\sqrt{61.2\times10^4}=780$

(C) $Z=\frac{0-590}{780}=-0.757$; Probability=0.224 approx. (from normal tables)

(D) L (0.757)=0.129 (from Appendix I)

$\therefore$ *EVPI=0.129×780=100

Example 11 Assume a discount rate of 0.10 and an investment with the following projected cash flows :

Year	Mean Cash Flow	σ of cash flow distributions
0	Rs. −1,600	Rs 700
1	2,000	1,000
2	1,000	1,500

The distributions are assumed to be normal and independent of each other.

(A) Compute the mean of the present value distribution. Using the mean value, would the investment be accepted ?

(B) Compute the standard deviation of the present value distribution.

(C) Compute the probability of the investment having a net present value of zero or less.

(D) Compute the expected value of loss (or the expected value of perfect information).

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*Please see that 780=bσ i.e. S.D. in rupees, σ having been pre-multiplied by the contribution per unit. In Study II, we used S.D. in units

Solution :

t	M_t	$(1+0.10)^{-t}$	$M_t(1+0.10)^{-t}$	σ_t^2	$(1+0.10)^{-2t}$	$\sigma_t^2(1+0.10)^{-2t}$
0	−1,600	1.000	−1,600	49×10^4	1.000	49.0×10^4
1	2,000	0 909	1,818	100×10^4	0 826	82.6×10^4
2	1,000	0.826	826	225×10^4	0.682	153.5×10^4
			1,044			285.1×10^4

(A) Since NPV mean is +ve the investment should be accepted.

(B) $\sigma=\sqrt{285\ 1\times10^4}=1,688$

(C) $Z=\frac{0-1044}{1688}=-0\ 62$, Probability=0 2676

(D) EVPI=1688×L=1688×0 1633=275

Example 12 Obtain expected DCF and the associated risk as measured by the standard deviation for project A, if the cost of capital equals 10% given probability distribution of cash flows as :—

Year	*Cash flows*	*Probability*	*Year*	*Cash flows*	*Probability*	*Year*	*Cash flows*	*Probability*
1	1,000	.10	2	1,900	.20	3	1,500	10
	1,500	20		2,500	.30		2,250	.70
	2,000	.40		2,750	.20		2,500	.10
	2,500	.20		3,150	.30		3,000	.10
	3,000	.10						

Solution ·

Means and standard deviations of various year's cash flow outcomes are calculated below

Year 1

Cash flow	*Prob. p*	*px*	*x—2000*	$\left(\frac{x-2000}{100}\right)^2$	*col 5×p*
(1)	(2)	(3)	(4)	(5)	(6)
1,000	0.10	100	−1,000	100	10
1,500	0.20	300	500	25	5
2,000	0.40	800	0	0	0
2,500	0 20	500	500	25	5
3,000	0.10	300	1,000	100	10
	Mean=2000			250	30

Variance=30×100²

Year 2

			$x-2625$	$\left(\frac{x-2625}{100}\right)^2$	
1,900	0 20	380	−725	52.56	10 512
2,500	0 30	750	−125	1 56	0 468
2,750	0.20	550	125	1 56	0.312
3,150	0.30	945	525	27 56	8.268
	Mean=2625				19 560

Variance$=19.560\times 100^2$

Year 3

			$x-2275$	$\left(\frac{x-2275}{100}\right)^2$	
1,500	0.10	150	−775	60 06	6 006
2,250	0.70	1575	−25	0 06	0.042
2,500	0.10	250	225	5 06	0 505
3,000	0 10	300	725	52.56	5 256
	Mean= 2275				11 809

Variance$=11.809\times 100^2$

The mean and variance are discounted below :

i	$(1+0.10)^i$	M^i	$(1+0\ 10)^{-i}M_i$	$(1+0.10)^{-2i}$	σ^{2i}	$\sigma_i^2(1+0\ 10)^{-2i}$
1	.909	2000	1818	.826	30×100^2	24.78×100^2
2	.826	2625	2168	.682	19.56×100^2	13.06×100^2
3	751	2275	1709	.564	11.809×100^2	6.69×100^2
		Expected DCF=5695				45.53×100^2

Standard deviation$=\sqrt{45\ 53\times 100^2}$
$=675.$

Example 13 An electronics company is considering investing in a new plant where it will produce either television components, or computer component for a new as of yet untested computer. Their economist predicts that the returns from an investment on the television component will be independent of each other and have the following mean and variances :

Year	Expected Value Rs.	Variance Rs.
1	100,000	20,000
2	80,000	15,000
3	200,000	30,000
4	200,000	40,000
5	200,000	50,000

The computer components returns on the other hand, are thought to be *perfectly correlated* with each other, as all of them will depend on the market's acceptance of the new computer. The returns on this investment have an expected value of Rs 130,000 and a variance of Rs. 40,000 each year. Assume that both investments can be made for Rs 500,000 each and that the firm's cost of capital is 10%. Find the expected value and the standard deviation of the net present value of (a) television project (b) computer project

Solution .

(a)

t	M_t	$(1+0.10)^{-t}$	$M_t(1+0.10)^{-2t}$	σ_t^2	$(1+0.10)^{-2t}$	$(1+0.10)^{-2t}\sigma^2$
0	-50×10^4	1 000	$-500,000$		1 000	0
1	10×10^4	0 909	90,900	20×10^3	0 826	$16\ 52\times10^3$
2	8×10^4	0 826	66,080	15×10^3	0 683	$10\ 25\times10^3$
3	20×10^4	0 751	150,200	30×10^3	0.565	$16\ 95\times10^3$
4	20×10^4	0 683	136,600	40×10^3	0 467	$18\ 68\times10^3$
5	20×10^4	0 621	124,200	50×10^3	0.386	$19\ 30\times10^3$
			67,980			81.7×10^3

Thus M 67,980 **Answer :** $\sigma=\sqrt{817\times10^3}$ 286 **Answer**

(b)

	M_t	$(1+0.10)^{-t}$	$M_t(1\times0.10)^{-t}$	σ_t	$\sigma_t(1+0.10)^{-t}$
0	-50×10^4	1 000	$-50,000$	0	0
1	13×10^4	0 909	118,170	200	181.8
2	13×10^4	0 826	107,380	200	165 2
3	13×10^4	0 751	97,630	200	150 2
4	13×10^4	0 683	88,790	200	136 6
5	13×10^4	0 621	80,730	200	124.2
			+492,700		758 0 **Answer.**
			$-500,000$		
			$-7,300$ **Answer**		

Exercise

1. Mercantile Motors, a large auto manufacture is considering an investment in a new plant that will expand the output of its most sneccessful madel, the Mercombile Since this vehicle has gained substantial consumer acceptance, management has reason to believe that the incremental cash flows from the investment will be independently normally distributed variables The plant is expected to cost Rs 101 5 million whilst the net cash flow will accrue over the next six years. Each flow has an estimated mean of Rs 30 million and a variance of Rs. 25 millions. If Mercantile's cost of capital is 10% what are expected value and variance of the net present value from investment ?

Answer :

Rs. 29.15 m and 81.17 m.

2. Assume that the proposed plant in exercise 1 above is to be utilised in the production of a new pollution-free vehicle, the cleanvair. Since this automobile will be expensive and will be marketed untested. Mercantile assumes that the net incremental cash flow will be perfectly correlated, with mean and variance as given in exercise above. What are the expected value and variance of the net present value of the investment.

Answer :

Rs. 29.15 m.

Moderate Correlation

We discussed in the previous section Hillier's models for (*i*) independent and (*ii*) perfectly correlated cash flows. Below, we straightway take up the case where cash flows are moderately correlated *i.e* the intermediate case, by means of Example 14.

Example 14. The investment in a project costing Rs 250 at time 0 were expected to generate the possible net cash flows shown in the table below :

Period 1		Period 2		
Initial Prob P (1)	Net cash flow	Conditional Prob. (2/1)	Net cash flow	Joint Prob. (1, 2)
0 25	--100	0.40	−400	0.10 (=0 4×0.25)
		0.40	−100	0.10
		0.20	200	0.05
0 50	200	0 20	−100	0.10
		0 60	200	0.30
		0.20	500	0.10
0 25	500	0.20	200	0.05
		0 40	500	0.10
		0.40	800	0.10

. Find expected NPV and its S.D. with a discount rate of 4%,

Solution : The decision tree is drawn below. In this problem, there are nine possible series of net cash flows. The first series is represented by cash flow of −100 in period 1, followed by a −400 cash flow in period 2. The probability associated with this path is 0.4× 0.25=0.10. The NPV of this series may be computed as below :

$$NPV_1 = -250 - \frac{100}{1.04} - \frac{400}{1.04^2} = -715.98$$

The second cash flow series is represented by a cash flow of−Rs. 100 in period 1, followed by a−Rs. 100 cash flow in period 2. The NPV of this series is

$$NPV_2 = -250 - \frac{100}{1.04} - \frac{100}{1.04^2}$$
$$= -\text{Rs. } 438.61.$$

3 4 **The certainty-equivalent approach** consists of modifying appropriately the numerator of the each flow stream in the NPV equations as below :

$$\text{NPV} = \sum_{t=0} \frac{\alpha_t A_t}{(1-r)^t}$$

where t stands for period. A_t for cash flows, α_t is the certainly equivalent eoefficient for period t and r is the interest rate. The value of the coefficient may vary between 0 and 1 and the variations are in relationship to the degree of riskness. This coefficient would be assigned by the management.

Hertz Simulation Model : Hertz developed a simulation model for the investment decision-making for introduction of a new product. It is discussed in the section on Simulation

Miscellaneous Examples

Example 1

The Alpha Co can invest in one of two mutually exclusive projects. The two proposals have the following discrete probability distributions of net cash flows for period, p :

A		B	
Prob	*Cash flow*	*Prob.*	*Cash flow*
0.20	Rs 1,000	0.10	Rs 1,000
0 30	2,000	0.40	2,000
0 30	3,000	0.40	3,000
0 20	4,000	0.10	4,000

(a) Without calculating a mean and a coefficient of variation, can you select the better proposal, assuming a risk-averse management ?

(b) Verify your intuitive determination.

(c) Suppose that Rs. 1,000, Rs. 2,000, Rs. 3,000, Rs. 4,000 have the util values of 0 008, 0.014, 0.018, and 0.020 respectively. Work with these and ascertain if this would change the decision.

Solution :

Since both the probability distributions are symmetrical and cash flows are the same following conclusions can be drawn without going into actual calculations :

(i) They have the same mean.

(ii) A is more variable.

Hence project B is to be preferred.

Below we compute the means as verification.

A	B
$.20\times1000 = 200$	$.10\times1000 = 100$
$30\times2000 = 600$	$.40\times2000 = 800$
$.30\times3000 = 900$	$.40\times3000 = 1200$
$.20\times4000 = 800$	$.10\times4000 = 400$
Mean $= 2500$	Mean $= 2500$

Below we compute the variances as verification.

A	B
$(2500-1000)^2\times 20=100^2\times45$	$(2500-1000)^2\times 10=22.5\times100^2$
$(2500-2000)^2\times.30=100^2\times7.5$	$(2500-2000)^2\times.40=10\ 0\times100^2$
$(2500-3000)^2\times.30=100^2\times7.5$	$(2500-3000)^2\times.40=10\ 0\times100^2$
$(2500-4000)^2\times 20=100^2\times45$	$(2500-4000)^2\times.10=22.5\times100^2$
105.0×100^2	65.0×100^2

Obviously A is more variable than B.

Utils (A) $= .20\times.008+30\times.014+.30\times018+.20\times0\ 020$

$= 0016+.0042+0054+0040 = 0152$

Utils (B) $= 10\times.008+.40\times0\ 014+.40\times0\ 018+10\times0\ 020$

$= .0008+.0056+.0072+.0020 = .0156$

Hence project B is still to be preferred

Example 2 :

The Theta company uses a certainty—equivalent approach in its evaluation of risky investments Currently the company is faced with two alternative proposals. The expected value of net cash flows for each proposals are as follow :

Year	A	B
0	−20,000	−15,000
1	+10,000	+ 8,000
2	+10,000	+ 9,000
3	+10 000	+10,000

(a) Risk analysis of each cash flow distribution has provided certainly equivalents as follows :

Year	A	B
0	1.00	1 00
1	.9	.95
2	.8	.90
3	.6	.50

If the after-tax risk free rate is 4% which of the two alternatives should be selected ?

(b) If the firm were to use risk-adjusted discount rate instead of a certainty-equivalent approach, what rates would be used in order to obtain an equivalent solution ?

Solution .

Under certainty equivalent approach,

$$\text{NPV} = \sum \frac{\alpha_t A_t}{(1+r)^t}$$

Where α_t is the certainty-equivalent coefficient for period t, A_t is the each flow for period t

Alternative A :

t	α_t	A_t	$\alpha_t A$	$(1+r)^{-t}$	$\frac{\alpha_t A_t}{(1+r)_t}$
0	1 00	−20,000	−20,000	−1.000	−20,000
1	.90	10,000	9,000	.962	8,658
2	80	10,000	8,000	925	7,400
3	60	10,000	6,000	.889	5,334
					+1392

Alternative B :

t	α^t	A^t	$\alpha^t A^t$	$(1+r)^{-t}$	$\frac{\alpha^t A^t}{(1+r)_t}$
0	1 00	15000	−15000	1 000	−15000
1	.95	8000	7600	.962	7311
2	90	9000	8100	925	7493
3	50	10000	5000	.889	4445
					+4249

Hence alternative B is more attractive.

(b) Let the risk adjusted discount rate be r,

Then $-20{,}000+10000\,(1+r)^{-1}+10000\,(1+r)^{-2}+10000\,(1+r)^{-3}=1392$

$$(1+r)^{-1}+(1+r)^{-2}+(1+r)^{-3}=2.1392.$$

Looking to the present value tables, r=20% gives L.H.S.=2.106. Thus the risk-adjusted discount rate is roughly 20%. For a more accurate solution, however, the student may sum up the L.H S as a G.P. and find r

Likewise, for alternative B,

$-15{,}000+8000\,(1+r)^{-1}+9000\,(1+r)^{-2}+10000\,(1+r)^{-3}=4249$

$$8(1+r)^{-1}+9\,(1+r)^{-2}+10\,(1+r)^{3}=19.249$$

Try $r=20\%$

L.H S. $=8\times.833+9\times.694+10\times.579$

$=6.664+6.246+5.790=18.70$

Try $r=18\%$

L.H S. $=8\times 847+9\times.718+10\times.609$

$=6.776+6.462+6.090$

$=19.328.$

Hence the risk-adjusted discount rate is roughly 18%.

Example 3 :

The Beta Co. has determined the following discrete probability distribution for net cash flows generated by a contemplated project

Period 1		Period 2		Period 3	
Prob.	Cash flow	Prob.	Cash flow	Prob.	Cashflow
0.10	Rs. 1,000	0 20	Rs 1,000	0.30	Rs 1,000
0 20	2,000	0 30	2,000	0.40	2,000
0 30	3,000	0 40	3,000	0 20	3,000
0 40	4,000	0.10	4,000	0.10	4,000

(a) Assume that the probability distribution of cash flows for future periods are independent. Also, assume that the after tax, risk-free rate is 4%. If the proposal will require an initial outlay of Rs. 5,000 determine the expected value of the N P V

(b) Determine the S.D. about the expected value.

Solution :

Mean cash flows (M_i) are computed below for the various years ,

Year 1			Year 2			Year 3		
Prob.	Cash flow		Prob.	Cash flow		Prob.	Cash flow	
1	2	3	4	5	6	7	8	9
		1×2			4×5			7×8
.10	1000	100	.20	1000	200	.30	1000	300
.20	2000	400	.30	2000	600	.40	2000	800
·30	3000	900	.40	3000	1200	·20	3000	600
·40	4000	1600	.10	4000	400	10	4000	400
		3000			2400			2100

(a)

Year	M_t	$(1+0.04)^{-t}$	$M_t\,(1+0.04)^{-t}$
0	−5000	1.000	+5000
1	3000	.962	2886
2	2400	.925	2220
3	2100	·889	1867

∴ Expected value of NPV = +1973 Answer.

(b) Below, we compute variance for each year.

Year 1 :—

$$(3000-1000)^2\times.10 = 1000^2\times 0\,4$$
$$(3000-2000)^2\times 20 = 1000^2\times 0.2$$
$$(3000-3000)^2\times.30 = \text{———}$$
$$(3000-4000)^2\times.40 = 1000^2\times.4$$
$$1000^2\times 1 = 100^2\times 100$$

Year 2 :

$$(2400-1000)^2\times 20 = 100^2\times 39.2$$
$$(2400-2000)^2\times.30 = 100^2\times 4\,8$$
$$(2400-3000)^2\times.40 = 100^2\times 14.4$$
$$(2400-4000)^2\times.10 = 100^2\times 25\,6$$
$$100^2\times 84\,0$$

Year 3 :

$$(2100-1000)^2\times.30 = 100^2\times 36\,3$$
$$(2100-2000)^2\times.40 = 100^2\times 0\,4$$
$$(2100-3000)^2\times.20 = 100^2\times 16\,2$$
$$(2100-4000)^2\times.10 = 100^2\times 36.1$$
$$100^2\times 89\,0$$

Computation of S.D

Year, t	Variances	$(1+r)^{-2t}$	Variance $\times (1+r)^{2t}$
1	100×100^2	0.925	92.500×100^2
2	84×100^2	0 855	71.820×100^2
3	89×100^2	0.790	70.310×100^2
			234.630×100^2

$$S.D. = \sqrt{234.630 \times 100^2}$$
$$= 1532$$
$$Z = \frac{0-1973}{1532} = -1{\cdot}3$$

Hence probability $=0{\cdot}1$ approx. (from Std. Normal Table)

Exercises

Q. 1. The Theta Corporation is faced with several possible investment projects. For each, the total cash out flows required will occur in the initial period. The cash out flows, expected net persent values and S.D.'s are given below. All projects have been discounted at the risk-free rate of 4% and it is assumed that the distributions of their possible NPV's are normal.

Project	*Cost* *Rs.*	*NBV*	σ
A	100,000	10,000	20,000
B	50,000	10,000	30,000
C	200,000	25,000	10,000
D	10,000	5,000	10,000
E	50,000	75,000	75,000

(a) Construct a risk profile for these each of these projects in terms of the profitability index.

(b) Ignoring size problems, are there some projects which are clearly dominated by others ?

(c) May size problems be ignored ?

(d) What is the probability that each of the projects will have a NPV$\leqslant$0.

(e) What decision rule would you suggest for adoption of projects within this context ? What (if any) of the above projects would be adopted under your rule ?

Q 2. Parameters of the probability distributions of the present value of net oash flows from an investment are given below. The distributions for each year are normal and independent of one another.

Year	Mean Rs.	S.D. Rs.
1	20,000	3,000
2	30,000	6,000
3	40,000	9,000

The investment involves a certain current cost of Rs. 94,000. Find the EVI

[Answer : Rs. 1353]

Q. 3 A firm is considering two proposals as a part of its capital budget. Proposal A requires a net cash outlay of Rs. 12,000 ; B requires Rs. 15000. Each project has an estimated life of 3 years. An analyst has estimated the returns after taxes.

For Project A

Year	outflow/chance
1	Rs 4,000/40% and Rs. 5,000/60%
2	Rs 8 000/30% and Rs. 5,000/70%
3	Rs 9,000/40% and Rs 5,000/60%

For Project B

Year	outflow/chance
1	Rs. 4,000/50% and Rs. 6,000/50%
2	Rs 12,000/40% and Rs. 8,000/60%
3	Rs. 7,000/30% and Rs. 10,000/70%

The firm's cost of capital is 10%

Questions :

(1) Which project has the higher weighted-average profitability index ?

(2) Which project has the larger S.D ?

(3) Which project has the larger coefficient of variation ?

Q 4. A firm is considering the introduction of a new product with returns expected over the next 5 years Because the product's market reception is uncertain management feels that if initial reception exceeds expectations, receptions in latter years, will also exceed expectations in about the same proportion i.e., the net marketing cash outflows can be treated as perfectly correlated over time.

On the other hand, estimates of the initial investment in the project and of production costs are reasonably reliable so that any deviation from expectations is assumed to be attributable to random fluctuations Consequently initial investment and net production cash flow are regarded as being mutually independent over time. The probability information for the introduction of the new product is shown in the table below Assume that each of the probability distributions involved can be regarded as normal. If 4% is used as the risk-free rate find the NPV and standard deviation of the NPV of the proposal.

Year	Source	Expected value of net cash outflows in 1000's Rs	S D Rs.
0	Initial investment	−600	50
1	Production cash outflow	−250	20
2	—Do—	−200	10
3	—Do—	−200	10
4	—Do—	−200	10
5	Production outflow-salvage value	−100	15
1	Marketing	300	50
2	—Do—	600	100
3	—Do—	500	100
4	—Do—	400	100
5	—Do—	300	100

Hint. This is a mixed situation involving an independent stream of cash flows and a perfectly correlated stream of cash flows.

$$\sum_{t=0}^{5} \frac{A_t}{(1\,04)^t}$$

$$=600+\frac{300-250}{1.04}+\frac{600-200}{1\,04^2}+\frac{500-200}{1.04^3}+\frac{400-200}{1.04^4}+\frac{300-100}{1.04^5}$$

$= $ Rs. 419.

$$\sigma=\sqrt{50^2+\frac{20^2}{1.04^2}+\ldots+\frac{15^4}{1.04^{10}}+\left(\frac{50}{1.04}+\ldots\ldots+\frac{100}{1.05^5}\right)^2}=398$$

Q 5. The Delta Company has invested Rs. 300 in new machinery with expected cash flows over two years as follows :

Year 1 cash flow Rs.	Initial Probability P(1)	Year 2 cash flow Rs.	Conditional Probability P(2)
200	0.3	100	0.3
		200	0.5
		300	0.2
300	0.4	200	0.3
		300	0.5
		400	0.2
400	0 3	300	0.3
		400	0.4
		500	0 3

The firm's cost of capital is 12%. Find expected NPV and its S D.

[**Answer :** NPV = Rs. 201
S.D. = Rs. 1450]

Q 6 Consider the following two investment proposals.

Investment A (Outlay Rs. 1,000) Return Rs.	Prob.	Investment B (Outlay Rs. 2,000) Return Rs	Prob.
600	0 30	1,500	0.30
1,050	0.40	2,100	0.40
1,500	0.30	2,800	0.30

Assuming that S.D. of return is useful measure of risk, which of the two investments is more risky. In reaching a conclusion, calculate S.D. of return both in rupees terms and in % age term.

4. Some Solved Examples on Annuities

Definitions : An *annuity* is a fixed sum paid periodically under certain stated conditions : the payment may be made either once a year or at more frequent interval.s

A *deferred annuity* or *reversion* is an annuity which does not begin until after the lapse of a certain number of years ; and when the annuity is deferred for n years, it is said to commence after n years, and the first payment is made at the end of $(n+1)$ year

Q 1 Find the amount of annuity of Rs. 100 in 20 years, allowing compound interest $4\frac{1}{2}\%$.

Solution : Let A be the annuity, R the amount of Re. 1 for one year, n the number of years, M the amount.

At the end of the first year A is due and the amount of this sum in the remaining $(n-1)$ year AR^{n-1}.

At the end of the 2nd year another A is due, and the amount of this sum in the remaining $(n-2)$ years is AR^{n-2}.

And so on

$$\therefore \quad M = AR^{n-1} + \quad ..+AR^2+AR+A$$
$$= A\{1+R+R^2+. \;...\text{to } n \text{ terms}\}$$

$$M = A\,\frac{R^n-1}{R-1} \qquad \textbf{FORMULA 1}$$

Substituting $n=20$, $R = 1+\frac{4\frac{1}{2}}{100}=1.045$,

A=100 in the above formula,

$$M=100\,\frac{1.045^{20}-1}{1.045-1}$$

[log $1.045^{20}=20$ log $1.045=20\times0.0191163=0.382326$. Antilog 0 382326 $=2\,4117$].

$$M=100\,\frac{1.4117}{0\,045}=\frac{141.17}{0.045}=\text{Rs. } 3137.99 \quad \textbf{Answer}$$

Q 2 A freehold estate is bought for Rs. 2750 , at what rent should it be let so that the owner may receive 4% on the purchase money.

[A freehold estate is an estate which yields a perpetual annuity $(n=\infty)$ called the rent ; and thus the value of the estate is equal to the present value of a perpetuity equal to the rent.

Let A be the annuity, R the amount of Re. 1 in one year, n the number of years, V the required present value.

The present value of A due in 1 year is AR^{-1} ; the present value of A due in 2 years is AR^{-2} ; the present value of A due in 3 years is AR^{-3} ; and so on.

N B The derivations of the formulas are, though, included here in the solutions the student should do them only if specifically wanted in a problem.

Now V is the sum of the present values of the different payments.

$\therefore \quad V = AR^{-1} + AR^{-2} + AR^{-3} + \ldots\ldots$ to n terms

$$= AR^{-1}\frac{1-R^{-n}}{1-R^{-1}} = A\frac{1-R^{-n}}{R-1}$$

Thus $V = A\dfrac{1-R^{-n}}{R-1}$ **FORMULA II**

For a perpetual annuity (syn, perpetuity) putting $n=\infty$ in Formula II.

$V = A/(R-1) = \dfrac{A}{R}$ **FORMULA III**

Substituting $V=2750$ and $r=0.04$ in Formula III.

$$A = 2750 \times 0.04 = 110 \text{ rupees (Answer).}$$

Q. 3. How many year's purchase should be given for a freehold estate, interest being calculated at $3\frac{1}{2}\%$.

[If mA is the present value of an annuity A, the annuity is said to be worth m year's purchase]

In the case of a perpetual annuity $mA = \dfrac{A}{r}$, hence

$m = \dfrac{1}{r}$ **FORMULA IV**

Substituting $r = \dfrac{3\frac{1}{2}}{100}$ in Formula IV

$$m = \frac{100}{3\frac{1}{2}} = 28\frac{4}{7} \text{ years.}$$

Q 4. If a perpetual annuity is worth 25 year's purchase, find the amount of annuity of Rs. 625 to continue for 2 years.

Solution · Let us find r by applying Formula IV.

$m = \dfrac{1}{r}$ $\quad\therefore \quad r = \dfrac{1}{m} = \dfrac{1}{25} = 0.04$

$\therefore \quad R = 1.04$

Applying Formula I

$$M = A\frac{R^n - 1}{R-1} = \frac{1.04^2 - 1}{1.04 - 1}$$

$$= 625\frac{0.0816}{.04}$$

$$= \text{Rs. } 1275 \text{ (Answer)}$$

Q. 5. If a perpetual annuity is worth 20 year's purchase, find the annuity to continue for 3 years which can be purchased for Rs. 2522.

Applying Formula IV, $r=\frac{1}{20}=0\ 05.$

Applying Formula II, $V=\frac{1-R^{-n}}{R-1}$

$$2522=\frac{1-1.05^{-8}}{05}A$$

$$A=\frac{2522\times\frac{5}{100}}{1-1\ 05^{-8}}=\text{Rs}\ \ 926\ 10\ \textbf{Answer)}$$

Q 6 If 25 year's purchase must be paid for an annuity to continue n years, and 30 year's purchase for an annuity to continue $2n$ year's find the rate per cent

Applying Formula II

$$25A=A\frac{1-R^{-n}}{R-1} \qquad \text{(i)}$$

and
$$30A=A\frac{1-R^{-2n}}{R-1} \qquad \text{(ii)}$$

From (i) $25R-25=1-R^{-n}$. (iii)

$\Rightarrow R^{-n}=24-25R$

From (ii) $30R-30=1-R^{-2n}$

$\Rightarrow R^{-2n}+30R-31=0$

Substituting $R^{-n}=24-25R$

$(24-25R)^2+30R-31=0$

$576+625R^2-1200R+30R-31=0$

$525R^2-1170R+545=0.$

The student may solve this quadratic equation himself and verify that $R=1\ 032$

i e. $r=3\ 2\%$.

Q 7. Raju has a capital of Rs. 20,000 for which he receives interest at 5%, if he spends Rs. 1800 every year, show that he will be ruined before the end of the 17th year.

Applying formula II, $V=\frac{A(1-R^{-n})}{R-1}$

$$20{,}000=1800\left(\frac{1-1\ 05^{-n}}{0\ 05}\right)$$

$$\Rightarrow \quad \frac{200}{18}\times\frac{5}{100}=1-1.05^{-n}$$

$$\Rightarrow \quad 1.05^{-n}=1-\frac{10}{18}=\frac{8}{18}$$

$\Leftarrow -n \log 1.05 = \log 8 - \log 18$

$$\Leftarrow \qquad n = \frac{\log 18 - \log 8}{\log 1.05}$$

$$= \frac{0\ 3522}{0.0212} \approx 17 \text{ years.}$$

Q. 8. What is the present worth of a perpetual annuity of Rs. 100 payable at the end of the first year, Rs 200 at the end of the second, Rs 300 at the end of the third, and so on, increasing Rs. 100 each year ; interest being taken at 5% per annum.

The present value of Rs. 100 due in one year is 100 R^{-1}

—Do— Rs 200 due in two years is 200 R^{-2}

—Do— Rs 300 due in 3 years is 300 R^{-3}

$$V = 100 R^{-1} + 200R^{-2} + 300R^{-3} + \ldots \infty \qquad (i)$$

[Formula for the Summation of the series on the R.H S, Let $x = R^{-1}$

Let $S = 1x + 2x^2 + 3x^3 + \ldots \infty \ . \ x < 1$

$Sx = x^2 + 2x^3 + \quad \infty$

$S(1-x) = x + x^2 + x^3 \ldots . \infty$

$= (1 + x + x^2 + \ldots\ldots \infty) - 1$

$= \dfrac{1}{1-x} - 1 = \dfrac{x}{1-x}$

$\therefore \qquad S = \dfrac{x}{(1-x)^2}$]

Using the formula.

$$V = 100 \left\{ \frac{R^{-1}}{(1-R^{-1})^2} \right\} = 100 \frac{1.05^{-1}}{(1-1.05^{-1})^2}$$

$$= \frac{100}{1\ 05 \times 0\ 48^2} = \frac{100 \times 10^4}{1\ 05 \times 4\ 8^2}$$

$$= \frac{100 \times 10^4}{1\ 05 \times 23\ 04} = \text{Rs. } 42{,}000 \text{ (Answer)}$$

Q 9 Give the present value discounted at 5% of an annuity of Rs. 300 for a period of 4 years commencing : (i) in 1 year's time, (ii) in 4 year's time ; (iii) in 7 year's time

Solution : [There is a problem of a *deferred* annuity to commence at the end of p years and to continue for n years, allowing compound interest

Let A be the annuity, R the amount of Re. 1 in one year, V the present value.

The first payment is made at the end of $(p+1)$ years.

Hence the present values of the first, second, third.. ... ..payments are respectively.

$$AR^{-(p+1)},\ AR^{-(p+2)},\ AR^{-(p+3)}\ldots\ldots$$

$\therefore$ $$V=AR^{-(p+1)}+AR^{-(p+2)}+AR^{-(p+3)}+\ldots\ \ldots\text{to } n \text{ terms.}$$

$$AR^{-(p+1)}\ \frac{1-R^{-n}}{1-R^{-1}}$$

$$=\frac{AR^{-p}}{R-1}-\frac{AR^{-p-n}}{R-1} \qquad \textbf{FORMULA V}$$

(N B. : The present value of a *deferred perpetuity* to commence after p years is given by)

$$V=\frac{AR^{-p}}{R-1} \qquad \textbf{FORMULA VI)}$$

Applying Formula V,

(i) $R=1.05$, $A=300$, $p=1$, $n=4$

$$V=\frac{300\times 1.05^{-1}}{.05}-\frac{300\times 1.05^{-5}}{.05}$$

$$=6000\ (1.05^{-1}-1.05^{-5})$$

$$=6000\ (0.952-0.784)$$

$$=6000\times .168=\text{Rs. } 1008 \text{ (Answer)}$$

(ii) $$V=6000(1.05^{-4}-1.05^{-8})$$

$$=6000\ (.823-.677)$$

$$=6000\times 0.146=\text{Rs. } 876 \text{ (Answer)}$$

(iii) $$V=6000\ (1.05^{-7}-1.05^{-11})$$

$$=6000\ (0.711-0.585)$$

$$=6000\times 0.126=\text{Rs. } 756 \text{ (Answer)}$$

Q. 10. a, b, c year's purchase must be paid for an annuity of continue for n, $2n$ and $3n$ year's respectively prove that $a^2-ab+b^2=ac$.

Solution : We are to prove that $a^2-ab+b^2=ac$

Or equivalently, $a^2+b^2=ab+ac$ ...(i)

Now L.H.S. of $(i)=a^2+b^2$

where $a^2=\left(\frac{1-R^{-n}}{R-1}\right)^2$ and $b^2=\left(\frac{1-R^{-2n}}{R-1}\right)^2$

Thus $$\text{L.H.S.}=\frac{(1-R^{-n})^2}{(R-1)^2}+\frac{(1-R^{-2n})^2}{(R-1)^2}$$

$$=\frac{1}{(R-1)^2}\{(1-2R^{-n}+R^{-2n})+(1-2R^{-2n}+R^{-4n})\}$$

$$=\frac{2-2R^{-n}-R^{-2n}+R^{-4n}}{(R-1)^2}$$

R.H S $=ab+ac$

$=a(b+c)$

Now $a=\dfrac{1-R^{-n}}{R-1}$; $b=\dfrac{1-R^{-2n}}{R-1}$; $c=\dfrac{1-R^{-3n}}{R-1}$

Thus R.H.S. $=\dfrac{1-R^{-n}}{R-1}\left\{\dfrac{1-R^{-2n}}{R-1}+\dfrac{1-R^{-3n}}{R-1}\right\}$

$$=\frac{(1-R^{-n})(2-R^{-2n}-R^{-3n})}{(R-1)^2}$$

$$=\frac{2-R^{-2n}-R^{3n}-2R^{-n}+R^{-3n}+R^{-4n}}{(R-1)^2}$$

$$=\frac{2-2R^{-n}-R^{-2n}+R^{-4n}}{(R-1)^2}$$

Thus L H S =R H S Q.E.D.

Exercises : A man borrows Rs. 5000 at 4% compound interest, if the principal and interest are to be repaid in 10 equal annual instalments, find the amount of each instalment

(Answer : Rs. 616 46 P.)

2. Find the sum of money received by a pensioner at 58 if he wants to commute his annual pension of Rs 1200 for a present payment when compound interest is reckoned at 4% p.a and the expectation of his life is assessed at 10 years only.

(Ans Rs 9717 app)

3. A wagon is purchased on instalment basis, such that Rs. 5000 is to be paid on signing the contract and four yearly instalments of Rs 3000 each payable at the end of the first, second, third and fourth year If interest is charged at 5% p a. what should be the cash down price ?

Ans Rs. 15644 (app)

4 A company sets aside for a reserve fund the sum of Rs 20,000 annually to enable it to pay off a debenture issue of Rs. 2,39,000 at the end of 10 years Assuming that the reserve accumulates at 5% p.a. compound, find the surplus after paying off the debenture stock

(Ans Rs. 500)

5. Find the present value of an annuity of Rs. 300 p.a. for 5 years at 4%. Answer to the nearest paisa, given that log 1.04=0.0170333, log 821.923=2.9148335

If the above annuity be a perpetual one, what will be the present value ?

(Ans. Rs. 1335.58, Rs. 7500)

6. On his 48th birthday a man decides to make a gift of Rs. 5000 to a hospital on his 60th birthday. He decides to save this amount by making equal, annual payments up to and including his 60th birthday to a fund which gives $3\frac{1}{2}\%$ compound interest, the first payment being made at once. Calculate the amount of each annual payment (answer to the nearest paisa).

(Ans. Rs. 310 90)

7. A man buys a car for Rs 16,000. He estimates that its value will depreciate each year by 20 per cent of its value at the beginning of the year Find the depreciated value (Rs. x, correct to the nearest rupee) of the car at the end of five years If the man sets aside at the end of each of the five years a certain fixed sum (Rs y) to accumulate at 4 per cent compound interest in order to be able to buy at the end of the five years another car costing Rs 22,000 (after allowing the above depreciated value Rs x, for the old car in part exchange) find to the nearest paisa, the value Rs. y of each payment

(Ans Rs 5242 , Rs 3103 33)

8 The accumulations in a Provident Fund are invested at the end of every year to earn 10% p a A person contributes $12\frac{1}{2}\%$ of his salary to which the employer adds 10% every month Find how much the accumulations will amount to at the end of 30 years of his service, for every 100 rupees of his monthly salary (Give the answer to the nearest rupee)

(Ans Rs. 44 442)

5. Replacement Theory

Replacement problem arise in the following types of situations :—

1 The existing units have outlived their effective life and it is not economical to continue their use because of increasing operating or maintenance costs etc. ; therefore, on economic considerations it becomes desirable to replace them with new units.

2. The existing units "died i e., perished all of a sudden or destroyed etc. The past probabilities of "death" of such units can be used to derive the optimal replacement interval.

The replacement problems are discussed under the following headings :

1. Replacement of equipment (viz a truck or a machine) which worsens with time.
2. Group replacement of items periodically (irrespective of how many actually failed during the period).
3. Staff Replacement Problems.

Replacement of items which deteriorate with time.

First we shall disregard the money value.

Let M_t = maintenance cost in period t.

C=Capital cost of item.

S=Scrap value of the item.

The maintenance cost may rise in a continuous or a discret (i e., stepped) fashion In the former case it will be described by some mathematical equation with M_t on the left side and a function of it on the right side. The latter case of the maintenance cost rising in a discrete fashion is typified by the following example which we do not propose to solve at the moment.

Example 1.

The cost of a machine is Rs. 6,100 and its scrap value is only Rs. 100 The maintenance costs (which can be seen as discrete) are found to be :

Year	1	2	3	4	5	6	7	8
Maintenance cost in Rs,	100	250	400	600	900	1250	1600	2200

When should the machine be replaced ?

Reverting to various costs symbolised above by M_t ; C and S, the total cost T (n) incurred on the item in *n* years will be

Continuous case

$$T(n)=\int_0^n M_t\,dt+C-S$$

Discrete case

$$T(n)=\sum_{t=0}^{n} M_t+C-S$$

Average cost of *n* years in the two cases is given below :

Continuous case

$$A(n)=\frac{1}{n}\int M_t\,dt+\frac{C-S}{n}$$

Discrete cass

$$A(n)=\frac{1}{n}\sum_{t=0}^{n} M_t+\frac{C-S}{n}$$

Without giving the proof, we shall make the following statement for the discrete case,

n is optimal at the least average cost.

The continuous case would not be discussed any more.

Example 1 is solved below with the intention of elucidating use of this statement.

Solution to Example 1.

1	2	3	4	5	6
Year t	Maintenance Cost, M_t	Cumulative Maintenance Cost $\sum_{t=0}^{n} M_t$	$C-S$	$\sum_{t=1}^{n} M_t + C-S$	Col. 5 ÷ Col. 1.
1	100	100	6000	6100	6100
2	250	350	6000	6350	3175
3	400	750	6000	6750	2250
4	600	1350	6000	7350	1838
5	900	2250	6000	8250	1650
6	1250	3500	6000	9500	1583 (Least)
7	1600	5100	6000	11100	1586
8	2200	7300	6000	13300	1663

Thus 6 years is the optimal replacement period

Example 2 :

A firm is considering replacement of [a machine, whose cost price is Rs. 12,200; and the scrap value is Rs, 200. The maintenance costs are found from experience to be as follows :

Year	1	2	3	4	5	6		8
Maintenance Cost (Rs.)	200	500	800	1200	1800	2500	3200	4000

What is the optimal replacement interval ?

Solution :

1 Year	2 Maintenance Cost	3 Cumulative Maintenance cost	4 Loss in purchase price	5 Col. 3+ Col 4	6 Col 5÷ Col 1
1	200	200	12,000	12,200	12,200
2	500	700	12,000	12,700	6,350
3	800	1500	12.000	13,500	4,500
4	1200	2700	12,000	14,700	3,675
5	1800	4500	12,000	16,500	3,300
6	2500	7000	12,000	19.000	3,167 (Least)
7	3200	10,200	12,000	22,200	3,171
8	4000	14,200	12,000	26.200	3,275

Hence 6 years is the optimal replacement period.

Example 3

A truck owner finds from his past records that the maintenance costs per year of the truck whose purchase price is Rs. 8000 are given below :

Year	1	2	3	4	5	6	7	8
Maintenance cost	1000	1300	1700	2200	2900	3800	4800	6000
Resale Price	4000	2000	1200	600	500	400	400	400

Determine at which time it is desirable to replace the truck.

Solution ;

1 Year	2 Maintenance Cost	3 Cumulative Maintenance Cost	4 (C—S)	5 Col. 3+ Col. 4	6 Col. 5÷ Col. 1
1	1000	1000	4000	5000	5000
2	1300	2300	6000	8300	4150
3	1700	4000	6800	10,800	3600
4	2200	6200	7400	13,600	3400
5	2900	9100	7500	16,600	**3320
6	3800	12,900	7600	20,500	3417
7	4800	17,700	7600	25,300	3614
8	6000	23,700	7600	31,300	3913

Scanning col. 6, 5 years is the optimal replacement period.

Example 4

A fleet owner finds from his past records that the costs per year of running a truck whose purchase price is Rs. 6,000 are as given below :

Year	1	2	3	4	5	6	7
Running cost (in rupees)	1000	1200	1400	1800	2300	2800	3400
Resale Value (in rupees)	3000	1500	750	375	200	200	200

Determine at what age replacement is due.

Solution : Table 1.

1 Year	2 Maintnance Cost	3 Cumulative Maintenance Cost	4 C—S	5 Col. 3+ Col. 4	6 Col. 5÷ Col. 1
1	1000	1000	3000	4000	4000
2	1200	2200	4500	6700	3350
3	1400	3600	5250	8850	2950
4	1800	5400	5625	11025	2756
5	2300	7700	5800	13500	2700
6	2800	10500	5800	16300	2717
7	3400	13900	5800	19700	2814

Hence 5 years is the optimal replacement interval,

Example 5

The truck owner of example 4 has three trucks, two of which are two years old and the third one year old. He is considering a new type of truck with 50% more capacity than one of the old ones at a unit price of Rs. 8000. He estimates that the running costs and resale price for the new truck will be as follows :

Year	1	2	3	4	5	6	7	8
Running Costs	1200	1500	1800	2400	3100	4000	5000	6100
Resale price	4000	2000	1000	500	300	300	300	300

Assuming that the loss of flexibility due to fewer trucks of no importance, and that he will continue to have sufficient work for three of the old trucks, what should his policy be ?

Table 2 *Costs of owning & running the new truck.*

Replacement at the end of year	Running Cost	Cumulative Running Costs	Total Capital Costs	Total Costs	Average Cost Per year
1	1200	1200	4000	5200	5200
2	1500	2700	6000	8700	4350
3	1800	4500	7000	11500	3833
4	2400	6900	7500	14400	3600
5	3100	10000	7700	17700	3540
6	4000	14000	7700	21700	3617

Hence 5 years is the optimal replacement interval.

Now, for 3 old trucks 2 new trucks are required. The average cost/year of old trucks is $3 \times 2700 =$ Rs, 8100. And the average cost/year of 2 new trucks= $2 \times 3540 = 7080$. Hence it is to be desired to replace the 3 old trucks with 2 new trucks But when ?

Yearly Costs for the old Truck

Year	Running Costs	Capital Expenditure	Yearly Costs
1.	1000	6000−3000=3000	4000
2.	1200	3000−1500=1500	2700
3.	1400	1500− 750= 750	2150
4.	1800	750− 375= 375	2175
5.	2300	375− 200= 175	2475
6.	2800	200− 200= 0	2800
7.	3400	200− 200= 0	3400

Hence the total costs *next year* for two old trucks aged two years and o smaller truck aged one year will be $2150 \times 2 + 2700 = 7{,}000$.

In the subsequent years the costs will be Rs. 6,500 Rs. 7125, Rs. 8075 f years 2, 3 and 4.

The average annual cost for two larger trucks will be Rs. 7080 so that the cost for the old trucks are less than the new ones until the third year hence. Thus all the 3 old trucks should be replaced two years from now.

Exercise: The following table gives the running costs per year and resale prices of a certain equipment whose purchase price is Rs 5,000.

Year	1	2	3	4	5	6	7	8
Running costs (in rupees)	1500	1600	1800	2100	2500	2900	3400	4000
Resale Value (in rupees)	3500	2500	1700	1200	800	500	500	500

At what year is the replacement due.

Example 6

(a) Machine X costs Rs. 9,000. Annual operating cost is Rs. 200 for the first year, and then increases by Rs. 2,000 every year i.e., in the fourth year operating *costs become Rs. 6200 Determine the best age at which to replace the machine.* If the optimum replacement policy is followed, what will be the average yearly cost owning and operating the machine ? Assume that the machine has has no resale value when replaced, and the future costs are not discounted.

(b) Machine Y costs Rs. 10,000. Annual operating cost is Rs. 400 for the first year and then increases by Rs. 800 every year. You have now a machine of type X which is one year old. Should you replace it with Y, and, if so, when ?

Sulution (a)

1	2	3	4	5	6
Year	Operating Cost	Cum-Operating Cost	C—S	Col. 3+Col. 4	Col. 5—Col. 1
1	200	200	9000	9200	9200
2	2200	2400	9000	11400	5700
3	4200	6600	9000	15600	*5200
4	(200	12800	9000	21800	5450

Average yearly cost of owning and operating X=Rs. 5200.

1	2	3	4	5	6
Year	Operating Cost	Cum-Operating Cost	C—S	Col. 3+Col. 4	Col. 5÷Col. 1
1	400	400	10,000	10,400	10,400
2	1200	1600	10,000	11,600	5,800
3	2000	3600	10,000	13,600	4,533
4	2800	6400	10,000	16,400	4,100
5	3600	10000	10,000	20,000	4,000
6	4400	14400	10,000	24,400	4,066

∴ Average cost=Rs. 4000.

Since the lowest average cost of Y is lower than that of X, X, should be replaced by Y.

Y should be purchased when the cost for next year of running X exceeds the average yearly cost for Y. From the first table, under col. 2 we observe that 4200 (against year 3) just exceeds 4,000, lowest average cost of Y. Year 3 is then the time for replacement. Since X is already one year *old it should be used for another year to come and then replaced.*

Example 7

A manual stamper currently valued at Rs 1,000 is expected to last 2 years and cost Rs 4,000 per year to operate. An automatic stamper which can be purchased for Rs 3,000 will last 4 years and can be operated at an annual cost of Rs 3,000 If money carries the rate of interest 10% per annum, determine which stamper should be purchased

Solution ·

The total discounted cost (purchased worth) of the manual stamper for two years is computed below :

Year	Cost	Disc Factor	Col. 2×Col. 3
1	1,000+4,000=5,000	1 0000	5,000
2	4,000	0 9091	3,636
			8,636

Total discount cost of the automatic stamper for 4 years is computed below:

Year	Cost	Disc. Factor	Col. 2×Col. 3
1	6000	1.0000	6000
2	3000	0.9091	2727
3	3000	0.8264	2479
4	3C00	0.7513	2253
			13459

Av. cost for 2 years=6729.

This shows that the apparent advantage is with the automatic stamper. But the comparison is unfair since the periods for which the costs are considered are different. So, if we consider 4 years period for the manual stamper also, then its total discount cost will be.

Year	Cost	Discount Factor	Col. 2×Col. 3
1	5000	1.0000	5000
2	4000	0.9091	3636
3	5000	0.8264	4132
4	4000	0.7513	3005
			15773

This proves conclusively that the automatic stamper should be installed.

[Note : Example 7 was a special case when the replacement period of one machine was exactly double that of the other. In general, this may not be so. For example, machine X may be replaced after every 13 years and Y every 17 years. To compare X and Y, then we should consider $13\times 17=221$ years. However, it is easier to consider infinite years instead for each machine. This matter will be taken up after the next example.]

Optimal Replacement Period in case of money value

Consider a series of time periods 1, 2, 3.. of equal length, and let the costs incurred in these periods be M_1, M_2, M_3, M_4 . . respectively. It is assumed that these costs are increasing monotonously. Assume that each cost is paid at the beginning of the period in which it is incurred, that initial cost of the equipment is C, and the cost of money 100 r% per period $\left(\text{so that } V=\frac{1}{1+r}\right)$. Then the discounted value K_n of all future costs associated with a policy of replacing equipment after each n period is given by

$$K_n=(C+M_1+VM_2+V^2M_3+\ldots\ldots V^{n-1}M_n)+$$
$$\{V^n(C+M_1)+V^{n+1}M_2+V^{n\times 2}M_3+\ldots+V^{2n-1}M_n)+\ldots$$

$$=\left(C+\sum_{i=1}^{n} V^{i-1}M_i\right)+V^n\left(C+\sum_{i=1}^{n} V^{i-1}M_i\right)+$$

$$V^{2n}\left(C+\sum_{i=1}^{n} V^{i-1}M_i\right)+\ldots$$

$$=\left(C+\sum_{i=1}^{n} V^{i-1}M_i\right)\left(1+V^n+V^{2n}+\ldots\right)$$

The second expression is an infinite G.P. Thus

$$K_n=\frac{C+\sum_{i=1}^{n} V^{i-1}M_i}{1-V^n}$$

K_n is the amount of money required now to pay all the future costs of acquiring and operating the equipment when it is renewed every n years. It is not suggested that any company would actually set up a fund of this size. However, if K_n is less than K_n+1 then replacing the equipment each n years is preferable to replacing each $(n+1)$ years Without giving the proof we shall state the following double inequality which holds good at n, the optimal replacement interval

$$\frac{1-V^n}{1-V} M_n < \left\{ (C+M_1)+M_2V+M_3V^2+\ldots+M_nV^{n-1} \right\}$$

$$< \frac{1-V^n}{1-V} M_n+1$$

Example 8 below is introduced to explain the use of this inequality for finding the optimal replacement interval. It is to be noted that $\frac{V^{n-1}}{V-1}=1+V+V^2+\ldots+V^n = \sum_{i=1}^{n} V_i$ Therefore, we don't have to compute $\frac{V^{n-1}}{V-1}$ directly but we simply accumulate the discount factors.

Example 8

A manufacturer is offered two machines A and B. A is priced at Rs. 5,000, and running costs are estimated at Rs. 800 for each of the first five years, increasing by Rs. 200 per year in the 6th and subsequent years. Machine B, which has the same capacity as A, costs Rs 2,500 but will have running costs of Rs. 1,200 per year for six years increasing by Rs. 200 per year thereafter. If money is worth 10% per year, which machine should be purchased ? (Assume that machines will eventually be sold for scrap at negligible price).

Solution :

Replacement Costs for Machine A

1	2	3	4	5	6	7	8
Year	Running Costs Rs.	V^{n-1}	Col. 2×Col. 3	Σ Col. 4 Rs.	Total Costs Rs.	$\frac{V^n-1}{V-1}$	Col 9×Col 7 as per* Rs.
1	800	1.0000	800	800	5800	*1.0000	*800
2	*800	0.9091	727	1527	6527	1.9091	1528
3	800	0.8264	661	2188	7188	2.7355	2189
4	800	0.7513	601	2789	7789	3.4868	2790
5	800	0.6830	546	3335	8335	4.1698	4170
6	1000	0 6209	621	3956	8956	4.7907	5749
7	1200	0.5645	677	4633	9633	5.3552	7497
8	1400	0.5132	718	5351	10351	5.8684	9390
9	1600	0.4665	746	6097→	11097	6.3349	11403

*Col 8 is not obtained by simply multiplying Col. 2 with Col. 7. Second figure of Col. 2 is multiplied by the Ist figure of Col. 7 to yield Ist figure of Col 8. Then, 3rd figure of Col. 2 is multiplied with 2nd figure Col 7 to give 2nd figure of Col 8 ; and so on.

Thus, this machine is best replaced every nine years (since Col. 8 < Col. 6 against year 9)

Replacement Costs for Machine B

1	2	3	4	5	6	7	8
Year	Running Costs Rs.	V^{n-1}	Col. 2×Col. 3	Σ Col. 4 Rs.	Total Costs Rs.	$\frac{V^n-1}{V-1}$	*Col. 1× col. 7 Rs.
1	1200	1.000	1200	1200	3700	1.0000	1200
2	1200	0 9091	1090.5	2290.5	4790.5	1.9091	2291
3	1200	0 8264	991.5	3282	5782	2.7355	3283
4	1200	0.7513	901 5	4183.5	6683.5	3.4868	4184
5	1200	0.6830	819 0	5002 5	7502.5	4.1698	5004
6	1200	0 6209	745 1	5747.6	8247.6	4.7907	6707
7	1400	0 5645	790 3	6523 9	9037.9	5.3552	8568
8	1600	0.5132	821.1	7345.0	9859 0	5.8684	10563

Thus machine B is best replaced after 8 years.

Now let us ascertain which machine is better. Towards this we shall make use their K_n's

$$\text{Machine A : } K_9 = \frac{C + \sum_{i=1}^{9} V^{i-1}M_i}{1-V^9}$$

$$= \frac{11097}{1-V^9} = \frac{11097/(1-V)}{(1-V^9)/(1-V)}$$

$$= \frac{11097}{6.3349(1-V)}$$

$$= 1751/(1-V)$$

$$\text{Machine B : } K_8 = \frac{C + \sum_{i=1}^{8} V^{i-1}M_i}{1-V^8}$$

$$= \frac{9859}{5.8684\,(1-V)} = 1680/(1-V)$$

Since $\frac{1680}{1-V} < \frac{1751}{1-V}$ machine B is more economical.

Exercise :—A manufacturer is offered two machines A and B. A is priced at Rs. 5,000, and running costs are estimated at Rs. 800 for each of the first five years, increasing by Rs. 200 per year in the 6th and subsequent years. Machine B which has the same capacity as A, costs Rs. 2500 but will have running costs of Rs 1200 per year of six years, increasing by Rs. 200 per year thereafter.

If money is worth 10% per year, derive separately the optimal replacement periods for the two machines

Group replacement (for items that fail suddenly)

Group replacement policy can be carried out at fixed intervals for such items that fail suddenly (*viz*) light bulbs Under this policy (i) at the end of the every period (*viz* a fortnight) all the bulbs are replaced whether or not they failed (ii) individual bulbs are replaced as and when they fail during this period Usually this policy is superior to the straightforward one under which individual bulbs are replaced as and when they fail without any periodic group replacement whatever. The problem then boils down to comparing (i) the group replacement which also includes replacement of individual bulbs as and when they fail at periodic intervals This would require determination of the optimal replacement period and (ii) the straightforward policy of replacing individual bulbs as and when they fail. The following numerical example should clarify the underlying concepts :—

Example 9

The following failure rates have been observed for a certain type of light bulbs in an installation with, 1,000 bulbs.

End of week	1	2	3	4	5	6	7	8
Probability of failure to date	0 05	0 13	0 25	0.43	0 68	0 88	0.96	1.00

The cost of replacing an individual bulb is Rs. 2 25, the decision is made to replace all bulbs simultaneously at fixed intervals, and also to replace individual bulb as they fail in service. If the cost of group replacement is 60 paisa per bulb what is the best interval between replacement ? Also prove that this optimal policy is superior to the straightforward policy of replacing each bulb only when it fails.

Solution :

Let us designate the probability of a new bulb failing in its nth week of life by p_n. Thus we derive the following values of $p_1, p_2 \ldots p_8$ (which are the probabilities of a new bulb failing in its first, second...eight week of life).

$$
\begin{aligned}
p_1 &= & &0\ 05 \\
p_2 &= 0\ 13-0\ 05 & &= 0\ 08 \\
p_3 &= 0.25-0\ 13 & &= 0.12 \\
p_4 &= 0.43-0.25 & &= 0\ 18 \\
p_5 &= 0.68-0\ 43 & &= 0\ 25 \\
p_6 &= 0.88-0.68 & &= 0.20 \\
p_7 &= 0\ 96-0\ 88 & &= 0\ 08 \\
p_8 &= 1.00-0.96 & &= 0\ 04 \\
\hline
& & &1.00
\end{aligned}
$$

Let N_i be the number of replacements made at the end of the i—th week.

$N_0 = 1{,}000$

$N_1 = N_0 \times p_1 = 1{,}000 \times 0.05 = 50$

$N_2 = N_0 p_2 + N_1 p_1 = 1{,}000 \times 0\ 08 + 50 \times 0.05 = 82\ 5$

$N_3 = N_0 p_3 + N_1 p_2 + N_2 p_1 = 1000 \times 0.12 + 50 \times 0\ 08 + 82.5 \times\ 05 = 128$

$N_4 = N_0 p_4 + N_1 p_3 + N_2 p_2 + N_3 p_1$

$= 1{,}000 \times 0.18 + 50 \times 0\ 12 + 82.5 \times 0.08 + 118 \times 0\ 05 = 199$

$N_5 = N_0 p_5 \times N_1 p_4 + N_2 p_3 + N_3 p_2 + N_4 a_1$

$= 1{,}000 \times 0\ 25 + 50 \times 0.18 + 82.5 \times 0.12 + 128 \times 0\ 08 + 199 \times 0\ 05 = 290$

$N_6 = N_0 p_6 + N_1 p_5 + N_2 p_4 + N_3 p_3 + N_4 p_2 + N_5 p_1$

$= 1{,}000 \times 0\ 20 + 50 \times 0\ 25 + 82\ 5 \times 0\ 18 + 128 \times 0\ 12 + 199 \times 0.08$
$+ 290 \times 0\ 05 = 273$

$N_7 = N_0 p_7 + N_1 p_6 + N_2 p_5 + N_3 p_4 + N_4 p_3 + N_5 p_2 + N_6 p_1$

$= 1{,}000 \times 0.08 + 50 \times 0.20 + 82.5 \times 0\ 25 + 128 \times 0.18 + 199 \times 0.12$
$+ 290 \times 0\ 08 + 273 \times 0\ 05 = 194\ 4$

$N_8 = N_0 p_8 + N_1 p_7 + N_2 p_6 + N_3 p_5 + N_4 p_4 + N_5 p_3 + N_6 p_2 +$
$N_7 p_1 =$
$1{,}000 \times 0\ 04 + 50 \times 0\ 08 + 82\ 5 \times 0\ 20 + 128 \times 0\ 25 +$
$199 \times 0\ 18 + 290 \times 0\ 12 + 273 \times 0\ 08 + 194\ 4 \times 0\ 05 = 198$

Since the replacement of all the bulbs simultaneously costs 60 paise per bulb and replacement of an individual bulb on failure costs Rs 2.25, cost of replacement at the end of the Ist week is given by

$1{,}000 \times 0.60 + 50 \times 2\ 25 =$ Rs. 712 5 ; Average cost = Rs. 712 5.

Similarly cost of replacement at the end of the 2nd week

$1{,}000 \times 0\ 60 + 50 \times 2\ 25 + 82\ 5 \times 2\ 25 =$ Rs. 898 12 ; hence, average cost = Rs. 449

Cost of replacement at the end of the 3rd week

$= 1{,}000 \times 0\ 60 + 50 \times 2\ 25 + 82\ 5 \times 2\ 25 + 128 \times 2.25$
$= 1186\ 12$

$\therefore$ Average cost = Rs 395 4 (NOTE)

Cost of replacement at the end of the 4th week

$= 1{,}000 \times 0\ 60 + 50 \times 2\ 25 + 82\ 5 \times 2\ 25 + 128 \times 2.25$
$+ 199 \times 2\ 25 = 1633.87$

Average cost = Rs 408 5

Thus the optimal group replacement interval is 3 weeks

Now let us prove that the 3 week interval under the group replacement policy is superior to the straightforward policy of replacing individual bulbs only when they fail.

Mean age of a bulb $= 1p_1+2p_2+3p_3+4p_4+. \; +8p_8$

$0.05\times1=0.05$
$0.08\times2=0.16$
$0.12\times3=0.36$
$0.18\times4=0.72$
$0.25\times5=1.25$
$0.20\times6=1.20$
$0.08\times7=0.56$
$0.04\times8=0.32$

4.62

Av Number of failures in a week $= \dfrac{1,000}{4.62}$
$=216.$

Cost of replacing the bulbs only upon failure
$=216\times2.25$
$=$Rs. 486 which is more than Rs 395.4, the weekly three group replacement policy costs,

Exercise : Following mortality rates have been observed for a certain type of light bulbs.

Week	1	2	3	4	5
% failing by end of week	10	25	50	80	100

There are 1,000 bulbs in use and its costs Rs 2 to replace an individual bulb which has burnt out If all bulbs were replaced simultaneously it would cost 50 paise per bulb It is proposed to replace all bulbs at fixed interval whether or not they have burnt out, and to continue replacing burnt out bulbs as they fail At what group replacement price per bulb would a policy of strictly individual replacement become preferable to the adopted policy ?

Staff Replacement Problems

Example 10. A researcher team is planned to rise to a strength of 50 chemists and then to remain at that level. The wastage of recruit is depends on their length of service and is as follows :

Year	1	2	3	4	5	6	7	8	9	10
Total % who have left upto the end of the period	5	36	56	63	68	73	79	87	97	100

(*a*) What is the recruitment per year necessary to maintain this strength ?

(*b*) There are eight senior posts for which length of the service is the main criteria. What is the average length of service at which a new entrant can expect to be promoted to one of these posts ?

Solution. Let us assume that the intake per year is 100.

Year	1	2	3	4	5	6	7	8	9	10
{% who left by the end of this year	5	36	56	63	68	73	79	87	97	100
{% who remain by the end of this year	95	64	44	37	32	27	21	13	3	0

Thus the 100 who join in the 1st year will become zero at 10th year,

and the 100 who join in the 2nd year (will serve for 9 years) and become 3 at the end of the 10th year :

and the 100 who join in the 3rd year (will serve for 8 years) and become 13 at the end of the 10th year ;

and so on.

Thus if we recruit 100 chemists every year we would be left with

$0+3+13+21+27+32+37+44+64+95+100=436$ when equilibrium is reached.

Thus an intake of 100 per year gives a total strength of 436 To maintain a strength of 50 requires intake of $50\times\frac{100}{436}=11.5$ per year (Roughly 12 as taken in part b)

Whe have assumed that all those who completed x years service but left before $(x+1)$ years service actually left immediately before completing $x+1$ years If we assume that they left immediately after completing x years service the total becomes 336 and the required intake is $50\times\frac{100}{336}=14.9$

In practice, however, the chemists may leave at any time in the year and $\frac{11\ 5+14\ 9}{2}\approx 13$ is a reasonable answer.

With 50 chemists in the team the distribution of the completed length of service of the chemists willl be :

Year	*Number of chemists*
0	12
1	$11=12\times 95\%$
2	$7=12\times 64\%$
3	5 and so on
4	4
5	4
6	3
7	2
8	2
9	0
10	0

Thus chemists can expect to be promoted to the senior posts after completing 5 and before completing 5 years of the service

Example 11.

A colliery recruits boys at 16 and trains them for face work immediately. At 16½ they do 3/4 stint and at 18 they do a full stint. Assuming the following probabilities of leaving the colliery and no recruitment from other collieries, what rate of recruitment for face training is necessary to keep a face manpower doing 100

stints ? Assume that those who leave do so at $16\frac{1}{2}$ or at the who years. (A stint is a unit of area to be faced).

Ages	*Probability of leaving*
$16-16\frac{1}{2}$	0.10
$16\frac{1}{2}-17$	0 05
17—18	0.10
18—19	0 01
19—20	0 01
20 – 25	0.05 (i.e. 0 01 per year)
25—30	0 05
30—35	0.05
35—40	0.05
40—45	0.10
45—50	0 20
50—55	0.20
55—60	0 03

Solution. Let us suppose that the intake of trainees is 100 *per half year*

When the system is in equilibrium, the colliery age—distribution will be as follows :

Age	*Number Employed*	*Stint Worked*
$16-16\frac{1}{2}$	100	Nil
$16\frac{1}{2}-17$	90 $=100-10$	$260\times\frac{3}{4}=195$
17—18	170 $=2(100-10-5)$; $\because$ 2 half years	
18—19	$150=2(100-10-5-10)$	150
19—20 to 35—40	Probability of leaving=0.01/year If 200 join in 1 year, number leaving each year$=200\times0.01=2$ $\therefore$ At age 19 we have $150-2=148$ recruitments At age 20 we have $148-2=146$ . . . upto 40th year (22 terms $S=148+146+\ldots$ (22 terms) is an A. P the sum of which is given by the formula $\frac{n}{2}(a+l)$ $=\frac{22}{2}\left[2\times148+(22-1)(-2)\right]$	2794
40—45	Prob of leaving=0 10/5=0 02/year $\therefore$ No. leaving each year$=200\times0.02=4$ $102+98+94+90+86=$	470
45—55	Pro. of leaving=0 20 $\therefore$ No leaving each year$=8$ $78+70+62+\ldots\ldots+6=$	420
55—60	Prob. of leaving=0.03/year. $\therefore$ No leaving each year$=200\times\frac{0.03}{5}=1.2$ $4.8+3.6+2.4+1.2=$	12
		4041

Therefore, to keep 100 strints going we need an intake of $100\times\frac{100}{4041}=2\ 47$ per half year. Or, say, 6 recruits per year.

Example 12. An airlines requires 200 assistant hostesses, 300 hostesses and 50 supervisors. Girls are requirted at age 21, if still in service retire at age 60. Given the following life table, determine.

(i) How many girls be recruited each year ?

(ii) At what age should promotions take place ?

Life Table

Age	21	22	23	24	25	26	27	28
No. in service	1000	600	480	384	307	261	228	206
Age	29	30	31	32	33	34	35	36
No. in service	190	181	173	167	161	155	150	146
Age	37	38	39	40	41	42	43	44
No. in service	141	136	131	125	119	113	106	99
Age	45	46	47	48	49	50	51	52
No of service	93	87	80	73	66	59	53	46
Age	53	54	55	56	57	58	59	
No in service	38	33	27	22	18	14	11	

Solution The total number of girls requirted at age 21 and those serving upto the age of 59 will be equal to 6480. We required $200+300+50=550$ girls in the airline

The recruitment every year is 1,000 when total number of girls are 6480 after 59 years Therefore, in order to maintain a strength of 550, hostesses we should recruit $550\ \times\frac{1000}{6480}\approx 85$ each year.

If we plan to promote the assistant hostesses at the age x, then upto age $x-1$ we need 200 assistant hostesses Amongst 550, there are 200 assistant hostesses. Therefore, out of a strength of 1,000 there will be

$$200\times\frac{1000}{550}=364 \text{ assistant hostesses.}$$

From the given life table this number is available upto the age of 24 years. Hence the promotion of assistant hostesses will be due in the 25th year.

Also, out of 550 recruitments, we need only 300 hostesses.

Therefore, if we recruit 1,000 then we shall require,

$$1{,}000\times\frac{300}{550}=545 \text{ hostesses.}$$

Hence the number of hostesses and assistant hostesses in a recruitment of 1,000 will be $364+545=909$.

We thus need 1 000 $909=91$ supervisors, whereas at the age of 46 only 87 will survive. Hence promotion of hostesses to supervisors will be due 46th year.

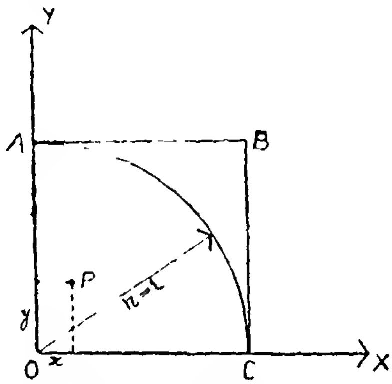
Simulation

Simulation refers, in business situations, to perform experiments on a model of a real life system (usually using a computer) with random numbers (and as such

it is synonymous with Monte-Carlo methods) or occasionally without the use of random numbers. Such experiments may be undertaken before the real system is operational so as to aid in its design, or to see how the system might react to change in its operating rules, or to evaluate the system's response to changes in its structure Simulation is particularly appropriate where (i) manipulations on a real system are either cumbersome or impossible (viz. the RBI wanting to know the effects of the change of the bank rate), or (ii) the real system is either not possible to be mathematically modelled or the mathematical model, if built at all, is not possible to solve analytically. In view of the enormous computations involved the computer is usually a necessary adjunct though below we shall deal with quite a few simple simulation problems towards its expostion.

Example 1. (on finding the value of π experimentally by simulation) ,

In the figure below is shown the arc of a circle of a unit radius in the first quadrant. Also shown is a square OABC of side of one unit.



The equation of the circle is given by $x^2+y^2=1$ Two* random numbers, each less than unity (viz , $x=0\ 1906$ and $y=0\ 3698$) are picked up and the point P corresponding to these is shown plotted. Obviously, if $x^2+y^2\leqslant 1$, P is inside or on the circle but if $x^2+y^2>1$, P is beyond the circle but within the square

In this manner, hundreds or thousands of pairs of random numbers are picked up and it is ascertained if the points corresponding to them fall in/on the arc or beyond the square suppose that n out of a total of N points fall in/on the arc.

Now the area enclosed by the arc$=\frac{\pi}{4}1^2=\frac{\pi}{4}$ and the area enclosed by the square$=1^2=1$.

$$\frac{\pi/4}{1}=\frac{n}{N} \qquad \text{or} \qquad \pi=\frac{4n}{N}$$

The experimental value of π is thus obtained Obviously larger the sample size, N closer we shall be to the true value of π

This way, the Monte Carlo methods can be applied to solve complex and stochastic/deterministic queuing, inventory control, production scheduling, etc pro-

*See the Appendix on an excerpt of random nos. at the end of the study How to use it is explained by an example below it. However, the nos. we are using have been taken from other excerpt.

blems. The methods have since recently been also applied in moon landing and studying galactic collisions, atomic behaviour and in military.

Example 2 Three points are chosen at random on the circumference of a circle. Estimate the probability by the Monte Carlo methods that they lie on the same semi-circle

Solution : The table random numbers (with a decimal on the left) lie between 0 and 1. (See the Appendix). Consider the circle of radius $\frac{1}{2\pi}$ Its circumference is $\frac{1}{2\pi} \times 2\pi = 1$.

Consider, three triplets of random numbers plotted along the circumference of the aforesaid circle shown in fig. 1 to 3.

A	B	C	
.25	.12	.08	Ist triplet
D	E	F	
50	69	.53	2nd triplet
G	H	I	
03	.67	97	3rd triplet

The three triplets are shown plotted in the three circles below ; the distance of a point from O along the circumference anticlockwise being the value of a random number. In each of the first two tripltes the distance between the maxima and the minima (0.25 −0 08 and 0 69 −0.50) is less than 0 50 Thus each of these two triplets obviouly lies on a semicircle In the third triplet, this difference is (0 97−0 03) which is more than 0 50, yet they lie on the same semicircle In this triplet, the distance between 0.67 and 0 03 (0.97 being in between these) is 1 −0 67 +0 03 = 1 03 −0 67 < 0.50 and therefore, this triplet, too, lies on a semicircle.

Hence, the rule for ascertaining if a triplet of random numbers lies on a semicircle is given by

"(i) Find the difference between the maxima and the minima If this difference is ⩽0.50 the triplet is on a semi-circle and

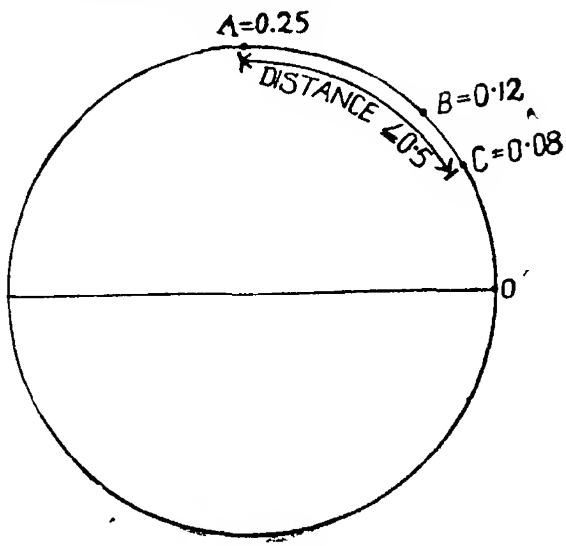


Fig 1

(ii) If this difference is >0.50 add unit to those random numbers in the triplet which is/are less that 0.50. Now find the difference between the maxima and minima. If the new difference <0.50 the triplet lies on a semi-circle, otherwise not."

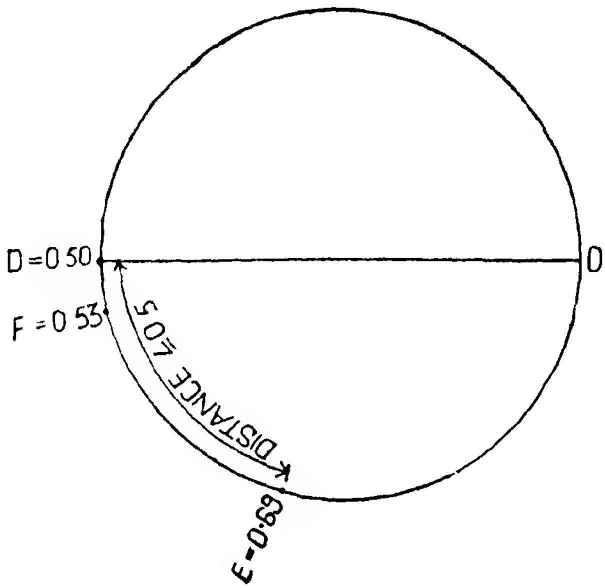


Fig. 2

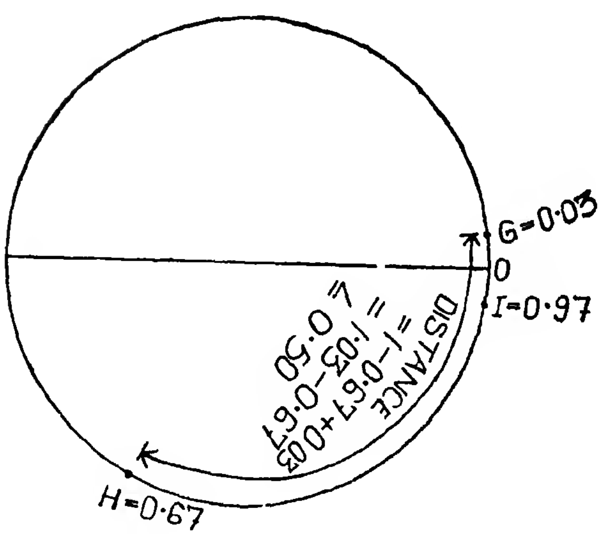


Fig 3

By this rule, it is ascertained for each of the following 17 triplets if it lies on a semi-circle or not. Each triplet represents three random number picked up from the random number tables. In this ✓ indicates that the triplet is on a semi-circle and x indicates the reverse.

			Difference between minima and maxima	Add to nos.> 0.50
.56	83	.55	0 83−0.55=0.28✓	
.47	.84	.08	0.84−0.08=0.76	1 08−0.47=0.61×
.36	.05	26	0.36−0 05=0 31	
.42	95	.95	0 95−0.42=0.53	1 42−0.95=0.47✓
.66	17	.03	0.66−0 03=0 63	1.17−0.66=0.51×
21	.57	.31	0.57−0.21=0.36✓	
.69	90	33	0.90−0.33=0.57	1.33−0.69=0.64✓
.15	.64	.23	0.64−0.55=0.49✓	
.92	.52	.74	0.92−0.52=0.40✓	
.04	.86	.41	0.86−0.04=0.82	1.41−0.86=0.55×
06	.39	89	0.89—0.06=0.83	1.39−0.89=0.50✓
61	.99	.95	0.99−0.61=0.38✓	
.35	.57	.86	0.96−0.35=0.61	1.35−0.57=0.78×
.52	60	.02	0.60−0.02=0.58	1.02−0.50=0.52×
.33	.79	.79	0.79−0 33=0.46✓	
30	.36	.58	0 58−0.30=0.28✓	
.43	65	.63	0 65−0.43=0 22✓	

Out of 17 triplets 11 lie on a semi-circle. Thus the required probability= 11/17=0 7 (which is close to the theoretical value of 0.75).

Exercise—Draw the computer program flowchart for this problem.

Example 2. Nine villages in a certain administrative area contain 720, 130, 150, 240, 960, 100, 52, 35, 532 fields respectively. Make a random selection of six fields using the random number tables.

Solution :

1 Village No.	2 No. of Villages	3 Cumulative No. of Villages	4 Random nos. fitted against the village as per column 3.
1	720	720	0091, 0684
2	130	850	0839, 0814
3	150	1000	
4	240	1240	
5	960	2700	
6	100	2300	
7	52	2352	
8	36	2388	2377
9	532	2920	2471.

The first random no. picked up from the random no. tables is 2377. Since it is immediately ⩽2388 in Col. 3 ; it is fitted against village 8 in col. 4. The next random no, 5997 is >2920 in Col. 4 ; therefore, it is dropped. In this manner, the

following random nos. area either fitted in Col. 4 or dropped ; 8269 (D=drop), 8385 (D), 6198 (D), 0091 (F1)=fitted against village no. in col. 4) 4829 (D), 3322 (D), 0684 (F1), 3267 (D), 8209 (D), 5166 (D), 0839 (F2), 0814 (F2), 7409 (D), 2471(F9) We stop here because 6 fields have been selected, two each from village nos 1 and 2 and each from village nos. 8 and 9.

Example 4 Ramu and Raju are workers on a two-station assembly line. The distribution of activity times at their stations is as follows —

Time in sec.	Time frequency for Ramu	Time frequency for Raju
10	4	4
20	6	5
30	10	6
40	20	7
50	40	10
60	11	8
70	5	6
80	4	4

(a) Simulate operation of the line for eight items. Use the random numbers given below :

Operation 1		Operation 2	
14	61	36	97
01	82	76	41
96	00	55	13
44	03	25	34

(b) Assuming Raju must wait unil Ramu completes the first item before starting work, will he have to wait to process any of the other eight items? Explain your answer, based upon your simulation.

Solution : Cumulative frequency distribution for Ramu is derived below. Also fitted against it are the eight given random numbers In parentheses are shown the serial numbers of random numbers.

10	4	01(2),	00(7),	03(8)
20	10			
30	20	14(1)		
40	40			
50	80	44(4)	61(5),	
60	91	82(6)		
70	96	95(3)		
80	100			

Thus the eight times are : 30, 10, 70, 50, 50, 60, 10 and 10 respectively.

Likewise the eight times for Raju are derived from his cumulative distribution below .

1	2 Frequency	3 Cumulative	4 2×col. 3	
10	4	4	8	
20	5	9	18	13(7)
30	6	15	30	25(4)
40	7	22	44	36(1), 41(6), 34(8)
50	10	32	64	55(3)
60	8	40	80	76(2)
70	6	46	92	
80	4	50	100	97(5)

(Note that cumulative frequency has been multiplied by 2 in Col. 4 in order that all the given random numbers are utilised).

Thus Raju's times, are ; 40, 60, 50, 30 80, 40, 20 and 40 seconds respectively.

Ramu's and Raju's times are displayed below to observe for waiting time, if any

1 Ramu	2 Cum. time	3 Raju	4 Raju's cumulative time with initial 30 seconds included
30	30	40	70
10	40	60	130
70	110	50	180
50	160	30	210
50	290	80	290
60	270	40	330
10	280	20	350
10	290	40	390

Since col 4 is consistently greater than col. 2 no subsequent waiting is involved.

Example 5.

A bakery chain delivers fresh cakes to one of its retail stores each day. The number of cakes delivered each day is not constant but has the following distribution :—

Cakes delivered per day	*Probability*
10	0.05
11	0 10
12	0 15
13	0 35
14	0.20
15	0 10
16	0.05

The number of constomers desiring cakes each day has the distribution ;

No. of customers	*Probability*
5	0.10
6	0.15
7	0 20
8	0.40
9	0.10
10	0.05

Finally, the probability that a customer in need of cakes wants 1, 2 or 3 takes described by

Cakes to a customer	*Probability*
1	0.40
2	0.40
3	0 20

Estimate by Monte Carlo methods the average number of cakes left over per day and the average number of sales lost per day owing to lacke of cakes. Assume that left over cakes are given away at the end of each day.

Solution

The first cumulative probability distribution is derived below.

Cumulative probability distribution of cakes delivered per day

Cakes per day	*Cumulative probability*	*Ten random nos fitted in*
10	0 05	.0153 (9)
11	0 15	.1342 (6)
12	0 30	2291 (4)
13	0 65	.5878 (2), .4391 (5), 5210 (7), .3411(8)
14	0 85	8136 (1), .7923 (10).
15	0 95	
16	1.00	.9655 (3)

Thus the cakes delivered in each of the 10 days are : 14, 13, 16, 12, 13, 11, 13, 13, 10, 14

Liekewise the no of customers per day are derived below from the cumulative probability distribution of the 2nd probability distribution.

No of customers	*Cumulative probability*	*Ten random nos. fitted in*
5	0 10	.0906 (9)
6	0 25	.2416 (4), .1934 (10)
7	0 45	.3501 (1), .3396 (3), .2587 (5), .3072(6)
8	0.85	.5054 (2)
9	0 95	.8511 (7), 8698 (8)
10	1 00	

Thus the number of customers for the ten days are 7, 8, 7, 6, 7, 7, 9, 9, 5, 6, and the total no. of the customers=71.

Thus cakes delivered per day and the no. of customers per day are laid out below.

		Days 1	2	3	4	5	6	7	8	9	10
No of Customers	1	1									
	2	2									
	3	3									
	4	3									
	5	2									
	6	1									
	7	2									
	8										
	9										
No. of cakes wanted		14									
No. of cakes delivered		14									
Left-overs		Nil									
Shortages		Nil									

This table has to be filled in by selecting no. of cakes demanded by each customer from the third cumulative distribution derived below In the first day there are seven customers. Thus 7 random nos have been fitted in the cumulative probability distribution below. These in turn, have been put under col. 1 of this table The student may fill in the remaining columns himself, as an exercise].

Cumulative probability distribution of the no. of cakes wanted by the customers.

7 random nos. fitted in below give the entries for the above table.

No of Cakes	*Cumulative probability*	
1	0 40	.23 (1), .00 (6)
2	0.80	.49 (2), .61 (5), .48 (7)
3	1.00	.82 (3), ·84 (4)

Thus the no. of cakes wanted by the 7 customers in the first day are 1, 2, 3, 3, 2, 1, 2. These are entered in column 1. Left-overs and shortages are also computed which may be done by student for the remaining columns.

Hertz Simulation Model: Hertz developed a simulation model for the investment decision making for introd iction of a new product. The following factors were considered for finding the net present value.

		Range (Expected value	
1.	Market size	100,000—340,000 (250,000)	
2.	Selling prices	385—575	(510)
3.	Market growth rate	0—6%	(3%)
4.	Eventual share of market	3—17%	(12%)
5.	Total investment required (for computing its cost)	7—10	(9.5)
6	Useful life of facilities	5—15	(10)
7.	Residual value	3.5—5.0	(4.5)
8.	Operating cost	370—545	(418)
9.	Fixed costs	250—375	(300)

It is obvious that these factors do have bearing on net present value. The discounted cash flow model would merely consider the expected value of each input variable (shown in brackets above) Its uselessness is to be seen from a simple

example Suppose that each of the expected values of these variables has 60% chances of being true (this is because these are estimates after all) the chances of all of them being true is only $(0\ 9)^9$ i e 1 in 100 To derive home this point take the case of rolling two six-faced disc Expected value is 7 no doubt, but 7 is the outcome with only 1 in 6 chances

1+6
2+6 2+5 1+5
3+6 3+5 3+4 2+4 1+6
4+6 4+5 4+4 4+3 3+3 2+3 1+3
5+6 5+5 5+4 5+3 5+2 4+2 3+2 2+2 1+2
6+6 6+5 6+4 6+3 6+2 6+1 5+1 4+1 3+1 2+1 1+1

(There is only a 1/6 chance or getting 7 by rolling 2 dice.)

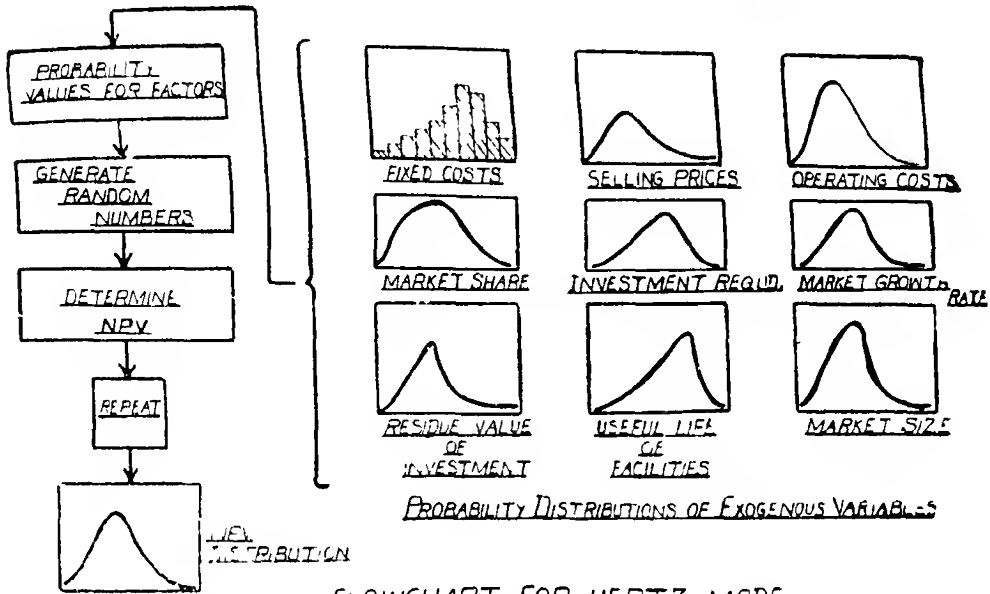
In business situations, by analogy, there may be 8 or 10 dices each with 100 faces! Hertz, therefore, considered range of each variable in determining the net present value, which, also, is described in a range

Hanssman defines investment as present commitment of major resources by a single decision to uncertain future returns Thus the entrepreneurial risk is inhersent. It is here that the executive wants more information. He knows that future is future after all and the expected value, a point estimate without any clear specification of the risk involved is not reliable. There have been attempts to construct sophisticated models of discounted cash flow that can give very precise estimates but behind the facade of refined mathematics there lurks data that is only crude There have been attempts from several other directions to give meaning to the point estimate but to no avail. It is suggested that estimates can be made realistic by empirical adjustments. In another context, for example, the project completion date by PERT may be Dec. 31, 1980. Someone may attempt to improve the estimate by extending the date of June 1981 on the ground that several such projects in the past had been delayed by so much time. This is liable to introduce serious bias Another attempt has been to make 3 estimates on the lines of activity durations in the PERT model. But this too does not clear the picture although it is a step in the right direction

Hertz proposes that the distribution be described for each variable. This may be on the basis of past data and/or by subjective estimates of the executives. The executives need to be aided by OR experts to enable them to describe the distribution and its parameters. First of all, the decision maker would be asked to pick up a value that he believes that there is the same chance of his estimate being too high as there is of its being too low This furnishes the mean. For an idea of the variability he would be asked to select 2 points, one on each side of the mean and equally distant from it, such that he believed that the probability of the true value of mean being between these two points was exactly equal to the probability of not being between these two points. From this a normal distribution be may be derived.

Having derived the distribution of all the input variables, i e. mean, standard

deviation and shape of distribution for each variable the simulation experiment may be performed on the lines of the diagramatic flow chart in the figure below.



FLOWCHART FOR HERTZ MODEL

An explanation on random number generation is in order at this stage; before the student concentrates on flowchart

Generation of Random Numbers. Simulation technique utilises random numbers to simulate a real-life situation But the random number tables (See Appendix with which the student is familiar cannot be feasibly stored in the computer memory, computer being necessary for even a moderate size simulation experiment They would occupy too much storage space Even on mathodological grounds it is objectionable to use the same set of random numbers contained in ordinary random number tables again and again. Therefore, several techniques of generating the random numbers internally by the computer have been evolved. The necessary formulas, as will be apparent soon, occupy relatively little space. We shall discuss only one of the these methods below :

Mid-square method of generating pseudo-random nnmbers: A four digit number is taken e g , $x=1212$. By squaring it a high-digit figure is obtained, in this case $x^2=01468944$ From this figure the middle 4 digits are extracted that is $x_1=4689$. By squaring it a four-digit figure is obtained $x_1^2=21985821$ from which $x_2=9888$. The process is repeated and the sequence of pseudo random numbers 1212, 4689, 9858 ..is obtained The method does not provide an indefinite series of random numbers. After a certain quantity of random numbers have been obtained one of the following contingencies may occur :

(*i*) The series vanishes because a random number obtained is 0000.

(*ii*) A random number reproduces itself, e.g., $x_9=7600$, $x_{10}=7600$, $x_{11}=7600$

(iii) A loop occurs, e.g. $x_{11}=6100$, $x_{12}=2100$, $x_{13}=4100$, $x_{14}=8100$, $x_{15}=6100$, x_{11} and x_{15} being the same and the process would therefore continue in this circle.

The random numbers generated by such formulas are known as 'pseudo-random numbers This is because they do not perfectly conform to the strictly true lottery process of radom numbers, and the computer will generate the identical sequence of digits each time it is run with the same initial values (e.g 1212 in the above example), constants and parameters of the formulas. Nevertheles the pseudo random numbers are required to pass several statistical tests before the underlying mathematical formula is accepted. The following are the statistical test of randomness which they ought to pass.

1 *Frequency test* Each digit (0 and 1 to 9) should appear in a sequence of random numbers with an expected frequency of $=\frac{N}{10}$, where N is the total number of decimal digits in these random numbers. Chi-square test may be performed to test the observed frequencies against the expected frequencies.

2. *Serial Relationship* In a true random sequence of N decimal digits the expected frequency of occurrence of every possible ordered pair of digits will be $\frac{N}{100}$. Chi-square test may similarly be performed here. Likewise, ordered triplets, quadruplets etc. can be examined.

3. *Gap test* Between two occurrences of the same digit in a sequence of digit theoretical frequency distribution of the number of intervening digits is geometric. Chi-square test may be performed here again to test the observed frequencies against the expected frequencies

Besides the above randomness tests the pseudo-random number should also :

(i) be reproducible.

(ii) be non-repeating for any desired length.

(iii) be capable of being generated at a high speed by the electronic computer.

(iv) require minimum amount of computer memory

Conversion of Random Numbers. The random numbers generated by the aforesaid formulas conform to the uniform distribution, i.e., each number has equal probability of occurrence. But, in simulation experiments we would generally want random numbers conforming to other theoretical distributions (viz, normal, gama etc) or empirical distributions. The problem of conversion of uniformly distributed random numbers to these distributions, then, arises. The student is also referred to the ogive conversion method explained in F.M. Study V,

For the empirical distributions this can be accomplished as illustrated by the following example.

Class	1	2	3	4	5	6	7	8	9	10
p_i	0 273	0 037	0.195	0 009	0 124	0.058	0 062	0.151	0.047	0 044

In each of the memory locations 1—1000 of the computer, store 273 locations with class 1, next 37 locations with class and so on. Then generate a pseudo-random number (viz. by the mid-square method) Extract, say last three digits and select appropriate class. If the extracted number is, for example, 274 2nd class would be selected. Selection of classes hundreds and thousands of times in this way would ultimately conform to the given empirical distribution. i e. each class would be selected with the given probability

For theoretical distributions, there exist mathematical formula for conversion. For the normal distribution, for example, the following procedure may be adopted :

(i) Generate 12 random numbers. Sum them up and subtract 6 from the sum. Designate the result by N.

(ii) Using the following formula, calculate x, random no. for normal distributions $x=\mu+\sigma N$, where μ and σ are respectively the mean and the standard deviation of the given distribution

Reverting to the flowchart now we have distributions for the 9 input (or exogenous variables in the simulation terminology). Whe need a random number for each This is obtained by the random number generation and conversion process discussed above. A value of NPV is computed Again another set of 9 random numbers is computed Another value of NPV is computed

Different levels of these factors are considered automatically. For example, in the first *set* very high operating cost with a low market share,etc may be used for computing net present value In the next *set*, it may be a moderate operating cost with a very large market size Ultimately when a very large number of *sets* have been made, say, 3000, the random values that were utilised for each input would conform to their distributions. The output is the NPV histogram or distribution.

This furnishes complete information (though not perfect) about the chances of NPV being between 0 to 1%, 1 to 3% etc In Hertz's original model for a company the internal rate was found to 13% as against the 25% of conventional discounted cash flow models.

Imperial Chemical Industries, Calcutta, are practising basically Hertz model investment decisions.

The Theory of Games (Competitive Processes)

In fact, a part of this theory has already been discussed in study II though under a different caption "Statistical Decision Theory". All the problems therein are games against the nature. In this chapter, we shall study games in which two

players or two firms and, in general, any two competitors are playing in an aggressive way.

Let us introduce an example straightway to explain the concepts of this theory. The example is on the "Two Finger Murra" game in which 2 players, A and B simultaneously show 2 or 1 fingers If the number of fingers shown by the two players is equal A wins and if unequal B wins The following matrix sets out the payoffs of A and with signs reversed they become payoffs of B

		Strategies of B	
		B_1	B_2
Strategies of A	A_1	1	—1
	A_2	—1	1

Thus each player has two strategies open to him which can be expressed in the equation form as below

$$S_1=(A_1, A_2)$$
$$S_2=(B_1, B_2)$$

S_1 and S_2 represent the sets of strategies.

[More generally,

$$S_1=(A_1, A_2 . \quad . \quad .A_n)$$
$$S_2=(B_1, B_2 \quad . \quad B_m)$$

Coming back to the two finger Murra payoff matrix above, if A adopts A_1 and B adopts B_1, A wins 1 and B loses 1 i e , the gains of one player are the loses of the other Such games are known as zero sum games i e , nothing is paid in the kitty.

The list of the characteristics of the games to be considered in the following pages is hereunder.

1 Each player has a finite set of strategies available to him.

2 Each player knows not only the strategies available to him but also those available to his opponent

3. A's gains are B's losses and *vice versa* If one player is interested in maximising his gains and/or minimising his losses the other player is interested in maximising his gains and/or minimising his losses

4. Each player acts rationally i e he knows the preference order of his moves and those of his opponent and is capable of selecting the best move with this knowledge

5. The moves by both the players are made concurrently *e g* , A and B would show one/two fingers simultaneously

It should seem that with the aforesaid restrictions we have greatly limited the scope of applicability of this theory to practical problems and this indeed is so.

Nevertheless, it can sharpen the decision-maker's intuition.

Let us take up an example for solution for further clarification of the concepts and terminology.

Example 1. Solve the following game.

		B	
		B_1	B_2
A	A_1	10	-3
	A_2	9	2

A circle is put around the *minimum* element of each *row*.

$$\begin{array}{c|cc} & B_1 & B_2 \\ \hline A_1 & 10 & -3 \\ A_2 & 9 & 2 \end{array}$$

-3 is the minimum payoff of A's strategy A_1 and 2 is the minimum payoff of A's strategy A_2 Thus the maximum strategy for A is A_2

Now squares are put around the *maximums* of the *columns* below

$$\begin{array}{c|cc} & B_1 & B_2 \\ \hline A_1 & \boxed{10} & -3 \\ A_2 & 9 & \boxed{2} \end{array}$$

The squared elements represent the maximum losses to B for adopting B_1 and B_2 Since he would want to minimise his maximum losses B_2 is *the* strategy for him

We note that the element at the intersection of A_2 and B_2 has both a circle and square around it and it is known as the *saddle point* or the *equilibrium point*

2 is the value of the game because it represents the best payoff for both the player and both of them would want to play the strategies against it *i e* A_2 for A and B_2 for B By not playing A_2, A will unnecessarily suffer a loss Likewise, by not playing B_2, B will suffer an unnecessary loss Games with a saddle point are known as *strictly determined games.*

The solution to a game consists of finding the appropriate strategies for the two players and the value of the game Not all games, however, have a saddle point as to be seen in the following example.

Example 2. Solve the following game :

		B	
		B_1	B_2
A	A_1	3	5
	A_2	4	1

Solution. By putting circles and squares in the manner of example 1 we find that this game does not have a saddle point.

	B_1	B_2
A_1	3	$\boxed{5}$
A_2	$\boxed{4}$	1

Thus none of the combinations. $A_1 B_1$, $A_1 B_2$, $A_2 B_1$ and A_2 B_2 is best for *both* the players. If A goes on playing A_1 with a view to maximise his minimum gains (3 out of 3 and 1) B would go on playing B_1. Seeing this A_1 would be tempted to play A_2 since it fetches him 4 (>3) and seeing that A is now playidg A_2, B would take to B_2 which inflicts on B a loss of just 1. Thus it can be seen that each player will be going round and round trying to lure the other player in wrong decision To avoid being lured, therefore, each player should apply a random process to select his strategy on each particular play The player, first, however, will need to know how to alternate between the choices in order to achieve his best overall payoff. In other words, a relative frequency for playing each alternative must be determined Playing both the strategies with certain probabilities (to be determined) makes it what is known as a *mixed game*

A : Let p stand for the probability with which A selects A_1 so that $(1-p)$ is the probability with which he selects A_2.

Now if B selects B_1, A's expected gains are :

$$3p+4(1-p) \qquad \text{(i)}$$

If B selects B_2, A's expected gains are :

$$5p+(1-p) \qquad \text{(ii)}$$

Equating (i) and (ii)

$$3p+4(1-p)=5p+1(1-p)$$

so that $p=3/5$ and $1-p=2/5$

Thus A should play A_1 3/5 times and A_2 2/5 times of the total plays. A's expected payoff$=3p+4(1-p)=3\times3/5+4\times2/5=17/5$.

Likewise, if B selects B_1 with a probability of p selection of B_2 has probability of $(1-p)$. Now if A selects A_1, B's payoff is $3p+5(1-p)$. If A selects A_2, B's payoff is $4p+1(1-P)$. Thus

$$3p+5(1-p)=4p+1(1-p)$$

$$p=4/5 \text{ and } 1-p=1/5$$

B' expected payoff$=3p+5(1-p)=3\times4/5+5\times1/5=17/5$.

Thus A should play A_1 and A_2 with probabilities 3/5, 2/5 and B should play B_1 and B_2 with probabilities of 4/5, 1/5. Value of the game$=17/5$.

Dominant Strategies

A game which has no pure strategy, is a game in which one player has more than two choices while the other player is limited to two choices only, is denoted by $2 \times M$ or $M \times 2$ game. For such a game, we have to eliminate certain strategies for reducing it to 2×2 game. One of the approaches for doing so is *Dominance Method* which states :

If all the elements in a column are greater than or equal to the corresponding elements in another column then that column is *dominated* Similarly, if all the elements in a row are less than or equal to the corresponding elements in another row, then that row is *dominated*. Dominated rows or columns or both may be deleted, resulting into the reduced size of the game. Dominance can be illustrated by the following example :—

$$\begin{array}{cc} & Y \\ X & \begin{pmatrix} 2 & 6 \\ -1 & -2 \\ 3 & 1 \end{pmatrix} \end{array}$$

Competitor X will not play row 2 since this will give Y his only chance to win It is evident that row 2 is dominated by row 1 or row 3 since these rows will always return to X a better payoff than the dominated strategy, regardless of Y's actian Hence row 2 can be eliminated and the resultant matrix is

$$\begin{array}{cc} & X \\ X & \begin{pmatrix} 2 & 6 \\ 3 & 1 \end{pmatrix} \end{array}$$

Example 3 Solve the following game :

		B		
		B_1	B_2	B^3
	A_1	4	5	2
A	A_2	10	−5	4
	A_3	6	6	3

Solution : We see that A_1 is dominated by A_3 because

$6>4,\ 6>5,\ 3>2$ *i e* gains with A_3 are more.

Thus A would never pay A_1 which is eliminated reducing the given games as below. .

		B		
		B_1	B_2	B_3
A	A_2	10	−5	4
	A_3	6	6	3

Now we see that B_1 is dominated by B_3 because

$4<10$ *i e* , losses with

$3<\ 6$ $\quad B_3$ are less than those with B_1

so that B_1 can also be eliminated reducing the game as below.

		B	
		B_2	B_3
A	A_2	−5	4
	A_3	6	3

This is now manageable since it is on the lines of the first two **examples.** Below, we realise that it does not have a saddle point *i e* it is mixed game.

	B_2	B_3
A_2	-5	$\boxed{4}$
A_3	$\boxed{6}$	3

Let p be the probability for A to play A_2, so that

$$-5p+6\,(1-p)=4p+3\,(1-p)$$

$\Rightarrow$ $p=3/12$ and $(1-p)=9/12$

Value of the game$=\quad -5\times 3/12+6\times 9/12=39/12$

Now let p be the probability for B to play B_2, so that

$$-5p+4\,(1-p)=6p+3\,(1-p)$$

$\Rightarrow$ $1/12=p$ and $1-p=11/12$

Value of the game$=-5\times 1/12+4\times 11/12=39/12$

The dominance criteria, however, can be extended, as to be seen in example 4

Example 4 Solve the following game:

	B_1	B_2	B_3
A_1	3	1	4
A_2	1	4	2
A_3	2	2	6

Solution : B_1 dominates B_3 Eliminating B_3, the game reduces to

	B_1	B_2
A_1	3	1
A_2	1	4
A_3	2	2

Now no row dominates ; but we see that the average of A_1 and A_2 dominates A_3 as set out below :

	B_1	B_2
$\dfrac{A_1+A_2}{2}$	2	$2\frac{1}{2}$
A_3	2	2

Thus we can delete A,

A		B_1	B_2
	A_1	3	1
	A_2	1	4

The student can now solve it as an exercise In general if any convex combination of two strategies (row or columns) dominates another strategy the latter can be eliminated.

Graphical Solution Procedure

Example 5. Solve the following game :

		B	
		B_1	B_2
	A_1	6	3
A	A_2'	4	8

Solution : This game does not have a saddle point. Proceeding with the previous method for a while to explain the basis of the graphical solution procedure let p be the probability of A playing A_1 so that his expected gain, if B selects B_1 is E_1.

$$E_1 = 6p + 4\ (1-p)$$

Also his expected gain E_2 if B selects B_2,

$$E_2 = 3p + 8\ (1-p)$$

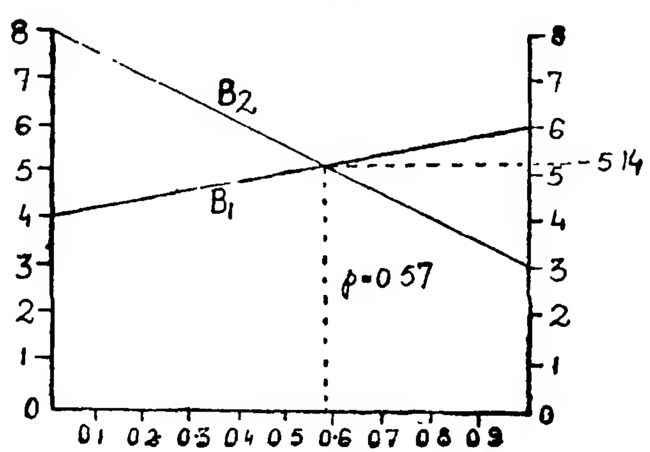
These two straight lines are plotted in the graph below, the points have been obtained as below

$E_1 = 6p + 4\ (1-p)$
If $p=1$ $E_1=6$
If $p=0$ $E_1=4$
↑
Note this is the Ist col of the given matrix

$E_2 = 3p + 8\ (1-p)$
If $p=1$ $E_2=3$
If $p=0$ $E_2=8$
↑
Note this is the 2nd Col. of the given matrix (Therefore, we can straightway proceed with plotting of the lines from the given matrix)

From the graph, $p = 0\ 57\ (=4/7)$ $\quad$ $1-p=0.43$

Value of the game$=5\ 14$.



This was too simple an example to graphically. The graphical method is particularly suited to solving $2\times n$ or $m\times 2$ games *i e.* one player has 2 strategies and the other more than 2. Also, though it is possible to solve such games (known as rectangular games) entirely graphically yet, in view of approximation introduced into the solution by the graph readings, we merely use the graphical method to reduce it into a 2×2 game *i.e.* we can once spot out the dominated strategies which can be eliminated. Then we solve the 2×2 game by the method which we used for the first four examples.

Example 6. Solve the following game graphically :

			B	
		B_1	B_2	B_3
A	A_1	1	3	11
	A_2	8	5	2

Solution : The three lines are plotted below

A should play A_1 with a prob of 0 28

and A_2 with a prob. 0 72

Value of the game is read as 4 44

It can be seen that the optimal solution has been obtained at the intersection of lines corresponding to B_2 and B_3 columns. Thus we have reduced the game as below.

		B	
		B_2	B_3
A	A_1	3	11
	A_2	5	2

Now let us find the probabilities for B. This can also be done graphically, however, usually, the graphical method is applied just to reduce the game as above.

Let p be the probability for B to play B_2. His payoff E_2 if A chooses A_1 is given by

$$E_2=3p+11\ (1-p).$$

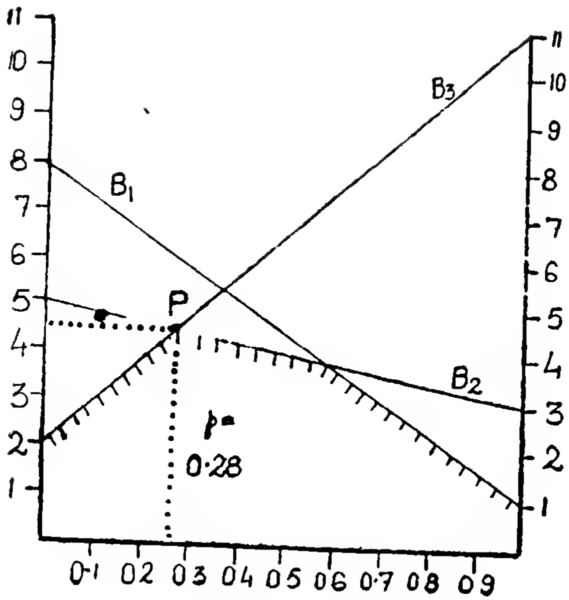
B's payoff, E_3 upon A selecting A_2 is given by

$$E_3=5p+2\ (1-p)$$

Equating E_2 and E_3

$$3p+11\ (1-p)=5p+2\ (1-p)$$

$$\Rightarrow\ p=9/11$$



Example 7. Solve the following game graphically :

		B			
		B_1	B_2	B_3	B_4
A	A_1	4	2	0	-4
	A_2	2	0	6	4

Solution : From the graph below, for A, p (Prob. of playing A_1)$=0.4$; therefore prob. of playing $A_2=0.6$. Value of the game may be read as 0.8. It can also be seen that B_1 and B_3 are dominated.

The reduced game is

	B_2	B_4
A_1	2	-4
A_2	0	4

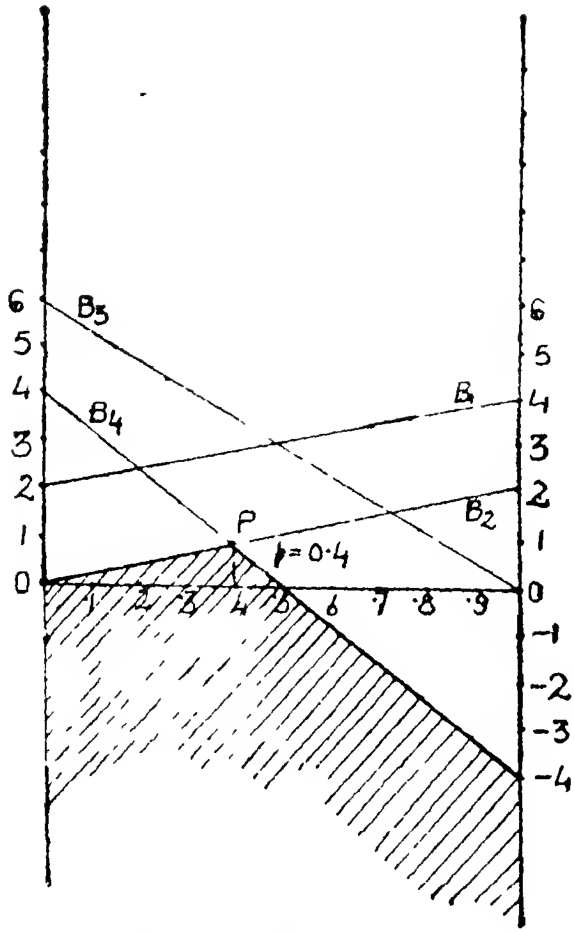
Let now p=prob. of B playing B_2 :

$$\therefore \quad 2p-4(1-p)=0p+4(1-p)$$

$$\Rightarrow \quad 10p=8 \qquad \therefore \quad p=8/10$$

Thus, prob. of B playing $B_2=8/10$

$\therefore \quad B_4=2/10$



Shaded region is the feasible region known as the lower envelope Lines not interfering this envelope represent dominated strategies

Example 8 Solve the following game graphically :

		B	
		B_1	B_2
	A_1	-2	5
	A_2	-4	3
	A_3	0	-2
A	A_4	-3	1
	A_5	1	-4

Solution : The game does not have a saddle point. In this case we have more than 2 rows ; therefore, we plot lines corresponding to rows in the graph below

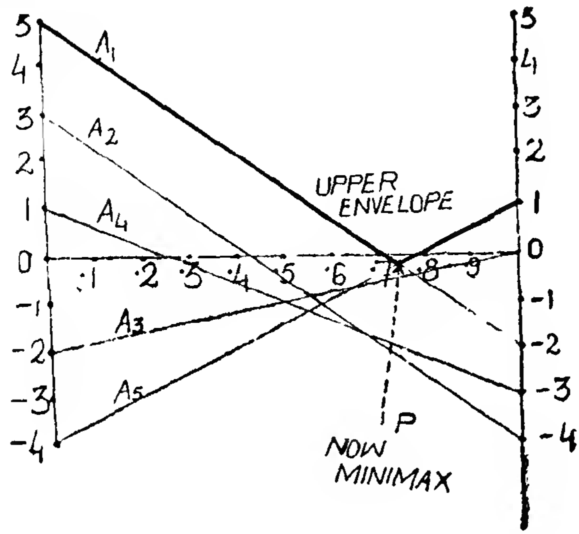
Also since we are plotting rows *i e.* making the graph from B's view we consider the upper polygon (or envelope) rather than the lower one. The two

dominating strategies A_1 and A_5 have their lines intersecting at point P (giving minimax for B) and these two lines constitute the upper envolope. Thus the game has been reduced as below.

		B	
		B_1	B_2
A	A_1	-2	5
	A_2	1	-4

The student may solve it as an exercise.

(**Answer.** A : 5/12, 7/12 ; B : 9/12, 3/12 : V=−1/4).



Games as LPP's

We saw the feasible region, etc. in the graphical solution to some games which should suggest the possibility of formulating a game as an LPP. This is possible and simplex method can be employed to obtain the solution. Considering, however, a great deal of computations involved the simplex method should be the last resort as set out below.

1. If there exists a saddle point to any $m \times n$ game solution can be read straightway, otherwise, step 2 below.
2. Apply dominance and the extended dominance criteria to reduce the game. If it is reduced to a 2×2 game solve it by the ordinary method, but if it is $2 \times n$ or $m \times 2$ game use the graphical method.
3. If it still remains an $m \times n$ game use the simplex method

Example 9. Solve the following game by formulating it as an LPP and then use the simplex method.

		B	
	B_1	B_2	B_3
A_1	1	2	3
A_2	3	2	1
A_3	4	2	0

Solution. B's Point of View

Let x_1, x_2 and x_3 be the probabilities with which B plays B_1, B_2, B_3.

E_1=Expected gain of B when he plays the three strategies with the aforesaid probabilities and A chooses A_1

Likewise, E_2 and E_3 represent B's expected gains when A chooses A_2 and A_3 respectively

Now, $E_1 = 1x_1 + 2x_2 + 3x_3$

$E_2 = 3x_1 + 2x_2 + 1x_3$

$E_3 = 4x_1 + + 2x_2 + 0x_3$

Now B's minimax payoff can be read as 2. Let us call it u. He can ensure that (*i e.*, he does not lose more than 2) by sticking to B_2; but he would endeavour to further reduce u by following a mixed strategy as discussed above.

Thus
$$\left.\begin{aligned} 1x_1+2x_2+3x_3 &\leqslant u \\ 3x_1+2x_2+1x_3 &\leqslant u \\ 4x_1+2x_2+0x_3 &\leqslant u \end{aligned}\right\} \qquad \text{I}$$
$$x_1,\ x_2,\ x_3 \geqslant 0$$

Define $X_1 = x_1/u$ — Since $x_1+x_2+x_3$

$X_2 = x_2/u$ — the sum of probabilities $=1$.

$X_3 = x_3/u$ — $X_1+X_2+X_3=1/u$.

(Assume u to be +ve More on this later).

The inequalities I can then be expressed as,

$$\left.\begin{aligned} 1X_1+2X_2+3X_3 &\leqslant 1 \\ 3X_1+2X_2+1X_3 &\leqslant 1 \\ 4X_1+2X_2+0X_3 &\leqslant 1 \end{aligned}\right\} \qquad \text{II}$$
$$X_1,\ X_2,\ X_3 \geqslant 0$$

B would want to minimise u or alternatively maximise $1/u = X_1+X_2+X_3$ which is the objective function for B and II gives all the constraints. For clarity the LPP is restated below.

Maximise $\quad X_1+X_2+X_3$

subject to
$$\begin{aligned} 1X_1+2X_2+3X_3 &\leqq 1 \\ 3X_1+2X_2+1X_3 &\leqslant 1 \\ 4X_1+2X_2+0X_3 &\leqslant 1 \\ X_1,\ X_2 &> 0. \end{aligned}$$

This is incidentally the symbolic model of the given game.

Introducing slack variables, the problem, became

Maximise $X_1+X_2+X_3+0S_1+0S_2+0S_3$

Subject to $1X_1+2X_2+3X_3+S_1=1$

$3X_1+2X_2+1X_3+S_2=1$

$4X_1+2X_2+0X_3+S_3=1$

$X_1, X_2, X_3, S_1, S_2, S_3, \geq 0.$

Solution :

Fixed Ratio	Programme	Profit	Qty.	1 X_1	1 X_2	1 X_3	0 S_1	0 S_2	0 S_3	Replacement Ratio
	S_1	0	1	1	2	3	1	0	0	1/3←
1/3	S_2	0	1	3	2	1	0	1	0	1
0	S_3	0	1	4	2	0	0	0	1	∞
1/3	NER			1	1	1↑	0	0	0	
1/12	X_3	1	1/3	1/3	2/3	1	1/3	0	0	1
2/3	S_2	0	2/3	8/3	4/3	0	−1/3	1	0	1/4
	S_3	0	1	4	2	0	0	0	1	1/4←
1/6	NER			4/3↑	1/3	0	−1/3	0	0	
	X_3	1	1/4	0	1/2	1	1/3	0	−1/12	
	S_2	0	0	0	0	0	−1/3	1	−2/3	
	X_1	1	1/4	1	1/2	0	0	0	1/4	
	NER			0	0	0	−1/3	0	−1/6	

Thus $X_3=1/4$

$X_1=1/4$

∴ $x_3=1/2$

$x_1=1/2.$

$$\text{Value of the game}=u=\frac{1}{X_1+X_2+X_3}$$

$$=\frac{1}{1/4+1/4}=2.$$

In fact the complete solution to the game is obtainable from the optimal table above ; but let us also view the problem from A's point.

A's point of view

Let y_1, y_2, y_3 be the probabilities with which A plays A_1, A_2, and A_3

E_1' = Expected gain of A when he plays the three strategies with the aforesaid probabilities and B choose B_1

Likewise, E'_2 and E'_3 represent A's expected gains when B chooses B_2, and B_3 respectively.

$$\text{Now}\quad \begin{aligned} E_1' &= 1y_1+3y_2+4y_3 \\ E_2' &= 2y_1+2y_2+2y_3 \\ E_3' &= 3y_1+1y_2+0y_3 \end{aligned}$$

Now A's row maximin payoff can be read as 1; call it u'. He can ensure it by sticking to either A_1 or A_2 But he endeavours to make more by following a mixed strategy

$$\text{Thus}\quad \left.\begin{aligned} 1y_1+3y_2+4y_3 &\geqslant u' \\ 2y_1+2y_2+2y_3 &\geqslant u' \\ 3y_1+1y_2+0y_3 &\geqslant u' \end{aligned}\right\}\qquad \text{III.}$$

$$\text{Define}\quad Y_1=y_1/u' \qquad\qquad \text{Since } y_1+y_2+y_3=1$$
$$Y_2=y_2/u' \qquad\qquad Y_1+Y_2+Y_3=1/u'$$
$$Y_3=y_3/u'$$

(Assume u' to be $+ve$ More on this later).

Now A would wish to maximise u' or

$$\left.\begin{aligned} &\text{minimise } 1/u'=Y_1+Y_2+Y_3 \\ &\text{subject to } Y_1+3Y_2+4Y_3\geqslant 1 \\ &\qquad 2Y_1+2Y_2+2Y_3\geqslant 1 \\ &\qquad 3Y_1+1Y_2+0Y_3\geqslant 1 \\ &\qquad Y_1,\ Y_2, Y_3\geqslant 0 \end{aligned}\right\}$$

LPP from A's point of view. This is the dual of B's LPP. Incidentally, this is also the symbolic model of the game

Since A's LPP is the dual of B's LPP we can real A's solution from B's optimal table wherein the last NER, we read below S_1 and S_3, $-1/3$ and $-1/6$ respectively which would be made $-2/3$ and $-2/6$ to correspond to x_1 and x_3,

Thus y_1 and y_3 are 2/3 and 2/6 = 1/3 respectively

Thus B should play B_1 and B_3 with probabilities of 2/3 and 1/3 respectively. The student may want to verify this by solving A's LPP directly.

Sign of the Game

We assumed u and u' to be non-negative

They are, in fact, so for this game because

A's maximin is 1,

B's minimax is 2

The value of the game must lie between 1 and 2, i e, the maximised u or minimised u' must lie between 1 and 2. Thus u and u' cannot be negative. So when A's maximin $\ntriangleleft$ B's minimax u and u' are non-negative

But in another game A's maximin may be, say, -1 and B' minimax may be say, $+6$ In this case u or u' is likely to go negative. Therefore, it would be necessary to add 1 to all the elements of the given matrix. This would require that the value of the game obtained by the simplex method is adjusted i e. 1 is subtracted from the computed value

Alternatively, the student may want to add the absolute value of most $-ve$ figure to all the elements and later adjust the game value for this.

Example 10. Formulate the general LPP for the following game

		B		
	strategies	1	2	3
	1	3	−4	2
A	2	1	−3	−7
	3	−2	4	7

Solution: By adding 7 to each element we make all elements $+ve$ below

		B		
		1	2	3
A	1	10	3	9
	2	8	4	0
	3	5	11	14

[Upon solving the mathematical model (LPP) formulated below the value of the game obtained from the solution by the simplex method would have to be adjusted by subtracting 7 from it]

Let x_1, x_2, and x_3 be the probabilities with which B plays B_1, B_2 and B_3 respectively.

E_1=expected gain of B when he plays the three strategies with the aforesaid probabilities and A choses A_1. Likewise, E_2 and E_3 represent B's except gains when A chooses A_2 and A_3 respectively.

Now $$E_1=10x_1+3x_2+9x_3$$
$$E_2=8x_1+4x_2+0x_3$$
$$E_3=5x_1+11x_2+14x_3$$

Now B's minimax payoff can be read as 10.

Let us call it u. He can ensure that (*i e.* does not lose more than 10 in the modified game) by striking to B_1; but he would endeavour to further reduce u by following a mixed strategy.

Thus
$$\left.\begin{aligned} 10x_1+3x_2+9x_3 &\leqslant u \\ 8x_1+4x_2+0x_3 &\leqslant u \\ 5x_1+11x_2+14x_3 &\leqslant u \end{aligned}\right\} \text{I}$$

$$x_1, x_2, x_3 \geqslant 0$$

Define $\quad X_1 = \frac{x_1}{u}, X_2 = \frac{x_2}{u}, X_3 = \frac{x_3}{u}$

Since $\quad x_1+x_2+x_3$, the sum of probabilities equal 1

$$X_1+X_2+X_3 = \frac{1}{u}.$$

The inequalities I can be expressed as,

$$\begin{aligned} 10X_1+3X_2+9X_3 &\leqslant 1 \\ 8X_1+4X_2+0X_3 &\leqslant 1 \\ 5X_1+11X_2+14X_3 &\leqslant 1 \\ X_1, X_2, X_3 &\geqslant 0 \end{aligned}$$

The objective function B would want to minimise u. Conversely, he would want to minimise $\frac{1}{u} = X_1+X_2+X_3$

Thus the objective function to be maximised is $X_1+X_2+X_3$

Exercises : Solve the following games

(i) $\begin{pmatrix} 6 & 8 & 6 \\ 4 & 12 & 2 \end{pmatrix}$ $\qquad$ (ii) $\begin{pmatrix} 2 & 7 \\ 3 & 5 \\ 11 & 2 \end{pmatrix}$

Answer A_1, B_3

$V=6$

Answer $\frac{9}{14}, 0, \frac{5}{14}$

$\frac{5}{14}, \frac{9}{14}$

$V=\frac{73}{14}$

2. Consider the following payoff matrix for two firms. What is the best mixed strategy for both the firms and also find out the value of the game.

	Firm II		
	No Advertising	Medium Advertising	Large Advertising
No advertising	60	50	40
Medium Advertising	70	70	50
Large Advertising	80	60	70

3 Solve the following games by linear programming

(i) $\begin{pmatrix} 2 & 4 \\ 2 & 3 \\ 3 & 2 \end{pmatrix}$

(ii) $\begin{pmatrix} 3 & -1 & -3 \\ -3 & 3 & -1 \\ -4 & -3 & 3 \end{pmatrix}$

Answer : $\frac{14}{15}, \frac{11}{45}, \frac{20}{45}$

$\frac{20}{45}, \frac{11}{45}, \frac{14}{45}$

$V=\frac{196}{45}$

(iii) $\begin{pmatrix} -1 & 1 & 1 \\ 2 & -2 & 2 \\ 3 & 3 & -3 \end{pmatrix}$

Answer. $\frac{6}{11}, \frac{3}{11}, \frac{2}{11}$

$\frac{5}{22}, \frac{8}{22}, \frac{9}{22}$

$V=\frac{6}{11}$

4 Use the relation of dominance to solve the rectangular game whose pay-off matrix to A is as follows

		I	II	III	IV	V	VI
	I	0	0	0	0	0	0
	II	4	2	0	2	1	1
	III	4	3	1	3	2	2
	IV	4	3	7	−5	1	2
A	V	4	3	4	−1	2	2
	VI	4	3	3	−2	2	2

Answer $\begin{array}{c|cc} & \text{III} & \text{IV} \\ \hline \text{III} & 1 & 3 \\ \text{IV} & 7 & -5 \end{array}$

5. In a game of matching coins two players, suppose A wins one unit of value. when there are two heads, wins nothing when there are two tails and loses 1/2 unit of value when there are one head and one tail. Determine the payoff matrix, the best strategies for each player and the value of game to A.

RANDOM NUMBERS

8481	5016	0080	4376	2570	8293	5950	1048	0650	4135
0744	3447	6173	3288	6378	6704	0966	9986	5202	1728
5558	7239	2976	4836	6134	5120	1541	6514	3581	2079
9371	1463	2164	2301	3142	3866	8707	9980	2011	5111
3033	1660	6365	9054	1155	8844	4085	9589	2924	1725
1053	7320	6532	7214	8972	6466	1217	0100	1458	9416
4389	3804	4086	9434	0136	5595	6876	7937	5476	3196
2158	8854	9534	1196	4941	2697	7497	1149	1952	3482
6749	3476	4943	1406	8614	2060	6433	1660	8875	3194
2878	3447	4804	6761	5309	0636	0522	2004	3207	4684
0591	6549	2206	6185	6188	2649	2389	9483	0924	1389
1025	3438	0546	2545	1089	1260	6701	9742	3453	5573
4244	9217	1628	4524	0163	9895	9586	2083	8459	0644
1331	9032	1388	5661	0472	7128	1902	4343	7724	6528
8853	3490	2589	8744	1221	4667	8396	4779	9937	7206
5059	4192	6331	5465	5922	0982	9390	8993	3621	2602
0821	4340	3194	0118	4773	8668	1591	7989	9190	2296
5262	1746	7108	6496	2570	4243	5029	6949	4989	5008
1210	1858	9365	6562	0269	9923	1796	6626	8591	1990
3642	6629	5775	3219	8801	4047	6861	0765	2379	3494
9598	5322	3747	0363	5995	5504	6804	7033	0957	9516
3894	3173	2853	9312	2498	8878	4956	8748	6247	0673
3603	3011	6762	0848	8316	3485	6388	8925	3799	6898
1141	2978	6313	5857	8457	1395	7240	8630	3895	6348
1930	4583	4227	4120	6893	7005	2264	6067	5621	7985
6309	9158	2830	3262	9809	4606	8669	1154	5841	7608
4460	3143	5383	0327	9668	1697	8335	0869	2188	1908
8371	5095	7271	1866	4193	4163	2035	2832	4996	7143
9397	5549	9298	9076	1299	6669	0088	2809	9631	3161
9304	1468	4013	7465	0861	6787	3581	7977	8409	4798
5606	2435	8546	3209	4802	6690	8527	2219	6706	1930
6693	8333	082	7546	2910	8553	8725	1237	4423	1570
0556	7715	8994	4245	1540	8159	3889	5273	6977	2703
6973	9299	4959	7146	1426	7086	8743	6982	5547	3394
4920	1223	5208	6661	4907	1102	0501	3625	8513	3192
0132	0928	8241	0858	7627	4174	1170	3142	2455	4891
4051	3101	9854	4488	6931	3266	3147	2560	8011	8848
0267	5612	5504	7917	7928	8034	9989	4353	2075	9497
0609	9469	3149	4086	8911	8547	3518	9349	1836	0548
2593	1666	5750	5105	4287	4380	7860	7792	1625	7659
8812	9491	2602	4100	4962	1037	9778	1778	4223	3193
3540	5985	0019	7155	1471	1851	8681	9957	3772	4706
9535	5375	1239	1624	55378	6803	7177	7911	4660	5669
3174	7677	8282	6669	879	7874	9931	6581	9784	2697
8864	4760	1129	6205	4949	4205	0222	7479	6470	8194
5245	7341	0593	5656	6799	3071	1751	4339	9630	9496
5468	6083	4511	1440	2135	5777	9903	1048	6726	8602
3951	7928	6818	4161	4840	1392	1323	5014	7538	9854
7319	4064	4024	5401	2834	75 8	3978	3742	1005	4619
5892	8731	6269	5189	2071	4084	9789	3620	9819	4548

Example on the use of this table Suppose we want five 3 digit random nos. We can enter any where in the table e g the top of the last column, first 3 digits of 5 consecutive nos., give us the answers 413, 172, 207, 511, 172. Thus we enter the table randomly but then on proceed serially

APPENDIX I

UNIT NORMAL LOSS INTEGRAL (.0^2 means 00 , etc.)

Z	00	01	.02	03	04	.05	06	.07	.08	09
0	3989	.3940	.3890	.3841	.3793	3744	3697	.3649	.3602	.3556
.1	3509	3464	.3418	3373	3328	3284	3240	3197	.3154	3111
.2	3069	3027	2986	2944	2904	2863	2824	2784	.2745	.2706
3	2668	2630	2592	2555	2518	2481	.2445	.2409	.2374	2339
.4	2304	2270	2236	2203	2169	2137	2104	.2072	2040	2009
.5	1978	1947	1917	1887	1857	1828	1799	1771	.1742	1714
.6	687	1659	.1633	1606	.1580	1554	1528	1503	.1478	.1453
.7	1429	1405	1381	1358	.1334	1312	1289	1267	1245	1223
8	.1202	1181	1160	1140	1120	1100	1080	1061	1042	1023
.9	1004	09860	09680	09503	09328	09156	08986	08819	08654	08491
1 0	08332	08171	.08019	07866	07716	07568	07422	07279	07138	06999
1.1	06862	06727	06595	06465	06336	06210	06086	05964	05844	05726
1 2	05610	05496	05384	05274	05165	05059	04954	04851	04750	04650
1 3	04553	04457	04363	04270	04179	04090	04002	03916	03831	03748
1.4	03667	03587	03508	03431	03356	.03281	03208	03137	03067	.02998
1.5	02931	02865	02800	.02736	02674	.02612	02552	02494	02436	02380
1 6	02324	02270	02217	02165	02114	02064	02415	01967	.01920	01874
1.7	01829	01785	01742	01699	.01658	01617	01578	01539	01501	.01464
1.8	01428	01392	01357	01323	01290	01257	01226	01195	01164	01134
1 9	01105	01077	01049	01022	09957	09698	$0^{2}9445$	$0^{2}9198$	$0^{2}8957$	$.0^{2}821$
2.0	$0^{2}8491$	$.0^{2}8266$	$0^{2}8046$	$0^{2}7832$	$0^{2}7623$	$0^{2}7418$	$0^{2}7219$	$0^{2}7024$	$0^{2}6835$	$.0^{2}6649$
2 1	$0^{2}6468$	$.0^{2}6292$	$0^{2}6120$	$0^{2}5952$	$0^{2}5788$	$0^{2}5628$	$0^{2}5472$	$0^{2}5320$	$0^{2}5172$	$.0^{2}5028$
2 2	$0^{2}4887$	$0^{2}47$ 0	$.0^{2}4616$	$0^{2}4468$	$0^{2}4358$	$0^{2}4235$	$0^{2}4114$	$0^{2}3996$	$.0^{2}3882$	$.0^{2}3770$
2 3	$0^{2}3662$	$0^{2}3556$	$0^{2}3453$	$0^{2}3352$	$0^{2}3255$	$0^{2}3159$	$0^{2}3067$	$.0^{2}2977$	$0^{2}2889$	$.0^{2}2804$
2 4	$0^{2}2720$	$0^{2}2640$	$0^{2}2561$	$0^{2}2484$	$0^{2}2410$	$.0^{2}2337$	$0^{2}2267$	$0^{2}2199$	$.0^{2}2132$	$.0^{2}2067$
2 5	$0^{2}2004$	$.0^{2}1943$	$0^{2}1886$	$.0^{2}1826$	$0^{2}1769$	$.0^{2}1715$	$0^{2}1662$	$.0^{2}1610$	$.0^{2}1560$	$0^{2}1511$

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
2.6	$0^{2}1464$	$.0^{2}1418$	$.0^{2}1373$	$.0^{2}1330$	$.0^{2}1288$	$0^{2}1247$	$0^{2}1207$	$.0^{2}1169$	$.0^{2}1132$	$0^{2}1095$
2 7	$.0^{2}1060$	$.0^{2}1026$	$.0^{3}9928$	$.0^{3}9607$	$.0^{3}9295$	$.0^{3}8992$	$0^{3}8699$	$.0^{3}8414$	$.0^{3}8138$	$.0^{3}7870$
2.8	$.0^{3}7614$	$.0^{3}7359$	$0^{3}7115$	$.0^{3}6879$	$.0^{3}6650$	$.0^{3}6428$	$.0^{3}6413$	$.0^{3}6004$	$.0^{3}5802$	$.0^{3}5606$
2.9	$.0^{3}5317$	$.0^{3}5233$	$.0^{3}5055$	$.0^{3}4883$	$.0^{3}4716$	$.0^{3}4555$	$0^{3}4398$	$.0^{3}4247$	$.0^{3}4101$	$.0^{3}3959$
.3.0	$.0^{3}3822$	$.0^{3}36\iota9$	$.0^{3}3560$	$.0^{3}3436$	$.0^{3}4316$	$.0^{3}3199$	$0^{3}3087$	$.0^{3}2978$	$.0^{3}2873$	$.0^{3}2771$
3.1	$.0^{3}2673$	$.0^{3}2577$	$.0^{3}2485$	$.0^{3}2396$	$0^{3}2311$	$.0^{3}2227$	$0^{3}2147$	$.0^{3}2070$	$.0^{3}1995$	$.0^{3}1922$
3.2	$.0^{3}1852$	$.0^{3}1785$	$.0^{3}1720$	$.0^{3}1657$	$0^{3}1596$	$0^{3}1537$	$0^{3}1480$	$0^{3}1426$	$0^{3}1373$	$0^{3}1322$
3 3	$.0^{3}1273$	$0^{3}1225$	$0^{3}1179$	$0^{3}1135$	$.0^{3}1093$	$0^{3}1051$	$0^{3}1012$	$0^{4}9734$	$0^{4}9365$	$.0^{4}9009$
3.4	$.0^{4}8666$	$.0^{4}8335$	$.0^{4}8016$	$0^{4}7709$	$0^{4}7413$	$.0^{4}7127$	$0^{4}6852$	$.0^{4}6587$	$0^{4}6331$	$0^{4}6085$
3.5	$.0^{4}5848$	$.0^{4}5620$	$.0^{4}5400$	$.0^{4}5188$	$.0^{4}4984$	$.0^{4}4788$	$0^{4}4599$	$.0^{4}4417$	$0^{4}4242$	$0^{4}4073$
3 6	$.0^{4}3911$	$.0^{4}3755$	$0^{4}3605$	$.0^{4}3460$	$0^{4}3321$	$0^{4}3188$	$0^{4}3059$	$0^{4}2935$	$0^{4}2816$	$0^{4}2702$
3.7	$.0^{4}2592$	$.0^{4}2486$	$.0^{4}2385$	$0^{4}2287$	$0^{4}2193$	$0^{4}2103$	$0^{4}2016$	$.0^{4}1933$	$.0^{4}1853$	$0^{4}1776$
3.8	$.0^{4}1702$	$.0^{4}1632$	$.0^{4}1568$	$.0^{4}1498$	$.0^{4}1435$	$.0^{4}1375$	$0^{4}1317$	$.0^{4}1262$	$0^{4}1208$	$0^{4}1157$
3.9	$.0^{4}1108$	$.0^{4}1061$	$0^{4}1016$	$.0^{5}9729$	$.0^{5}9307$	$.0^{5}8908$	$0^{5}8525$	$.0^{5}8158$	$0^{5}7806$	$.0^{5}7469$
4.0	$.0^{5}7145$	$.0^{4}6835$	$0^{5}6538$	$.0^{5}6253$	$.0^{5}5980$	$0^{5}5718$	$0^{5}5468$	$0^{5}5227$	$0^{5}4997$	$0^{5}4777$
4.1	$.0^{5}4566$	$.0^{5}4364$	$.0^{5}4170$	$.0^{5}3985$	$.0^{5}3807$	$.0^{5}3637$	$0^{5}3475$	$.0^{5}3319$	$.0^{5}3170$	$0^{5}3027$
4.2	$.0^{5}2891$	$0^{5}2760$	$.0^{5}2635$	$0^{5}2516$	$.0^{5}2402$	$0^{5}2292$	$0^{5}2188$	$0^{6}3085$	$0^{5}1992$	$0^{5}1901$
4.3	$0^{6}1814$	$0.^{5}1730$	$.0^{5}1650$	$.0^{5}1574$	$.0^{5}1501$	$.0^{5}1431$	$.0^{5}1365$	$.0^{5}1301$	$0^{5}1241$	$.0^{5}1183$
4.4	$.0^{6}1127$	$0^{6}1074$	$.0^{6}1024$	$.0^{6}9756$	$.0^{6}9296$	$0^{6}8857$	$0^{6}8437$	$.0^{6}8037$	$0^{6}7655$	$.0^{6}7290$
4 5	$.0^{6}6942$	$.0^{6}6610$	$.0^{6}6294$	$.0^{6}5992$	$.0^{7}5704$	$.0^{6}5429$	$0^{6}5167$	$0^{6}4917$	$0^{6}4679$	$.0^{6}4452$
4.6	$.0^{6}4236$	$.0^{6}4029$	$.0^{6}8833$	$.0^{6}3645$	$.0^{6}3467$	$0^{6}3297$	$0^{6}3135$	$0^{6}2981$	$.0^{6}2834$	$.0^{7}2694$
4.7	$.0^{6}2560$	$.0^{6}2433$	$.0^{6}2313$	$.0^{6}2197$	$0^{6}2088$	$0^{6}1984$	$0^{6}1884$	$0^{6}1790$	$.0^{6}1700$	$.0^{6}4615$
4 8	$.0^{6}1533$	$.0^{6}1456$	$0^{6}1382$	$.0^{6}1312$	$.0^{6}1246$	$.0^{6}1182$	$0^{6}1122$	$.0^{6}1065$	$0^{6}1011$	$.0^{7}9588$
4.9	$.0^{7}9096$	$.0^{7}8629$	$0^{7}8185$	$.0^{7}7763$	$.0^{7}2362$	$.0^{7}6982$	$0^{7}6620$	$.0^{7}6276$	$0^{7}5950$	$.0^{7}5640$

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FINAL COURSE

Supplementary study material on the subjects of operations Research and S A &DP

This bookelt contains the study material for the additional topics included in th new syllabus of the Final Examination This should be read in conjuction with th following existing study materials—Operations Research 2, 6/re-numbered 5 in De '84 print Intermediate Statistics 6, and SA & DP 2, 5 In the new syllabus, certai portions of the existing materials have been deleted and therefore the students ai advised not to go through the materials dealing with the topics which have been delete from the syllabus For facility of reference and use, the relevant details concerning th additions and deletions to the syllabus are furnished below

Operations Research

Additions – Exponential smoothing ; Dynamic E.O.Q. models, Methodology c Simulation; Applications of Simulation to problems of Financial Plannin and Investment Decisions

Statistical quality control ; control charts for variables an attributes ; Acceptance Sampling for Attributes and variables.

Sampling & Posterior distributions : Normal and Binomial.

Deletions – Value analysis and cost/benefit analysis Techniques, Input-outpu Analysis as a means of measuring productivity, Concepts c productivity measurement of Productivity, Game Theory.

Work study and time and motion study.

* **System Analysis and Data Processing**

Additions – Applications of data processing devices : purchases and productio scheduling, Cost accounting Use of outside consultants and servic bureaus, Budgeting for systems development

*No material included in the existing studies has been deleted.

Supplementary Study Material on OR & SA.

Contents :

Statistical quality control

Control charts for variables and attributes

Acceptance sampling for Attributes and variables.

Posterior probability

Posterior analysis.

Exponential smoothing.

Applications of simulation (to be read in conjuction with study VI of OR & SA)

—Investment decisions

—Financial planning.

Students are advised not to go through Study papers consisting of following topics

Work study
Uses of Measurements
Value analysis
Productivity
Productivity & work study
Soul cost-benefit analysis

Statistical Quality Control (S.Q C.)

The mass production of an identical item consists of a production process that requires a set of inputs, *viz*, equipment, raw materials, men, etc. Variations in these factors occur, over time, and due to these it becomes impossible to produce items that are exactly identical in all respect *i e*, there is always some degree of variation in the dimensions of two identical objects, measured under precision To ensure the quality of the product, tolerance limits are set If the product falls within the tolerance limits it is considered to be of acceptable quality : otherwise the quality of the product is questioned Though it is possible to ensure the quality of the product by measuring the dimensions of each product, it is time consuming as well as expensive Moreover, if the no of finished products, those do not meet the specifications are too large in number, an unnecessary wastage of the inputs stands as one of the barrier for optimising profits Therefore, it would be advantageous if the process itself could be controlled so as to produce a known minimum level of defective articles The statistical technique that is employed in achieving this objective is known as *Statistical Quality Control* (*S Q C.*).

TYPES OF VARIATIONS

There are two types of variations in any mass production System.

A class of variations in the input factors, that can be identified, are, sometimes, responsible for the variations in the outputs. For example, variations in the product dimensions may be observed due to maladjustment in the machine, or a change in the characteristics of the raw materials, or worker fatigue, etc. Such variations are called *assignable variations*, or variations due to assignable cause.

Sometimes the variations in the output are due to the resultant effect of many minor causes, which vary randomly independent of each other, and, it is difficult or uneconomical to detect or eliminate these causes For example, slight variations in temperature, humidity and a few other similar factors may result slight variations in the quality of the output Variations of this type are called *random variations* or *chance variations*. These are considered to be the characteristics of the production process, and are not eliminated · but predicted statistically.

Type of Control

There are two broad ways of statistical control of the quality of a product, viz , process control and the product control.

(a) Process control—

This is concerned with controlling the quality of the goods manufactured in the process of production. Process control detects whether the production process is going on in the desired fashions. It other words it controls quality of the goods to be produced. In ensures that the machines are turning out the product of a requisite standard. This is achieved largely through control chart device.

(b) Product control—

This is concerned with classification of raw materials or finished goods into say acceptable, non-acceptable or whether another sample has to be tested It is concerned with the inspection of the goods already produced, whether these are fit to be dispatched.

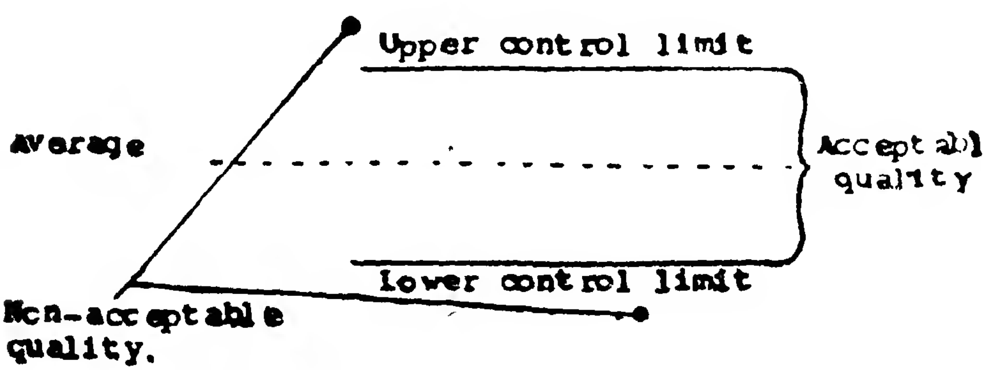
Techniques used

The following chart shows the various techniques used for statistical quality control (S Q C)

- Techniques
  - Process control (by control chart Device)
    - Variables
      - X-chart
      - R-chart
    - Attributes
      - C-chart
      - np-chart
      - p-chart
  - Product control (by acceptance plan)
    - Attributes
    - Variables

CONTROL CHARTS

The most widely used statistical tools of quality control are the *Shewhart Control Chart.* It was introduced by Walter A Shewhart. A control chart consists of three horizontal lines as shown below.



The upper line represents the upper control limit (UCL), the lower line the lower control limit (LCL), and the middle line an acceptable average for the process based on specifications. The control limits are based on the distribution of sample

statistics. The data are collected for a series of samples during a production process at different time intervals and are plotted on the graph. As long as the points fall within the control limits, the products are of acceptable quality, but, if a point falls outside the control limits, the production supervisor tries to determine whether this is due to chance or some assignable cause. Different variables control charts are maintained for each variable of interest If the control chart indicates that the observed variation is due to chance alone, the process is said to be *in control* (*i e.*, statistically stable), otherwise it is said to be out of control (*i e*, in a state of statistical instability) Once the process is found to be out of control, it is halted and set right by removing the possible causes.

(1) The idea of control chart can be extended to the comparison of budgetary figures with actual performance. Such comparison may either point to the need of improving actual perpormance or altering the budget figure.

(2) With the help of control charts one can easily detect whether or not a change in the production process results in a significant change in quality. We can decide about the change in the method of productions and separating good products from the bad ones.

Types of Control Charts

Basically, there are two types of control charts

1. Control Charts for Variables.
2. Control Charts for Attributes.

1. Control Charts for Variables

Control charts for variables are based on measured quantities and find its use, in case, the specifications for the quality are given in terms of variates. The most common charts that come under this category are the

(*a*) Control charts for sample means ($\bar{x}$ chart)

(*b*) Control charts for sample ranges. (R chart).

Following are the steps for the construction of control charts ;

(*i*) A random sample of size n is selected during the manufacturing process and the measurements $x_1, x_2, \ldots\ldots, x_n$ are noted. The most frequently used sample size is either 4 or 5.

(*ii*) The sample mean $\bar{x}$ and the sample range R are computed using

$$\bar{x}=\sum_{i=1}^{n}\frac{x_i}{n}, \text{ and } R=x_{max}-x_{min}.$$

(*iii*) If the process is found to be stable, $k(20 \leqslant k \leqslant 30)$ successive samples are selected and $\bar{\bar{x}}$, and $\bar{R}$ an estimate of the sample mean, and the sample range are made by using

$$\bar{\bar{x}} = \frac{\sum_{i=1}^{k} \bar{x}_i}{k} \quad \text{and} \quad \bar{R} = \frac{\sum_{i=1}^{k} R_i}{k},$$

respectively.

(*iv*) The upper and the lower control limits are set using
$UCL_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R}$ and $LCL_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R}$,
where A_2 is found from the following table

Factors Useful in the Construction of Control Charts

	Chart for Averages	*Chart for Ranges*		
	Factor for control limit	*Factor for central line*		*Factors for control limits*
n	A_2	d_2	D_3	D_4
2	18 80	1 128	0	3 267
3	1 023	1 693	0	2·575
4	0 729	2 059	0	2·282
5	0 577	2 326	0	2 115
6	0 483	2 534	0	2 004
7	0 419	2 704	0 076	1·924
8	0 373	2 847	0 136	1 864
9	0 337	2 970	0 184	1 816
10	0 308	3 078	0 223	1 777
11	0 285	3 173	0 256	1 744
12	3 266	3·258	0·284	1·716
13	0·249	3 336	0·308	1 692
14	0·235	3 407	0·329	1·671
15	0·223	3·472	0·348	1·652
16	0·212	3·532	0·364	1·636
17	0 203	0 588	0 379	1 621
18	0 194	3 640	0 392	1 608
19	0 187	3 689	0 404	1 596
20	0 180	3·735	0·414	1 586
21	0·173	3·778	0 425	1·575
22	0·167	3·819	0·434	1·566
23	0·162	3·858	0·443	1·557
24	0·157	3·895	0 452	1·548
25	0·153	3·931	0·459	1·541

(*v*) **The Upper and the lower control limits for R are set using.**
$UCL_R = D_4\bar{R}$, and $LCL_R = D_3\bar{R}$,
where D_4 and D_3 are found from the above table.

(vi) The upper and the lower process tolerance limits (known as natura tolerance limits) for individual values of x are computed by usin

$$UTL_x = \bar{\bar{x}} + \frac{3\bar{R}}{d_2}, \quad \text{and} \quad LTL_x = \bar{\bar{x}} - \frac{3\bar{R}}{d_2},$$

where d_2 is found from the above table. In these natural toleranc limits fall within the manufacturer's specifications, most of the product are accepted to be of a satisfactory quality, and the production proces is said to be in good control.

Note 1 The control limits for an $\bar{x}$ chart are based on an estimate of $3\sigma_{\bar{x}}$ and that for a R chart are based on an estimate of $3\sigma_R$.

2 If the population from which the samples are drawn is normally distri buted the distribution of sample means are normally distributed, bu the distribution of sample range is neither normal nor symmetrical However, control limits for ranges are assumed to be symmetrical.

Choice between $\bar{X}$ and R chart

The choice between the $\bar{X}$ and the R chart is a managrinal problem. $\bar{X}$ cha is used to show the quality averages of the samples drawn from a given process, whereas R chart is used to show quality dispersions (variabilities) of the samples. If the presence of an assignable course is noticed on both type of charts, only one type of chart should be used.

In practice, R charts are constructed first. If R chart indicates that the dispersion of the quality by the process is out of control it is better not to construct $\bar{X}$ chart until the quality dispersion is brought under control.

Interpretation of $\bar{\bar{X}}$ and R charts

1. If all the sample points $(\bar{X})$ or R, as the case may be, are scattered within the control limits, the process is under control and is satisfactory. Even when all the points are within control limits or there is a clear upward or downward bias of points, it should be checked whether the machine required adjustment, it is also likely that the measuring gauge or the central line needs adjustment.

2. Assuming the population to be normal, the probability of any point falling outside the control limits $\pm\sigma$ or $\pm 3R$ is only 0027 yet, if a point falls just outside the limits it will be desirable to take another bigger sample. If the mean of the combined sample still lies out of the control limits, there should the be search for assignable cause.

3. If one or more of the points in any or both the charts go out of the control limits, we say that the process is out of control *i.e.*, it is not in the state of statistical control.

Example : A pharmaceutical manufacturing company intends marketing [redacted] 15 ml. Samples of four vials are drawn randomly every

half-hour during the production and results are produced below. Draw a control chart for sample means and determine whether the process exhibits statistical stability

Sample number	*Vol of each sample observed*					
	x_1	x_2	x_3	x_4	*Mean* $\bar{x}_i$	*Range* R_i
1	15 58	15 82	15·45	15·71	15·64	0 37
2	15 32	15 47	15 60	15 33	15 43	0·28
3	15 94	15 07	15·02	15·81	15 46	0 92
4	15 26	15 98	15 04	15·21	15 37	0 94
5	14 79	15·01	15 04	14·92	14 94	0 12
6	15 31	15 25	15·27	14 97	15·20	0 34
7	14 89	14 91	14 97	15 15	14 98	0·26
8	15·18	15·40	15 34	15·36	15 32	0 22
9	14 93	15 03	14·87	15·17	15·00	0 30
10	15 15	15 56	15 43	15·14	15 32	0·42
11	15 02	15·04	15 23	15·19	15·12	0·21
12	15 17	15 08	14·81	15 02	15 02	0·36
13	14·85	15 01	15·82	14 76	14 86	0·25
14	15 68	15 95	15 59	15 74	15·74	0 36
15	15 15	15 12	15 23	15·14	15·16	0·11
16	15·48	15 52	15 60	15·56	15 54	0·12
17	15 61	15 72	15 46	15·41	15 55	0·31
18	14 91	14·54	14 78	14 81	14·76	0 37
19	15 19	15 34	15·24	15·27	15 26	0 15
20	15 63	15·67	15 60	15·54	15·61	0 13

Solution :

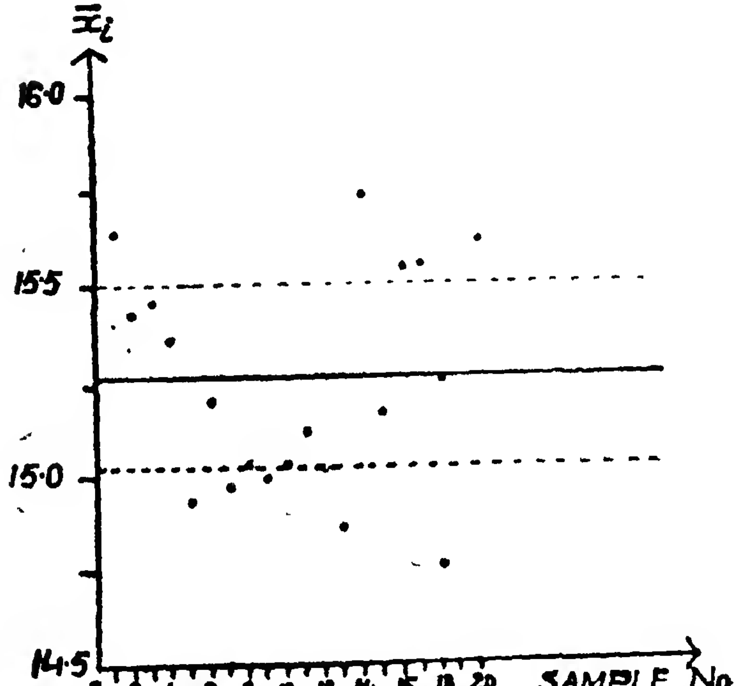
$$\bar{\bar{x}}=\frac{\sum_{i=1}^{20}\bar{x}_i}{20}=\frac{305{\cdot}28}{20}=15{\cdot}264$$

$$\bar{R}=\frac{\sum_{i=1}^{20}R_i}{20}=\frac{6{\cdot}54}{20}=0{\cdot}327$$

The lower and the upper control limits for $\bar{x}$ are respectively given by

$$LCL_{\bar{\bar{x}}} = \bar{\bar{x}} - A_2\bar{R} = 15\cdot264 - 0\cdot729 \times 0\cdot327$$
$$= 15\cdot264 - 0\cdot238 = 15\cdot026$$

$$ULC_{\bar{\bar{x}}} = \bar{\bar{x}} + A_2\bar{R} = 15\cdot264 + 0\cdot729 \times 0\cdot327$$
$$= 15\cdot264 + 0\cdot238 = 15\cdot502$$



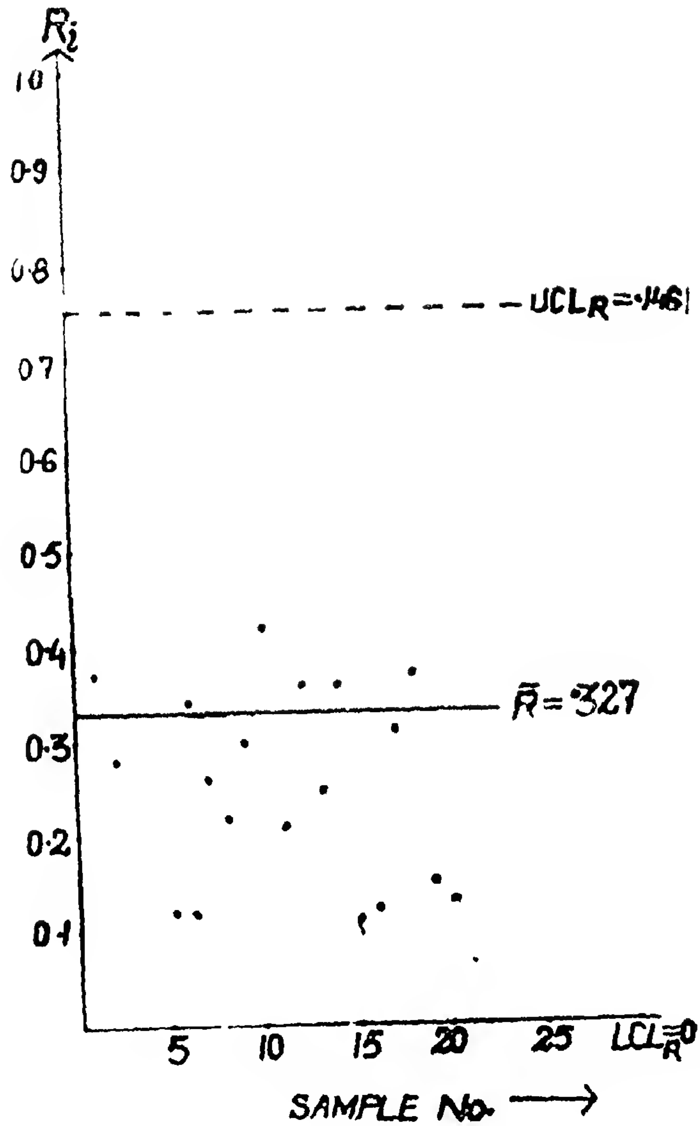
$\bar{x}$ - CHART

The means of sample nos. 1, 5, 7, 9, 12, 13, 14, 16, 17, 18, and 20 lie beyond the control limits, indicating statistical instability.

Example. *Using the data of the previous example draw a control chart for* R.

Solution :

$$UCL_R = D_4\bar{R} = 2\cdot282 \times 0\cdot327 = 0\cdot746$$
$$LCL_R = D_3\bar{R} = 0 \times 0\cdot327 = 0.$$



R-CHART

It seems the process variation has changed at the time of drawing sample nos. 3 and 4. We assume that an assignable cause was found and was corrected.

2. Control Charts for Attributes

Sometimes it becomes impossible or difficult to determine the quality [redacted]

the product may be classified as either acceptable or not-acceptable, *i e.*, if the attribute determines the quality of a product, control chart for attributes are used during producting the process The most common control charts that come under this category are the

(*a*) Control charts for fraction defective (p chart)

(*b*) Control charts for number of defects (c chart)

(*a*) Control Charts for Fraction Defective

The control charts for fraction defective is based on the distribution of sample proportions, and it is assumed that the products follow Bernoulli process

In constructing *p chart*, we define

$$p = \text{process average fraction defective} = \frac{\text{no of defective items in a sample}}{\text{total no of items in the sample}}$$

and $$\bar{p} = \frac{\text{total no of defective items in several samples}}{\text{total no. of items sampled}}$$

Corresponding to the statistic p, $\bar{p}$ is used as an estimate of p', the population defective The control limits are set using.

$$\text{UCL}_p = p' + 3\hat{\sigma}_p \text{ and } \text{LCL}_p = p' - 3\hat{\sigma}_p,$$

$$\text{where } \hat{\sigma}_p = \frac{p(1-p)}{n} \text{ is an estimate of } \sigma_p.$$

The central line (CL) in this case is $\bar{p}$.

A proportion that falls within the above control limits is assumed to possess the particular attribute and that falls outside these limits is supposed to be due to an *assignable cause*

(*b*) Control Charts for Number of Defects

When the production process involves complex assembly of number of components, *viz*, production of TVs, Computers, Cars, etc a knowledge of the number defects per unit is useful in maintaining a satisfactory level of quality. Such a situation, where the likelihood for no of defects is large, yet, the actual occurrence is found to be small, and a single unit with one or more defects is not necessarily rejected follows the Poisson distribution rather than the binomial distribution. A control chart based on the Poisson distribution is called ***a control chart for defects*** or ***c-chart***.

Since the mean and variance of a Poisson distribution are equal, the **standard deviation of the process** is given by

$$\sigma_c = \sqrt{\bar{c}}.$$

where $\bar{c}$ = the expected number of defects per unit.

The $3\sigma_c$ control limits for a *c chart* are given by

$$ULC_{\bar{c}} = \bar{c} + 3\sigma_{\bar{c}} \quad \text{and} \quad LCL_{\bar{c}} = \bar{c} - 3\sigma_{\bar{c}}$$

Example : The following is a table showing the number of defective items in sample of size 50 drawn from a manufacturing process. One sample was drawn from each of 25 time periods

Sample No.	*No. of deflctives*	*Sample No.*	*No. of defectives*
1	0	14	2
2	2	15	4
3	1	16	5
4	3	17	4
5	1	18	3
6	0	19	3
7	1	20	2
8	2	21	1
9	2	22	1
10	3	23	0
11	4	24	6
12	1	25	2
13	0		

Construct the *p-chart.*

Solution

Sample No.	No. of defectives (p)	p	Sample No	No. of defectives (d)	p
1	0	0·00	14	2	0 04
2	2	0·04	15	4	0·08
3	1	0·02	16	5	0·01
4	3	0 06	17	4	0 08
5	1	0·02	18	3	0 06
6	0	0 00	19	3	0 06
7	1	0 02	20	2	0 04
8	2	0·04	21	1	0 02
9	2	0 04	22	1	0 02
10	3	0 06	23	0	0 00
11	4	0·08	24	6	0·12
12	1	0 02	25	2	0 04
13	0	0 00			

$\Sigma d = 55$

$\overline{p} = \frac{53}{1250} = 0{\cdot}0424,$ [since the total no. of items sampled $= 50\times 25 = 1250$]

$$\hat{\sigma}_p = \sqrt{\frac{\overline{p}(1-\overline{p})}{n}}\sqrt{\frac{0\ 0424(1-0{\cdot}0424)}{50}}$$

$$= \sqrt{\frac{0\ 0424\times 0\ 9576}{50}} = \sqrt{\frac{0{\cdot}04060}{50}} = \sqrt{0{\cdot}0008}$$

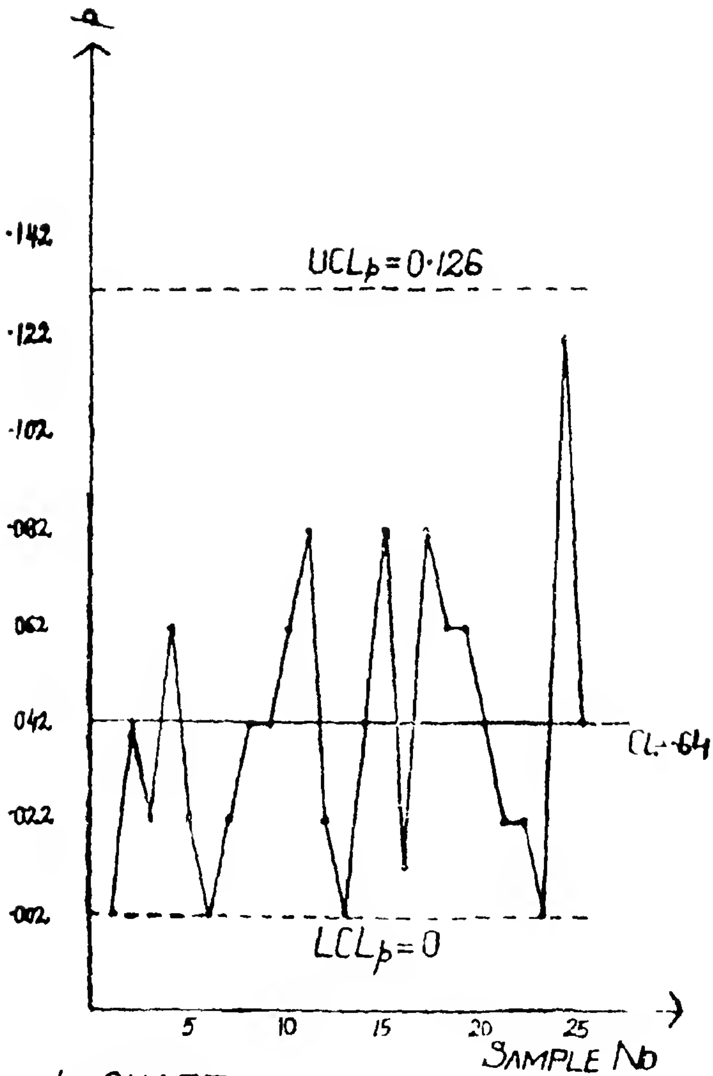
$= 0{\cdot}028$

$\therefore\ 3\hat{\sigma}_p = 3\times 0{\cdot}028 = 0{\cdot}084$

$UCL_p = p' + 3\hat{\sigma}_p = 0{\cdot}0424 + 0\ 084 = 0\ 1264$

$LCL_p = p' - 3\hat{\sigma}_p = 0{\cdot}0424 - 0{\cdot}084 = -\ 0{\cdot}0416$
$= 0$ [since negative]

CL $= 0{\cdot}04$



·142
UCLp = 0·126
·122
·102
·082
·062
·042
CL = ·64
·022
·002
LCLp = 0
5
10
15
20
25
SAMPLE No

p-CHART

Example : The following table shows the number of missing rivets detected during the final inspection of the aircraft

Aircraft No.	No of missing rivets	Aircraft No	No. of missing rivets
1	8	14	26
2	16	15	15
3	14	16	8
4	19	17	9
5	11	18	14
6	15	19	11
7	8	20	9
8	11	21	10
9	21	22	22
10	12	23	7
11	22	24	28
12	16	25	9
13	9		
	182		168

Compute trial control limits for number of defects What value of $\hat{c}$, an estimate of the expected no of defects per unit would you suggest for the future ?

Solution :

$$\bar{c} = \text{Expected no of defects per unit} = \frac{350}{25} = 14.$$

$$\sigma_c = \sqrt{\bar{c}} = \sqrt{14} = 3\ 74, \qquad 3\sigma_c = 11{\cdot}22$$

$$UCL_{\bar{c}} = \bar{c} + 3\sigma_c = 14 + 11 = 25$$

$$LCL_{\bar{c}} = \bar{c} - 3\sigma_c = 14 - 11 = 3$$

The points corresponding to Aircraft no 14 and 24 are beyond the control limits. Removing these two observations,

$$\bar{c} = \frac{350 - 26 - 28}{(25 - 1 - 1)} = \frac{296}{23} = 12\ 86$$

$\therefore$ Recommended $\hat{c}$ for the future $= 13$.

ACCEPTANCE SAMPLING FOR ATTRIBUTES

By the use of control charts the manufacturing process is controlled, and, hence, this is essentially a process control technique. There are situations that do not involve any manufacturing process, yet require, assurance on the quality. Though inspection of each item under consideration is possible in some cases, and inference may be drawn on whether it is to be accepted or rejected, it is time consuming as well as expensive Moreover, if the inspection is destructive, we have to resort to sampling When the decision on the acceptability of a lot is based on sampling, the technique is known as *acceptance sampling* Because a decision is taken on whether the sample processess a particular attribute or not it is also known as *attribute sampling*. The actual sampling procedure is known as ***sampling plan***. Though there are various sampling plans, the three standard sampling plans that are in use are

(a) Single sampling plan

If a decision on the acceptability of the lot is based on one random sample from the lot, the sampling plan is known as *single sampling plan*. This plan is completely defined by three numbers N, n and c, where

N = the size of the lot,

n = the size of the sample,

and c = the maximum number of allowable number of defectiver for acceptance.

As an example, let us consider N = 500, n = 50, c = 1. This may be interpreted as,

(1) draw a sample of size 50 out of a lot of 500 ;

(2) if the number of defectives in the sample be $\leqslant 1$, accept the lot, otherwise reject the lot

(b) Double sampling plan

A lot may be accepted if the first sample is good enough or rejected if the same is bad enough But, if the first sample is neither good enough nor bad enough, a decision on the acceptability of a lot is taken by drawing two samples from it This is known as *double sampling plan* This plan is completely defined by five numbers N, n_1, c_1, n_2, c_2, where

N = the size of the lot,

n_1 = the size of the first sample,

c_1 = the maximum number of allowable number of defectives for acceptance on the basis of the first sample,

n_2 = the size of the second sample,

c_2 = the maximum number of allowable number of defectives for acceptance on the basis of two samples.

As an example, let us consider

N = 1000, $n_1 = 30$, $c_1 = 1$, $n_2 = 40$, $c_2 = 3$

This may be interpreted as,

(1) draw a first sample of size 30 from a lot of 1,000 ;

(2) if the number of defectives in the sample be $\leqslant 1$, accept the lot ;

(3) if the sample contains more than one defective, reject the lot ,

(4) inspect a second sample of size 40 if the first sample contains 2 or 3 defectives ,

(5) if the combined sample of size 70 contains 3 or less defectives accept the lot ;

(6) if the combined sample contains more than 3 defectives, reject the lot

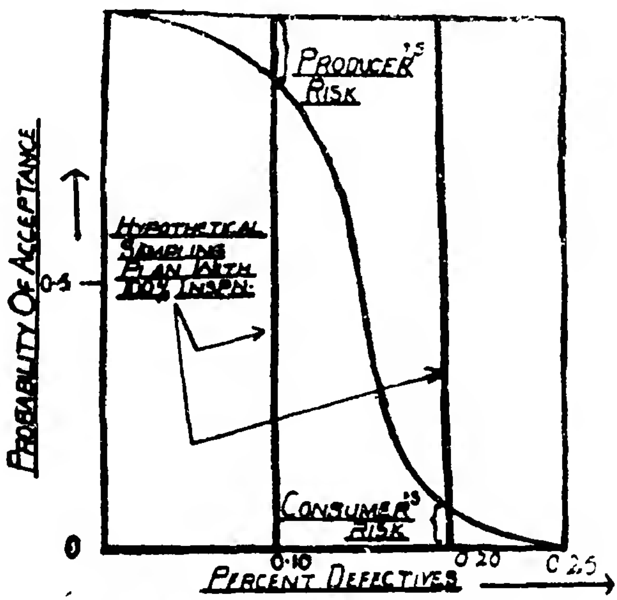
The double sampling plans are usually more efficient in that less inspection is required for the same protection afforded by a single sampling plan Also, the producer derives the satisfaction from the fact that each of his lot is being given another chance, before being rejected.

(c) **Multiple sampling plan**

This is an extension of the double sampling plans.

Operating Characteristic Curve (OC curve)

In acceptance sampling both supplier and consumer face the inevitable risks willingly If the sample shows good results but the lot is bad the producer bears the risks of sampling. If the sample shows poor results but the lot is good the supplier bears the risk Assigning different values of the variables the sampling plans (N, *n*, *c*) are infinite in number Which plan should be selected for the given situation then ? Or, in other words, what are the values to be assigned to N, *n* and *c* ? There are quite a few considerations in devising the right plan for the given situation First, the risk should be equitably shared by both the parties. Second, the given situation means precisely the *per cent average defectives* of the supplier's production process and the *desired lot quality* by the consumer Some plans would be good vis-a-vis % average defectives and others bad. But this average cannot be maintained at a constant level This should be obvious after



Operating Characteristic Curve
Single Sampling Plan

discussion on control charts The supplier's production or output is bound to vary in quality What we have to consider, then, is how the plan would operate against the output quality varying from its average

A curve may be drawn by plotting the probability of accepting the lot $p\alpha$ as a function of the fraction defective p', of the lot. This curve is known as operating characteristic curve The right OC curve takes into account the protection desired by both the supplier and the customer, and the inherent variability of the supplier's process

Posterior Probability

The collection of events A_1, A_2, A_3, A_n are referred to as causes, hypothesis and states of nature The unconditional probabilities $P_r(A_1)$, .. $P_r(A_2)$, $P_r(A_n)$ are called the *prior probabilities* for the hypothesis These are termed as prior probabilities since these probabilities are assessed before the occurrence of the event B (say) Suppose now, we want to derive the probability of A when B has occurred written as P(A/B) or B when A has occurred as P(B/A) We are mainly concerned here with finding out the probabilities of the dependent events when the results of the other event is known. This is called a *Posterior probability* or *revised probability*

Example A ball is selected at random from two urns by first choosing an urn and then drawing the ball from the selected urn The first urn contains four black balls and six white balls The second urn contains eight white balls and two black balls If the chosen ball is black, find the probability that it comes from first urn

Solution

Let B be the event that the chosen ball is black and A the event that it comes from the first urn Since either urn is likely to be chosen, a priori probability of A is 0 5 (This is the probability of A before new information is obtained) Using Baye's theorem

$$P(A/B) = \frac{P(B/A)\,P(A)}{\Sigma P\,(B/A)\,P(A) + P(B/A')\,P(A')}$$

$$= \frac{(0\ 4)\,(0.5)}{(0\ 4)\,(0.5) + (0\ 2)\,(0.5)} = \frac{.2}{.2 + .1}$$

$$= \frac{.2}{.3} = \frac{2}{3}$$

P(B/A) is the *a posterior probability*

Based on the above definitions, we will now present the method of analysis for choosing among alternative acts in the field of formal decision making under uncertainty. The method is known as *Posterior analysis* based on posterior probabilities.

Posterior analysis

Consider that the decision maker has reached a decision to sample, carried out the survey, and made the results available. In addition to the sample results, pay off and prior probabilities if the decision maker has the conditional probability

analysis is the method whereby we combine sample information with prior information to obtain revised probabilities for the various events.

The best act using posterior analysis is found as follows .

Step 1 . Find the posterior probabilitie s for each state conditional upon the sample result by using Baye's theorem

$$P_1(S/s) = \frac{P_0(S)\, P(s/S)}{\Sigma P_0(S)\, P(s/S)}$$

where S = a state of nature, s = a specific sample result,

$P_0(S)$ = the marginal probability of the state s (a prior probability),

$P(s/S)$ = the conditional probability that the sample result, s, is based upon the state S.

$P_1(S/s)$ = the posterior or conditional probability that the state S exists given the sample result s

Step 2 Multiply the payoff associated with every state of nature by the posterior probability P_1 corresponding to the state and add the results to compute the expected payoff

Step 3 : Compare the expected payoff E_1 of all acts and choose the course of action corresponding to

(i) the largest when payoff are to be maximised.

(ii) the smallest when payoff are to be minimised

Example : A company wishes to decide whether or not a shipment of items should be accepted From the past experience, it is known that the percentage of defective items in a batch is either 6, 10 or 15 The probability of occurrence of these states are 0 8, 0 3 and 0.1 Shipment with 6 per cent defective items are acceptable. The cost of accepting and rejecting good and bad shipments are given by the following payoff table.

Payoff table

State	*Act*	
percentage defective	*Accept*	*Reject*
6	0	150
10	300	0
15	400	0

In addition to the information given above, assume that a sample of 10 items has

result (s 2), the conditional probabilities of the result are given in the following table

State	Conditional probabilities $P(s/S)$
6	0834
10	1847
15	2645

Posterior analysis · The above information is summarized in the following table .

S (State)	$P_0(S)$ (Prior Prob)	$P(s/S)$ (Conditional Prob)	Cost	
			Accept	Reject
6	0 8	0834	0	150
10	0 3	1847	300	0
15	0 1	2645	400	0

In order to choose the best act using the additional information (i e conditional probabilities), P(s/S), we perfom the following steps .

Step 1 . We first find the posterior probabilities $P_1(S/s)$. The necessary computation are given in the table below

S	$P_0(S)$ (1)	$P(s/S)$ (2)	$P_0(S)P(s/S)$ (3) = (1) × (2)	$P_1(s/S)$ (4) = 3 − 0.1485
6	.8	0834	0667	4492
10	.3	.1847	0554	.3731
15	.1	.2645	0264	.1778
			.1485	

Step 2 : Compute the expected payoff for each act, using the posterior

[redacted]

$$E_1 \text{ (accept)} = .4492\ (0) + .3731\ (300) + .1778\ (400)$$
$$= 0 + 111.93 + 71.12$$
$$= 183.05$$
$$E_1 \text{ (reject)} = .4492\ (150) + .3731\ (0) + .1778\ (0)$$
$$= 67.38$$

Step 3 : When comparing the two expected costs we find that Rs 67 38 is less than Rs. 183.05. Since the lower of the two values corresponds to the second alternative, we conclude that shipment should be rejected on the basis of posterior probabilities.

Sampling, Binomial and Normal distribution.

Students are advised to refer to study VI, Statistics of intermediate course.

Exponential Smoothing

Exponential smoothing refers to a family of forecasting models. The exponential smoothing model estimates average smoothed demand for the upcoming period F_t by adding or subtracting a fraction of the difference between actual current demand and the last smoothed average F_{t-1} The new smoothed average F_t is then given by

New smoothed average = old smoothed average + α (new demand-old smoothed everage) ;

When symbolized, it takes the form

$$F_t = F_{t-1} + \alpha\ (D_t - F_{t-1}) \qquad ...(1)$$

or

$$F_t = \alpha D_t + (1-\alpha)\ F_{t-1} \qquad ...(2)$$

D_t = actual current value of variable being forecast in period t,

and α = smooting constant

The value of α lies between 0 and 1 It determines the degree of smoothing that takes place and indicates how responsive the model is to fluctuations in the forcast variable. The setting of α is usually done by trial and error and is not a scientific method. The commonly used values of α are between 0 01 to 0.30.

It is evident from the above discussion that the major advantage is that it is *not necessary* to store all the historical data for the forecasting model and compute average each time. Instead, exponential smoothing requires only the previous forecast and the actual value of the previous period.

To illustrate, Govind Ram & Sons Company uses exponential smoothing to forecast energy demand on a monthly basis. Experience has shown that the appropriate constant to use is $\alpha = 0.10$.

If $D_t = 110$ and $F_{t-1} = 100$, then the new smoothed average is

$$F_t = 0.1 \times 110 + 0.9 \times 100 = 11.0 + 90 = 101.0$$

Since the new demand figure D_t includes possible random variations, we are discounting 90 per cent of these variations Obviously then, small values of α will have a stronger smoothing effect than large values Conversely, large values of α will react more quickly to real changes (as well as random variations) in actual demand As an example, if α = 0 4 and the previous data remain the same, the new smoothed average would be

$$F_t = 0\,4 \times 110 + 0\,6 \times 100 \times 44\,0 + 60 = 104.0$$

Note that the component of currents actual demands and the old average are now weighted quite differently, giving considerably more weight to current actual demand The equation (2) actually gives weight to all past actual demand data, though this is not obvious This weighting occurs through the chain of the periodic calculations to produce smoothed averages for each period Thus, the smoothed averages are based on a sequential process representing all previous actual demands

Generally, exponential smoothing is considered superior to the other forecasting methods Its cost is equivalent and its accuracy especially in the short term is better Owing to their relatively small computational cost and computer storage requirement, exponential smoothing models are probably the most widely used of the time series techniques For longer term forecast, however, exponential smoothing is also considered a poor technique

Trend Effects

The model, given by (2) above, does not take into account the seasonal or trend component There exists other exponential smoothing models that compensate for the various components of a time series such as the seasonal or trend component To illustrate more complex exponential smoothing models, let us consider a model that has an adjustment for trend built into it If trend exists in either a positive or a negative form, there will be a lag if we make use of the simple exponential smoothing model just decribed (see figure .)

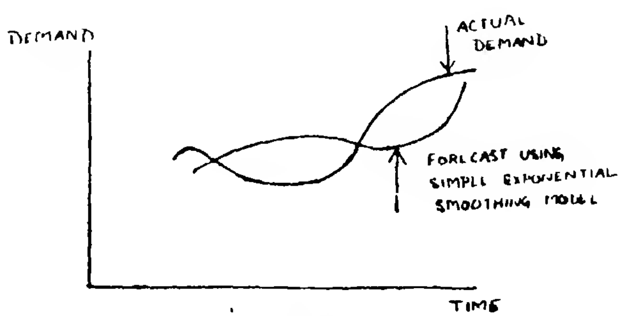


Fig. 1

The basic idea behind the trend-adjusted model is to calculate a simp exponentially smoothed forecast and adjust the forecast for a trend lag

Mathematically, the trend-adjusted model can be described as follows :

$$F_t' = F_t + \frac{1-\beta}{\beta} . T_t$$

Where F_t' = trend-adjusted forecast for time period t,

F_t = simple exponential smoothing forecast for time period t,

and β = trend smoothing factor.

T_t is computed using the following formula :

$$T_t = T_{t-1} + \beta(t_t - T_{t-1}), \qquad \text{where } t_t = F_t - F_{t-1}$$

Computation of a trend-adjusted forecast is a four step process.

Step 1. Compute F_t, a simple forecast for time period t.

Step 2 Compute t_t by using $t_t = Ft - F_{t-1}$.

Step 3. Calculate the exponentially smoothed trend by using

$$T_t = T_{t-1} + \beta\,(t_t - T_{t-1})$$

Step 4. Finally, calculate a trend-adjusted forecast by using

$$F_t' = F_t + \frac{1-\beta}{\beta} T_t.$$

Table 1

Period	*Demand*
1	13
2	18
3	20
4	17
5	23
6	25
7	31
8	30
9	34
10	35
11	38
12	39

$$F'_t = F_t + \frac{1-\beta}{\beta} T_t.$$

Let us illustrate how to compute a trend adjusted exponentially smoothed forecast using the demand data in Table 1 and smoothing constants α and β equal to 3 and 25 respectively Let the initial forecast be 12 5. The trend-adjusted forecast for period 2 is computed as follows :

The first step is to compute F_2

$$F_2 = F_1 + \alpha (D_1 - F_1) = 12,5 + .3 (13 - 12\ 5)$$
$$= 12\ .\ 65.$$

The next step is to calculate t_2

$$t_2 = F_2 - F_1 = 12.65 - 12.50 = .15$$

T_2 can now be calculated assuming the initial trend adjustment is 0 :

$$T_2 = T_1 + \beta (t_2 - T_1) = 0 + .25 (\ 15) = .0375$$

Finally, the trend adjusted forecast for time period 2 can be computed ·

$$F_2' = F_2 + \frac{1 - 25}{.25} T_2 = 12\ 65 + 3 (.0375) = 12.7625.$$

Table 2

Time period	*Actual Demand*	F_t	t_t	T_t	F_t'
1	13	12.5000	0	0	12 5000
2	17	12 6500	0 15000	0.037500	12 7625
3	20	14.2550	1.60500	0.429375	15.5431
4	17	15 9785	1.72350	0.752906	18.2372
5	23	16.2849	3.06450	0.641292	18.2088
6	25	18 2995	2 01451	0 984598	21 2533
7	31	20.3096	2.01016	1.24099	24.0326
8	30	23 5167	3.20711	1.73252	28.7143
9	34	24.4617	1.94498	1.78563	30 8186
10	35	28.0232	2.56149	1.97960	33.9620
11	38	30 1162	2.09304	2.00796	36.1401
12	39	32.4814	2.36513	2.09725	38.7731

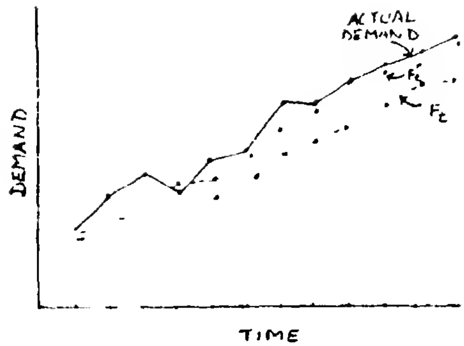


Fig 2

Doing the same calculations for the remaining time periods for the time series data shown in Table 1 results in trend adjusted forecast are reflected in Table 2 Fig 2 graphically contrasts the unadjusted and adjusted forecast for the time series in Table 2

Ex Consider the set of data values in Fig 2,

x	y	x	y
100	57	60	46
70	43	50	45
30	35	20	26
40	33	10	26
80	56	90	53

Now assume that the variable referred to as x is really time and that we associate the x values 10, 20, 30 and so on with the values 1, 2, 3 and so on.

(*a*) Would you expect the exponential smoothing model

$F_t = \alpha D_t + (1-\alpha)F_t$ to produce a good fit for these data ? Why ?

(*b*) What relationship would you expect to hold between S_t and Y_t ?

(*c*) Suppose α was originally set equal to 0.5, would you increase or decrease α to get a better fit solution.

Solution : Consider the scatter diagram for the given data.

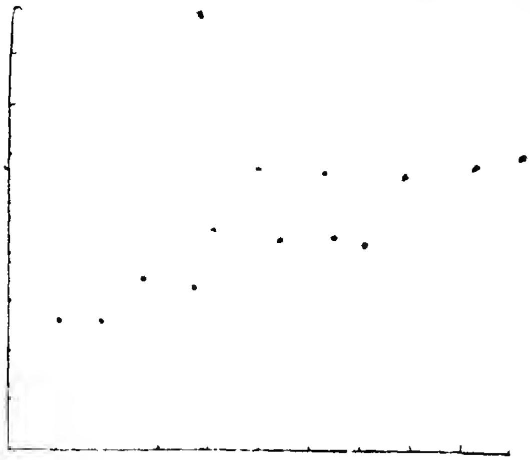


Fig. 3.

(*a*) The scatter diagram shows that Y appears to increase with *t*. Since the model $F_t = \alpha D_t + (1 - \alpha) F_{t-1}$ is designed for systems in which variations in Y_t occur at random (that is, where there is not a regular pattern of change) the model will probably not produce a very good fit.

(*b*) One would expect F_t to be smaller than D_t because F_t is a weighted sum of terms, most of which are less than or equal to Y_t.

(*c*) One would expect a better fit by increasing the value of α since this puts more weight on the most recent, and thus larges, values of Y

Investment Decision through Simulation

One of the critical areas of decision making by business Executive is choosing among alternative capital investment opportunities. It is not the problem of projecting return on investment under any given set of assumptions that is very critical but to evaluate the degree of uncertainty and risk involved in these assumptions which when taken together multiply into a total uncertainty of critical size. Thus the management should have a clear picture of the relative risks and the probable odds of coming out ahead or behind in the light of uncertainty. Prof. David B. Hertz while evaluating the risky investments has proposed the use of a simultation model to obtain the expected return for an investment proposal. To carry out the analysis, Prof.—Hertz proposed following three steps :

(*i*) Estimate the range of values for each of the factors like selling price, sales growth rates and so on and within that range, the likelihood of occurrence of each value *i.e.* develop the probability distribution of uncertain factors in the system.

(ii) Select at random from the distribution of values for each factor or particular value i e generate to a set of random numbers and associa each one of them with the respective probability distribution to dete mine the expected value for each factor in the model Then combir the values of all the factors and compute the rate of return (or presei value) from that combination.

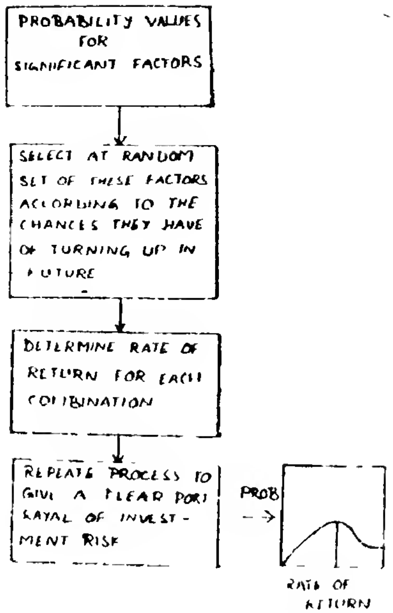
(iii) Repeat the above procedure a number of times to define and evalua the odds of the occurrence of each possible rate of return. The resu will be a listing of the rates of return that one might achieve, rangir from a loss (if the factors are not favourable) to whatever maximu gain that is possible with the estimates that have been made

The variability of outcome values from the average is determined. This w help the management to select investments having the same return but with low variability When the expected return and variability of each of a series investments have been determined, the same procedure may be used to study tl effectiveness of various combinations of them in meeting management objectives

In evaluating an investment proposal, the following factors are to l considered

(i) Market size
(ii) Selling price,
(iii) Market growth, rate,
(iv) Market share, (in physical sales volum
(v) Investment required,
(vi) Residual value of investment,
(vii) Operating cost,
(viii) fixed costs, and
(ix) useful life of facilities

For each of these factors, probability distributions are developed bas upon management's assessment of the probable outcomes. The next step in t simulation approach is to determine the returns that will result from rand combinations of the factors involved This requires realistic restrictions such not allowing the total market to vary more than some reasonable amount from ye to year A computer can be used to carry out the trials for the simulation proc so as to obtain more accurate results. The process is repeated a number of tim each time we obtain a combination of values for the nine factors and the corr ponding return on investment When sufficient number of trials are carried o we can plot a graph for the rates of return and also obtain a frequency distributi From this frequency distribution, we can evaluate the expected return and t variability of this expected return or risk. In other words, we can determine t probability that an investment will provide a return greater than or less tha certain amount. By comparing the probability distribution of the rates of retu



the management is in a position to evaluate the respective merits of different investments involving risks The block diagram for the simulation procedure is given below

Simulation and Financial Planning

A modern business corporation faces major financial decision making problems in varying situations and at different stages of its development The preparation of company budget for the management decisions, is one of the most complex problem with which the planning department has to deal Following are the critical quantities to be decided by the President in the decision making process

1. Amount of money to borrow
2. Amount of existing debt to pay.
3. Amount of dividend to pay.
4. Amount of money to allocate to production.
5. Amount of money to allocate to research.
6. Amount of money to allocate to marketing.
7. Amount of money to allocate to capital investment in production and research facilities, resp.

The president realizes that expenditure today must be motivated by the desire to enhance future as well as current returns. Therefore, he must take a "long-term" viewpoint. Thinking in terms of, for example, a 15-year time period, he wants to make his decision in such a way that not only the 15 years profits

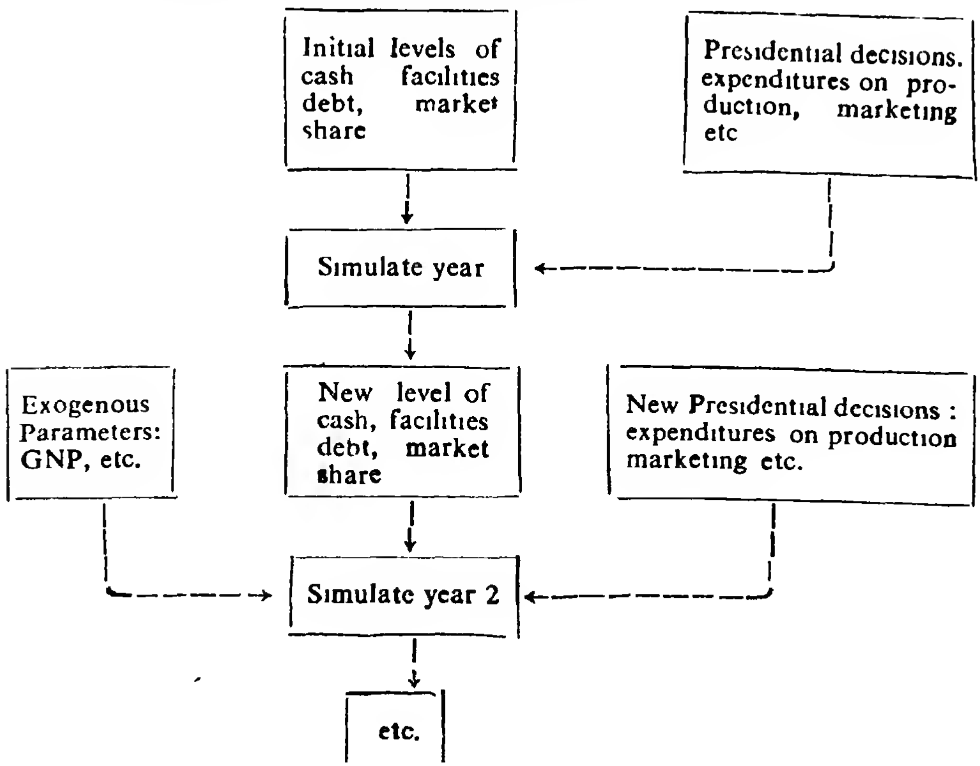
will be high but also the future prospects for the firm, at the end of 15 years are good. Consequently his goals are three-fold, namely.

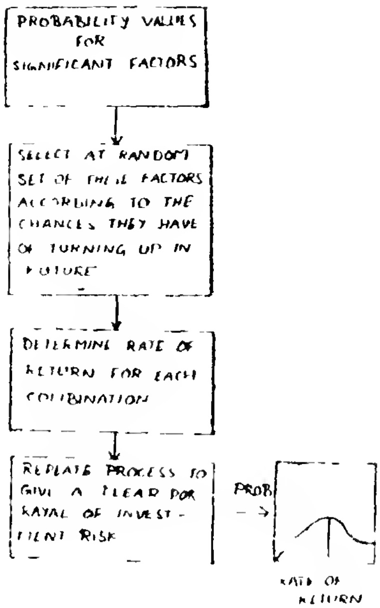
1 to maximize the discounted streams of profit during time horizon
2. to maximize the firms market share at the end of the time horizon
3 to maximize the firm's physical capital in the form of production and research facilities *i e*, maximize the capital existing at the end of the time horizon

The President realizes that enhancing any one of these objectives can only leave a smaller budget to further the other two objectives. Thus, given a budget constraint, he cannot liberally try to maximize all three simultaneously Hence it will be very difficult to construct and solve an optimization model In order to assist the President, the planning department develops a *strategic corporate model* which operates by *simulating* the President's problem on a period-by-period basis. The output of one year's activities forms part of the input for the next year. This general idea is illustrated in Fig 5 below As this figure shows, the simulator can be though of as a *black box* that receives inputs and generates outputs. In any oneyear (say *t*) it receives the following input :

1 External inputs
(*a*) The set of exogenous parameters
(*b*) The set of presidential decisions

The simulated output from the previous year (*t*—1)





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3 Amount of dividend to pay
4 Amount of money to allocate to production.
5. Amount of money to allocate to research.
6. Amount of money to allocate to marketing.
7. Amount of money to allocate to capital investment in production and research facilities, resp.

The president realizes that expenditure today must be motivated by the desire to enhance future as well as current returns. Therefore, he must take a "long-term" viewpoint. Thinking in terms of, for example, a 15-year time period, he wants to make his decision in such a way that not only the 15 years profits

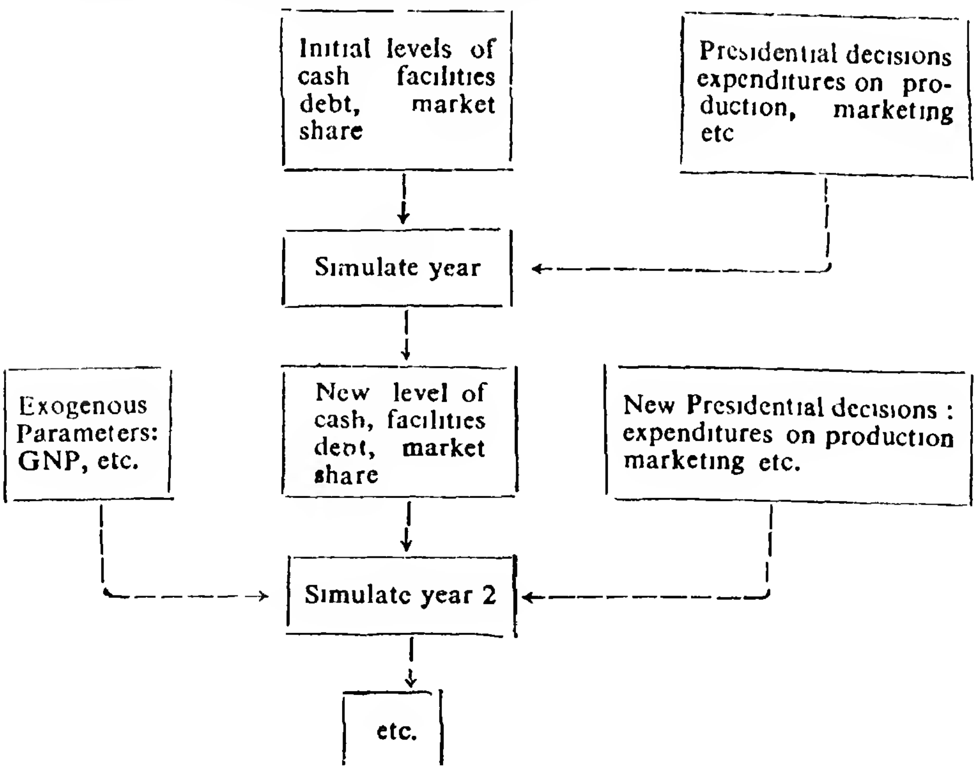
will be high but also the future prospects for the firm, at the end of 15 years are good Consequently his goals are three fold, namely.

1. to maximize the discounted streams of profit during time horizon
2 to maximize the firms market share at the end of the time horizon
3. to maximize the firm's physical capital in the form of production and research facilities *i e*, maximize the capital existing at the end of the time horizon

The President realizes that enhancing any one of these objectives can only leave a smaller budget to further the other two objectives. Thus, given a budget constraint, he cannot liberally try to maximize all three simultaneously. Hence it will be very difficult to construct and solve an optimization model In order to assist the President, the planning department develops a *strategic corporate model* which operates by *simulating* the President's problem on a period-by-period basis. The output of one year's activities forms part of the input for the next year This general idea is illustrated in Fig 5 below As this figure shows, the simulator can be though of as a *black box* that receives inputs and generates outputs. In any oneyear (say t) it receives the following input :

1 External inputs
   (*a*) The set of exogenous parameters
   (*b*) The set of presidential decisions

The simulated output from the previous year (t—1)



Since year 0 does not exist, it is obvious that an intitial set of "outputs" ist be provided to do the simulation in year 1 The simulator itself contains odels (a simulation model is often a collection of submodels) that reproduce and late pertinent aspects These models use the inputs to make various endogenous cisions like price and the quantity produced

The overall model then simulates the behaviour of the system under the nditions that would exist if these inputs and endogenous decisions had actually cused It does so by using random number to select an "actual demand" from statistical d stribution and then by determining what would happen if this mand had o curred The appropriate distribution of demand is itself determined some combination of external inputs (exogenous parameters and management visions) and en logenous decisions For example, the distribution of demand ight be determined by the general state of the economy (an exogenous rameter), the level of marketing activity (a presidential decision) and e price (an endogeneous decision) Given the price, quantity produced and he actual demand ' the simulator can calculate the gross revenue and profits iese quantities determine new financial position, which is part of the internally nerated input to the next period of activity In this next period, new values for e exogenous parameters are input Financial results from the previous time riod, augmented possibly by a decision to borrow at the beginning of this new riod, are now used to determine the available funds for a new set of budget locations The model continues to operate in this manner over a sequence of me periods Thus with a given sequence of exogenous parameters and presidential cisions the model unfolds the future, and discounted future profits over any ven time horizon can be obtained In this way, the model is used as a tool to plore some of the interactions between various presidential policies and ogenous conditions

Supplymentary Study Material on SA & DP.

Contents

Applications of data processing devices (in conjuction with Study II run flowcharts)

—Purchasing

—Production scheduling

—Cost accounting

Electronic Data Processing budgets (in conjuction with Study V)

Financing the use of computers

—Renting

—Purchase for manufacturer

—Leasing

Outside consultants and service bureaus

PURCHASE ORDER

In addition to its stock and inventory syytem, the company C has mechanised its purchase order system Inputs to the system are

(i) Items to be ordered tape of step 17 from Inventory control application
(ii) Vendor history file,
(iii) Open purchase order master file
(iv) Inventory receipts

Outputs to be generated are

(i) Purchase orders
(ii) Cash flow commitments
(iii) Overdue deliveries report
(iv) Vendor performance report

Draw a run flow chart and explain the process

Explanation

1 This process continues from step 17 of inventory control file The 'Items to be Ordered' file is sorted by vendor code This step is appropriate because several items might be ordered from the same vendor, and such items should all be included in one purchase order

2 The above order list is then processed together with the *vendor history file* and *open purchase order* file to prepare purchase orders

3 For each purchase order, the vendor's name and address, shipping arrangements and credit terms are fatched from the *vendor history file* The order list includes all data on items ordered for each purchase order,

4 A purchase order number is assigned sequentially according to the next highest number available on the *open purchase order* master An original and four copies of each new purchase order are printed out

5 The new purchase order is also added to the *open purchase order file*

6 Once a week a report of short run cash flow commitments according to the *open purchase order master* is printed This report is provided to the controller for planning short-run cash flows

Weekly reports These operations are performed once a week to generate reports for use in purchasing department

7 Inventory receipts are sorted by purchase order number

8 The open purchase order master file is updated using these receipts Quantity received are subtracted from quantities on order in the file Orders, which have been completely supplied, are removed from the file

9 The above computer run also generates a report on overdue deliveries The *overdue deliveries* report lists for which the expected delivery date has passed without the order having been received This report is provided to the purchasing department for following up on these vendors

10 The magnetic tape file of *vendor performance report* is also generated with performance data tape contains information on each vendor concerning such matter runs The vendor as compliance with requested delivery dates, discrepancies between quantity ordered and quantity received, price quoted and prices billed,

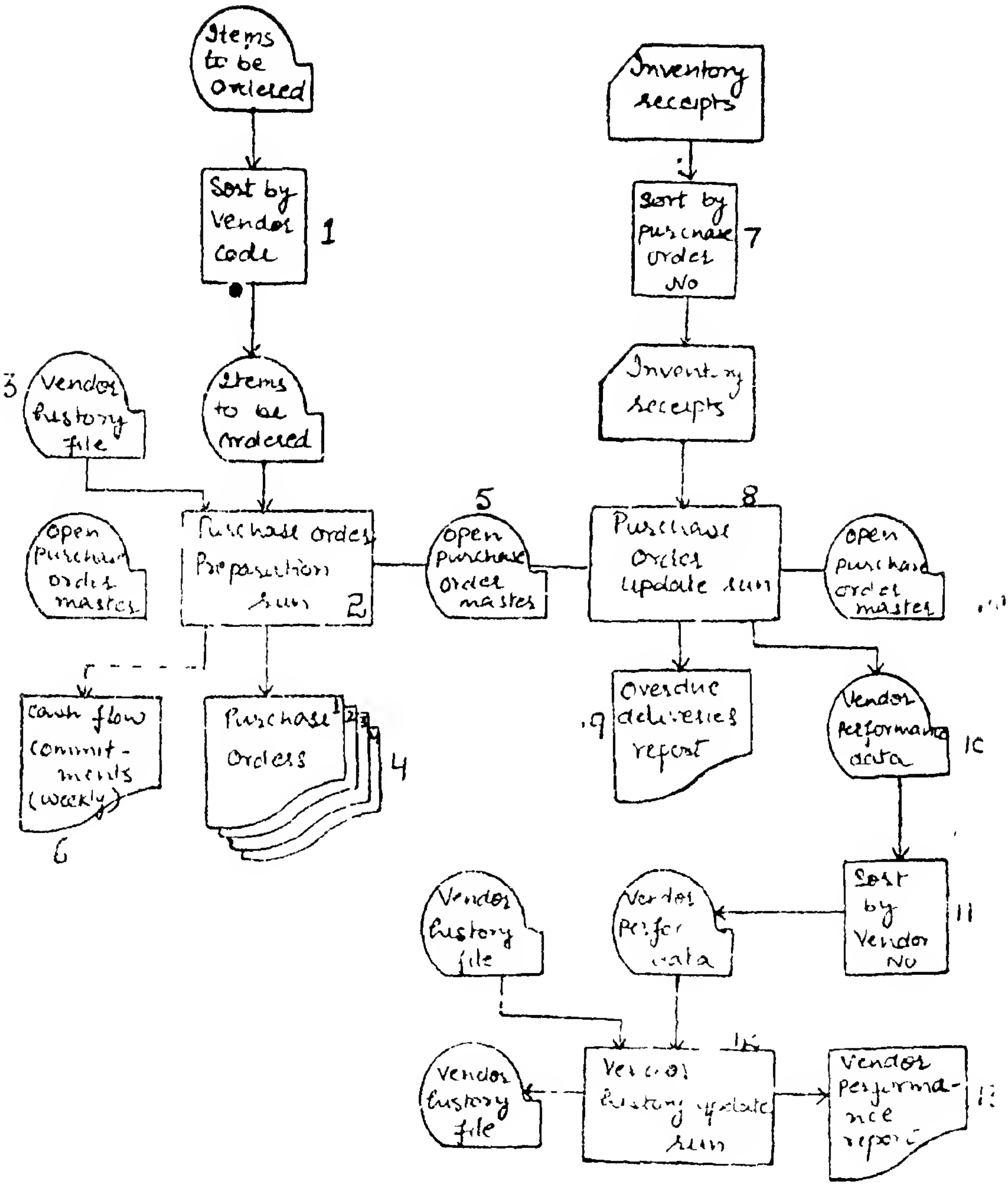


Fig 1

11 Vendor performance tape is sorted by the vendor code number

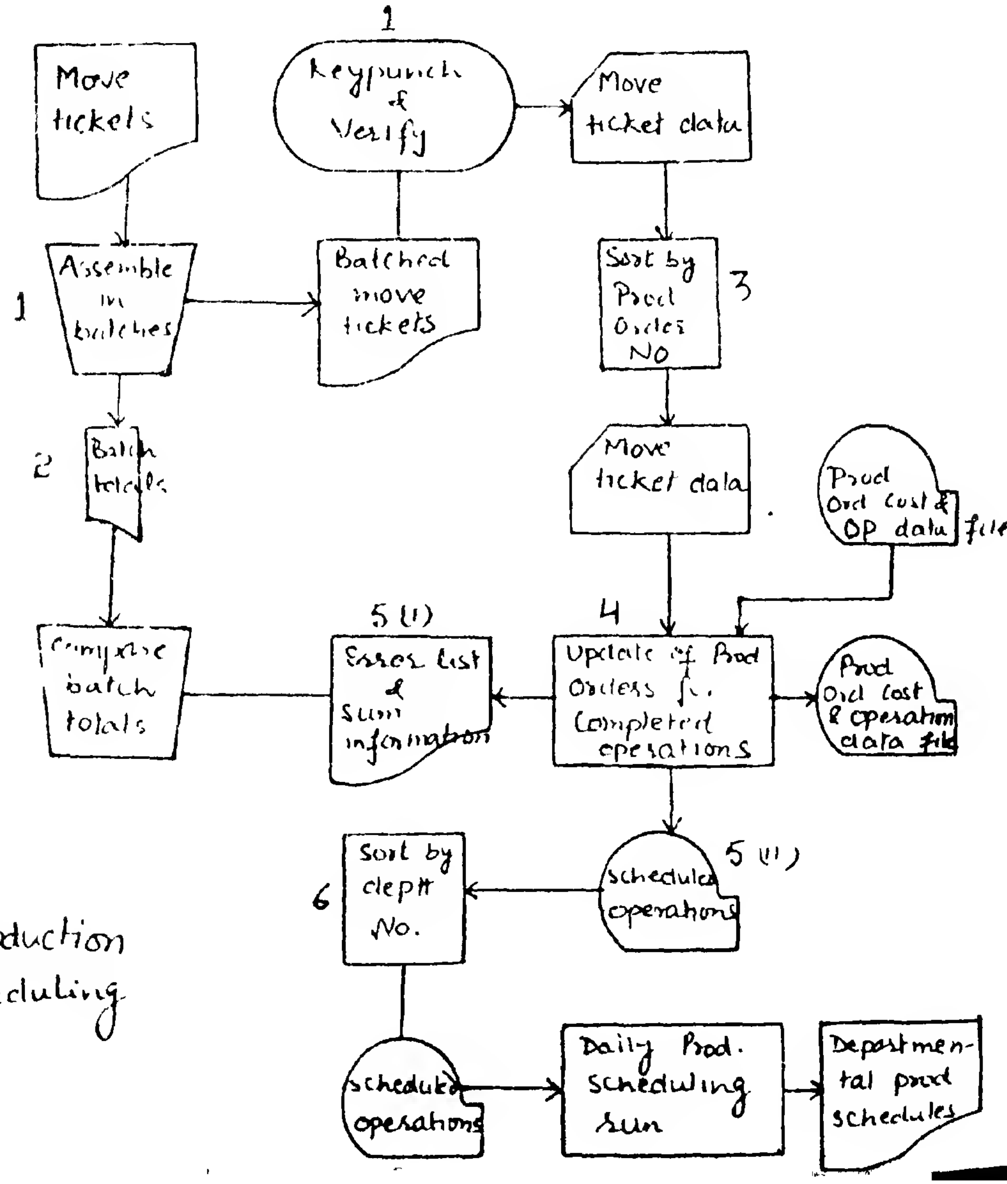
12. This tape is processed to update the vendor history file for the current performance factors

13 This process generates a report of vendor performance, summarizing for each vendor historical and current performance in regard to the factors described

[redacted]

Production Scheduling

One of the major functions of a production planning department is to prepare daily production schedules for all factory production department. The production schedule for each department lists all operations to be performed in the department each day, including the production order number, the machine number, total time required, start and stop time, priority of the order, the location from which the work in process is to arrive, and the location to which it must be sent when completed.



Production Scheduling

This activity is assumed to be performed daily in the computerised system. The basic file used in this system is the "*production order cost and operations data file*" which is the integration of *open production order* and *work in process* data This file contains both physical and cost data for all operations and material connected with the order In addition, it is assumed that standard cost data are recorded on this file The run flow chart is depicted in Fig 2 vide page 34 —

1 The process begins with keypunching verification of move ticket data. These tickets serve as an evidence of completion of an operation at one work station and transfer of the work to the next scheduled work station Move tickets are received during the shift from the factory and assembled into batches at the completion of the shift

2 Batch totals taken include a record count as well as hash totals of production order number and quantity of units completed The batched tickets are then keypuched and verified

3 This batch of move tickets is then sorted by production order number

4 Sorted move tickets data is processed against '*production order cost and operations data file*' to update the record of operations performed on that file Hence current record of operations still remaining to be performed for each production order is maintained

5 Run 4 generates two outputs :

(i) Printed listing of error transactions and summary information

(*ii*) Tape listing of operations scheduled for performance during the forthcoming shift Each record of this tape includes the production order number, operation number, department number, work station number, quantity to be completed, standard time required for each operation, indication of priority of each operation and the appropriate sequence of performance of operations

6 This tape is sorted by department number and processed by a special program which prepares department production schedules for all production departments This program contains data on machine and labour capacity of each department and is usually very sophisticated one

Cost accounting

A run flow chart of daily computerized operations for cost accounting appears in Fig 3 vide page 36 and 37 The chart has been divided into three separate areas : application materials costing, labour costing, and finished goods file update

Materials Costing

1 Input to material costing operations is the material issues tape generated as an output of Inventory control application step 19 Each record on this tape contains the requisition number, production order number, department code to which the materials were issued, quantity issued and unit cost

2. This tape is sorted by production order number

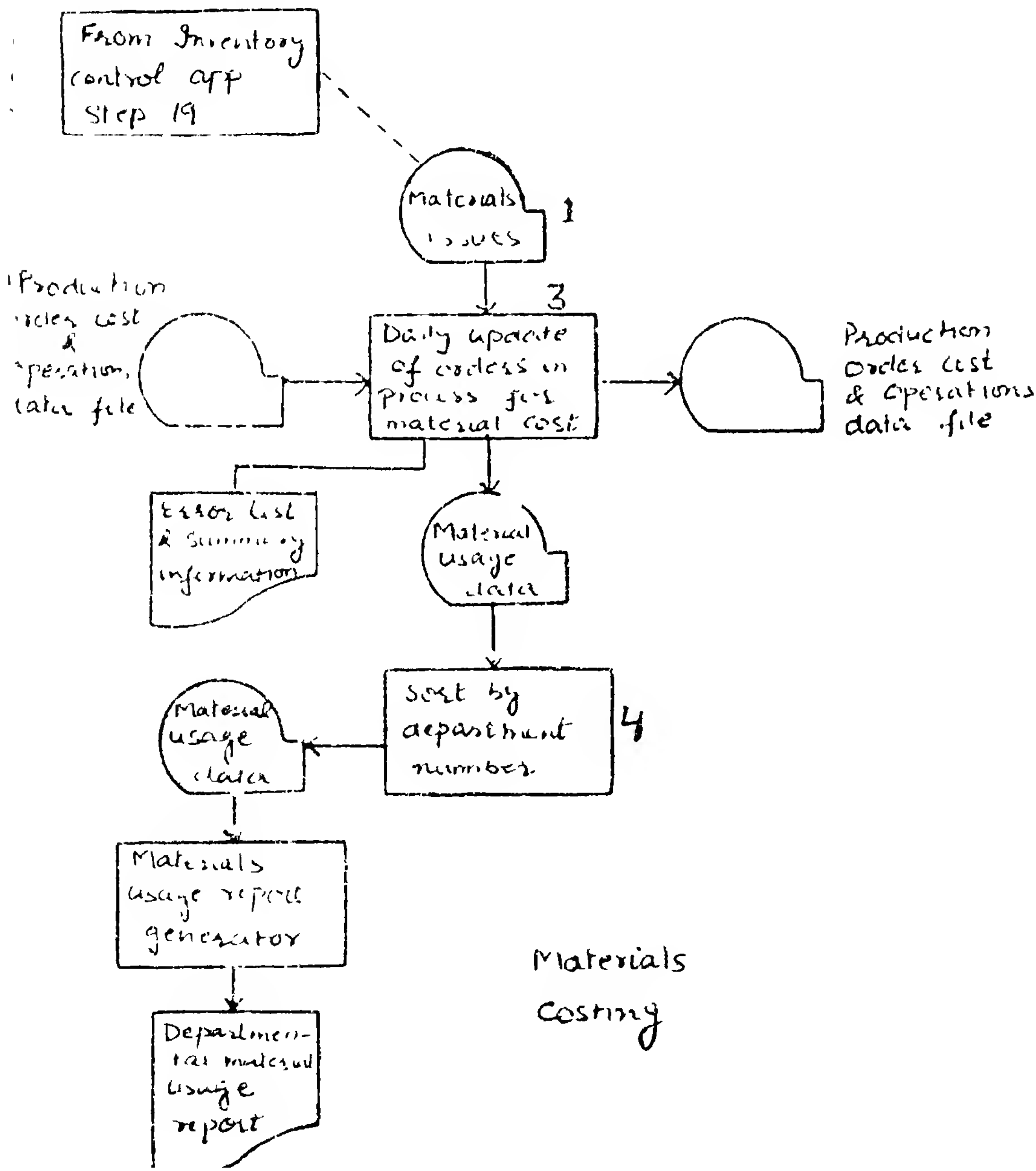
3 Sorted material issues tape is processed against the *production order cost and operations data file.* This computer run updates the material usage and cost records in this file. As an output of the run emerge :

(i) a printed list of error transactions and summary information, which includes the summary journal entry debiting work in process and crediting raw materials inventroy, and (ii) a tape of material usage date containing actual vs. standard usage and resulting cost variances for all completed operations

4 The latter tape is sorted by the department no and processed to generate daily material usage reports for each production department.

Labour costing

5 Input to the system are job time tickets of which a specimen format is given in Fig 5 vide page 38



Materials Costing

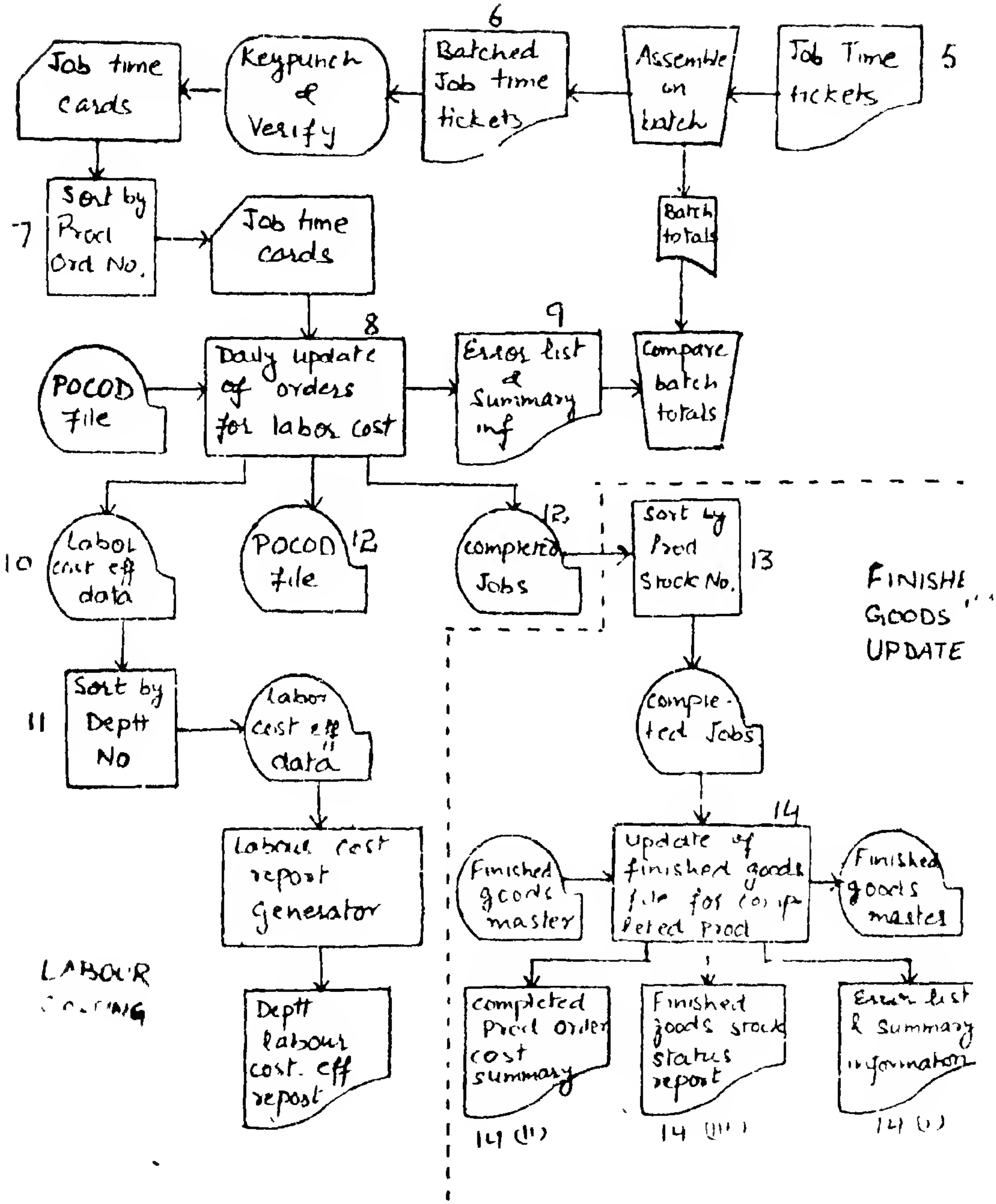

6
Job time cards
Keypunch & Verify
Batched Job time tickets
Assemble on batch
Job Time tickets
5
7
Sort by Prod Ord No.
Job time cards
Batch totals
8
9
POCOD File
Daily update of orders for labor cost
Error list & Summary inf
Compare batch totals
10
Labor cost eff data
POCOD File
12
12
Completed Jobs
Sort by Prod Stock No.
13
FINISHED GOODS UPDATE
11
Sort by Deptt No
Labor cost eff data
Completed Jobs
14
Labour cost report Generator
Finished goods master
Update of finished goods file for completed Prod
Finished goods master
LABOUR COSTING
Deptt labour cost eff report
Completed Prod Order cost summary
Finished goods stock status report
Error list & Summary information
14 (ii)
14 (iii)
14 (i)

Fig. 4

Job Time Ticket			
Date	Dept No	Department name	
Prod Ord	Oper no	Operation description	
Emp no	Name	Hourly rate	
Start time	Stop time	Total hours	Qty completed
Approved by ____ Department foremam			

Fig. 5

6 These job tickets are assembled into batches and batch totals computed which include record count and hash totals of employee number, pay rate, and hours worked Job time records are then keypunched and keyverified

7 Job time records file is then sorted by production order no

8 Next operation is to process this file against production order cost and operation file The time of completion of operations and labour rate data are posted to the production order cost and operations data (POCOD) file. This process also calculates applied overhead costs and records these costs in the file

9 The first output of this process is a printed listing of error transactions and summary information which includes the summary journal entry debiting *work-in process* and crediting the *payroll and manufacturing overheaad control accounts.* It also contains the batch totals which are compared with those compiled manually.

10 A second output of this processing is a tape *of labour cost efficiency data* This tape contains the actual and standard time and standard labour rate of all operations performed by all production employees during the day

11 This tape is sorted by department no. and employee code and is processed to generate a daily cost efficiency report for each production department A specimen of this report is given in Fig. 6.

Daily Labour cost efficiency Report

Dept. No.			*Formeman :*					*Date :*		
Employee		*Order No.*	*Operation*		*Stand Rate*	*Hours*		*% efficiency*	*Total Cost*	
No.	*Name*		*No.*	*Description*		*Actual*	*Stand*		*Actual*	*Stand*
	Dept. Totals									

Fig 6

12 The third output of this process is a tape listing of all cost data for production orders which are completely finished as a result of operations represented by the job time card All records of such orders can be eliminated from the POCOD file, since they no longer represent work-in-process inventory, nor do they require any further production scheduling

Finished Goods Update

13 The completed job tape is sorted by finished goods inventory stock number

14 The sorted file is processed to update the finished goods inventory master for the completed stock The outputs which are generated from this run are

(*i*) Printed listing of *error transactions* and summary information containing summary journal entry debiting *goods* and crediting *work-in process* for the cost of manufacturing the items

(*ii*) Printed report in the form of a series of completed production cost summarise for each product completed Each of these summary reports details all production cost for the particular order

(*iii*) The *status report of finished goods stock* This report is assumed to be prepared once each week rather than once a day

EDP budgets

The cost of a computer is substantial Consequently, it is important to use good management policies when considering a first computer or replacing an existing computer system The smallest third generation computer being offered in India today rents for about Rs 30,000* per month and can be purchased for approximately Rs 15 lakhs However the computer harwdare is only the tip of the iceberg as far as costs are involved Apart from the direct costs of the computer, an installation attracts a number of other expenses, which a novice buyer does not anticipate

How much should a company spend on data processing ? This is a critical question often asked by EDP managers and non-EDP managers alike and is quite difficult to answer. There are a variety of criteria and yardsticks by which to evaluate EDP budgets One can use industry statistics as a base point for measuring expenditures against the average for a company of about the same sales volume within the same industry Obviously this brings up the question of what economic return or benefits are afforded by the computer. If one uses the return on investment tools and economic evaluations, the company with the highest EDP budget could be the most effective Another consideration is the level of application saturation Two companies might be spending equal amounts on data processing While one company has the major administrative and logistical application areas computerized within an MIS framework, the other company only

*Figuers (or rates) quoted here and in the subsequent material are indicative and need not be authentic

has administrative and clerical operations running on the computer Suffice it to say that direct comparison of EDP budget is not the most realistic base point from which to view the effectiveness of data processing

With this disclaimer in mind, let's take a look at a survey conducted by Booz Allen and Homilton' in 1967 who analysed the average expenditure on data processing of 108 U S Companies The average expenditure over all companies was found to be 0 51% of sales With this standard, an Indian company with a turnover of Rs 1 crore should spend Rs 57,000 per year on data processing It must be realized that in the U S a large fraction of the expenditure is for applications that replace clerical labour In India, these applications are not so important due to the lower manpower costs Thus, even if computers were applied in India with the sophistication of the west, the expenditure should be considerably lower This study also showed that the percentage of sales spent on EDP increased with the number of years of experience the company had with EDP This is not surprising if viewed from an organisational point of view The computer department, once established, tend to proliferate and expand to more sophisticated and expensive hardware, larger and costlier staff and multifarious applications

Now let us study a recent analysis of data processing budgets made by Datamation Magazine Although these indicators are broad averages, they are interested in that a company can at least determine whether it is spending more or less than the industry average. These figures can be used as a starting point to a more comprehensive evaluation of data processing effectiveness The average data processing installation spends the following percentage of its budget on the functions listed.

Function	Percent of total budget
Personnel Expense	52 0%
Hardware & Maintenance	38 0%
Media, Supplies & Accessories	7 0%
Packaged software	1 0%
Communication Lines	0 7%
Outside Personnel Services	0 6%
Outside Data Processing	0 4%
Other Costs	0 3%
	100 0%

It emanates from the above table that the computer hardware constitutes only a small part of total cost A large initial investment is involved in starting a computer department Even if the computer is rented, a one-time installation charge has to be paid to the manufacturer This covers import duty, if all or part of the computer is imported, freight, insurance, etc Apart from the direct costs of the computer, the computer room has to be specially prepared with air-conditioning and false flooring Some furniture has also to be acquired for work facilities Typical initial costs for a small computer installation are shown in table 1 1

Typical initial costs for a Small, Business Computer Installation.

Table 1 1

	Cost	Typical magnitude
1	One time charge covering freight, packing, insurance and customs and excise duty	Rs 90,000 to Rs. 4,00,000*
2	Site preparation including airconditioning false flooring and accessories for the computer room (storage racks, tables, card trolley, large waste bin etc)	Rs 45,000 to Rs. 1,00,000
3	Purchase of magnetic tapes and disks	Rs 30,000 to Rs. 50,000
4	One time change for off-line card-punching equipments (12 punches and 8 verifiers)	Rs 10,000 to Rs. 25,000

In addition to the initial cost, there are number of recurring costs which a computer installation has to bear These costs include rental for the computer and card-punching machines, overheads for the floor space, electricity, air-conditioning etc The cost of cards and stationery and a large expenditure on operations and programming staff are other recurring costs Table 1 2 shows typical recurring costs for a small computer installation

Typical recurring costs for a small computer Installation

Table 1·2

	Cost	Typical magnitude per month
1	Computer rent	Rs. 30,000 to Rs. 60,000
2.	Cards	Rs 5,000 to Rs 12,000
3	Stationery	Rs 5,000 to Rs. 10,000
4.	Other ancilliary computer equipments like printer ribbon	Rs 2,000 to Rs 3,500
5.	Purchase of magnetic tapes and disks	Rs. 3,000 to Rs 15,000
6.	Electricity (for computer, air-conditioning, card punching equipments and lighting etc)	Rs 3,000 to Rs 3,500
7.	Maintenance of air-conditioning.	Rs. 1,000 to Rs. 2,500
8.	Floor rental, maintenance and other overheads	Rs. 10,000 to Rs. 20,000
9.	Staff costs (management, systems, programming and operations) and overheads	Rs. 25,000 to Rs. 50,000
		Rs 84,000 to Rs. 1,76,500

***Figures quoted above are merely indicative and need not be authentic.**

These charges assume a rented computer If the computer is purchased, then the one time charge in table 1 1 will increase by the purchase price In table 1 2, the rental will be replaced by a much lower maintenance charges Some manufacturers also charge extra for training courses for customer personnel and for technical manual supplied to them These costs have not been included in table 1 1, 1.2 above

Service Bureaux

We have discussed above that installing inhouse computer is a costly affair Thus a small user faces the problem of justifying such a large initial and recurring expenditure Also, he may not want to get involved in the setting up and staffing of a computer department until a few applications have shown results For such a user, a number of consulting and service bureaux exist that provide computer time and services to clients on a fee basis This service is available either on an as-needed basis or on a continuing contract basis

There are a large number of such service bureaux operating in India Their charges very from Rs 150 to Rs 1200 per hour over a wide range of computer capability The quality of service which is offered by such centres varies widely. Some bureaux are run by companies who have acquired a computer for their own use and rent out the spare time. Their rates are low and times inconvenient and customer service non existent The user has to arrange for his own machine operator, tapes, disks, cards and stationery This type of bureaux is good for persons having experience in EDP and wanting cheap computer time On the other hand, some service bureaux provide operators, cards, tapes digks etc if needed Such bureaux are customer oriented and it is better to start the work with such firms, though they are somewhat more expensive than the spare-time centers

Alternatively, the user can entrust his entire job to a service bureau Charges of such bureaux vary considerably depending on the responsibilities under taken and the standing of the firm. The charges are made up of fixed cost for systems development work and also variable cost which depends on the volume of the data processed. For example, the financial accounting for a company with a turnover of about 1 crore can be done for less than Rs 4000 a month These figures are of course, merely indicative, costs for individual system will depend on the particulars systems requirements and service bureaux

Financing the use of computers

After deciding which computer to acquire, the basic configuration and a general plan for its expansion, a further decision has to be taken as how to finance the use of a computer. There are three major approaches to financing the use of computer, renting or leasing from the manufacturer , purchasing the computer ; or leasing it from an independent leasing firm. We will now discuss these three alternatives in brief.

Renting

This is the most common financing method Until a few years ago, most computers were rented on the assumption that technological progress would soon make them obsolete In renting method, the ownership of the equipment rests with the manufacturing company and hence the client is relieved of the risk of ob solescence The rent is charged on a monthly basis All basis for rent is the re-covery of purchase cost in three to four years The rental price also includes maintenance for using each piece of hardware There are differences in the rental agreement that different manufacturers use, but the most common agreement calls for a base rental for a certain number of hours (say 200 hours) of actual use per month This represent in 8 hour shift per day for an average month of 25 working days Hours above 200 are usually charged at a lower rate (e g 30 per cent of the implied hourly rate for the first 200 hours)

The user can most often cancel a standard rental agreement on fairly short notice (usually about two or three months) Manufacturers offer reduced rental rates for companies that are willing to sign *binding*, long term leases (three to five years) for hardware Users may also get the hardware on rent with options to purchase

The major advantages of renting are, the absence of a large capital outlay, the manufacturer's assumption of all maintenance of hardware, and the removal of the risk of hardware obsolescence Of course, the latter risk is not entirely removed, since the user has large amounts of funds invested in such things as site preparations, employee training and systems and programming work that may be partly 'lost' if the computer is replaced by a different manufacturer's computer

The major disadvantage of renting a computer is the higher total cost Manufacturers generally base rentals on a five-year life for the hardwhere, but modern computers are more likely to have productive "lives" of closer to eight or nine years

Purchase from the Manufacturer

The user buys the data processing equipments and the equipment becomes his own property The major incentive for purchasing a computer is the savings to be had over the life of the equipment There is also a tax advantage because of an investment credit provision of the income tax law Also, managers have begun to realize the enormity of the problems associated with changing computers A purchased computer that has been performing satisfactorily for say, three years has already paid for most of its cost and there would seem to be little reason for changing over The operating costs of computers are small compared to their purchase price or rental Even if new computers are not introduced the computational capabilities still exist, and one can carry on the work for many years until the requirements substantially expand or alter Besides, the cost and

retraining involved in a change over are avoided In very special cases the new equipment may provide the possibility of radically different and more effective systems and this may justify replacement.

Purchasing, rather than renting equipment can also carry some financial benefits In India, a development rebate is allowed in the form of a concession on purchased equipment provided some other conditions are met Also, import licences and investment grants, in the form of Government loans at low rates make purchase look more attractive.

On the other hand, the problem with purchasing is that the purchase assumes risks related to obsolescence, hardware reliability, and maintenance of the hardware. An organisation may either arrange for a separate contract for maintenance of the equipment, or, if the computer department is too large, the maintenance personnel of its own The latter method involves stocking of components and all the associated attendent problems Further, a plan for gradually increasing the power of the configuration, however, discourages purchase Rented equipment is generally much easier to replace Companies dealing in used computer equipment will, however, buy used peripherals and replace them by faster ones at very good terms Another solution is to purchase the part of configuration that is supposed to be usable for, say, three or four years like the central processor and on rent the rest that is expected to be replaced sooner. However, in any case, the break even point is dependent on such factors as number of shifts that the hardware will be used

Leasing from third-party Organization

A company that has examined the obsolescence and reliability problems and has concluded that purchase is the best alternative, may not have funds available, or, perhaps using the funds for purchasiag computers will restrict future sources of funds Under these circumstances, the possibility of leasing from an independent leasing company should be examined

A lease typically provides for the purchase by a leasing company of the specific computer configuration that the user desires The leasing company, then leases it to the user Lease agreements vary, but usually they have provisions such as the ones discussed now The operating lease generally covers a period of two to five years An operating lease can be cancelled, or it terminates when the lease payments equal to purchase price Under the lease principle, an agency, or a third party assumes a longer leasable life for the equipment and offers it at a rental lower than that offered by the manufacturers Another form of lease, called the financial lease, covers a longer term, and can not be cancelled by the lessee The lessee makes the lease payments over to lease period although such payment have exceeded the purchase price of the equipment. The financial lease method guarantees the full return on the equipment and, for this reason, many leasing companies prefer financial lease to operating lease. In a financial lease, the monthly rental

payable by the lessee includes a portion towards interest charged by the lessor In other words, the lessor charges the lessee for providing equipment as well as the funds

The advantage of leasing is that the total lease payments are usually lower than the total payments on the rental basis The user can choose an equipment and arrange for its lease by a leasing company This will allow him to spread his investment over the lease period The main disadvantage is that the user has to enter into a maintenance contract with the manufacturer and cannot hold the leasing company responsible for its maintenance

Some user buy some devices while certain other devices are rented or leased Such a combination offers increasing flexibility to the user, he can virtually choose different manufacturers for different items of equipment, such as CPU, input/output peripherals, terminal devices, controller, etc, and acquire them on different basis, *e g*, CPU on rental basis, input/output peripherials through lease and terminals by outright purchase This has become possible with the emergence of those specialising in the manufacture of controllers

Outside Consultants and Service Bureau

If an organisation can benefit from computer processing but cannot justify spending money on keeping its own computer, a data processing service bureau may be the answer These service centres located in most cities, are businesses that provide data processing services to the users Their service can range from processing on unit record equipment to sophisticated computer systems operating on a time-sharing or batch processing mode A service bureau can provide 'total' service to a user This includes system development, programming, data preparation and execution of jobs on the equipment The fees are charged separately for each of these tasks.

A user may avail himself of the services from a service bureau in place of manual processing in case computational or data processing complexities can justify the use of facilities from a service bureau Service bureau facilities are provided by many leading computer manufacturers Some organisations which have installed computer system also offer facilities during their spare time In some cases, these organisations allow the use of a computer and its associated peripherals requiring the users to have their own programs, operators and other supplies (*e g* cards, stationery, tapes, etc). These are organisation established as service bureaus, having systems and programming and operating staff Some of these service bureaus specialise in the type of applications and also by industry

In a typical service bureau, three distinct functions can be identified (*a*) selling, (*b*) system development and programming, (*c*) production The salesman makes a preliminary study of the user's requirements and estimates the computer time, recurring and non-recurring, as well as the efforts required for system development and programming. Based on these estimates, he makes a quotation

which will cover both one-time fees for the system development and recurring fees for regular production runs The salesman may involve the systems analyst at the time of preliminary study if the system is a complicated one After the client or the user has accepted the quotation, the service bureau proceeds with the work of detailed systems design and programming Once th systems and programs are tested, the user's files are converted to the computer system and the results are observed for some time to detect any system or program faults The outputs are examined by the user's internal audit staff or any other representative to check the completeness and accuracy of the results produced by the system.

Instead of going for complete service from a service bureau a user can buy time on hourly basis Under this arrangement, (called block time) the user must have his own systems and programming personnels, develop his own programs and equip himself with all the supplies In other words, he must have all these things required in an in-house computer system, except the computer equipment Thus, the selling organisation provides the hardware on a rental basis for a fixed time, no services or programs are provided

The advantage to the user is that this method gives him experience in implementing the computer-based information system covering one or more areas in his organisation The use of block time allows the user to have absolute control and security over the data and programs An organisation intending to install a computer will profit by experience it acquires by the use of block time

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THE INSTITUTE OF CHARTERED ACCOUNTANTS OF INDIA

STUDY MATERIAL F.S P S A & DP—1
Combination—B

FINAL COURSE (N)

SYSTEM ANALYSIS & DATA PROCESSING

STUDY—I

Contents ·

[redacted] ... Paper has been prepared by the Board of Studies of the

What is Data Processing

The word "data" is the plural of "datum" though "data" is commonly used as both in the singular and plural forms Data can be defined as any fact, observation, assumption or occurrence They are useful knowledge or information of value to an individual or business They are factual material used as a basis for discussion, decision, calculation, or measurement Data are compiled to form reports, letter, figures, or documents etc

Data can take the form of numerical or alphabetical characters or special symbols (viz =, +, %, Rs etc.) Some important data definitions are given below

Digit A simple numeric character 0, 1, 2, . 9.

Character A single alphabetic, numeric or special symbol

Data item (field) A set of characters which are used together to represent a specific data element *e g* , a name item contains the alphabetic characters in a name, and an amount item contains the numerics in an amount

Record: Consists of a group of data items related to an object of data processing *e g*, a payroll record for an employee contains such data fields as name, age, qualifications, sex, wage rate etc

Data file is a compilation of related data records maintained in some prearranged order Files are usually created function wise *e g* , a payroll file might consist of 1000 individual employee's data records and the records might be arranged in file by employee number

The relationship of a character, data item, record and the file is shown in figure—1 below

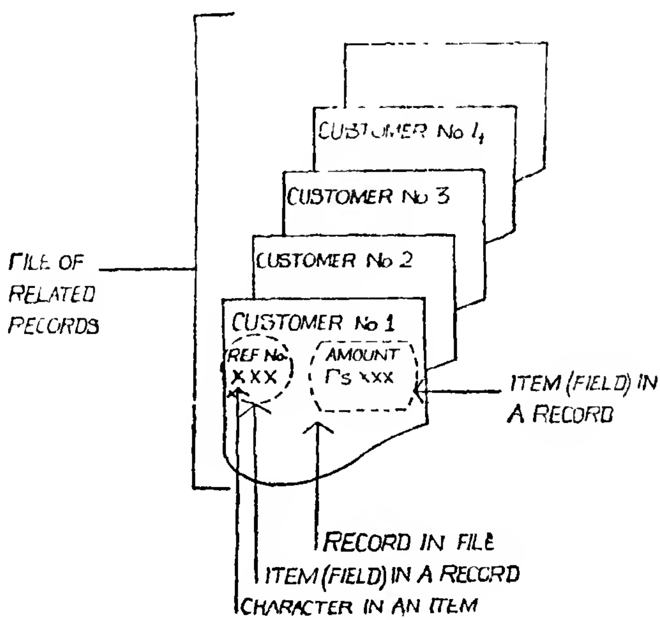

Fig. 1

Information Information is organised or classified data so that it has some surprise value to the receiver. For example, the daily production figures are raw data for the managing director to whom they have little value as such If however, they are aggregated for the month and the shortfalls computed against the plan, it becomes information that would prompt him to apply some corrective action so that production rate is accelerated to match the plan

Data Processing is the restructuring, manipulating, or reordering of data by people or machines, to increase their usefulness and value for some particular purpose Data processing can be performed manually with the aid of such simple tools as paper, pencil and filing cabinets or electro-mechanically with the aid of unit record machines or electronically with the aid of a computer which is the major concern of this and following study papers

Regardless of whether the system used to process the data is manual, mechanical or electronic, certain fundamental operations must be performed. These operations are :

1 Recording
2 Classifying
3 Sorting
4 Calculating
5 Summarising
6. Reporting

Recording : Recording is the transcribing of data into a permanent form *e g*, writing the quantities of goods received on manual ledgers or Kardex cards. As we shall see later, in electro-mechanical or electronic data processing, writing by clerks on ledgers is replaced by codification on machine readable file media

Classifying :— Classifying involves grouping of like items or transactions Data are generally classified according to a code in the form of an alphabetic or numeric abbreviation In charting accounts, for example one may assign numeric codes to the various classes 1. asset account 2. liability account, 3 proprietorship account, 4 income account 5 cost account or 6 expense account. There could be further sub-classes within each class For example, the asset account may have a sub-class of CASH which may be assigned, the code 11 to signify that it is the first sub-class. Thus classification may be carried down to the lowest level sub-classification

Sorting :— Sorting is the arranging of data or transactions in an ascending or descending sequential manner Sorting may be by numeric or alphabetic code For example, the Indian States may be arranged in the descending order of population Like-wise, the sales vouchers may be sorted by voucher number The data field by which the transactions are arranged in a sequential order is known as the key; thus voucher number constitutes the key in the above example As an example of alphabetic sorting, the students in a class may be arranged alphabetically in the attendance register The dictionary in which words of a language are arranged alphabetically provides another example At times, sorting may involve more than one field For example, you may want to have all the cities in Andhra in alphabetic order, then all the cities in Assam, and so on This allows ease in reading a report. Arranging data or transactions in this type of order is known as major and minor sort The category that is the sub-division is known as the minor field, while the other field is known as the major field In the above example, the cities are the minor field, and the states are the major field

Sometimes, you may have to sort three fields to produce desired results. You may want voucher number within date within account head Or you may want villages within districts within states Thus there are three sorting fields : minor, intermediate and major

The procedure for this type of sorting is to sort with the minor field first followed by sorting with the intermediate field and finally sorting with the major field

Calculating A calculating is adding, subtracting, multiplying or dividing data to produce usable results Updating of stock balances or computing interests on amounts provide examples Advanced calculations (viz square roots) are also included

Summarising Summarising involves the consolidating of data emphasising main points and tendencies It usually involves deriving totals, results percentage etc

Reporting The summarised data, which would have been derived after recording, classifying, sorting, calculating etc constitutes information and is presented to the management in the form of reports, reporting being the ultimate of all-data processing activities All these steps can be performed manually as well electro-mechanically or electronically as to be discussed at a considerable length in the following pages Graphs, histrogrames, etc are also included

The purpose of this study is to provide an understanding of the meaning of data processing systems We have discussed data and its processing in this regard. Let us now take up the word "system"

A *System* can be defined as a group of interrelated components that seeks the attainment of a common goal by accepting inputs and processing them into outputs in an organised process Thus, a production system accepts raw materials as inputs and produces finished goods as output with a goal of, perhaps, maximising return on investment and the components of this system men, machines and materials interact with one another in an organised manner

A *data processing system* can be viewed as a system that accepts data as input and processes it into information as output a and is sought to be depicted in Figure 2 The components in the manual data processing system would be men, *i e*, the clerical staff who perform data processing in an organised way with the goal of producing meaningful information Men would be using such simple tools as paper, pencils and filing cabinets Mechanical data processing systems utilise such mechanical devices as typewriters, calculating machines and book keeping machines The term non automatic data processing systems includes both manual and mechanical data processing systems

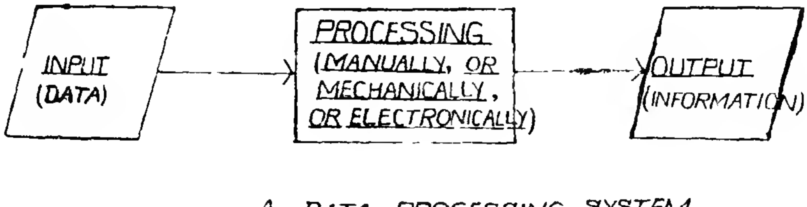

Fig. 2

Automatic data processing implies the use of machines for data processing. It could either be performed electro-mechanically by the unit record method or electronically by the computer

Unit record method The *unit record method* relies on human operators and electromechanical devices to process data It is sometimes called the EAM electrical accounting machine) process In the unit record, or EAM system, data are stored in coded form on punched cards Data from each transaction are punched on a separate card (that is why they are termed as unit records) These unit records are manipulated by unit record machines (discussed in appendix I) to input, process, and output the data

The advantages of using this method of data processing include simplicity, relatively low cost of equipment and reliability The system uses ordinary punched cards for storing and transmitting data Punched cards are easily filed, stored However, unit record accounting is practical for small to medium size business firm. Its accuracy and speed far exceeds that of the manual system.

Electronic data processing -- Computers were a natural and logical outgrowth of the unit record system. With the grwth of the company, it becomes inefficient to perform data processing through punched cards EDP offers better method of data processing at a low cost per unit processing as it relies on the computer and principles of electronics for processing data Data to be processed by this method can be recorded on one of the several media for input Once the data are entered in the computer, they are manipulated by electronic means Calculations are performed at a tremendously high speed with unsurpassed accuracy and output is accomplished on high-speed printers and other output devices

For large firms, the computer is the most practical means for processing data. Minicomputers and microcomputer bring these advantages to the smaller firms also.

A knowledge of the role, capacity, and, limitations of data processing systems is of value to everyone In fact, it is almost impossible to go through a day without encountering data processing in some form Government agencies, schools, hospitals, courts and law firms, and business of all sizes use data processing in some way The character and development of modern business is influenced by data processing Business decisions depend on the quality and accuracy of available data

Management Information Systems (MIS)

The terms data processing system and MIS are often and wrongly used interchangeably but there is a basic distinction between the two. An information system can be defined as a system which collects and processes data and disseminates information in an organisation. It is thus easy to see that the data processing system is a part of MIS (Fig 3). In other words, it is a sub-system of the management information system Information systems are organisational and application oriented. The marketing information system, for example, caters to the information needs of the marketing department of an organisation The management information system provides the necessary information to the managers and supervisors at various levels to enable them to discharge their functions of organising, planning and control and decision making A data processing system, in contrast, is hardware oriented i e., a manual data processing system, a mechanical data processing system, and electronic data processing system, etc. By reading these study papers you are gathering data, and you are storing information i e , you are engaged in data processing or you can

be considerd as one engaged in data processing systems that make up the education information system Thus, in MIS, electro-mechanical or electronic equipment

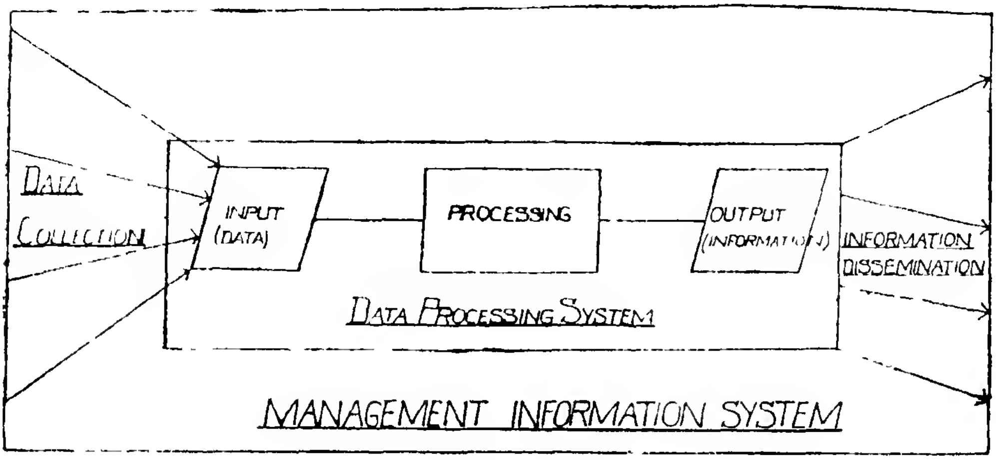

Fig. 3

constitutes its sub-system and in addition men may be engaged in an organised way in collection of data and dissemination of information distilled and derived from data by the aforesaid system MIS may then be formally defined as an integrated man-machine system for providing information to support the operations, management, planning and control and decision making functions of the management of an organisation

Three levels of planning and control and their information needs. One of the management's major duty is planning and control. This activity of planning and control is carried out at three levels . top management, middle management and supervisory level whose information needs naturally vary and the extent to which data processing can be automated also vary Let us then discuss these three levels in a little detail since computer, being a data processing sub system of the MIS, is acquired mainly to support the planning and control activities at these three levels in any business organisation For fuller details, however, the student should refer to study VII

Level I : Strategic Planning Strategic planning is undertaken to set the objectives for an organisation Its primary concern is to deal with major threats and opportunities for which obviously there cannot be a time table , therefore, it is not periodic and is only undertaken as and when major threats or opportunities arise. Its time horizon is long, usually five years to 15 years and is the domain of the top management and the corporate staff Though internal capabilities and the progress of an organisation are appraised its major thrust is towards the external environment viz , competitors, political situation, economic undercurrents, etc The task of collecting the necessary information is not continuous but just a one time affair. Even in highly computerised organisations these tasks remain eminently human tasks The computer will help in many areas but it is more difficult to provide computerised information or assistance at this level for two reasons. First, the level of thinking is often complex and ill-structured and second, the needs are unpredictable

Strategic Planning tasks

Determination of markets

Longrange forecasts

Directing research

Choosing new product lines

Setting financial policies

Setting personnel policies

Level Two Management control is the process by which managers assure that resources are obtained and used effectively and efficiently in the accomplishment of organisation's objectives It is characterised by the following :

(i) Its time horizon is usually one year and it concerns functional planning (and allied budgeting) viz , marketing, production purchasing, finance etc It is undertaken within the framework of strategic planning and the plans that are complied are supposed to support the objectives set therein

(ii) Unlike strategic planning, it is a continual activity , it is periodic and rhythmic and concerns the middle management. Since the external environment does not figure so importantly, there is much less uncertainty than at the strategic planning level and consequently the plans are fairly realistic

(iii) Appraisal is constant and not too difficult.

(iv) The emphasis is on both planning and control as more or less a continuous activity unlike in the case of strategic planning where the control is virtually non-existent Both planning and control are in the same hands. Basically, the management control tasks (listed below) remain human tasks but computer can provide a variety of information and analysis to assist the manager to compile realistic plans and exercise stringent control.

Management control tasks

Setting work budgets

Planning Working capital

Determining prices

Choosing suppliers

Sales management

Annual forecasts

Production scheduling

Maintenance management

Routine personnel administration

Formulating rules for routine operation.

Level 3 Operational Control : To see that profits are not lost through inefficient handling of men and machines, it is necessary to closely watch costs and the volume of work In other words, annual operations of production and distribution have to be kept under a close surveillance The control activity at this level is known as the operational control In operational control, the emphasis is on day to

day or weekly planning and control There is hardly any uncertainty and the plans that are compiled are in exact terms. Standards are set in exact quantitative terms and performance is appraised against these standards This enables an exact measurement of efficiency Standard costing is of immense value at this level. The decision-making involved in planning and control at this level tends to be repetitive For example, setting the economic order quantities, reorder levels over thousands of inventory items are repetitive tasks to be carried out daily, weekly etc Obviouly, it would be a tremendous task manually to derive these *ab initio.* The decision-making can be progammed for these in the sense that decision rules for these can be derived once and for all by means of operational research techniques. etc , and the decision rules entrusted either to junior staff or to the computer. Therefore, the data processing involved is quite amenable to automation. The actions at the operational control level can be likened to reflex actions in a living creature that can be handled by a low level mechanism that do not involve thinking in the central brain.

Routine Operations or Reflex Actions :

Recording of customer's orders.
Break down into parts and sub-assemblies.
Determining net requirements of parts and materials

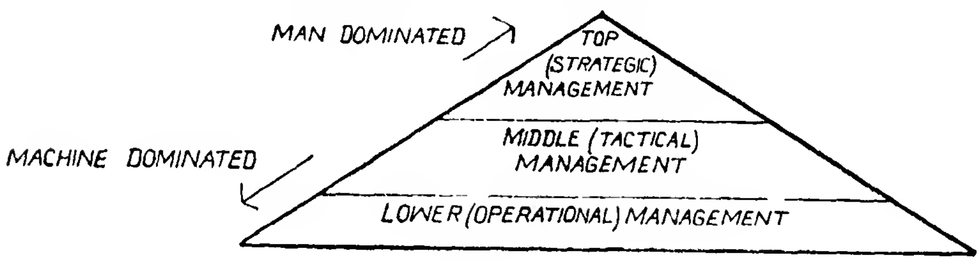

Fig 4

Shop floor data collection
Preparation of work tickets
Maintaining inventory records
Reordering parts and materials
Production of purchase orders
Goods receiving
Payment to suppliers
Accounts payable
Goods shipping
Invoicing
Accounts receivable
General ledger
Budget accounting
Costing
Payroll
Quality control.

MIS can also be visualised as comprising such functional information sub-system as finance, marketing, manufacturing, purchasing, research and development, etc Marketing information system, for example, would generate the information necessary for carrying out the marketing operations Figure 5 depicts the functional classification in the MIS pyramid.

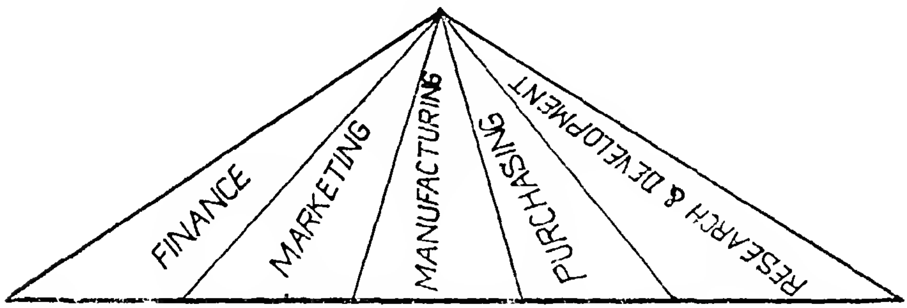

Fig 5

Figure 6 superimposes the 3-tier planning and control structure of figures 4 on to the functional structure of figure 5 There would be as many plans as the number of sections in this figure For deriving each plan and exercising control against it the MIS would provide the support by generating the necessary information.

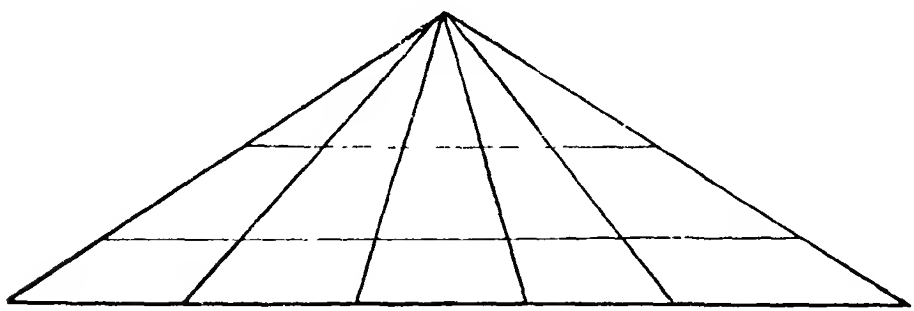

Fig. 6

Figure 7 depicts roughly how the information is generated for the three levels of management planning and control activity. Obviously, the computer would be put to use in the data processing for such information generation.

Introduction to the computer : The electronic computer was first created as an exotic calculating device designed to solve arithmetic problems for scientists and mathematicians. Today it has become an indispensable tool, helping to shape the society it serves. Electronic computers are now used in supermarkets, hotels, banks, hospitals, factories, government offices, schools and, research organisations.

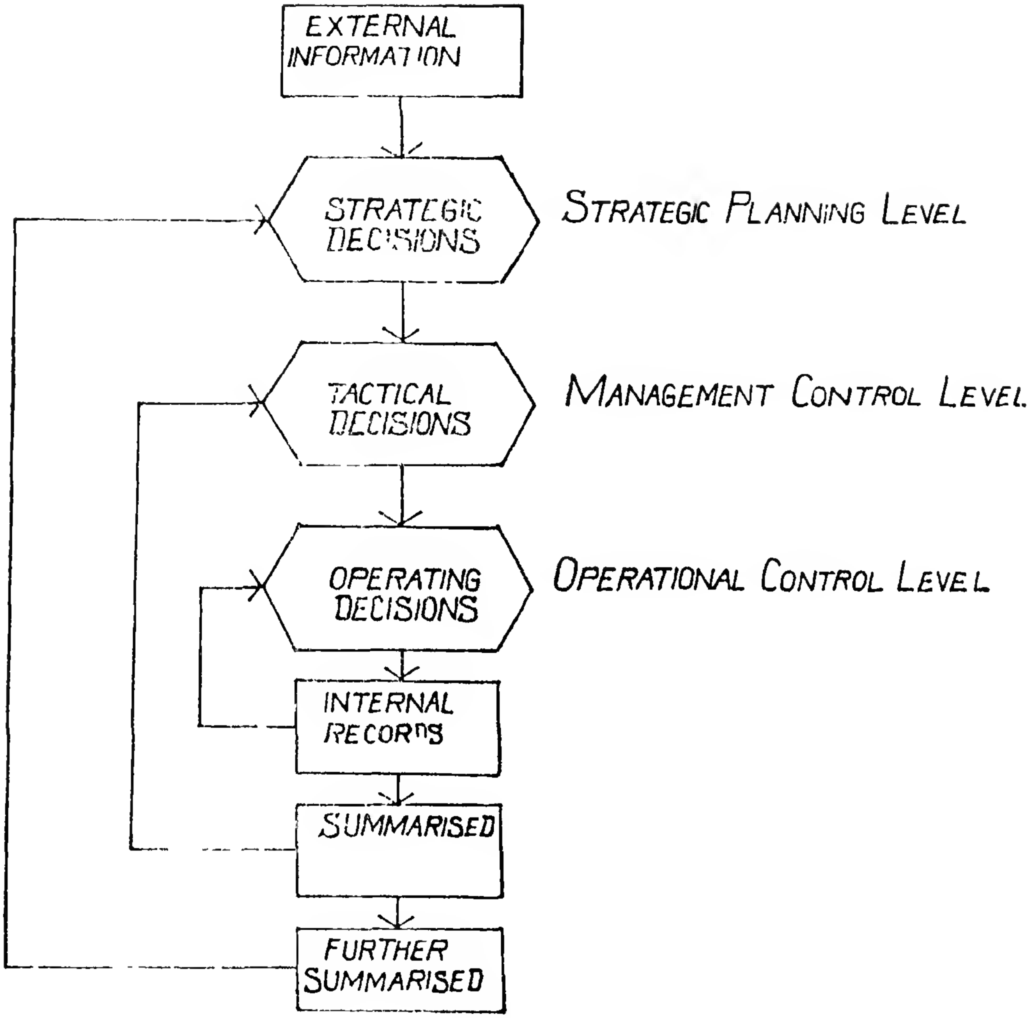

Fig. 7

What are computers ?

The term "computer" can logically be applied to any calculating machine. However, in common usage, the definition of a computer has become more limited in contemporary usage We now define a computer as an *electronic data processing device* capable of receiving input, storing sets of instructions for solving problems and generating output with high speed and accuracy. Computers are composed of switches, wires motors transistors, and integrated circuits, assembled on frames. The frames form components such as typewriters, line printers, card readers, card punches, magnetic tape drives, and central processing units. These components are wired together into a network called a computing system, often called, a *computer*

The size of computer system varies from small hand-held devices with limited capacity to huge machines occupying several large rooms. They may be constructed as a single integral device, or as many separate pieces of equipment remotely connected but functioning as a unit The individual parts may be located within the same building or scattered across the country, connected by telephone lines.

Computers can perform a variety of mathematical calculations, ranging from simply adding and subtracting to solving complicated math equations that involve thousands of steps. They can repeat a complicated calculation million of times without error They can also print out whole paragraphs of text, write letters, draw pictures, or plot curves and draw graphs They can sort data, merge lists, search files and make logical decisions and comparisons. However, computer is devoid of any original thinking It does nothing what it is not told to do Computer is provided with a set of instructions by man professionally known as the *programmer* These instructions or statements are recorded in a form that can be understood by a computer These instructions direct the computer through a series of steps to solve a problem. This set of instructions is called the program which controls all operations of the computers In other words, a computer obeys man in all its doings and hence any notion of mysteriousness about the computer should be dismissed It is no doubt a revolutionary machine but the fact remains that it is a machine sub-servient to man.

Types of computers . Computers are distinguishable by the way in which they receive and process data Some systems process data of a continuous nature, others process numeric values represented by discrete electronic pulses

Analog computers process data input in a continuous form Data such as voltage, resistance or temperature and pressure are represented in the computer as a continuous, unbroken flow of information In engineering and scientific applications where quantities to be processed exist on waveforms or continually rising and falling voltages, pressures and so on, analog computers are very useful For example, they are used to control processes in the food, and the petroleum industries As the measurement in an analog computers are carried out by a few single purpose devices, the analog computer offers low cost and ease in programming The main disadvantage of an analog computer is its accuracy factor, and the limited storage capacity Hence they are not suitable for processing business data

Digital computers on the other hand count and accept letters or numbers through various input devices that convert the data into electric pulses, and performs arithmetic operations on numbers in discrete form In addition to performing arithmetic operations, they are also capable of (1) storing data for processing, (2) Performing logical operations, (3) editing or deleting the input data and (4) printing out the result of its processed routine at high speed One of the main advantage in the use of digital computers is that any desired level of accuracy can be achieved by considering as many places of decimal as is necessary and hence are most suitable for business applications The main disadvantage of digital computers is their high cost and the complexity in programming

Since we are concerned mainly with business data processing in this and the subsequent studies, *computer* will now be understood to mean the *digital computer*

Fundamentals of computer system : A system as mentioned earlier, is an assembly of methods, procedures or techniques that interact in a regulated manner

to form an organized whole for accomplishing specific functions. The human organism is one of the nature's most perfect systems The components of this system could be called input, processing, output and memory Input is accomplished through five senses viz ear, eyes etc Processing and memory occur in the brain and output through various actions However, people are also capable of goal setting and self direction

Computers are also systems, as such, are similar in many ways to the human organism. Input, processing output, and storage are also elements of the computer system The system is capable of logical deductions and to a limited extent self direction The block diagram in Figure 8 illustrates these functions

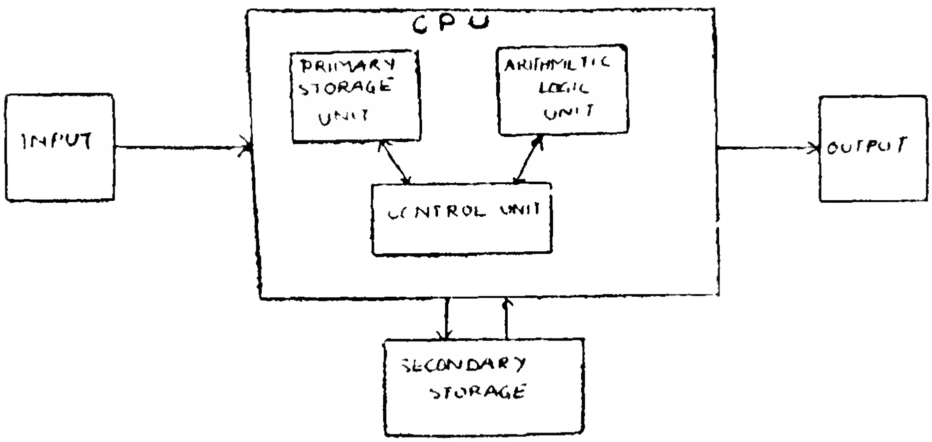

Fig 8

Each of these units performs specific functions which are enumerated below .

Input : A computer must receive both program statements and data to solve problems The entry of program statements and data into a computer occurs by means of an input device Some of the more common input devices are punched cards, floppy discs, punched tape recorders, magnetic tape. Regardless of the type of device used, they are all instruments of interpretation and communication between people and the computer

Central processing unit : The heart of any computer is the central processing unit (CPU) It is this central processor that makes comparisons, performs calculations, reads, interprets and controls the execution of the instructions. It consists of three separate sub-units.

(1) The logical and arithmetic unit,

(2) The control unit,

(3) Primary memory/storage.

Control Unit , The control unit, as the name implies, supervises the operations of the entire computer. It selects the program statement from the storage unit, interprets the statement and sends the appropriate electronic impulses to the arithmetic/logic and storage units to cause these units to carry out the operations required Thus, the control unit does not perform the actual processing operations on the data Rather, its function is to maintain order and direct the flow of sequence of operations and data within the computer The control unit also instructs the input device, when to start and stop transferring data to storage unit, and it tells the storage unit when to start and stop transferring data to output devices. Hence it acts as a central nervous system for the component parts of the computer.

Arithmetic Unit : Arithmetic and logic unit performs mathematical calculations, compares numeric and non-numeric values and makes decisions The data flows between this unit and the storage unit during processing Specifically, data is transferred as needed from the storage unit to the arithmetic/logic unit, manipulated and returned to the storage unit

Storage : The storage, or primary memory section of the computer consists of the devices used to store the information which will be used during the computations The storage section of the computer is also used to hold both intermediate and final results as the computer proceeds through the program Common storage devices are magnetic cores, magnetic disk etc.

Secondary storage Because the primary storage capacity of most computers is limited, it is both expensive and not always possible to hold a large volume of data and instructions in the primary storage Hence it becomes necessary to have secondary or auxiliary storage for holding data and programs not currently in use The various secondary storage devices are the magnetic tape, magnetic disk and magnetic drum Punched cards or paper tape can also be used as a medium of secondary storage but it is very slow compared to the magnetic tape or disk The magnetic tape and disk offer fast access next to primary storage For this reason, these are widely employed for secondary storage

Output devices ' Output devices like input units are instruments of communication between people and machine They are used to record the results obtained by the computer and present them to the outside world They take information in machine coded form from storage unit and convert it typically into a form that can be used (i) by human (e g printed) form, (ii) as machine input in another processing operation e g magnetic tape The most commonly used output devices are line printers, magnetic tape, graph-plotters, magnetic disk, etc

Historical development of computers ,

The modern computer with the power and speed of today was not a solitary invention that sprang completed from the mind of a single individual It is the end result of countless inventions ideas, and developments contributed by many people throughout the last several decades The history of automatic data processing begins with Charles Babbage's attempt to build an automatic mechanical calculator at Cambridge, England, in 1830 By the 1930's, punched cards were in wide use in large business, and various types of punched card handling machines were available In 1937, Howard Aiken, at Harvard, proposed to IBM that a machine could be constructed which would automatically sequence the operations and calculations performed. This machine used a combination of electro-mechanical devices, including relays.

First Generation computers

UNIVAC (Universal Automatic Computer) was the first general purpose electrical computer to be available and marks the beginning of the first generation of electrical computers The first generation electrical computer employed vacuum tubes These computers were large in size and required air conditioning The input and output units were the punched card reader and the card punches, Because of the inherently slow speed of these input/output units, the power of the CPU was subjugated to their speed IBM-650 was however, the most popular first generation computer and was introduced in 1950 with magnetic drum memory and punched cards for input and output It was intended for both business and scientific applications

The second generation computers employed transistors and other sol d state devices Their circuits were smaller than the vacuum tubes, and generated less heat Hence the second generation computers required less power, were faster and more reliable IBM 1401 was the most popular second generation computer There were two distinct categories of the second generation computers for business and scientific applications They employed magnetic tape as the input/output media Second generation computers successfully displaced the unit record equipment on cost benefit grounds in many an installation

The Third generation computers employed integrated circuit in which all the elements of an electronic circuit are contained in a tiny silicon wafer The third generation computers are much cheaper and more reliable than the second generation computers They are more speedy with much vaster capacity and admit connection of a wide variety of peripherals particularly magnetic disk units They are based on the principles of standardisation and compatibility The core storage of a given model of a computer can be expanded by adding modules and it still permits the use of older program The third generation computers can be used for both scientific and business applications

The third generation computers permit multi-programming* which is inter leaved processing of several programs to enhance the productivity of the computer, time sharing* which is the use of the computer by several customers at a time, operating systems which optimise the man-machine capabilities and such data communications facilities as remote terminals They also permit use of such high level languages as FORTRAN and COBOL. The mini computers are the latest development in the third generation computers

The on facing page table shows the distinguishing characteristics of three generations :—

Each generation of computers has an effect on the MIS centralisation and decentralisation issue The first generation computers were high in costs and large in size , therefore information system were sought to be centralised to serve benefits of hardware economies The second generation computers were substantially cheaper and the trend was towards MIS decentralisation Third generation however, offered communication capabilities and the use of remote terminals and the trend was reversed to centralisation

Charactristic of computers :

Computers can be characterised by functions *A special purpose computer* is designed to perform an especial task and, theirfore, the instructions are inbuilt in the computer circuitry by the manufacturerso that the given task is performed

Note : *These terms have been explained in the study subsequently)

quickly and efficiently. They are generally used for applications such as processing airline reservations or for solving navigational problems, etc

Characteristics Function	Scientific oriented 1st generation Scientific computation	Business oriented 2nd generation Data processing	General Purpose 3rd generation Scientific or business data processing
Basic hardware component	Vacuum tubes	Transistors	Integrated circuits
Access speeds (Seconds)	10^{-3}	10^{-5}	10^{-6}
Primary Storage Device	Magnetic drum	Core memory	Integrated circuits
Programming Language	Machine	Symbolic	Problem oriented
Internal Memory size (No of characters)	2×10^{3}	up to 1 6X 10^{4}	10^{8}

However, the special purpose computer lacks completely in versatility *i e*, it cannot be used for any other task

General purpose computers. General purpose computers can be used for many business, scientific, educational, social and other applications They are capable of storing different programs of instructions and offer a fairly extensive instruction repeatories to write programs with Not only a wide variety of jobs can be handled by these but also the old programmes can be amended or dropped

Scientific and Business computers : This distinction was important in the early computers but it stands very much blurred now-a-days

The scientific and business applications do differ greatly in their data processing characteristics Scientific problems involve enormous amount of complex computation on a relatively small input data and their output, too, may rarely exceed a line or two on the continuous stationary Since input/output volume is low in scientific applications the speed of the input/output peripherals is more or less irrelevant but that of computations is highly important since there do arise scientific problem that may take several man-months or even man-years of computations Reverse is the case with the business applications The input-output volume is very high The arithmetic is neither voluminous nor usually complex. Therefore, business applications would call for fast peripherals Another difference is in the storage requirements The scientific applications would have usually long programs but handle not much input data to be stored in the primary memory , therefore the storage requirements are modest as compared to the business applications where not only the program is longish but also

considerable input data may have to be stored in primary memory. Therefore ; the internal storage requirement of business applications are usually high.

It is a fact, however, that many companies used business oriented computers for scientific applications and vice-versa but this dual use led to inefficiency

The third generation computers have blurred greatly the distinction between the two categories of the computer Certain software and hardware control features together with a wide range of peripheral equipment permit many central processors to flexibly and efficiently serve both types of applications.

Concepts of Computer files

As we mentioned in the beginning of this study, a file can be defined as a collection of related information Just as there are files (accounts ledger and Kardex cards) in manual systems, there are files in computer systems too , but the media is usually magnetic so that the computer can itself read the files As mentioned above, computer performs arithmetic computations and is extremely fast in this function Upon reading the files and the details of transactions, it can update the file Besides, it can produce such output as replenishment orders in the works and vendors in an Inventory control application and pay slips in a payroll application Data Files used in business can be divided into seven types, such as (a) master files, (b) transaction files. (c) reference files (d) table files, (e) report files, (f) historical files, and (g) back up files

Master files : A master file contains relatively permanent records for identification and summarising statistical information A product file, customer file and employee file are examples of master files The descriptive information in a master file may include such items as product code, description, specifications, etc Statistical information, in a customer master file, contains amount outstanding, age of outstanding etc

Transaction files : Transaction files are created from source documents used for recording events or transactions These are detail files, and the information is used for updating the master files If the processing is of the batch type, the transactions are accumulated for a period and a transaction file is created at the end of the period, The typical source documents, from which transaction files are created, are : purchase orders, job-cards, invoices etc

Reference files : These files contain keys of records in other files In order to retrieve a record from a file, the reference file is first searched to find out in which file a record can be located.

Table files : These are in the nature of catalogues or price lists

Report files · A report file is created from records in other files in a meaningful and concise form A sales performance report and a report on materials rejected are examples of report files.

Historical files These contain statistical information of past periods. These files are used to analyse trends or make comparisons of one period with another and so on.

Back-up-files : These are copies of currently used master files kept in the computer library (*i e* collection of all computer files) as a measure of security.

Input and output device and media.

No matter how powerful the computer is, it is useless unless it is fed data and programs. Communication of data (stored in the form of file) is accomplished by input and output devices. Except when direct entry keying is involved, all data-transfer methods use some type of media (*e g* punched cards, magnetic tape, magnetic disk or machine readable source documents). We will now discuss some of these media and I/O devices in detail.

Punched Card: The oldest and still the most popular method of encoding information in a machine sensible or machine readable form is by punching holes in specially designed cards. The punched cards were devised by Hollerith during his servicas in U S Census Bureau to cope with enormous calculation work inherent in any census These became so popular that he left his job and undertook the manufacture of these cards and the associated processing machines His company, through a series of mergers, is today the formidable international combine, the IBM, the manufacturers of a wide ranging business machines. There is another card marketed by Remington Rand which, however, has found only limited acceptance; therefore, that is not discussed in these study notes

Physical details The standard IBM punched card is made from paper or plastic, is $7\frac{3}{8}''$ long, $3\frac{1}{4}''$ wide and 0 007" thick One of the corners usually, the top right one, is cut to help keep the cards right end in processing The card has 80 columns equally spaced in the length of $7\frac{3}{8}''$. Each column is divided into 12 equally spaced rows which are numbered 9, 8, 7, 6, 5, 4, 3, 2, 1, 0, from bottom to top and rows 11, and 12 above 0 in this order. But they (11 and 12 rows) are not printed on the cards whereas others are These figures are intended to be punched to represent digits, 0 to 9, alphabets A to Z and some special symbols as explained

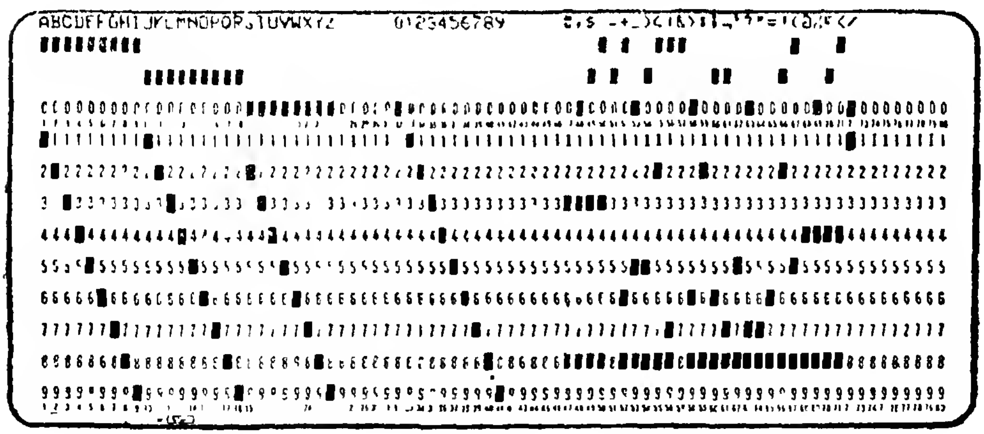

Fig. 9

in the following paragraph. Usually, under 0 and 9 rows are printed the column numbers 1 to 80 to avoid any visual confusion about the column numbers and serve no other purpose whatever A punched card is inexpensive, can be had in different stripes or colours for differentiation viz, payroll, stock control, etc. The layout **disussed below may also be got specially printed on them.**

Hollerith's Code permits coding of data in the punched card.

A digit is simply *written* by punching a rectangular hole in the appropriate number in the column For example, 2 in a column is represented by punching the 2 position in that column. An alphabet is represented by a combination of 2 punched holes in a column The following table provides the Hollerith's codification scheme for alphabets.

Punch Positions for representation of alphabets

Punch position	*Character*	*Punch positions*	*Character*	*Punch position*	*Character*
12,1	A	11,1	J		
12,2	B	11,2	K	0,2	S
12,3	C	11,3	L	0,3	T
12,4	D	11,4	M	0,4	U
12,5	E	11,5	N	0,5	V
12,6	F	11,6	O	0,6	W
12,7	G	11,7	P	0,7	X
12,8	H	11,8	Q	0,8	Y
12,9	I	11,9	R	0,9	Z

. Fig 10 shows the name of student and a date (21/05/1977) encoded on a punched card, 12, 11 and 0 rows are called Zone punches

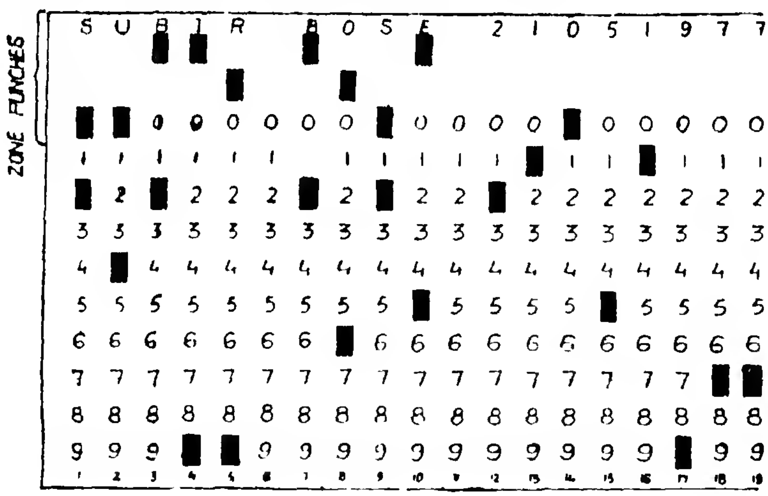

Fig. 10

Punch Position for representation of some special symbols

N.B. The student ought to memorise the Hollerith's codification scheme for alphabets above. He can, however, just have a look on the following table for representation of some special symbols each of which may require punching in one, two or three positions in a column.

Punch position	*Special symbol*
11	—
12	&
12, 3, 8	.
0, 3, 8	,
3, 8	±
11, 4, 8	≠
0, 4, 8	%
4, 8	@
0, 1	/

Card Layout

A punched card usually carries one transaction record or one master record Since there are 80 columns in the card the record, if more than 80 characters long, should be sought to be reduced to 80 characters or less by, perhaps, abbreviating some of the data items in the record. But where absolutely necessary more than one card may be employed for one record.

The assignment of column numbers to the various data item of each record in a file leads to layout of the card for that file For example, a stock record may have the card layout shown in figure 11. All the records in the stock control file would be accommodated on a card each according to this layout.

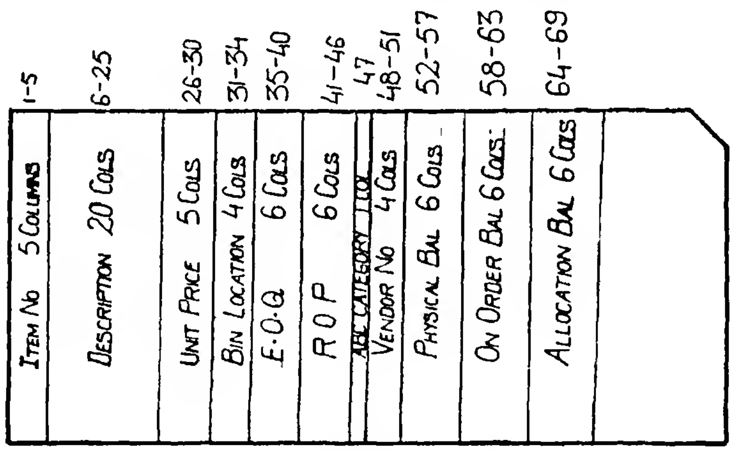

Fig. 11

The top right edge of the punched card is chamfered as shown in figure 12. This helps to detectat once a wrongly placed card with its chamfered corner protruding out of the deck of punched cards.

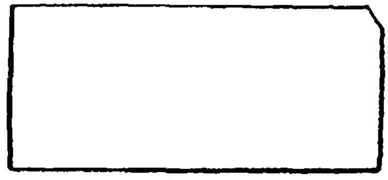

Fig. 12

Type Code

Consider a transactions file of stock transactions containing two types of transaction cards, receipts and issues. One column may by assigned to punch R for receipts and I for issues for differentiation by machines.

Left-justified and Right-justified Fields

Alphabetic fields are left-jnstified For a left-justified, data field, name of an employee (Column 4 to 20) the following is the correct way of encoding the three given names :

Column No. 4 5 6 7 8 9 20
R A M U
S U B I R
S A M

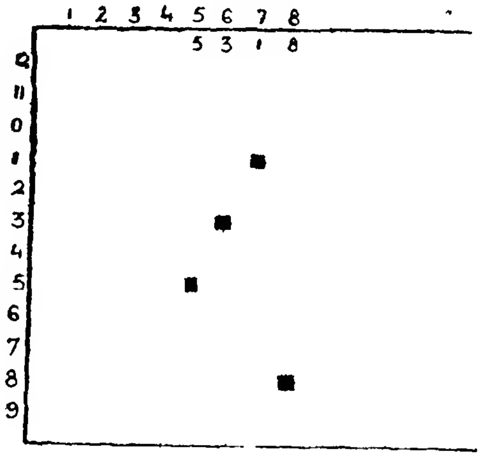

Fig 13

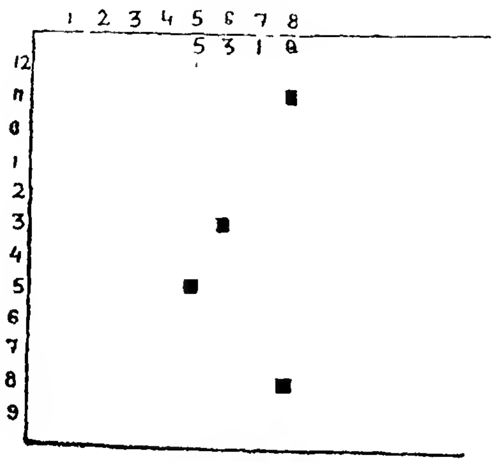

Fig. 14

The debit/credit or ± Code.

The positive quantities are enclosed as such without any code to suggest that they are +ve. In figure 13 is shown punched +5318.

The negative quatities are encoded with an 11 punch over the right most digit. In figure 14 is shown encoded −5318 which would literally read as 531Q but machines would take it as −5318 as intended.

***Off-line Punched Card Machines**

The records from the manual ledgers would be put on to punched cards by a machine known as the key punch. The punched cards would be verified for correct punching on another machine known as the card verifier. The punched and verified cards may then be sorted by the key data field on a machine known as the card sorter. These machines are operated off-line i e , without any connection with the Computer These are discussed below briefly.

The keypunch : The key punch is the usual means of initially coding the data from source documents viz , for putting the records of manual ledgers on to punch cards (i e , file conversion) at the initial stages of computerisation or preparing transaction cards for receipts, issues, adjustments etc. viz , in inventory control from such source documents as goods received notes; etc The key punch is shown in figure 15 vide page 22. Its key board (Fig. 16) is not much different from an ordinary typwriter keyboard.

It carries keys for all the numeric digits, alphabets (only upper cases letters) and special characters The card hopper can carry a maximnm of 500 blank cards. The bottom most card in it travels beneath the punching station where it is punched column by column by the key punch operator who knocks the key board as per the information read by him from the source douments. If the last column punched is not column 80 the card may be advanced by pressing a special key. The punch card in the reading station and the blank card in the punching station then move together in a synchronised way This synchronisation feature permits data to be duplicated in the blank card automatically if desired and programmed. For duplication the operator would have to depress a special key (known as DUP key) so that the data sensed in the completed card will be automatically reporoduced in the same columns of the following card. Of course, if the data for the various cards is entirely different this feature would not be put to use. The punched cards travel to their ultimate destination, the stacker that, too, can hold a maximum of 500 cards.

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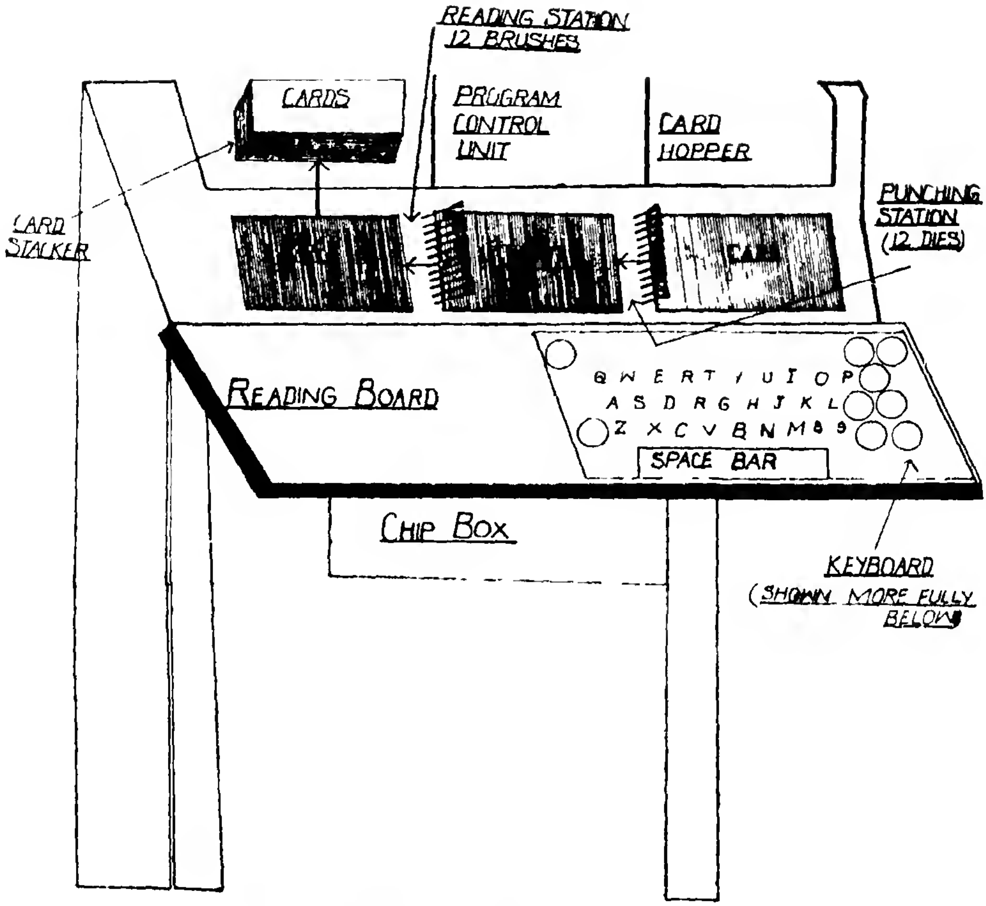
*Synonymous with auxiliary or ancillary



Schematic Of The Keypunch

Fig. 15

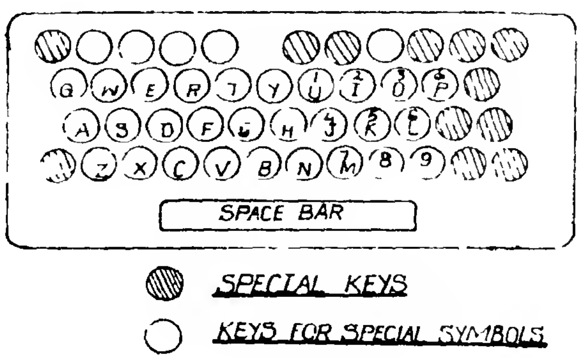
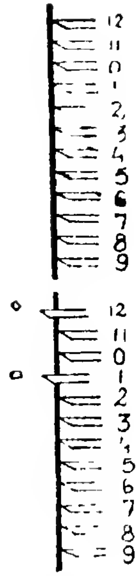


Fig. 16

A set of 12 dies (corresponding in the 12 punch positions in a column of the card).

The 12 and 1 dies project to punch holes in a column of the card to encode upon operator knocking A on the Keyboard.

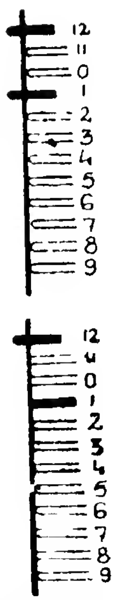


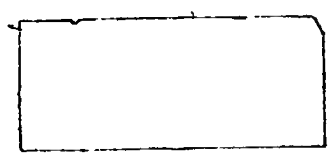
Operation of the Keypunch Dies.

Fig. 17

The 12 and 1 needs project to sense A (in a column upon the operator knoeking A on the keyboard

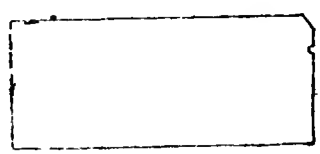
Needle 1 gets jamped whilst sensing incorrectly punched A





Rejected Card

Fig 18-A



Okayed Card

Fig. 18-B

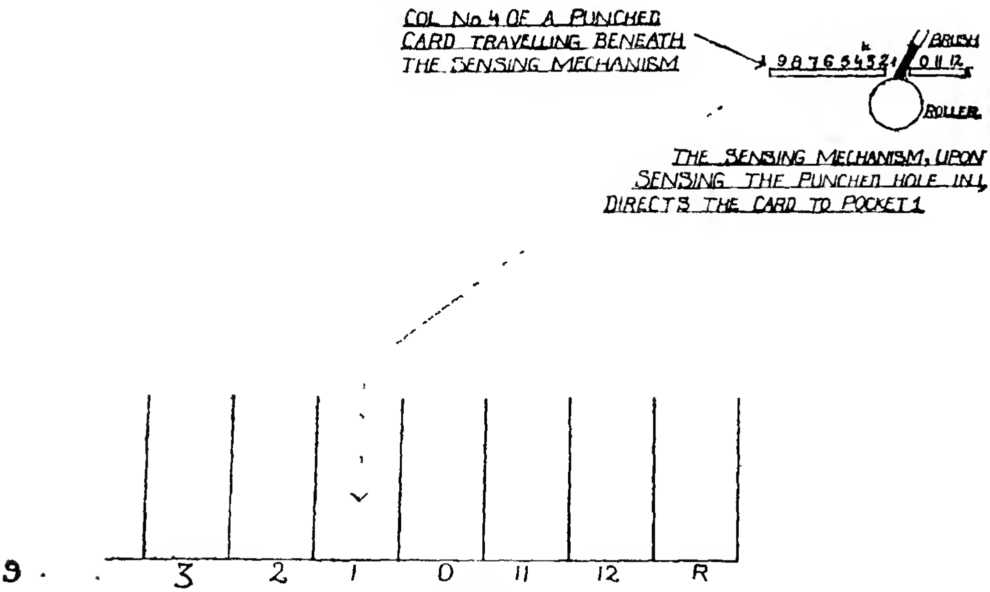
Operation of the Verifier Needles.

Fig. 18

The verifier The verifier looks very much like the keypunch, the major difference being that instead of punching station there is verifying station which is used to verify the accuracy of data punched by the keypunch operator. The cards punched in the keypunch are put in the hopper of the verifier and their path of travel is identical to the one on the keypunch. The verifier operator tries to key the data using the same source document The verifier station consists of thin metal needles that can pass through the punch holes If the key depressed correesponds to the holes in the card column the needles will pass through the holes allowing the card to advance one card column. If the holes differ from the depressed key some needles will be jammed, the machine will lock and a red light will go on The operator can unlock the machine and depress a key two or three or more times If this is still wrong the card will be notched over that card column, and indication that column has not been punched correctly (see figure 18A) . the cards for correct card upon verification would be okeyed by a notch at the right edge of the card (as shown in figure 18 B)

Card Sorter. Cards are sorted in a machine known as the card sorter Its schematic sketch is shown below Its essential components are:

1 *The hopper* which can hold a maximum of 1200 punched cards



Schematic of Card Sorter

Fig. 19

2. Sensing Mechanism that consists of a single wire brush. This brush can sense a hole in a column of the card that travels beneath it The principle of sensing or reading is the same as employed in the card reader discussed Page 26-271 *i.e.,* upon encountering a hole the brush contacts the roller upon which the card travels, generating thereby an electrical impulse.

3. *Selection Switch* that is used to select the card column that will travel beneath the sensing brush. It is set manually prior to the operation of the card sorter. If, for example, the key field by which a deck of punched cards are to be sorted encompasses columns 23 and 24 the selection switch, in the first pass (i e , set up), would be set at column 24 and in the second pass at column 23

4. Thirteen stackers (or *pockets* more appropriately) numbered as R. 12, 11, 0, 1, 2...9. Each pocket can hold a maximum of 450 cards.

In the figure, the card column 4 is being read by the sensing brush which, upon encountering the hole at 1, directs it to pocket 1.

Consider an example. The key field in an application encompasses columns 25 and 26. The 9 punched have 10, 1, 6, 15, 3, 21, 69, 32 and 48 as the data in this field The cards are to be arranged in an ascending sequence. The column selector is set at column 26 (the rightmost in the key field). The start key of the sorter is pressed. The cards fall in the 9 hoppers as shown in the figure 20 on p. 26 The cards in the 9 pockets are then merged in a deck with the cards of 0 pocket at the bottom, cards of pocket 1 at its top and so on This deck is put in the sorter hopper The selector switch is given a turn to set the sorter brush at column 25 now. The start key of the sorter is pressed and the cards are sorted as in figure 11. The card deck is made as above and this is the final sorted deck

Major and minor sort. The procedure for this type of sorting is to sort lowest level fields first, starting with the right most column and then the next higher level field again starting with its right most column In an application, having only two fields you would first sort the minor field and then the major field.

If there were three fields you would first sort the minor, then the intermediate and finally the major field. For example, in a financial accounting application, the following types of transactions on punched cards may have been sorted by voucher number ; then date within account head :

(i) Cash receipts and payments,

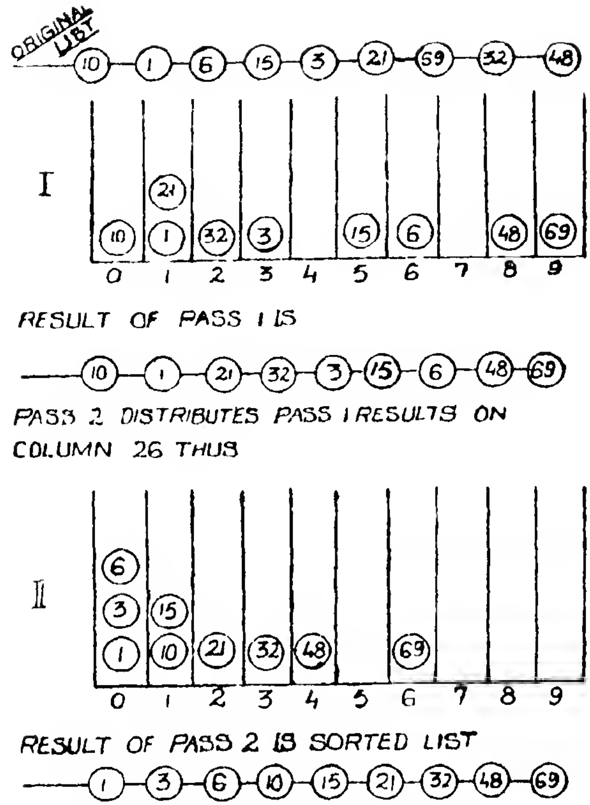
(ii) Bank receipts and payments,

(iii) Sales bills and

(iv) Journal vouchers.

Assuming that the voucher number is assigned columns 1 to 3, date is assigned column 4 to 9 and account head is assigned columns 17 to 20 the card would first be sorted by voucher number (columns 1 to 3), then by date (columns 4 to 9), and finally by account head (columns 17 to 20).

Alphabetic Sort. There are times when one sorts alphabetic fields, for example, name, product description, locations, etc. If we had to sort once on each column during a numeric sort, how many times do we sort in an alphabetic sort ? Twice, because there are 2 punches to represent a letter It may be noted that in the first pass on a column the cards would fall into pockets 0 to 9 and in the second pass on the same column the cards would fall into 12, 11, 0 pockets To the reject pocket, in general, would be consigned those cards that have unacceptable punches For example, a card carrying no punch whatever in the numeric zone would be consigned to this pocket in the very first pass.



SORTING ON A CARD SORTER

Fig. 20

We have discussed three machines : The key punch, the verifier and the card sorter. These machines are ancillary or preparatory in nature in that they are used to prepare the punched cards to be subsequently used in the computer. And these machines are operated off-line i e , they are not connected to the computer. All the master files would be converted from manual ledgers to the punched cards on the key punch and the verifier. They would then be sorted on the card sorter by their respective keys The punched card files are usually stored in racks Transactions, too, would be put on punch cards either as and when they arise i.e , singly or in batches on the key punch, verified on the verifier and sorted on the card sorter.

Card Reader

This is a device which 'reads' the holes punched in cards and they are transmitted to the input area in the storage of computer's CPU.

There are basically two kinds of punched card readers :—

(i) Wire brush type.

(ii) Photo electric type.

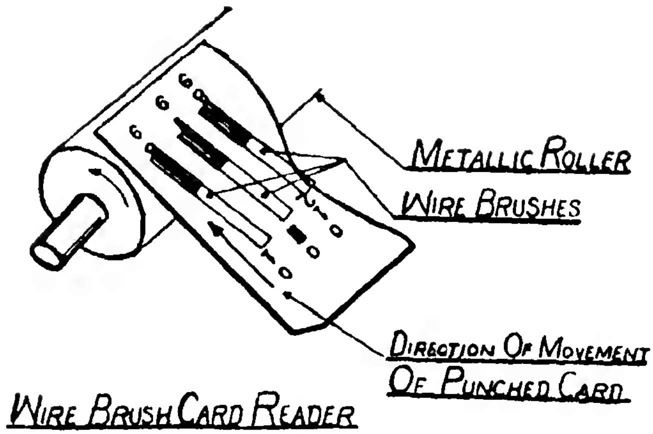


Fig. 21

The functioning of the former is sought to be depicted above. The card is sandwiched between a metallic drum and a series of 80 wire brushes and as such travels row wise. In case of punched holes, the corresponding brushes contact the drum below, thereby permitting electric impulses to pass. This enables reading or sensing of the card row by row The student might wonder at this stage why the card is read row-wise when the data has been coded in it column-wise. Well this could indeed be done with the set of 12 wire brushes but then the card would have to travel 80 columns making reading slower than in case of 80 wire brushes in row-wise reading

Generally, however, there are two sets of 80 wire brushes through which a card has to travel In its first pass, a mere count of the holes is taken for the purpose of verification later In the second pass, the card is read as well as the count is verified

Photo electric card reader reads the card column wise and a light source and light sensing unit are used to detect punch holes.

A card reader consists of the following units :

(a) *Input hopper.* Cards to be read out loaded in the hopper by the computer operator.

(b) Reading station The cards travel from the input hopper to the reading station one by one and pass beneath 80 wire brushes or the photo electric. Sensing unit constituting the read station. The cards are read row-wise and not column-wise to make the reading faster

(c) *Output stacker.* After a card is read it is ejected and falls down into the output stacker. After all the cards have been read, they are removed by the computer operator.

(d) *Decoder.* The contents of the cards are transferred into the CPU to which the card reader is electrically connected The cards are punched in Hollerith code which the CPU does not understand The CPU operates in binary mode to be explained later.

Thus, there is a translation problem from Hollerith code to binary codes The decoder unit fitted in the card reader does this translation.

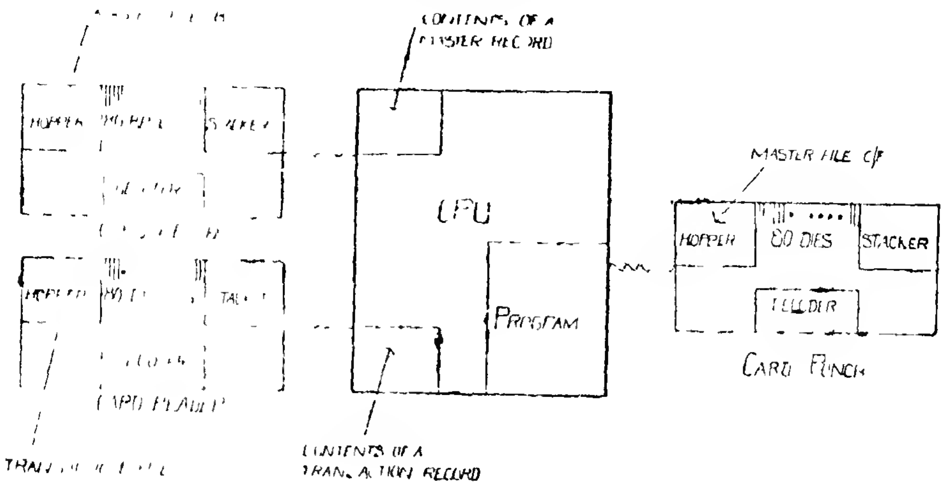


Fig 22

It would be useful to give an idea of the speed of the card readers Different models of the card reader have different speeds Models are available which read at speeds of 300, 600, 900, 1200 and 2000 cards per minute These speeds are followed if all the 80 columns of the card are punched and have to be read The speed would be proportionately higher if less columns are actually punched Consider now, a model with the speed of 600 cards per minute If all the columns of the cards are punched the characters that can be read per second $= 600 \times 80 - 60 = 800$ characters per second which is the speed with which data is transferred from the card reader into the CPU

Card punch (see to the right of fig 22) is similar to the keypunch But the keyboard would not exist Since the cards are not to be punched manually but by the CPU The CPU electrically transfers the contents of the up-dated records into the card punch which punches a blank card for the updated contents. It also has a decoder unit that translates the contents in binary codes to Hollerith code. It has a hopper that holds the blank cards travelling one by one beneath the punching station having 80 dies that punch cards row-wise and the stocker into which the punched cards are finally ejected.

Alternative Computer Media

Having given the overview of the computer operation by means of punched card files, we now take up several alternative file media.

2 **Punched paper tape** : It is also one of the earlier media. It is available in continuous length, and is ½" to one inch in width. It has 7 tracks (or channels or rows) and columns (or frames) running through its length. One column can carry one character in BCD (information is stored in the CPU using BCD number system explained in the later part of study). A round hole in a row of a column represents the 1-bit and no hole in a row of a column represents the o-bit. In the figure 23 is shown the name "RAMU" punched in BCD in a paper tape. The smaller holes are used merely by the drive sprocket mechanism for moving the tape. Typically, the

tape density is about 10 frames per inch Tapes with more or less channels are also available for different codification scheme

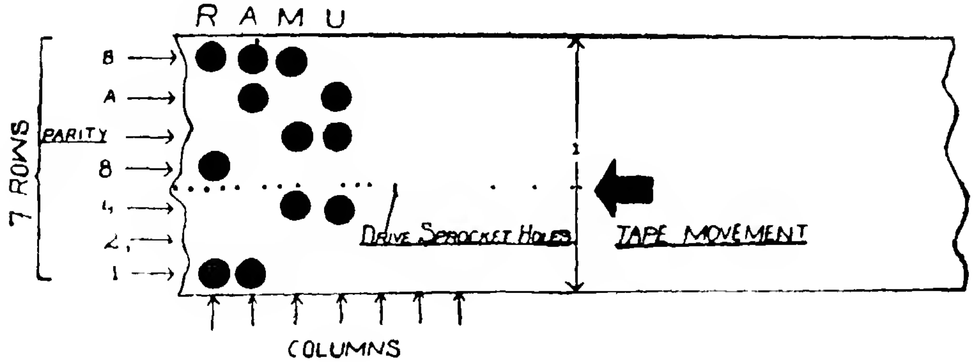


Fig 23

Akin to the keypunch there are tape-writers with paper tape attachment which can be used for putting the records on to the paper tape directly from source documents manually off-line In addition, paper tape obtained as a by product of the output of cash registers, book-keeping machines etc may also be used for subsequent input to the computer.

The punched paper tape readers and punches operate on the same principles as their punched card counterparts i e., the wire brush mechanism and photo electric sensing , however, they are much lighter in size

Modern punched paper tape readers operate at speeds ranging from 200 to 1000 characters per second, those utilising the photoelectric principles being the fastest Information recorded on paper tape can be easily and quickly transmitted over telephone or telegraph wires at speeds of 115 characters per second Should it be necessary to transmit data over long distances and where the data are already recorded on another input medium (e g , punched cards), the data can be converted by the conversion machine from the punched card to paper tape and transmitted over telephone or telegraph wires to produce a duplicate punched paper 'tape (See figure 24) The punched paper tape is then converted to punched cards or other media Though punched cards can be transmitted directly; it is more economical to first convert the data on to punched paper tape and transmit the paper tape.

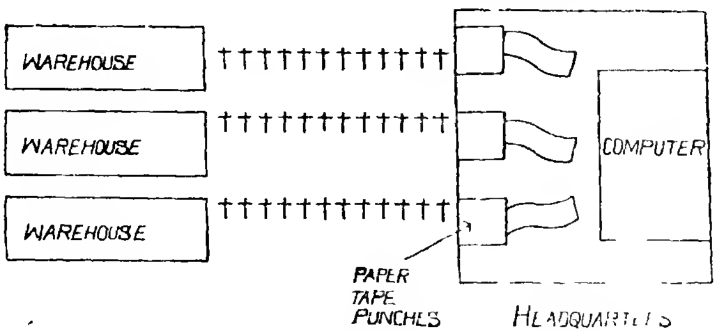


Fig. 24

With the punched card one is constrained to accommodate a record in 80 columns If the record is longer it may have to be abbreviated, if it is smaller the card space would go waste With the punched paper tape there is no such limitation i e , the records can be of any length Also since recording of data is much more compact on the paper tape reading is faster Because of the continuity of the length of the paper tape the following recording possibilities exist.

Fixed length fields Consider, for the data field for a name as 20 characters long If a name occupie 4 columns the remaining 16 columns would be left blank i e , each name would be occupying 20 columns

Variable length fields In this scheme, the data items are assigned the number of columns exactly equal to their length, In the above example, the name of four characters would be assigned just four columns However, for the program to distinguish the end of a data field, a field marker would be used The field maker may be such special character as % For example, the two data fields : RAMU and 28 (age in years) would appear on the paper tape as RAMU %28, of course, codified in BCD (See figure 25) Upon encountering the field marker % the program would know that the name data field has ended and the age data field begins.

Fixed variable Length fields This method is a hybrid of above two methods of recording data on the paper tape Such data fields as product stock number, account number etc are treated on the fixed length field basis and others as the name of the customer, the name of an item etc. are tieated on the variable length field basis

The paper tape, however is rarely used for the mastes files and its use is mainly confined to transaction files. It contends with the punched card in this respect and pros and cons of using a paper tape are discussed in detail below.

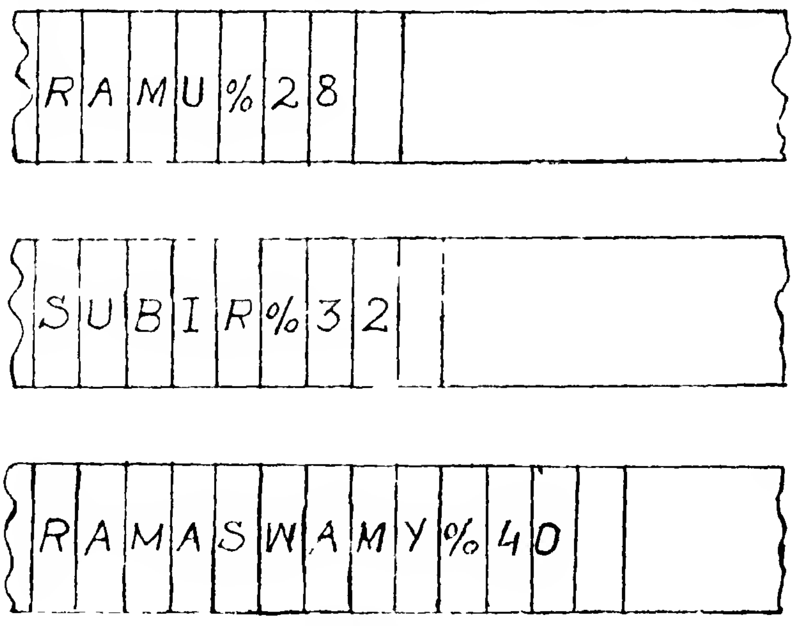


Fig. 25

Advantages of punched cards :

(i) The use of standard 80 column is ubiquitous In contrast there is a conversion problem from a wide range of formats in 5, 6, 7 and 8 channel tapes.

(ii) Record can be inserted or deleted merely by inserting new cards or removing the existing cards.

(iii) For program input use of cards is easier.

(iv) Cards can be loaded in the card reader hopper whilst the computer is operating. In changing reels of paper tape time may be lost.

(v) Cards can be sorted off-line and can also be processed off line by auxiliary equipment.

(vi) card files are tidier than unreeled tape

(vii) Error correction is easier—just pull out the incorrect card and insert the corrected one in its place.

(viii) Cards are more robust than paper tape

Advantage of paper tape,

(i) It is easier and cheaper to produce paper tape as a by-product to the production of hard copy, viz on typewriters or accounting machines.

(ii) It is less bulky

(iii) Records cannot be lost or get out of sequence

(iv) Variable length records are more easier to accommodate

(v) Parity check can be provided

(vi) It is telex compatible.

(vii) It is cheaper.

(viii) Its equipment is cheaper.

(ix) It can be read in reverse

(x) Errors during punching can be designated and ignored during processing. Cards, in contrast, have to be re-punched

(xi) Paper tape reel is easier to the computer than its card equivalent.

3. Magnetic Ink Character Recognition (MICR)

MICR employs a system of printed characters which are easily decipherable by human beings as well as a machine readers. There is used a special printing font to represent characters In this font, each character is basically composed of vertical bars (see "2" of figure 27) The characters are printed in special ink which contains a magnetizable material In "2" of figure 27 there are four small gaps and two big gaps When a character is subsequently read it is passed beneath a reading head and big and small gaps send in different types of impulses represented by 1 bit and 0 bit respectively Some of the characters are represented by the following codes, for example :

0	0	0	1	1	0	0
1	1	0	0	0	1	0
2	0	1	1	0	0	0
Y	0	1	1	0	0	1
Z	0	0	1	1	0	1

(The student need not memorise these codes).

Data can be input to the computer by reading a line on a document encoded in MICR (See cheque of figure 26) MICR is mostly used for processing cheques in U S A.

Advantages of MICR . MICR processes a very high reading accuracy Cheques may be smeared, stamped, roughly handled yet they are accurately read, (ii) Cheques can be handled directly without transcribing them on punched cards, paper tape etc (iii) Cheques can be read both by human beings and machines

Disadvantages: (i) MICR has not found much favour from business, (ii) Damaged documents, cheques not encoded with amount etc have still to be clerically processed

Optical Character Reading (OCR)

OCR also employs a set of printing characters with standard font that can be read by both human and machine readers The machine reading is by light scanning techniques in which each character is illuminated by a light source and the reflected image of the character is analysed in terms of the light-dark pattern produced Key-board devices are used to give the required print quality OCR has the potential of reading even handwritten documents straightway

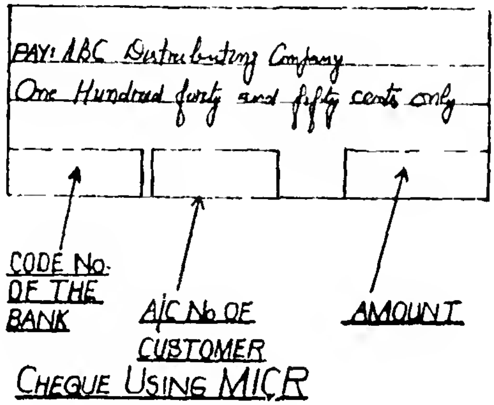


Fig. 26 and Fig. 27

Optical Character Readers can read upper and lower case letters, numerics and certain special characters from handwritten, typed and printed paper documents. The specific characters that can be read and whether the character must be

handwritten, typed or printed depends upon the type of OCR being used. Obviously then OCR annihilates the time consuming step of transcription Before we delve into the common type of the optical reader we shall discuss and especial (though rather primitive) kind of optical character recognition known as optical mark recognition

Optical Mark Recognition (OMR)

Optical marks are commonly used for scoring tests. The below diagram shows part (just the social security number in the U S) of a typical test scoring sheet. It it marked by the person taking the test, and can be read by the optical mark page reader. The optical mark reader when on-line to the computer system, can read upto 2,000 documents per hour. Seemingly this rate is slow but the fact that transcription has been eliminated the overall time is less than those of the conventional file media

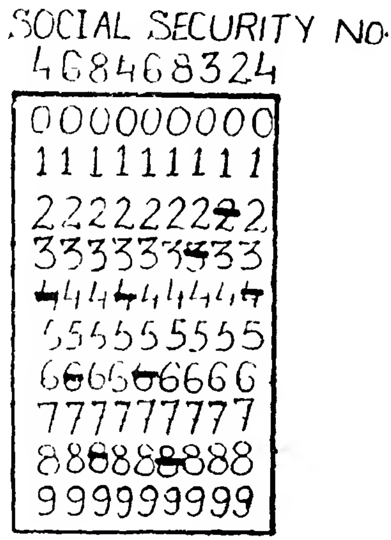


Fig 28

OMR can also be used for such applications as order writing, payroll, inventory control, insurance, questionnaires, etc. However, it is to be noted that designing the documents for OMR is rather a tough task. They should be simple

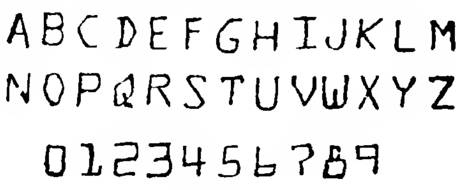


Fig. 29

to understand otherwise errors may result more perhaps than would occur in using a traditional source documents and keypunching from them Reverting now to the optical character readers they have a slightly irregular type face as shown

in the figure 29 They can read the characters printed by computer printers, cash registers, adding machines and typewriters Some readers can also read handwritten documents Figure 30 shows some handwritten characters that can be read by the recognition devices, most handwritten readers can be used to sort cut forms as well as to read data into computer storage

Optical character readers can read from both cut forms and continuous sheets

Technique of Optical Scanning . In all optical readers, the printed marks and/or characters must be scanned by some type of photo-electric device, which recognises characters by the absorption or reflectance of light on the document (characters to be read are non-reflective) Reflected light patterns are converted into electric impulses, which are transmitted to the recognition logic circuits—there they are compared with the characters the machine has been programmed to recognise, and, if valid, are then recorded for input to the CPU. If no suitable comparison is possible, the document may be rejected.

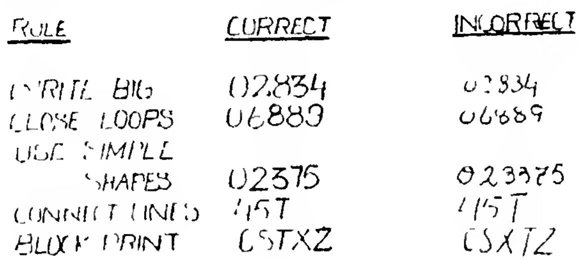


Fig 30

Advantages of OCR

(i) OCR eliminates the human effort of transcription

(ii) Paperwork explosion can be handled because OCR is economical at a high rate of input

(iii) Since documents have only to be typed or handwritten not very skilled staff (like the keypunch operators) is required

(iv) Furthermore, these input preparation devices (typewriters etc) are much cheaper than the keypunch or the key-to tape devices

Limitation of OCR

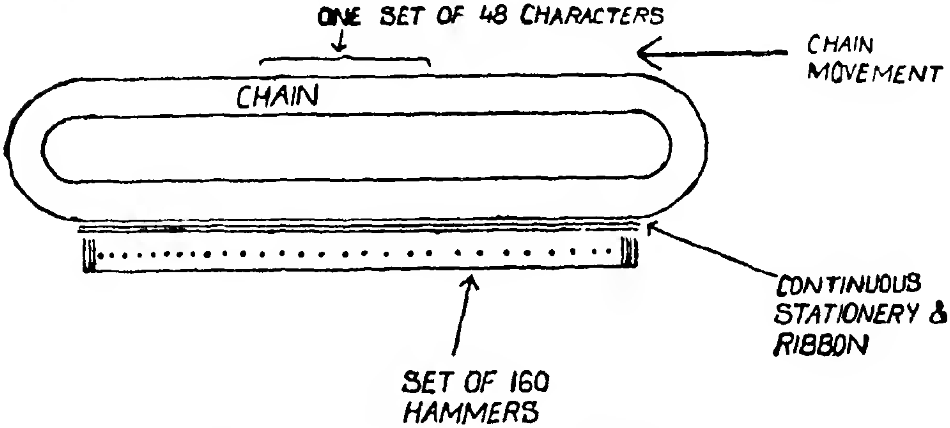
(i) *Rigid input requirements* There are usually specific (and rather inflexible) requirements for type font and size of characters to be used In typing there is always the scope for strike-overs, uneven spacing, smudges and erasures, and the form design, ink specifications, paper quality, etc become critical and have to be standardised

(ii) Most optical readers are not economically feasible unless the daily volume of transactions is relatively high However, future developments in OCR are likely to make optical readers much cheaper

5 **PRINTER** is one of the most common output devices It provides the user with a parmanent visual record of the data output from the computer Printers can print on ordinary paper or on specially prepared forms such as despatch notes, invoices or packing slips Printers have been developed that are capable of printing from 150 to 2 500 lines per minute, each line consisting of as many as 150 characters Printers can broadly be subdivided into two categories impact and non-impact printers. The former are the most common. The impact printers print in the style of

type-writer with the difference that the various characters are cast on a wheel that strikes the paper together with the ribbon after a particular character has been revolved into position. They can be divided in two categories viz. chain printers and drum printers.

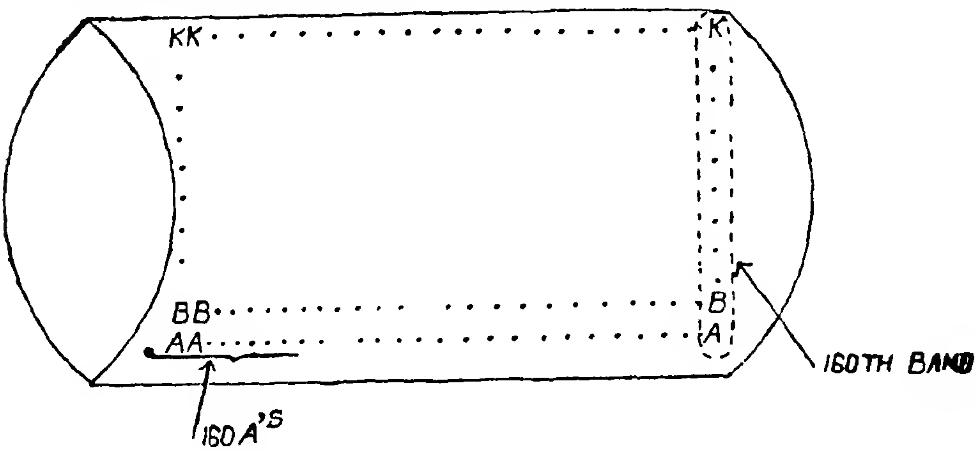
Chain Printer : (Fig. 31) It has a chain that revolves at a constant speed in a horizontal plane The complete chain has a complement of the 48 numerals, alphabets and special symbols cast on it 5 times over. It is confronted by a set of as many hammers as the number of print positions say, 160 These hammers are magentically controlled The continuous stationery and a ribbon are interposed



CHAIN PRINTER SCHEMATIC

Fig. 31

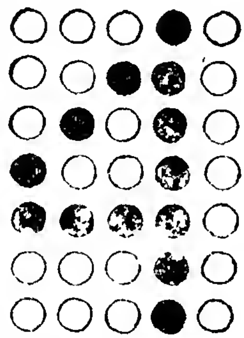
between a segment of the chain and the set of hammers. When a required character on the chain faces its print position the corresponding hammer is actuated



DRUM PRINTER SCHEMATIC

Fig. 32

Drum Printers (Fig 32) use a solid cylinder There are as many bands on it as the number of print positions Each band has cast on it the complement of 48 numerals, alphabets and special symbols The drum rotates at a constant speed confronted by a set of as many hammers as the number of bands with the inked ribbon and continuous stationery interposed In one rotation of the drum there would be appropriate strikes by the set of the hammers In the first strike A's are printed in the appropriate print positions, followed by B,C, Z, 0, 1 .... 9 and special symbols one by one



WIRE MATRIX DOT PATTERNS

Fig. 33

Non-impact Printer is the wire-matrix printer (Fig 33) It prints characters made of pattern of dots formed by the ends of small wires arranged in a five by seven rectangle For the character to be printed its pattern is pressed against an inked ribbon to print the character There are 47 usable dot patterns Non-impact printers however are not commonly used for the following reasons :

(i) Special and more expensive paper is required

(ii) Only one copy can be printed at a time

(iii) The print is not as sharp or clear as with the impact printers.

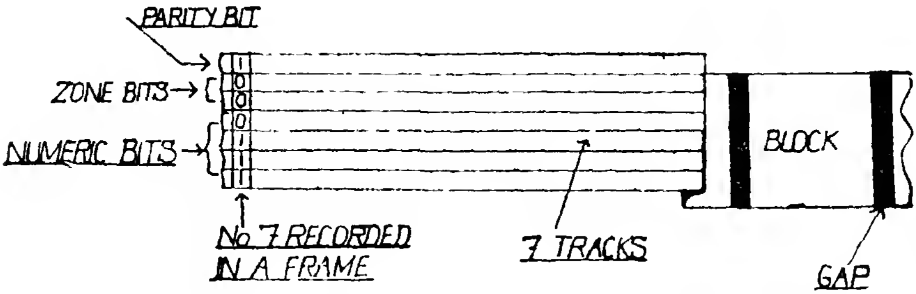
(iv) Output is difficult to copy on office machines

Auxiliary Storarge Devices

6 **Magnetic Tape** : Magnetic tape is a continuous length sequential file medium The tape contained in a typical reel would be 1/2″ in width and 2,400 ft. in length Other sizes are available Because of relatively fast transfer rate (the speed at which data is transferred into the CPU and *vice versa*) the magnetic tape is preferred to both punched cards and paper tape It is made of a plastic ribbon that is coated one side with a magnetisable material By electromagnetic pulses, data are recorded in the form of tiny invisible spots on the iron-oxide side of the tape just as sound waves form magnetic patterns on the tape of a sound recorder. Just as the sound can be played back from the tape recorder as many times as desired data can be read over and again from the magnetic tape Also, magnetic tape, once written can be rewritten i e , data previously stored is automatically erased when new data is deposited in it

The data is encoded in B C D in the magnetic tape, therefore, it has 7 rows (or channels) The columns (or frames) run through its length An intersection of a row and a column has a magnetisable spot. Thus a column has 7 spots which can

hold one character in B C.D, a magnetised spot designating the 1-bit and the demagnetised spot designating the 0 bit The decimal number 7 is shown recorded in BCD in a frame of the magnetic tape in figure 34



RECORDING OF DATA ON MAGNETIC TAPE

Fig 34

There could also be 9 channel coding for the 8-bit alphanumeric codes, in which case the tape would have 9 rows or channels

The number of frames per inch are usually not fixed would depend upon the type of tape Tape density typically ranges from 200 to 1600 frames per inch

We likened the magnetic tape above to the tape of a home tape recorder. Just as the latter runs at a constant speed for being recorded or played back the magnetic tape also has to run at a constant speed for being read or written There is however, an important difference between the two Whereas the tape of the home-tape recorder moves continuously the magnetic tape moves intermittently (Fig 35)

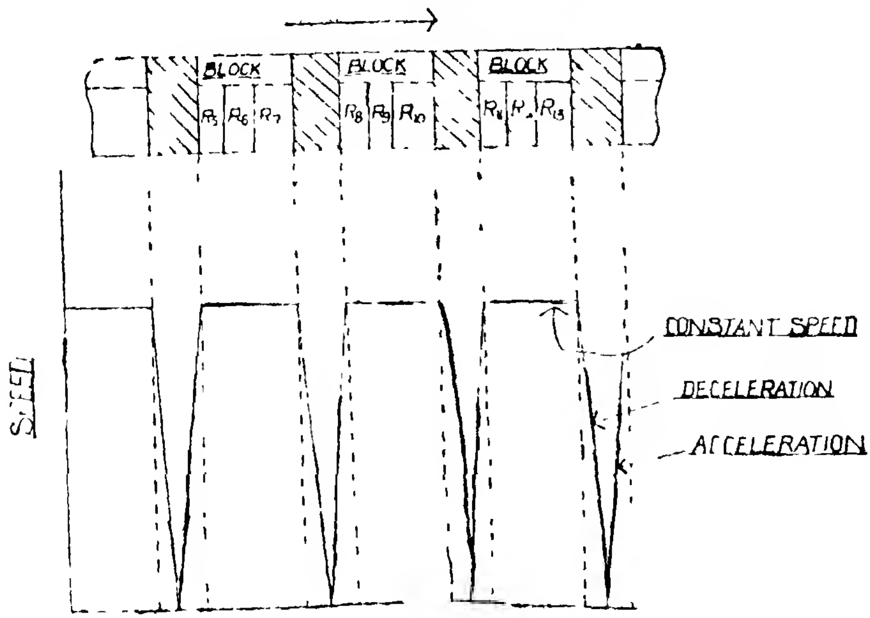


Fig. 35

i e. it alternately runs and stops. If we were to run the magnetic tape continuously we would have to read it in entirety into the CPU Since commercial files are fairly long this would require the CPU to be very large which is very expensive indeed ; therefore, we read a portion of the tape (this portion is known as the block), process it in the CPU, then read the next block and process it in the CPU and so on. Whilst a block is being read the tape would be moving at a constant speed. Whilst the read block is being processed in the tape CPU the tape should not be moving at a constant speed i e , it should stop Since however, anything running cannot stop all of a sudden and having stopped cannot pick up a constant speed all of a sudden, the magnetic tape, therefore, moves in cycles of constant speed—declaration—acceleration—constant speed, necessitating the separation of blocks by gaps (known as Inter-record gaps, IRG's in abbreviation) which are blank i e , they do not hold any data

A block would usually hold several records but if the records are exceptionally long each of them may occupy more than one record. If the records are of fixed length each block would hold the same number of records. It is a simple matter for the program to distinguish one record from another because each record is accommodated in a fixed number of frames.

If, however, the records are of variable length, the program would not discern where a record ends and the other starts if the adjacent records are merely juxtaposed together. Therefore, the variable records within a block are separated by a special symbol (viz ,') that signifies the end of one record and the start of the next. Obviously one frame would be occupied by this special symbol and it would be used as many times as the number of records in the block. This mark is known as the end of the record marker or the record separator The use of the record marker is depicted in the figure below. RM's may also be employed in fixed length records

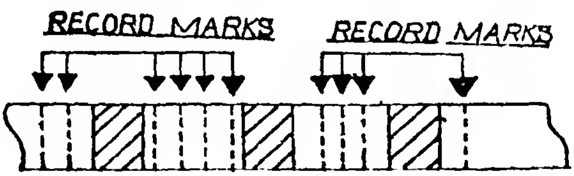


Fig. 36

File Layout : A data block may consist of

1 A fixed number of fixed length records (fixed-fixed block).

2 A fixed number of variable length records (fixed-variable block).

3. A variable number of fixed length records (variable-fixed block).

4. A variable number of variable length records (variable-variable block).

To give an example for each of these :—

1. Block A (450 Characters)	Record A1	150 characters	FIXED number of fixed length records.
	Record A2	—Do—	
	Record A3	—Do—	
2. Block B (450 Characters)	Record B1	—Do—	
	Record B2	—Do—	
	Record B3	—Do—	

*These are also known IBG's (Inter-block gaps) which, in fact, is a more fitting name.

1. Block A (408 Characters)	Record A1	150 Characters	Fixed number of variable length records ERM=End of record marker occupying Character ERM's are necessitated by variability of the length of the records.
	ERM	1 Character	
	Record A2	130 Characters	
	ERM	1 Character	
	Record A3	125 Characters	
	ERM	1 Character	
2. Block B (389 Characters)	Record B1	140 Characters	
	ERM	1 Character	
	Record B2	123 Characters	
	ERM	1 Character	
	Record B3	123 Characters	
	ERM	1 Character	

1. Block A (450 Characters)	Record A1	90 Ch.	Variable number of fixed length records.
	Record A2	90 Ch.	
	Record A3	90 Ch.	
	Record A4	90 Ch.	
	Record A5	90 Ch.	
2. Block B (360 Characters)	Record B1	90 Ch	
	Record B2	90 Ch.	
	Record B3	90 Ch.	
	Record B4	90 Ch.	

1. Block A (428 Ch)	Record A1	160 Ch.	Variable number of Variable length records.
	ERM	1 Ch.	
	Record A_2	120 Ch.	
	ERM	1 Ch.	
	Record A3	145 Ch.	
	ERM	1 Ch.	
2 Block B (447 Ch)	Record B1	165 Ch.	
	ERM	1 Ch.	
	Record B2	280 Ch.	
	ERM	1 Ch.	

It can be noticed that such a data flexibility would be almost impossibly hard to obtain in either punched card or paper tape

File Reel Ralationship. Earlier we mentioned that reels up to 2,400 ft are most common This is, however, arbitrary and this length is chosen because of convenience of handling Reels of 3600 ft. are in use Because of convenience in handling, however, a longish file may have to be put on more than one reel Several smaller files may be put on one reel though to avoid confusion, it is desirable to have one reel for each file If a reel has many files all the files together are known as physical files and individually as logical files. [Similarly if a punched card contains more than one record the individual records are called logical records and the entire card as a physical record].

Control character There is another character, control character, that is used by the program for distinguishing the master record from the detail records For example, in the punched card file, a customer's order for 10 items may be put in 11 punched cards. The leading card carrying the details of the customer is known as

the master card (Fig 37) It would be followed by cards carrying the stock data, one for each of the item on order. These following cards are known as the detail cards The master card carries a 'X' punch in a specified card column so that the program can distinguish it from the detail cards. The 'X' punch is then the control character in the case of the punched card files Likewise to distinguish the master record from its detail records a control character (viz. %) is put in the frame at the end of the master record in a magnetic tape file This is shown in the figure below

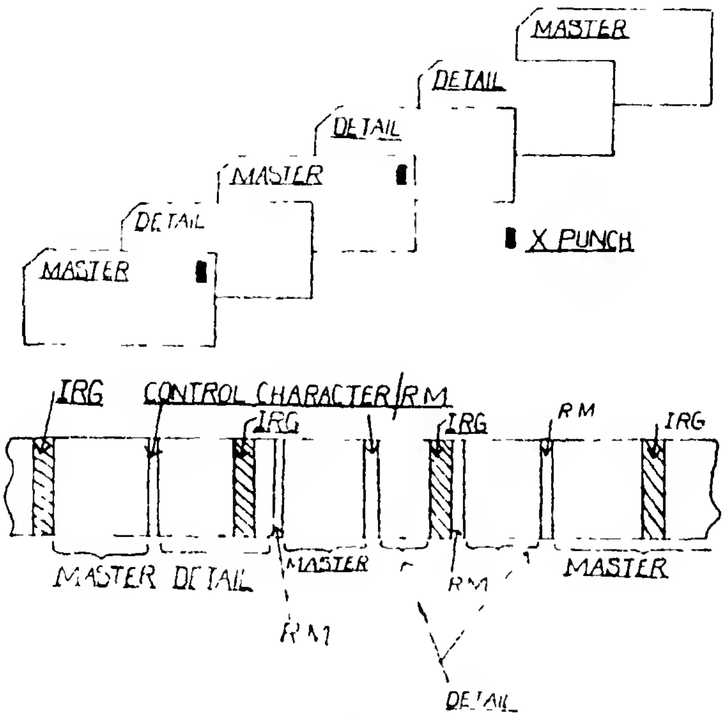


Fig. 37

Tape Mark As we mentioned earlier, two or more files of records can be placed on one reel of a magnetic tape Each file in this case is separated by a tape mark (viz *) which will signify the end of that particular file. Shown below are 3 separate files on one reel of mangetic tape : Parts, Payroll and customer listings.

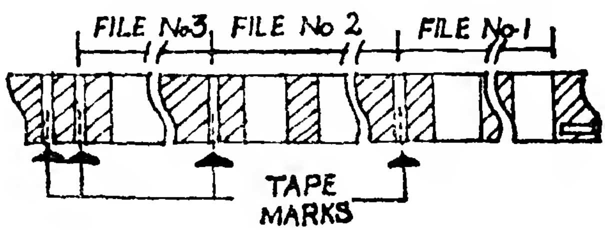


Fig. 38

Blocking. Placing several data records into one larger record is called blocking.

Deblocking is, hence, obtaining the next record from a multi-record block (by the program) Or it might consist of the function of transmitting a block of information suitable for the output medium after having assembled it from multiple records or after having suitably fractionated a single internal record

Beginning and End of the Tape A reflective marker, called the *Load Point,* marks the beginning of usable tape Likewise, a reflective marker called the *end of reel* marks the approximate ending of the usable tape These are shown in figure 39. The markers are placed on opposite edges of the tape to allow the magnetic tape (read/write) unit to distinguish between them.

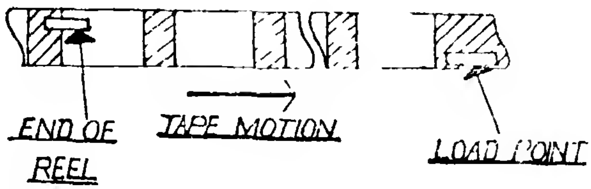


Fig 39

Lables. A reel of magnetic tape which has been recorded generally contains a handwritten description on the face of the plastic reel This description is written by the operation personnel at the time the tape was generated (written). The label bearing this description is affixed on the reel and is used during its subsequent handling

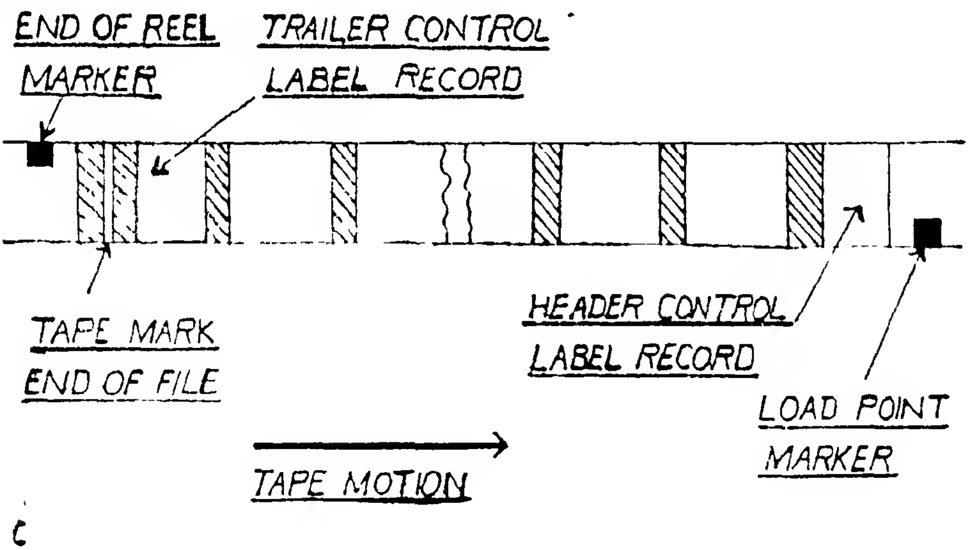


Fig. 40

Aside from having a hand written description on the face of a reel, most written tapes contain a label record at the beginning and a record at the end of a given file of records The label record at the beginning of a magnetic tape file of records is called a header label The one at the end of tape file of records is called the trailer label The layout of these labels is discussed in study VI under file control section.

The header label contains the file details. These are printed out by the program for visual verification (*the* file) by the operator before processing begins. The trailer label contains the control totals to be discussed later. It is hoped that the student would get a better understanding of these in study VI and at the moment he may be content with the hazy idea formed about these now.

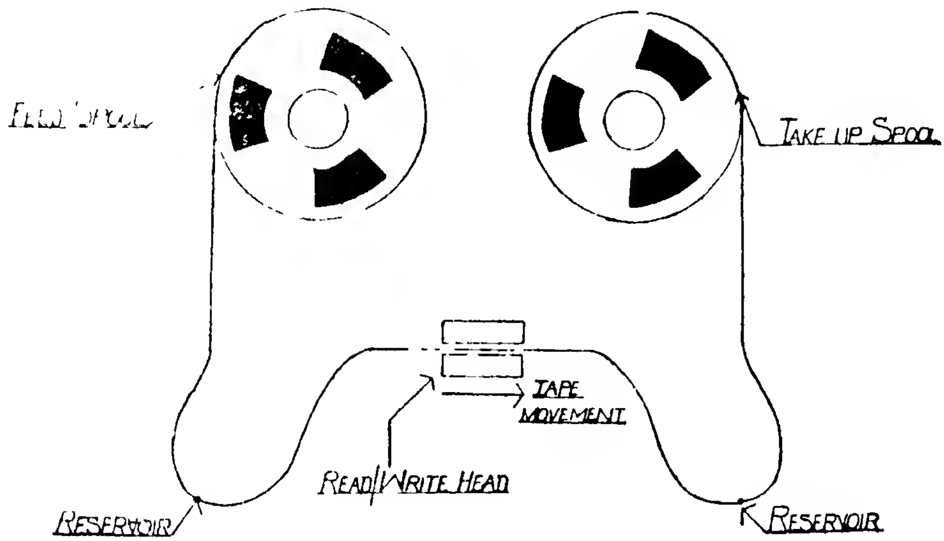


Fig. 41

The Read/Write Unit. Its sketch is shown in fig 41

The tape moves across from the feed spool to the take up spool via the read/write heads The reservoirs are provided to keep the tape tension within fixed and close tolerance in between the heads. They also allow very fast acceleration and deceleration of the tape, to and from the speed required for the data transfer without the inertia of the tape reels causing the tape to snap

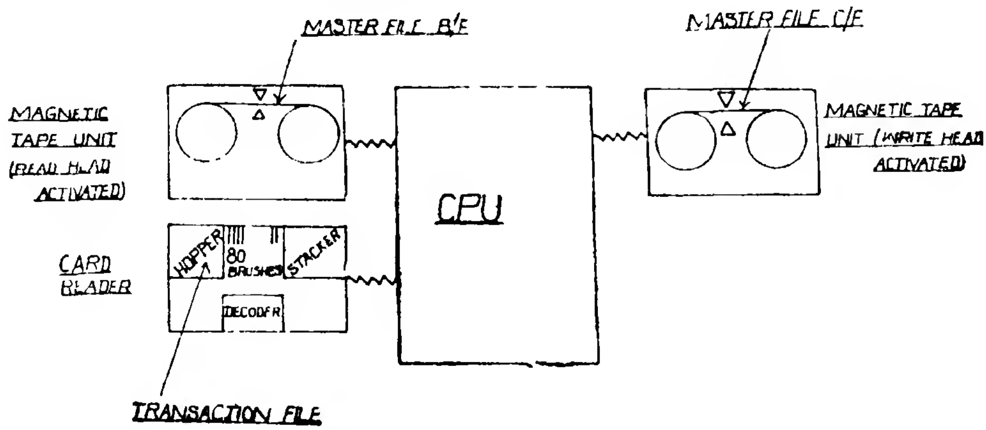
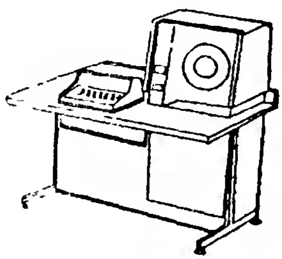


Fig. 42

The figure 42 depicts a computer configuration with a megnetic tape unit on each of the input and output sides for reading the B/F file and writing the C/F file and a card reader on the input side for reading the transaction file on punched cards. It is noted that constructionally the magnetic tape input and output units are akin i e., there are no differences In the former, the read head is activated and in the latter the write head is activated. Magnetic tape is more commonly used for master files than the punched cards.

Magnetic Tape Encoder. (See figure 43) Like the keypunch, the magnetic tape encoder is used manually off-line via a keyboard attached for encoding the data from source documents into magnetic tapes The encoder itself may be used for verification as well.



MAGNETIC TAPE ENSCRIBER

Fig. 43

Conversion Devices. There are conversion devices that can transfer the data from punched cards or paper tape on the magnetic tape These devices are operated off line Very often, data are first punched into cards and input to the computer in random order, they are then validated (i e , checked) and written to magnetic tape for further processing to take advantage of magnetic tape transfer speeds and flexibility.

Uses of Magnetic Tape. Magnetic tape is used for (i) storing transaction files (ii) storing master files, (iii) storing programs, (iv) storing intermediate processing results (this will be more clear later in the discussiou on sorting), (v) storing output data to be subsequently printed out.

Advantages of the Magnetic Tape

(i) The data transfer speed of the magnetic tape is much higher than that of the punched cards or the paper tape. The transfer speed refers to the speed with which a block of data can be read from the magnetic tape into the CPU. The following table is intended to give an idea on the transfer speeds of these and other media, some of which will be discussed later

	Typical	Maximum
	(Thousands of characters/second)	
Console Tapewriter	0 015	
Paper Tape Reader	1	(2)
Card Reader	0.3	(1)
Magnetic Tape	60	(340)
Magnetic Drum	1000	(5000)
Magnetic Disc	150	(300)
Line printer	1	(4)

(ii) The magnetic tape stores the data far more compactly than the punched cards or the paper tape Also, it is much more easy to handle than the punched cards The information in a stack of punched cards over a ceiling high can be stored in a single reel of the magnetic tape And it can indeed be reused ! The reels can also be stored efficiently in racks that can be kept in the fire proof vaults and maintained by the tape librarian The punched cards and the paper tape are liable to be damaged and moistened The paper tape is also liable to be torn There is an ever present hazard of dropping a pack of punched cards The magnetic tape is immune from these hazards

(iii) If desired the magnetic tape can be automatically backtracked. The punched cards would have to be shuffled manually in order to move back.

(iv) The magnetic tape employs the internal code of the CPU and, therefore, there is no translation problem It is to be noted that there is special decoding unit fitted in the card reader and the card punch to perform translation from Hollerith's code to the CPU code and *vice versa*

(v) A reel of magnetic tape is much cheaper than its punched card equivalent Also, for sequential application it is much cheaper than the magnetic disc

(vi) It finds use even in predominantly magnetic disc installations where the contents of the magnetic discs are periodically dumped on magnetic tapes for purposes of file reconstruction in case of fire, sabotage etc This point will be more clear during the discussion on overlaying in the disc later

Disadvantages

(i) The punched cards and the paper tape can be interpreted by sight though with difficulty but the magnetic tape affords no such possibility whatever i e., it is not at all human readable

(ii) In the case of the punched cards insertions or deletions of records are very easy. All that one has to do is to take out the records required to be deleted and put the new cards in the sequential order

(iii) The magnetic tape is highly susceptible to damage by dust and magnetic fields in the vicinity (e g , electric motors nearby)

(iv) The magnetic tape is a sequential media and is not good for handling inquiries for which the magnetic disc is ideal.

Updating and Amendment Runs

The only viable method for comparing records on two or more magnetic tape files is by first sorting them into the same sequence, i e , the transaction file has to be sorted into the same sequence as the master file Occasionally there may be a double sort This applies, for example, to stock updating and allocation. In this application some movements have priority and therefore the sort must be into date sequence within stock number sequence

Brought forward-carried forward file method (Figure 44). In this method, both the transaction file and b f master file are input in the same sequence Un-matching records are copied into the c.f , file whilst matching records are updated and then written on the c f file

Father-son Technique

The brought forward tape and carried forward tape at any period are known as the father and son tapes respectively The brought forward tape of previous period is known as the grandfather tape Referring to figure 45, for week 6, it can be seen that there are three tapes in existence during an updating run .—

(i) The son tape (S_8) which is in the process of being created on a fresh reel. This is, of course, the carried forward tape

(ii) The father tape (F_6) which was previous week's son tape This is of course the brought forward tape

(iii) The grandfather tape (G_6) which was the previous week's father tape

By retaining the previous week's father tape (now grandfather tape) and the movement tape it is possible to reconstruct the previous week's son (now father) tape in case of any damage to the previous week's son or this week's father tape.

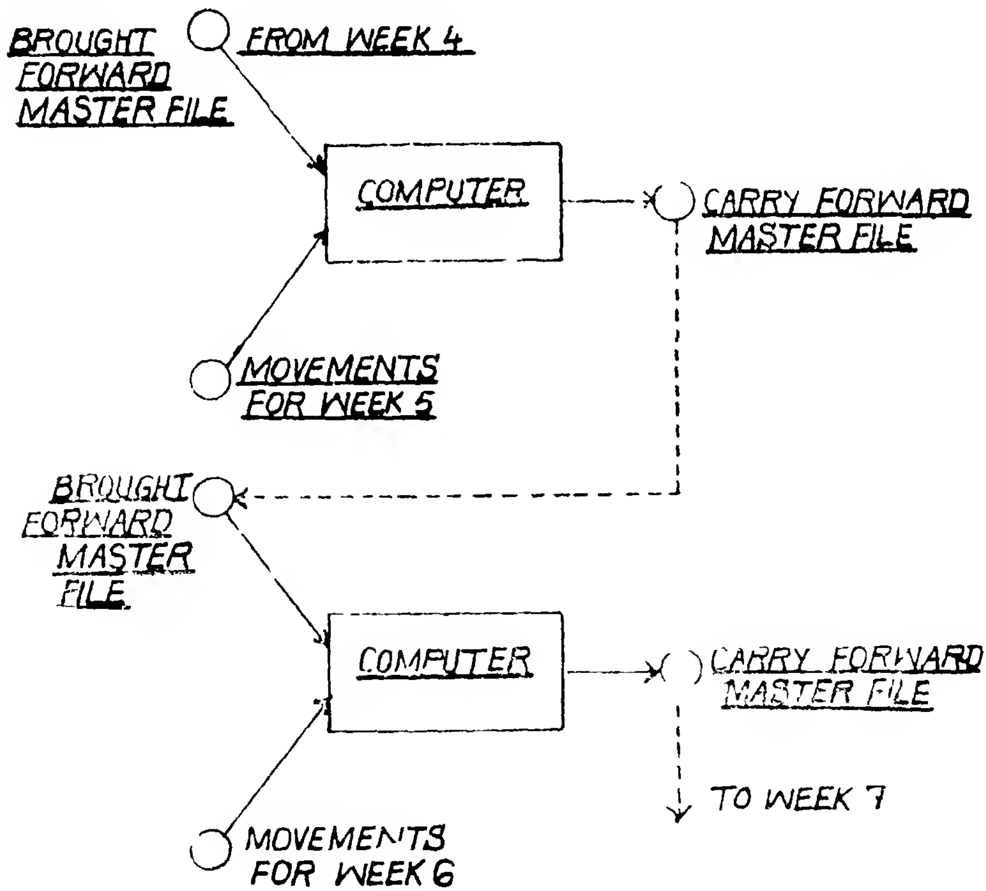
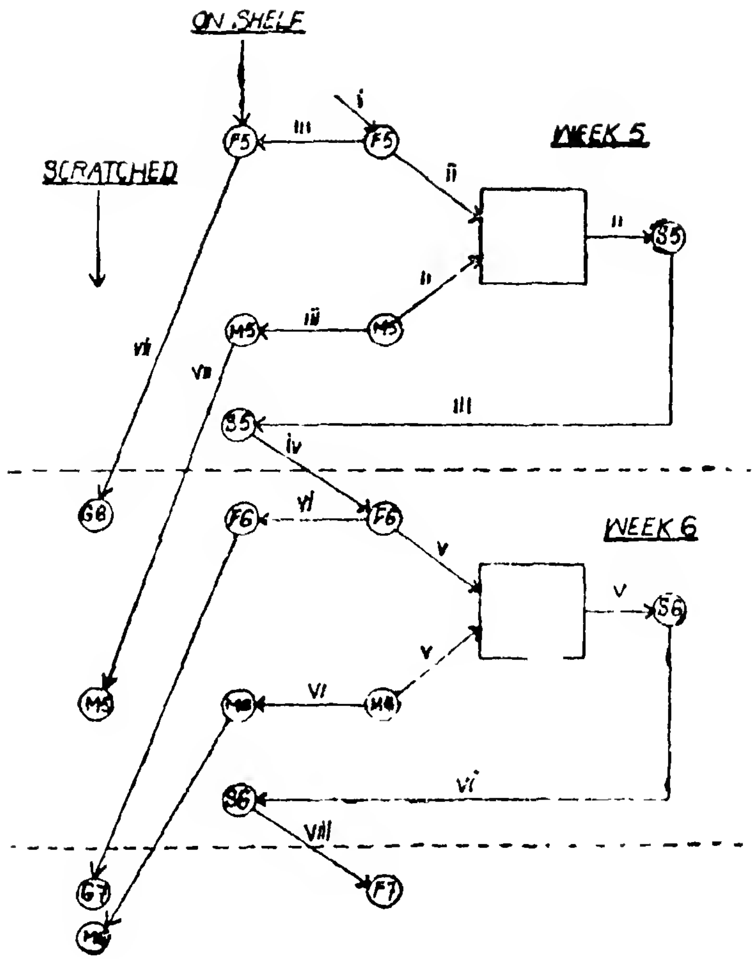


Fig. 44



Notation : F6=Father tape fos week 6
G6=Grandfather tape for week 6
S5=Son tape for weeks 5
M5=Movement (transactiou) tape for week 5

Roman numerals indicate sequence of action.

Fig. 45

7. Magnetic Disc

Magnetic discs are the most popular direct access medium By *direct access we men that a record can be accessed without having to plod through the preceding records The other direct access media, magnetic drum (which is much more expensive) and magnetic ledger cards (which are much more slow) would be discussed later. The magnetic discs are made out of thin metal plates which

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The media discussed upto now are sequeential media in that records in them are sequenced in the ascening descending order of a key.

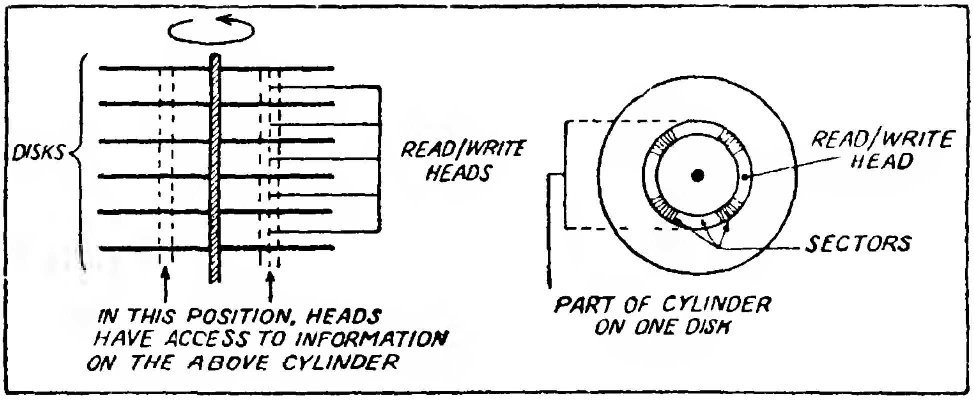


Fig. 46

can be coated on one or both the faces with a magnetisable material Typically several discs are permanently mounted on a shaft as in figure 46 which shows a pack of six discs The upper face of the upper most disc and lower face of the lower most disc are not usable i e , they are not magnetisable, the intermediate discs, however, having both the faces as usable, meaning that there are ten faces to this disc pack which can be used. The shaft and, therefore, the discs rotate at the constant speed for the data to be read from them or written on them For reading and writing the data there are as many read write heads as the number of usable faces The read/write heads can move in and out to access the data from a particular region of the magnetic disc faces.

The magnetic disc can be likenned to a gramophone record for elucidation of the concepts of data storage and retrieval involved Just as the gramophone record has concentric tracks so is the case with the magnetic disc each face of which has 200 tracks A song on a gramophone record occupies many tracks, but a record of data on a magnetic disc can be stored in just a portion of a track, therefore, the tracks of a magnetic disc are further subdivided into a number of sectors or buckets, usually 8 to 10 Further, each of the bucket or sector can hold about 8 to 10 records Consider a gramophone record cantaining six songs It is not difficult to see that we can have direct access on the gramophone record in the sense that the desired song can be listen straightway by just placing the needlehead over its first track This way we do not have to listen the preceding songs But if we desire we can also listen all the songs one by one sequentially. Thus, on the gramophone record we can have both direct (or random) access and sequential access In the magnetic disc too we can have both random and sequential access It is to be carefully noted that in the gramophone record we do have an index of the song in its inner portion by referring to which we can access the desired songs The principle of indexing applies to direct access on the magnetic disc also At the time of data storage in the magnetic disc pack we have to compile an index which sequentially lists the keys of all the records and the addresses of buckets in which they are stored Just as we could move the needlehead to the desired track on the gramophone record for random access, the CPU can command the read/write heads to move in or out to access the wanted record directly.

As we stated above the magnetic disc can be used for both direct as well as sequential access and this would also be the case with the magnetic drum and the

magnetic ledger cards to be discussed later But in the sequential access media that is punched cards, paper and magnetic tape it is not posible to have direct access. For example, consider a payroll file of 2000 records on the magnetic tape The manager wants to know the qualifications of 1992nd employee This would necessitate moving the records of all the preceding 1991 employees before this particular employee's record can be accessed and the wanted information supplied to the manager concerned

As depicted in the figure below only one magnetic disc pack is connected to the CPU and serves both as the input and the output unit Thus is in contrast to the punched cards which have a card reader on the input side and a card punch on the output side of the CPU The implication of this being that a record picked up from a bucket and updated in the CPU is deposited back in the same place in that

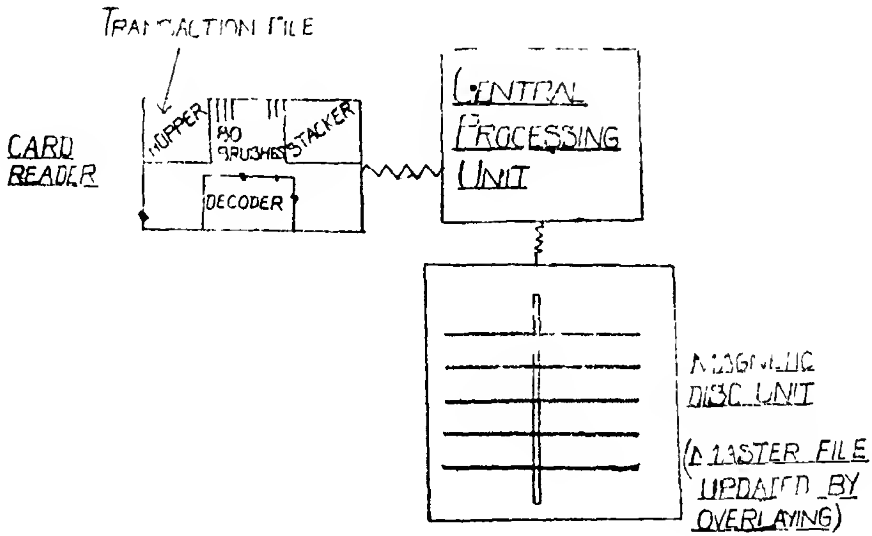


Fig. 47

buchket This mode of storing updated data is known as overlaying i e , original data is automatically erased when the new up lited record is being deposited in its place This leads to economics in the sense th it fewer discs are needed but as a disadvantage it becomes difficult to trace any errors as unlike the magnetic tape files, it is not possible to reconstruct the latest file from the previous version.

The time required in accessing a record has a generally two components. (i) The time required to move read write head to its track (ii) the time required for the disc to spin its bucket under it

The total of these two components is known as the seek time and typically ranges from 200 to 600 miliseconds

As we emphasised earlier, the magnetic disk, like gramoponne record offers both direct and direct/sequential access Each of these has two further categories as depicted in the chart below.

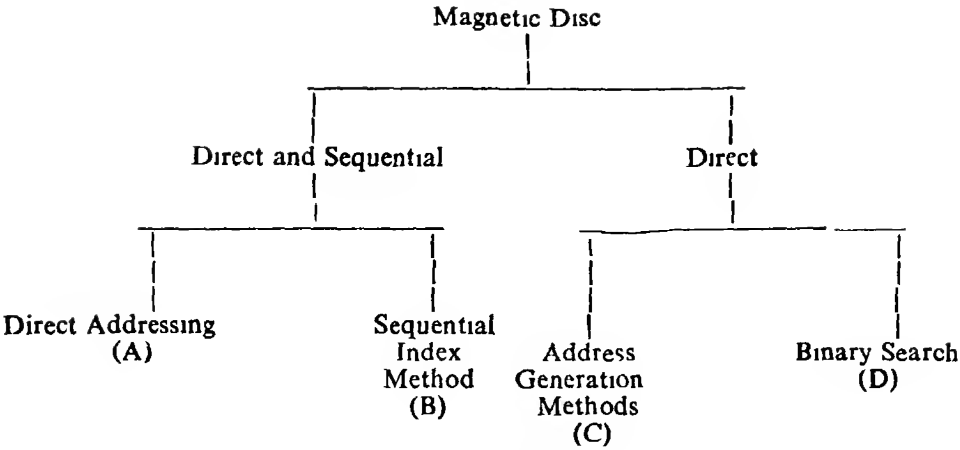


Fig 48

We shall discuss each of these in detail. Prior to that, however, some points of practical importance are discussed below.

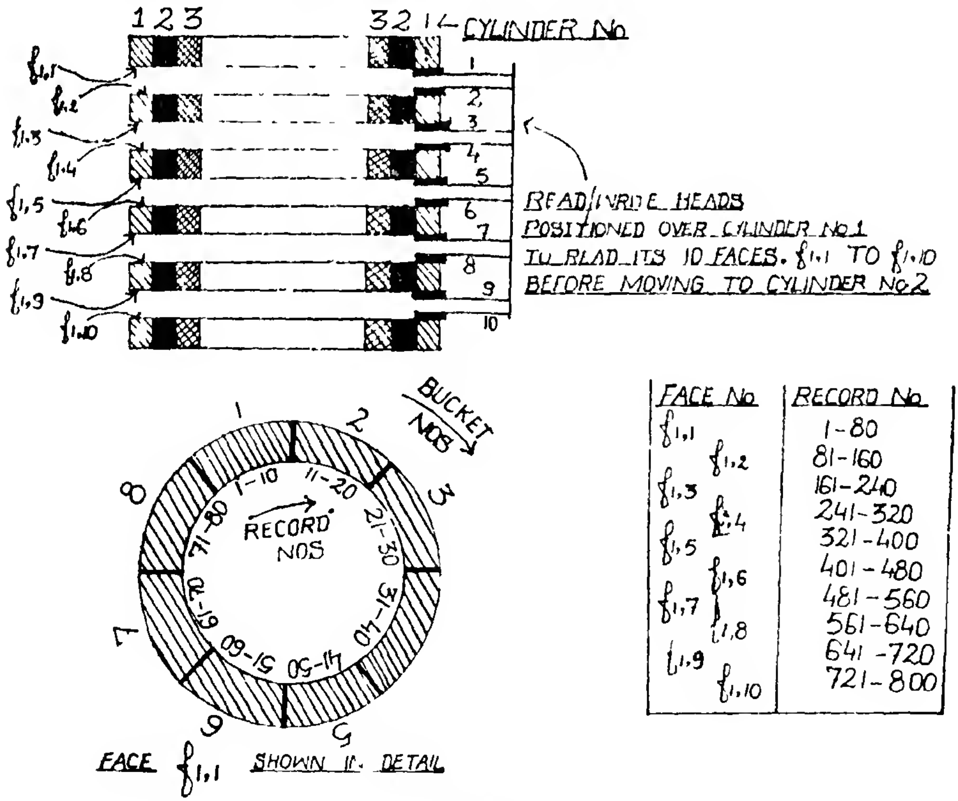


Fig. 49

The keys of the files in commercial situations do have gaps. Payroll file, for

example, may have gaps because of the employees leaving the organisation on retirement, resignation etc As another example, the inventory file too has gaps because items may be rendered obsolete with time In fact, in the case of stock items the designers deliberately tend to leave gaps For example, if there is a bolt in ten sizes of a particular type the designer may assign 10 codes to these and the following ten codes may be left unassigned for the time being so that in the future, when further sizes of this type of bolt come into being they can be fitted in. This fact that the files have gaps has a lot of bearing on the file organisation on the magnetic disc

The other point of importance is the fact that read/write heads move in and out simultaneously Referring to figure 48 there are 10 magnetisable faces and there are, therefore, 10 read/write heads Even though, a record on the first disc face were to be accessed not only the first read/write head would move but the other nine read/write heads would also move in unison, but only the first read head would be activated, other remaining' inactive As a consequenec of this, if the read/write heads have once been moved all the ten tracks vertically above and below each other should be read or written before any further movement of the heads take place. This eliminates one component of the seek time i e , horizontal movement of the read write heads This has led to the concept of cylinders (synonymous : seek areas) Any ten tracks vertically above and below each other constitute more or less a hollow cylinder, there, therefore, being 200 cylinders. Because of the simultaniety of the movement of the read/write heads it is to be desired that the records are arranged sequentially in cylinders so that when the first cylinder (i e , the first ten tracks) has been read the heads move to the next cylinder i e , reading or writing is performed cylinder wise

Each of the 200 cylinders in figure 48 has 10 faces Let us designate the j-th (j=1, 2 10) face of the i-th (i=1, 2 200) cylinder by f_{ij} Thus the faces of the first cylinder would be designated by $f_{1,1}$ $f_{1,2}$ $f_{1,3}$, $f_{1,4}$ $f_{1,5}$, $f_{1,6}$, $f_{1,7}$, $f_{1,8}$ $f_{1,9}$, and $f_{1,10}$ Now say that to start with the read/write heads are positioned over the first cylinder and it is required to read the disc pack in entirety To do this as efficiently as possible we should first read all the 10 faces of the first cylinder then move the read/write heads in by a track and read all the 10 faces of the next cylinder and so on until all the 200 cylinders have been read. This would minimise the movement of read/write heads and this component of the seek time. Thus, in sequential organisation of records on the magnetic disc the records should be arranged cylinder wise Consider towards elucidation of this point a pay-roll file without any gaps and containing 1,60,000 records Assuming that there are 10 faces, 200 tracks/ face, 8 buckets/track and 10 records/bucket, the first 800 records (1 to 800) should then be stored in the first cylinder, the next 800 records (801 to 1600) should be stored in the second cylinder and so on When this file has to be later processed sequentially this storage scheme would minimise the movement of the read/write heads and the corresponding seek time component

Sequential Access Methods (Direct Access also possible)

A. Self (Direct) addressing

This method is suitable for determining the bucket address of fixed length records in a sequential file, and in which the keys form a complete or almost complete range of consecutive numbers In a way, we have actually discussed this **method in the above paragraph where we stored the 1,60,000 payroll records**

cylinder-wise in the magnetic disc pack of 6 discs. The first cylinder carries the first 800 records, the 2nd cylinder carries the next 8000 records, and so on. For periodic processing of the file the read/write heads would move cylinder by cylinder in which the records have been sequentially arranged. For example, the ten faces in the first cylinder would carry the first 800 records as below :

f 1, 1	1 to 80
f 1, 2	81 to 160
⋮	
f 1, 10	721 to 800

How do we have direct access then in such a file organisation? There are a total of 16,000 buckets Let the bucket address range from 10,001 to 26,000 And the keys of the records range from 1 to 1,60,000 We wish to know where record of the key 1,49,892 is to be found i e in which bucket it is stored The following arithmethic computations would have to be performed towards this purpose :

1. Divide the wanted record's key by the number of records per bucket.

$$\frac{149892}{10} = 14989 \text{ and a remainder of } 2.$$

2 Add the first bucket number to the quotient to give the wanted record's bucket :

$$14989 + 10001 = 24990.$$

3 The remainder (2) is the record's position within the bucket The remainder 0 would mean that it is the last record of the preceding bucket Thus if a manager wishes to know the qualification of a particular employee (say, no 149892) i e , makes a random inquiry, the above computations would be performed to derive the bucket number, command the read/write heads to move to that bucket and supply the wanted information.

But this method is highly impractical because files do have gaps in the keys and this would leave too many empty buckets i e., storage would not be compact.

Advantages of Self-addressing :

(i) No index has to be stored.

Disadvantages of Self-addressing :

(i) The records must be of fixed length.

(ii) If some records are deleted their storage space remains empty.

B. Index Sequential Organisation

This method is the most popular because it covers nearly all the key ranges. For the buckets, an index is compiled and stored in the first bucket Within the bucket the records may or may not be sequentially arranged Bucket index indicates the highest key. This is shown in the lower part of figure 50 The wanted record 4302 is to be found in buckets No. 2 which has the highest key as 4384. It may be possible to hold the index permanently in core store if there is sufficient space. If there are several seek areas for a file a seek area index is also compiled

and held permanently in the core store. This index contains the highest key within each seek area, and thereby enables the appropriate seek area to be found This is shown, for example, in the upper portion of figure 50.

It can be seen that this organisation permits both sequential as well as random access For the former, the index is not put to use. If there arises the necessity of expanding additional records can be stored in a special overflow area discussed in a section below The other possibility is in keeping some space in the buckets empty for new insertions upon expansion Indexed sequential organisation combines the best features of sequential organisation with best features of direct access by means of the index

Disadvantages are the extra storage requirements to accommodate any subsequent insertions, the extra storage for the indexes and the extra time needed to process the indexes and the need to periodically reorder the file.

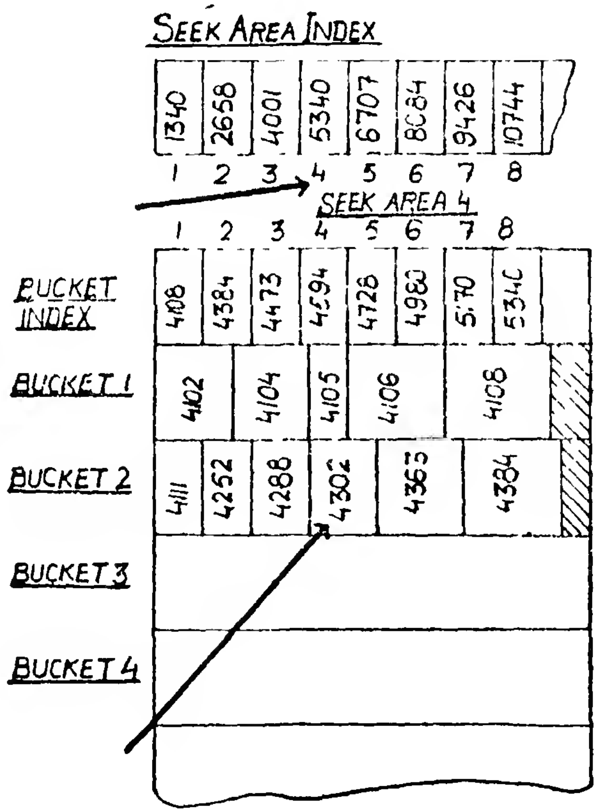


Fig. 50

C. Random processing

Random processing is characterised by the fact that records are stored in such a way that there does not exist any simple relationship between the keys of the adjacent records.

D. Address Generation Techniques

In order to store a record initially and then retrieve it on a routine basis it is necessary to calculate its address i e , its bucket number from its key Several address generation techniques for such calculations are discussed below. We shall introduce the student with the *Extraction Method* Consider, for example, a stock record with the key 467325. The extraction alogarithm consists of extracting the alternate digits of the key starting with the first digit. Thus 472 is obtained and it is then the address of the bucket in which this record would be stored In this manner, the bucket addresses for all the records would be computed, and they would be stored in those buckets Now, suppose that a salesman wishes to know the status of item 467325 The program would apply the extraction alogarithm on this key and obtain 472 as the bucket in which this record would be found The salesman's inquiry can now be answered There is, however, a problem with this and all the other address generation techniques. Consider another record with the key 48,7923. This would also generate 472 as the bucket address Such records as generate the same bucket number are known as *synonyms.*

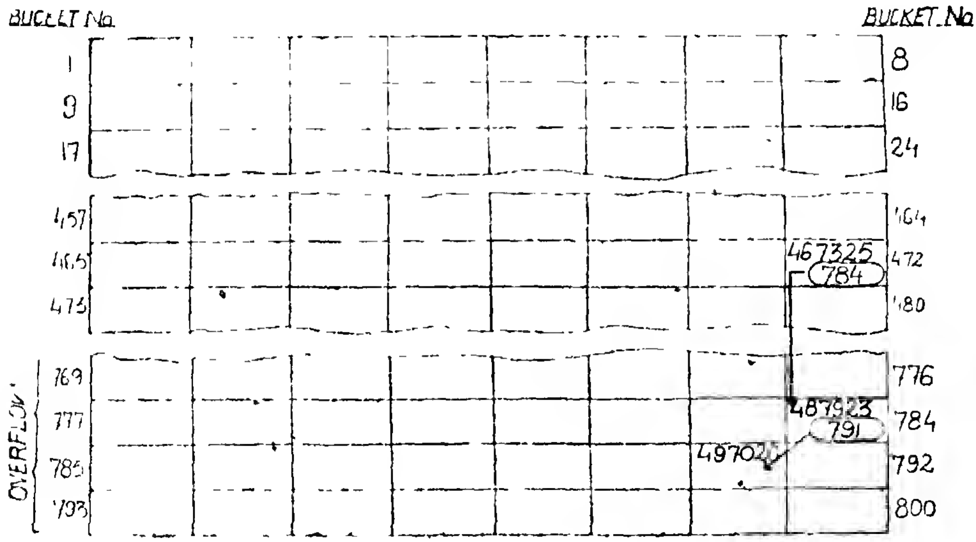


Fig. 51

Assuming that one bucket can hold one record only this record (with the key 487923) cannot be stored in bucket 472 which is storing record 467325. In the technical jargon the bucket 472 is overflowing. Where then to store the synonym 487923 ? Overflowing synonyms are stored in a special area in the disk pack e g., the last disc face) The bucket 472 would contain not only the original record but also a pointer which means the physical address in the overflow area where the synonym is stored. This is illustrated in figure 51 The original record 467325 in bucket 472 is linked by a pointer in bucket No 472 to record 487923 in bucket 784 which, in turn, is shown linked to record 497020 (in bucket No. 791 in the overflow area) by a pointer in bucket No 784

There is, however, another side to the problem. Some of the bucket addresses

may never be generated owing to gaps in the files. Thus some disc storage is liable to be empty because of this problem. This is true of the other address generation methods discussed below too.

(1) **Prime Number Division.** The key number of a record is divided by the number of available buckets. The remainder so obtained is added to a constant, yielding the bucket number in which the record is to be stored. Obviously the remainders should be evenly distributed to attain highest packing density It has been empirically established that prime number divisors yield fairly even distribution Thus instead of using the number of buckets as a divisior the prime number nearest it may be employed.

(2) **Folding** The keys are split into two or more parts, these are then added together in order to obtain a higher degree of randomness

Example 467325 may be split as below :

46+73+25=144 (three parts)

467+325=792 (two parts)

472+635=1107 (using alternate positions)

Squaring The key is squared and the portion of the result used as a bucket number.

Example, 428 squared is 183184. Using the 2 middle digits the bucket number is 31.

(3) **Radix tranformation** Consider records number 4532. It is of radix (base) ten. Using 11 instead of 10 we obtain the following number

$4 \times 11^3 + 5 \times 11^2 + 3 \times 11^1 + 2 \times 11^0$

$= 5324 + 605 + 33 + 2 = 5964$

Extracting the penultimate three digits gives bucket number 596.

(4) **Alphabetic Keys** . The *alphabets are first converted to some numeric equivalents* and then one of the *above methods is employed* to derive the bucket number

Advantages of Random Processing by address generation techniques

(i) Sorting of transactions is obviated

(ii) No index is necessary This saves storage space.

(iii) Determination of bucket number takes less time than searching the index.

Disadnantages of Random Processing

(i) If file activity is high this method is less efficient than sequential processing.

(ii) Some buckets do remain empty and others overflow. For the latter storage space is needed as described later.

(iii) A change in the range of keys may call for re-organisation of the addresses.

D. Index Random File Organisation

Instead of attempting to locate addresses by randomising, separate index may be maintained in which the address of record being sought is found The index is by record key The index may therefore be searched sequentially or by binary search explained below :

Binary search · Suppose there is a disc file of 16 items stored under index random file organisation. The items numbers run from 101 to 159 as below That they are not consecutive represents a typical practical situation in which records are frequently purged leaving gaps The corresponding index is given below.

PHYS. ADDR.	203	208	207	188	186	196	219	225	220	190	196	201	200	193	195	204
ITEM No.	101	102	107	109	127	129	131	133	134	136	148	149	150	153	156	159
No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
COMPARISON No.								1		3	4	2				

BINARY SEARCH OR CHOPPING

Fig. 52

Let us say that item 148 is to be updated. The index which is stored somewhere in the disc itself is brought into the computer memory The program would see if the 1st item has its key as 148 Then it would move to the 2nd item and so on until after 11 comparisons it has got item 148 and its physical address as 196. This is ordinary sequential search which suits human capabilities but not the computer's capabilities There is a binary search that suits computer's capabilities Item No 8(=16−2) is checked if it has key 148, which it has not Then it is checked if it has kcy greater than 148 or less than 148 As we can see it is less than 148 ($133 < 148$). Obviously, search now needs to be carried out in the lower half i e., from item nos. 9 to 16 Item no. $\left(\frac{16+8}{2}=12\right)$ is now checked if it is greater than, equal to or less than 148 etc , etc. The procedure of binary search is depicted in figure 52. It can be seen that item no. 11 has been located in only 4 comparisons. Binary Search is thus quite efficient. For example, to locate a record with a given key out of 5,000 records it requires only 11 records to be accessed and compared.

Distribution of Records. As discussed above the distribution of records in buckets under random processing tends no be uneven. Some buckets therefore remain empty and others tend to overflow This happens at the file creation stage itself under random processing In sequential organisation there is no such problem during creation But subsequently buckets may overflow if more records are sought to be added or the existing records are expanded in content How to deal with overflow?

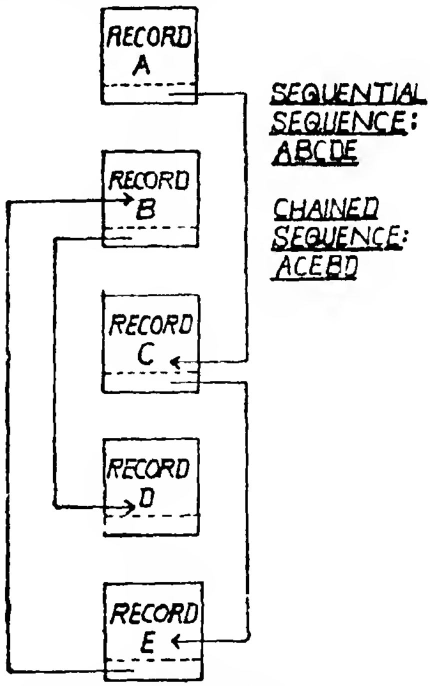
Overflow areas are empty storage spaces either in various cylindrical seek areas or in an independent area. For each cylinder the overflow records (or synonyms) are arranged sequentially in the overflow area. The advantage of

having overflow area in the various cylinders is that read/write arms need not be moved for accessing the synonyms but this arrangement requires more area than an independent area Compromise between the two arrangements can also be had. This is achieved by having overflow area in all the cylinders as well as an independent area The overflow from cylinders can be accommodated in the independent area.

Progressive Overflow. The synonym is stored in just the next higher bucket that can accommodate it and this means, when searching for a record, the home bucket is examined first and if this does not hold it, successive buckets are examined until the wanted record is found. Obviously this method is rather inefficient, and applies to random processing alone

Using pointers Pointer is a noting of the physical address of its synonym in a record During processing, when a record is not found in the home bucket the pointer is read and the synonym accessed.

List Organisation for information Retrieval. In simple list organisation, each record has a pointer for the next record by some key, the contents of the pointers being the physical storage address of the next record Thus the entry in a simple list organisation is to the first record the pointer of which leads to the next record and so on An example would be the payroll application where the employees of a particular department are to be accessed The department index is accesssed for the required department number which has against it mentioned the physical storage address of the first record. Obviously there could be multiple pointers e g, in a payroll application ; such pointers may pertain to :



LIST ORGANISATION

Fig. 53

Department

Trade of the employee

Age group

Sex

Salary range, etc.

8. **Magnetic Ledger Cards** combine the advantages of magnetic tape and magnetic disc, and are discussed under the Visible Record Computer

9 **Magnetic Drums** were an early means of primary storage. but now-a-days they are also used as a secondary storage device The access time is much less than that of magnetic disc, and therefore they are preferred where fast response rather than greater capacity is the criteria.

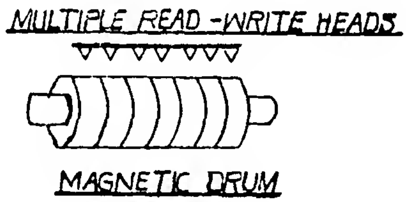


Fig. 54

The stored data is arranged in bands or tracks which may further be sub-divided into sectors. There is usually a read/write head for each head though a single head that can move across all the bands is also used to cut down expenses The direct access time depends upon (i) the number of heads, (ii) the position of the sector to be accessed with respect to the position of the corresponding head and (iii) speed of the drum.

10 *Flexible (Floppy) disk systems* :—The floppy disk is a thin flexible plastic disk with a coating of ferromagnetic material. It is approximately 10 inches in diameter and is housed in a square plastic envelop with an aperature that allows a read/write head to access the disk with the envelop in place. Floppy disk are similar to phonograph records. They can be removed from the disk drive, filed, mailed, or otherwise easily handled. Their low cost makes them competitive with other storage media. Concentric tracks on one side of a floppy disk can store approximately 1.6 megabytes (1 megabyte$=10^6$) of data.

For recording of data/reading the recorded information, the flexible disk is inserted in a flexible disk drive having a provision for mounting the disk The disk drive rotates the disk within the envelop under a read/write head at a speed of several hundred revolutions per minute The data are recorded (or read from) disk by this read/write head through the aperture in the envelop Typically data may be transferred at the rate of 500,000 bytes per second The average access time is approximately 6 milliseconds. Records and files on a flexible disk are organised and processed in the same way as with rigid disk systems. Flexible (floppy) disk drives are generally smaller and more economical to manufacture than rigid disk systems

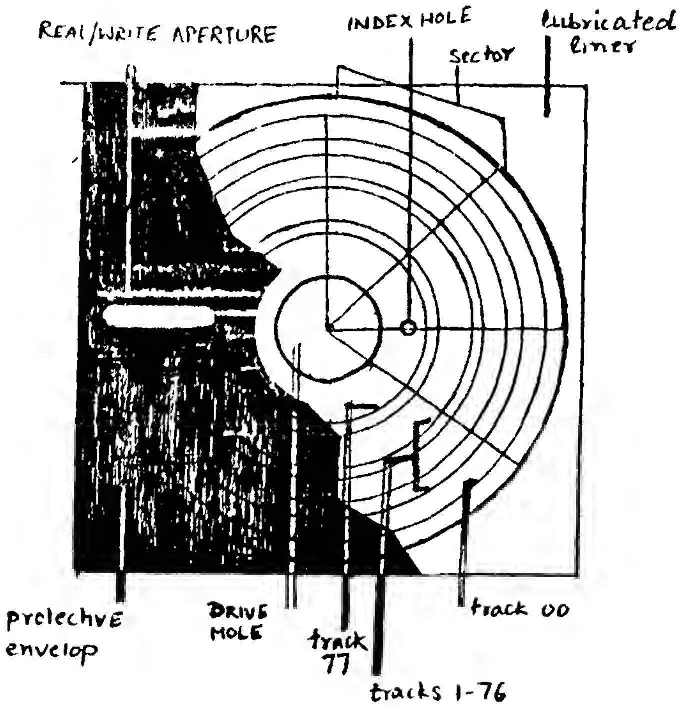


Fig 55

Hardware. Computer data processing involves equipment which can perform the following functions · data preparation; input to the computer; computation, control and primary storage; secondary storage and output from the computer Equipment which is connected directly to the computer is termed "on-line" whereas equipment which is used separately and is not connected is called off line The following is a list of the *hardware* i.e. various computer equipment :

Function	Types of Equipment Used
Data Preparation	Key-driven card punch Key-driven card verifier Paper tape punch Magnetic Tape encoder Magnetic ink enscriber Optical character enscriber Data collection devices with keyboard plastic card sensor, to etc. transcribe data on to some machine-readable medium. Devices to prepare cards, paper tape as a by-product of another operation. Conversion devices, such as paper tape to punched card converter and paper tape to punched card converter.
Input	Card reader. Magnetic tape unit Paper tape reader Magnetic ink-character reader Optical Scanner Floppy disk drives Console typewriter (discussed later) Terminals (discussed later)
Computation, control and primary storage	Central Processing Unit (CPU)
Secondary storage	Storage devices using the following file media : Magnetic tape Magnetic disc Magnetic drum Magnetic card or strip Punched Cards, etc.
Output	Printer Card punch Paper tape punch Floppy disks Terminals (discussed later)

Fig. 56

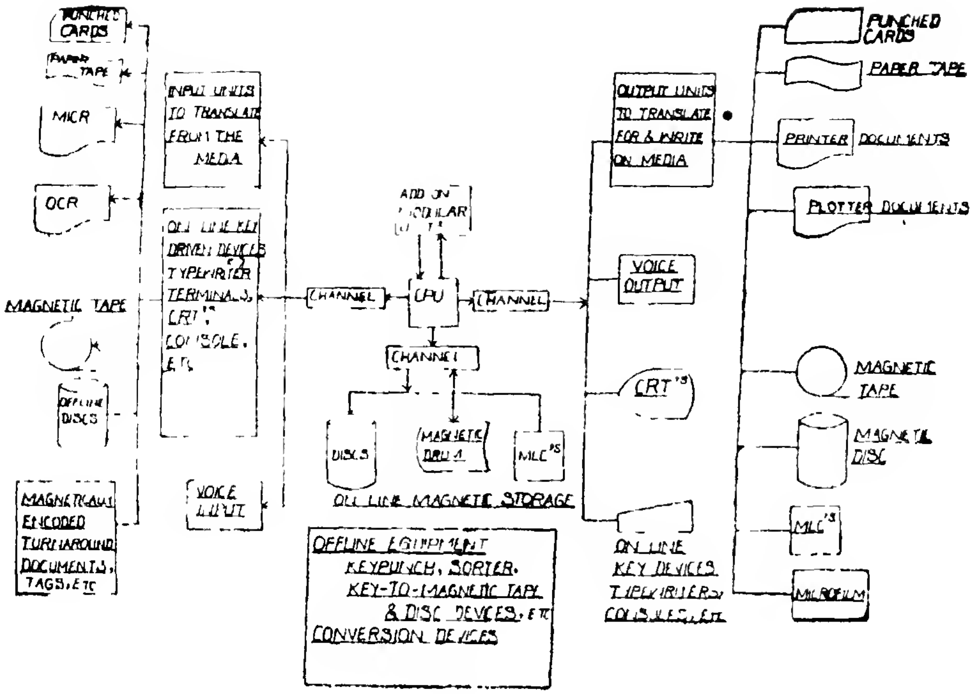


Fig 57 (Hardware)

Central Processing Unit

As has been discussed earlier, the CPU (Fig. 58) may be considered as composed of three functional sub-units :

(i) Storage unit (Main memory)

(ii) Control Unit

(iii) Arithmetic and logic unit (including Registers)

Main memory in the modern computer, physically consists of thousand of tiny doughnut shaped ferrite cores (Fig. 59) arranged in arrays of 64×64. The cores are strung out on thin wires about the size of human hair. Electric current passing through the wires can magnetise the cores in two directions of politary A clock-wise direction indicates 0 state (bit) and an anti-clock-wise direction 1 state (Fig 60) The diagonal wires through each core "sense" which core are magnetized and which are not. Using a memory made up of these cores, a computer can remember thousands of numbers and letters during a program The ferrite core memory is fairly fast, being capable of transferring 2-4 million characters per second The student need not, however, delve further into the technical aspects except for noting the fact that newer solid-state technologies have begun to produce memory devices that are cost-competitive with core, while offering substantially better performance at transfer rate of over eight million character per second and in the near future they are likely to render the ferrite core obsolete.

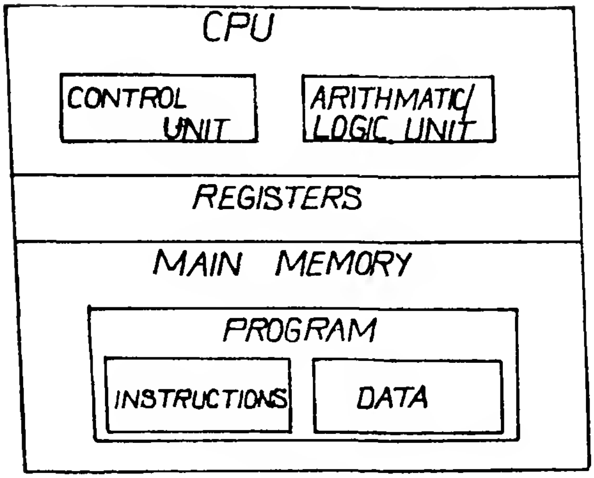
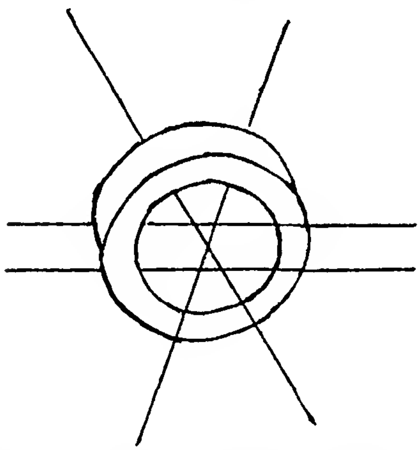


Fig. 58



Magnetic core with wires sturng for read and write.

Fig. 59

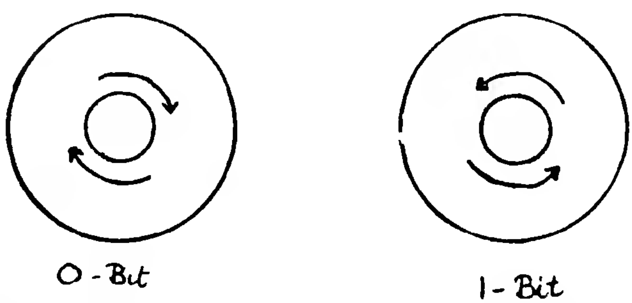


Fig 60. Clockwise and anti-clock wise direction of polarity to indicate 0 state and 1 state (bit)

Fig. 60

In order to identify one number and keep it separate from all the others numbers, a core memory is divided into separate locations, with each one storing one number. Rather than being a large basket which just stores numbers, the memory is more like a file cabinet where everything is stored in a specified place. To identify all numbers and separate them, each location in the memory has a unique address The address is therefore a way of telling the computer where to put a number, and where to look for it later. The capacity of the primary storage is expressed in terms of the number of addressable storage locations. Often, the capacity will be a multiple of 4096 (64×64) locations. The number, 4096 represents the 64×64 core array Commercially 1024 words are designated by K and the capacity of 4096 words is taken as 4K.

Since the addressable storage locations or words vary substantially in capacity, a straight forward comparison of different computers is not to be made simply on the multiples of K The following table is intended to guard against making comparisons on the basis of the primary storage capacity in terms of K's.

Table of Comparison of Computer capacities

Computer	Orientation	Code	Capacity in K's	Max storage if all devoted to alphabets/Numeries		Locations to encode one instruction
IBM 1620	Scientific	BCD	40K	20,000	40,000	12
IBM 1130	—do—	16 bit binary	8K	16,000	32,000	1 or 2
IBM system 360	General Purpose	Byte binary	16K	16,000	32,000	2, 4 or 6
Honywell 200	Medium Commercial	BCD	16K	16,000	16,000	1 to 12
UNIVAC 1108	General Purpose	36-bit binary	65 K	300,000	30,000	1

What does the capacity signify ? It signifies the complexity and lengths of problems and records that can be handled by a computer. If a company wants to computerise merely the payroll functions, 1K capacity computer may perhaps be sufficient

Access to data stored in primary storage is instantaneous at the speed of light or electricity, but ferrite core is a highly expensive medium ; therefore the need for such secondary (syn : auxiliary) storage media as magnetic disc, magnetic tape etc. arises. It is to be noted that primary storage is synonymous with core storage, main storage and computer memory.

Comparison of Primary and Secondary Storage.

	Characteristic	*Primary storage*	*Secondary storage*
1	Location w r t. CPU	Within CPU	Outside of , but connected to CPU
2.	Cost	Most expensive	Less expensive than primary storage
3.	Capacity	Up to several million bits	Billion of bits
4.	Access time	In billionths of a second	In millionths of a second
5.	Data can be processed directly from storage	Yes	No, must first be moved into primary storage
6	Means of storing information	Magnetic core (There are alternative media mostly under development)	Magnetic disc, magnetic drum, magnetic tape, magnetic ledger cards, etc

The magnetic core memory is called a *random access memory* (RAM) ; that is, each separate location (address) inside the memory is as easy to reach as any other location and takes the same amount of time We can reach into the memory at random and inserts or remove numbers in any locations at any time A random access memory is extremely fast but can also be quite expensive

Another type of computer memory is the *ready only-memory* (ROM). It is used for miscorprograms not available to normal programmers The term read only means that the storage cannot be altered by regular program instructions. The information is stored permanently in such memory during manufacture. The information from the memory may be read out but fresh information cannot be written into it The microprograms in read-only-memory may be used for a variety of purposes, but a common use is to hold a set of instructions that are needed frequently, for executing small, extremely basic operations, which are not otherwise available in the computer circuitry

Storage addresses : As discussed above, the primary storage area of CPU may be considered as composing of storage locations, each of which can hold a word of data A word may be of fixed length e g , 16 digits The length depends upon the computer make. Different makes usually employ different lengths. Figure 61 shows memory as composed of storage locations which are addressable

There are 32 locations Actual computers have many more locations, but that is enough as representation Each location may hold a number of characters. Let us say the length of a word is 10 characters How data is stored in these locations ? Consider that a stock control record with the following particulars has been read into the CPU by an input unit.

Item Code	04223
Description	Split pin
Unit Price	Rs. 8.50
Physical Balance	40
Order Quantity	60
Recorder level	30

The data may be held in first six locations (locations addressed 001 to 006) as Fig. 62).

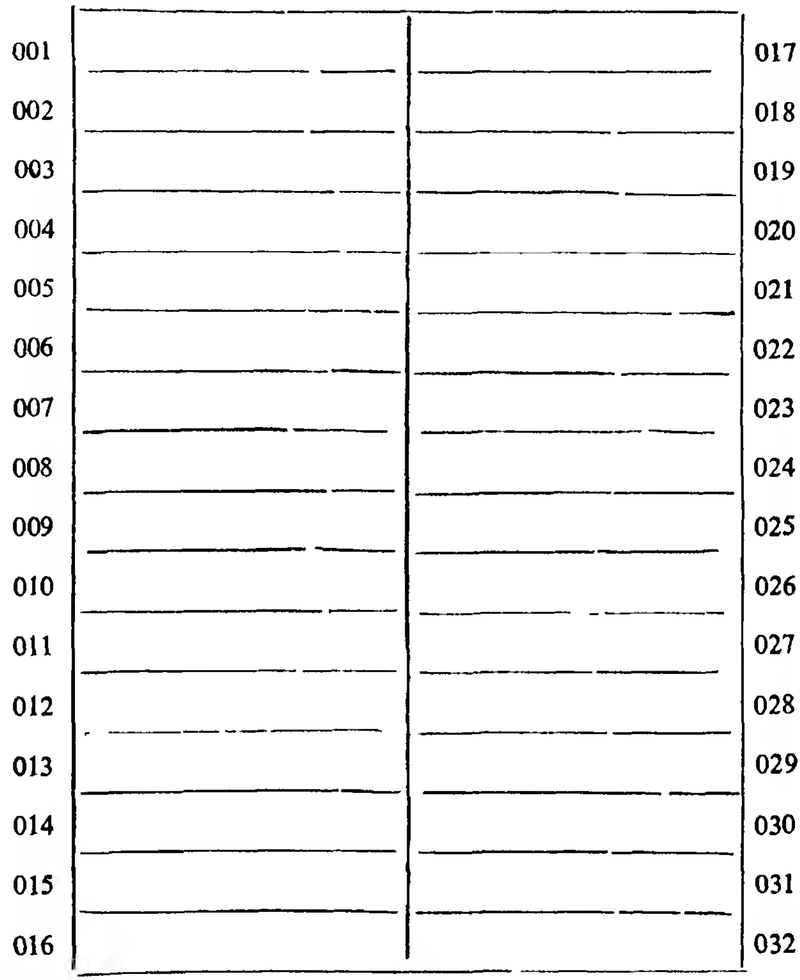


Fig. 61. Concept of storage as composed of addressable Storage Locations.

Likewise, the transaction record containing the following data is read into CPU by another input peripheral.

Item code	04223
Quantity received	5.

This data is stored in the storage location 007 and 008. How is the master record updated ? This is done by pre-storing a program of such instructions as below :

1. Are the item code numbers of the master record and the transaction record same ?
2. If they are unequal read the master record.

001						0	4	2	2	3
002	S	P	L	I	T		P	I	N	
003								8	5	0
004									~~4~~ 4	~~0~~ 5
005									6	0
006									3	0
007						0	4	2	2	3
008										5
009										
010										

Fig 62

3 If they are equal add the contents of location 008 to those of location 004 and store the results back in 004, i e , update the physical balance.

4 Punch a card for the updated master transaction

Such instructions are codified appropriately and pre-stored in the CPU say from locations 029 to 032

Upon pressing the appropriate computer switch the instructions are executed sequentially. This is in brief how the computer operates It is immaterial where i e. in which particular locations the program is stored or the master record or and transaction data stored. This is completely the discretion of the programmer It is to be noted that the computer follows instructions pedantically and, if there is an error in the program it is incapable of spotting it It would blindly follow the wrong instruction and shall produce absurd results as a consequence The program error has technically come to be known as a bug A beginner in programming is often frustrated by computer producing absurd and unbelievable results because of these bugs. It is, therefore, not infrequent to find a placrad in computer installations, bearing "GIGO" which mean "Garbage In, Garbage Out"

Variable Word Length

It can be seen in Fig 62 that there is a great deal of unused space Confining ourselves to the first six locations, the contents can be packed as in Fig 63. More

0	4	2	2	3	S	P	L	I	T
	P	I	N	8	5	0	4	0	6
0	3	0							

0	4	2	2	3	!	S	P	L	I
T		P	I	N	!	8	5	0	!
4	0	!	6	0	!	3	0	!	

Fig. 63.

than half the storage space has been released Under variable word length, computer has an address for each set of bits that encode one character If it requires 6 bits to encode a character, each of the 6 bits will have an address assigned to it For example the item code 04223 will have five addresses The scheme would apparently look quite cumbersome To retrieve 04223, 5 addresses and as many instructions would have to be used This however can be circumvented by one of the following two ways :—

(i) Specify the length of each word e g , 04223 is 5 characters long 04223 can be retrieved character by character until 5 characters are retrieved

(ii) Use word end marker Six signs of exclamation are used to show the end of each field of information 04223 will, for example, be retrieved character by character until the exclamation designating the end of the item code field is encountered

Comparison of fixed length words and variable length words .

1 **Storage efficiency** · Variable length word scheme is quite compact and efficiently utilises the storage space

2 Machines operating with fixed word lengths have faster calculating capability But this faster speed be paid for since the necessary circuitry is more complex and thus more expansive

Some modern computer employ a flexible combination of two schemes

Virtual Memory

A programmer has to take into account the size of the memory to fit all his instructions and the data to be operated in the primary storage If the program is large, then the programmer has to divide the program into segments in such a way that, at a time, one segment resides in the primary storage and the remaining segments in the secondary storage After the first segment of the program, which is in the primary storage is complete, the other segments from the secondary storage are written into the primary storage This is known as *overlaying*

The virtual memory refers to the hardware and software* features, which provide for automatic segmentation of the program, and for moving the segments from secondary storage to primary storage when needed The segments of the program are thus spread through the primary and secondary (on-line) storage, and track of these segments is kept by using tables and indices So far as the programmer is concerned, the virtual memory feature allows him to consider unlimited memory size, though not in physical term .

Internal Computer Codes

Data Representation

As mentioned earlier, the central processing unit in a computer system uses electronic elements such as switches, pulses, etc , which are represented in one of the two states viz on or off, high or low etc These two states are indicated as 0 and 1 If a switch is on, it is said to be in state 1 and if it is off, it is state 0. In other words, a computer operates with only two digits 0 and 1, called binary digits and the data is represented as a string consisting of 0s and 1s Hence the

***Soft-ware refer to techniques or means of controlling the computer equipment.**

ordinary decimal system of data representation is not suited to CPU. It works with simple but rather longish binary system.

Binary Systems : Binary systems work with only two bits : 0 and 1 unlike the decimal system that works with 10 digits, 0 through 9 Let us take the number 1267 of ordinary decimal system Its binary equivalent is computed below by successive division by 2. And, then (on the right hand side) 1267 is got back by conversion by weightage of 2^x.

Number—2 Cols : 1		Quotient 2		Remainder 3	Weightage, 2^x 4			Remainder $\times 2^x$ $5=3\times4$
1267—2	=	633	+	1	2^0	=	1	1
633—2	=	316	+	1	2^1	=	2	2
316—2	=	158	+	0	2^2	=	4	0
158—2	=	79	+	0	2^3	=	8	0
79—2	=	39	+	1	2^4	=	16	16
39—2	=	19	+	1	2^5	=	32	32
19—2	=	9	+	1	2^6	=	64	64
9—2	=	4	+	1	2^7	=	128	128
4—2	=	2	+	0	2^8	=	256	0
2—2	=	1	+	0	2^9	=	512	0
1—2	=	0	+	1*	2^{10}	=	1024	1024
		*(Senior digit)						1267

1267≡10011110011.

The equivalent is obtained by writing the content, of Col 3 starting with the senior digit i e , from bottom to top

Binary Arithmetic : 2^x, in the above example gives the various positional values : 1, 2, 4, 8, 16, 32, 64, Just as there is a decimal point there is binary point too The positional values on both sides of the binary point are given in the table here-under

Position	4	3	2	1	0	−1	−2	−3	−4
Position Value	2^4	3^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}
Quantity represented by position value	16	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$

The table can, of course, be extended indefinitely on both sides The decimal equivalent of a number written in binary is found by adding the products of the absolute and positional values as illustrated in the above example where 1267 was got back from 10011110011 This procedure also applies to binary numbers containing binary point The student may want to verify as an exercise that the binary number 1001 011 is $9\frac{3}{8}$ in the decimal system by using the above table of positional values

We saw that a whole decimal number can be converted to binary by successive division by 2 The remainder juxtaposed starting from bottom to top yield the binary equivalent Below the conversion of a decimal fraction 0 3125 is given to show the procedure which consists of successive doubling of the fraction (i e, multiplying by 2), retaining the integers and stopping when we get 0

Integers resulting from doubling		3125 ×2
	0	6250 2
	1	2500 2
	0	.5000 2
	1	0000 Stop.

Thus, 3125= 0101, the bits having been arranged from top to bottom.

And 9 3125≡1001 0101

Binary Addition is performed in way of decimal addition i e, we proceed from right to left with a carry over, if any. The following rules are for the carry-over.

$1+1=0$ and carry 1 to add to next column.

$1+0=1$

$0+1=0$

$0+0=0$

Example :

	Binary
Carry over	11111 11
	1101010011
	10011110011
	100001000110

We shall stop here and would not take up binary substraction, multiplication and division since computer performs these operations too in addition mode For example to multiply 9 by 5 it would add 1001 (equivalent of 9) 5 times Therefore, computer is sometimes simply defined as a fast adder.

Binary Coded Decimal (BCD) System . The pure binary system discussed above is more appropriate for scientific applications where the bulk of work consists of arithmetic computations Business applications entail reading and writing of voluminous amount of plain English, viz , names and addresses of customers and suppliers, employees, descriptions of stock items, etc. and comparatively less arithmetic. Therefore, most business computer employ a version of the pure binary system, the BCD system that is a hybrid of the pure binary system and the decimal system.

In this system, the decimal weighing is maintained, but the digit is represented by a combination of the binary digits 0 & 1 Since ten digits 0 9 have to be represented, a minimum of four bits must be used to encode each digit. Hence, each digit is represented by its binary cquivalent using four bits e g , the digit 5 is equivalent to binary 0101 and the digit 9 is equivalent to binary 1001 By this method, the decimal number 59 is represented as 0101 1001 The pure binary equivalents of the decimal digits 0 through 9 are given below.

Digit	Pure Binary Equivalents (or BCD's)
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001

The BCD equivalents of longer numbers are simply derived by appropriate juxtaposition of the equivalents in the above table. For example, the BCD equivalent of 951 is got by juxtaposing the individual equivalents of 9, 5 and 1 from this table as below :

BCD equivalent of 951 : 100101010001.

This codification scheme has been extended to cover the alphabets and special symbols by adding two more bits (known as the zone bits) on the left of the 4 bit sets in the above table. By permuting the two zone bits the representations for alphabets and special symbols can be obtained, those for the alphabets being as hereunder : –

Character	*BCD representation*	*Character*	*BCD representation*
0	00 0000	J	10 0001
1	00 0001	K	10 0010
2	00 0010	L	10 0011
3	00 0011	M	10 0100
4	00 0100	N	10 0101
5	00 0101	O	10 0110
6	00 0110	P	10 0111
7	00 0111	Q	10 1000
8	00 1000	R	10 1001
9	00 1001		
A	11 0001		
B	11 0010	S	01 0010
C	11 0011	T	01 0011
D	11 0100	U	01 0100
E	11 0101	V	01 0101
F	11 0110	W	01 0110
G	11 0111	X	01 0111
H	11 1000	Y	01 1000
I	11 1001	Z	01 1001

With the 6 bits, it is possible to have $2^6=64$ codes and this suffices to represent all the digits (10), alphabets (26, only upper case) and special symbols (around 15)

Parity bit is an additional or redundant bit that is used to provide a check on the integrity of a representation Consider, for example, Z which is represented by 011001 in the 6 bit BCD code The parity bit is derived from this representation itself Since the number of 1's in this representation is 3 that is an odd number, 1 is placed usually to the left of this representation so that the number of 1's in the 7-bits are even Likewise, another representation with even number of 1's (viz., 100001 for J) would have a 0 bit to its left Thus by the addition of the parity bit the representations for Z and J in 7-bits would appear as follows :

Z	1011001
J	0100001

The parity bit is put to use each time the computer processes a representation The number of 1's in the 6-bits set of a representation are computed and is ascertained if they correspond to the parity bit placed on the left. If they correspond the representation is accepted and processed, otherwise, an error signal is given by the computer meaning that there is something wrong with the computer circuitry. For example, suppose that because of corruption in the computer circuitry *Z* is being mis-represented as 1011000. In the 6-bits of the mis-representation there are two 1's meaning that parity bit should be 0 whereas actually it is 1.

The above type of parity bit is known as the even parity bit since the number of 1's in the 7-bits set for a representation is always even Similarly, the odd parity *check can be employed but the two checks are mutually exclusive i e , only one of them is performed.

Fig 64 **Common methods of Representing Data**

Character	Hollerith Code	BCD	EBCDIC	ASCII-8
0	0	**00 1010	1111 0000	0101 0000
1	1	00 0001	1111 0001	0101 0001
2	2	00 0010	1111 0010	0101 0010
3	3	00 0011	1111 0011	0101 0011
4	4	00 0100	1111 0100	0101 0100
5	5	00 0101	1111 0101	0101 0101
6	6	00 0110	1111 0110	0101 0110
7	7	00 0111	1111 0111	0101 0111
8	8	00 1000	1111 1000	0101 1000
9	9	00 1001	1111 1001	0101 1001
A	12—1	11 0001	1100 0001	1010 0001
B	12—2	11 0010	1100 0010	1010 0010
C	12—3	11 0011	1100 0011	1010 0011
D	12—4	11 0100	1100 0100	1010 0100
E	12—5	11 0101	1100 0101	1010 0101
F	12—6	11 0110	1100 0110	1010 0110
G	12—7	11 0111	1100 0111	1010 0111
H	12—8	11 1000	1100 1000	1010 1000
I	12—9	11 1001	1100 1001	1010 1001
J	11—1	10 0001	1101 0001	1010 1010
K	11—2	10 0010	1101 0010	1010 1011
L	11—3	10 0011	1101 0011	1010 1100
M	11—4	10 0100	1101 0100	1010 1101
N	11—5	10 0101	1101 0101	1010 1110
O	11—6	10 0110	1101 0110	1010 1111
P	11—7	10 0111	1101 0111	1011 0000
Q	11—8	10 1000	1101 1000	1011 0001
R	11—9	10 1001	1101 1001	1011 0010
S	0—2	01 0010	1110 0010	1011 0011
T	0—3	01 0011	1110 0011	1011 0100
U	0—4	01 0100	1110 0100	1011 0101
V	0—5	01 0101	1110 0101	1011 0110
W	0—6	01 0110	1110 0110	1011 0111
X	0—7	01 0111	1110 0111	1011 1000
Y	0—8	01 1000	1110 1000	1011 1001
Z	0—9	01 1001	1110 1001	1011 1010

*The parity check, however, is not infallible e g , it fails if 2-bits get corrupted at the same time, probability of this though being very low

**This is an alternative (and superior) representation of 0

Hexadecimal System : Although the BCD system increases the flexibility and ease of programming numeric data, it does require more computer space to record each number The hexa decimal (hex) system, using base 16, incorporates the convenience of the BCD system with the full storage capability, It is a base-16 system that uses the symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F. Each hex place value is expressed in binary by one group of four digits Therefore a hex number with two place values requires eight binary digits and the one with three place value requires 12 binary digits For example

Hex	Binary	Decimal
A5	10100101	165
2E7	001011100111	743
Foo	111100000000	3840

and

1110	0010	0000	1101	1010	1011	=E2 EODA B
E	2	0	D	A	B	=237,899,179

8-bit Alphanumeric Codes To permit greater flexibility in data representation, equipment designers have extended the 6-bits aphanumeric BCD code to 8-bits codes $2^8=256$ representation are possible They can then also cover a lower case letters and greatly expand special symbols The table of page 71 provides the equivalents of the 10 digits and 26 alphabets in the Hollerith card code, (standard) BCD (interchange) code, extended BCD interchange (EBCDIC) and American Standard Code for Information Interchange (ASCII-8) code. Hollerith code stands apart in that it is used only for the punched card and not the internal working of the CPU. The parity bit has been ignored in the table 64 on page 71.

Byte We have seen above that numeric digits and alphabets may either be represented in 6 bits in BCD code or 8-bits in the extended codes. The 6 bits or the 8-bits moved together constitute what is known as a byte. However, a variation of these methods is quite common in the modern computers, in which the use is made of an 8-bites which can represent either two numeric digits or one alphabet/special symbol. For encoding alphabets/special symbol, one of the 4-bit sets encodes the zone and other the numeric whilst both 4 bit sets can encode a numeric each. The term "word" stands for a CPU location. A word can hold one or more bytes i.e., a byte is always shorter than or equal to a word.

The Control Unit

The control unit co-ordinates the various parts of the computer system—the arithmetic/logic unit, the main memory and the peripheral units In addition, the control unit also controls the flow of data to, from and within the main shortage as per the program instructions The control unit comprises the following four components .

(*a*) Instruction register

(*b*) Decoder

(*c*) Address register

(*d*) Instruction counter.

The instruction register receives one by one the instructions to be executed in the required sequence. The operation code (op code) of the instruction in the instruction register is transferred to the decoder which after decoding the operation code (e g whether addition, subtraction, multiplication or devision etc) activates the appropriate circuits of the arithmetic/logic unit to perform that operation The address register enables the data in the location specified in the instruction to be transferred to a specified accumulator for the arithmetic/logic unit.

It can be seen from above discussion that there are two distinct phases in the execution of each instruction : (1) The control unit receives an instruction in its instruction register, decodes its op code and activates the circuits of the arithmetic/logic unit to perform this operation and causes the data of the instruction operands to be transferred to the working registers of the arthmetic/logic unit; (2) the arithmetic/logic unit, having been appropriately activated by the control unit performs the operation upon the data as depicted in figure 65 The two time components of this cycle of instruction execution are known as the I-time and the E-time. That is how the computer works The control unit figures out what is to be done and turns control over the arithmetic/logic unit, and on and on in a cyclic pattern until the entire program has been executed.

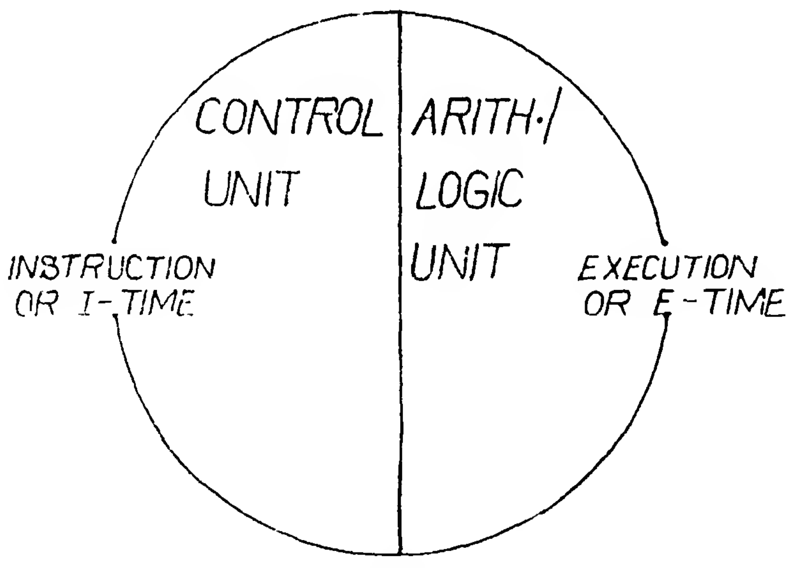


Fig. 65

Registers , In addition to the registers of the control and arithmetic/logic units mentioned above, there is a series of registers interposed between these units on the one hand and the main storage on the other. A register acts as a path of conduit to connect these two components of the computer. An instruction counter is used for recording the number of instructions executed, and is incremented by 1 after completing each instruction

Operator's console is a typewriter-cum-visual display unit (akin to a television set-cum-panel of lights and switches) which is electrically connected to CPU. Just as the driver of an automobile uses steering of the car to control its operation

the computer operator uses the console to control the operation of the computer. The computer is started or shut by the operator by depressing the appropriate switches on the control panel Besides, it aids the operator in the following ways :

1 If programmed, the operator can key in data into the CPU via the console typewriter For example, the details of a receipt transaction in inventory control application can be keyed in via the console However, though possible in theory, the transaction files are seldom input via the console since the operation being manual tends to be very slow and hence expensive. Instead, these files are input into the CPU via such peripherals as card readers, magnetic tape units etc Nevertheless inputting such general details in small amount of data like today's date the operator would invariably make use of the console

2. *Programs* can also be input into the CPU via the console typewriter if desired Again, for the reasons mentioned above this would be done only for smaller programs Usually larger programs are input via the punched cards or the magnetic tape But the console typewriter can be particularly useful during debugging * For example, consider one of the many instructions of a program .

   20 00 000 000 000 503 which needs to be modified to 20 00 000 000 000 305 If this were to be done via the punched cards, the card for this instruction would be taken out from the original deck of punched cards and replaced by the one in which the modified instruction has been punched by the operator. The complete deck including the replaced card would now be read into the CPU In contrast the operator can modify just this instruction simply by keying in the modified version, the instruction number etc via the console

3 The console unit can also print out or display the intermediate computation of the part of a program Consider for example, the problem .

   $\frac{2^7 + 7}{96} =$ to be discussed later If the operator/programmer wishes to know the intermediate processing results he can get these on the continuous stationery or the visual disply unit of the console This facility is again of great assistance in debugging the program

4. Operator/programmer can inquire about the contents of certain storage areas which would be printed out on the continuous stationery or display on the visual display unit

5 The data contents of specific storage locations can be altered, if desired

Multiprogramming

In order to utilize the computer more effectively, a technique known as multiprogramming has been developed This is a module that is aveilable in an operating system** that handles scheduling Multiprogramming is defined as the execution of two or more programs that both reside in primary storage

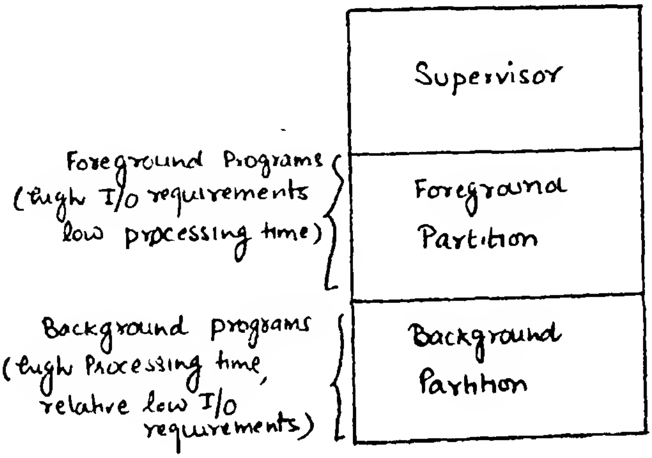
Since the CPU can execute only one instruction at a time, it cannot simultaneouly execute instructions from two or more programs However, it can

---

*Debugging means removing errors

**Operting system : a set of program modules which supervises the overall operations of the CPU and controls the I 10 functions of the computer.

execute instructions from one program, then from a second program, then from first again, and so on. This type of processing is referred to as "concurrent execution". This is desirable because input and output operations are much slower than internal CPU operations are. If only one program is being processed, the CPU is often idle while it waits for an input/output operation to be completed. However, in a multiprogramming environment, the CPU can execute one program's instructions while a second program is waiting for I/O operations to take place.*



A Typical Stock Application On Unit Record Equipment

Fig. 66

In one system of multiprogramming, storage is allocated for each program. The areas of primary storage allocated for individual programs are called "partitions" Each partition must have some form of storage protection and priority-protection to ensure that a program in one partition will not accidentally write over and destory the instructions of another partition, and priority (when two or more programs are residing in primary storage) because both programs will need access to the CPU's facilities (e g , the arithmetic and logic section) A system of priority—some method that will determine which program will have first call on the computer's facilities—is normally determined by locating the programs in specific partitions.

Programs that have the highest priority normally are stored in an area called the "foreground partition". Programs with the lowest priority are stored in an area called the "background partition".

Multiprocessing

The term multiprogramming is sometimes loosely interchanged with the term multiprocessing, but they are not the same. Multiprogramming involves concurrent execution of instructions from two or more programs sharing one CPU and controlled by one supervisor In *multiprocessing*, two or more CPU are required. Instructions are executed simultaneously, because the available CPU can execute different instructions of the same program or of different programs at any given time.

Multiprocessing offers data-processing capabilities that are not present when only one CPU is used. Many complex operations can be performed at the same time. CPUs can function on complementary units to provide data and control

for one another. Multiprocessing is used for nation's major control application such as railroad control, traffic control of airways etc

Overlapped Processing and Buffer storage The CPU processing speeds are enormously higher than those of the input peripherals Whereas the times are measured in milliseconds 1,000th of the seconds) for transfers to and from peripherals central processor times are calculated in micro second (10,000,000ths of seconds or even nanoseconds (1,000th of a 10,00,00,000ths of a seconds) A good card reader, for example, is capaple of handling 1,000 cards per minute, if a card holds 80 characters the rate is approximately, 1,333 characters per second A good tape drive can transmit 3,00,000 characters per second. In contrast, main memory is capable of moving 40,00,000 characters per second and modern non-core memories are capable of doing even better. The speed disparity between the CPU and its peripherals is great

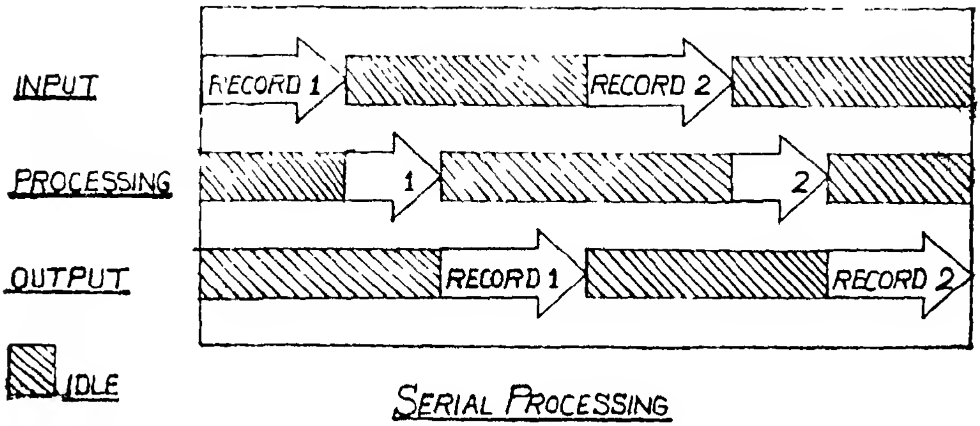


Fig. 67

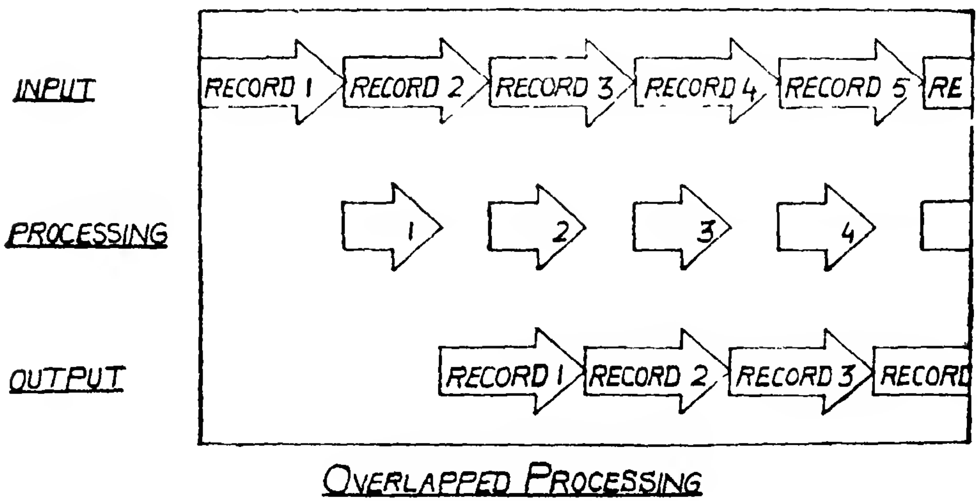


Fig. 68

Earlier computers employed serial processing to complete a task. Data was first read in, was then processed and finally output all in series as depicted in figure

67. It can be seen that the CPU is busy only a fraction of the total time i e, the slowest peripheral sets the pace of the entire computer system If therefore, one or more of the peripherals are card readers and card punches the computer system would work on their speeds and not that of the central processor In the second generation computers, this problem was mitigated to some extent by the technique of spooling which takes advantage of the fact that reading/writing of the magnetic tape is faster than the reading, writing of the punched cards. The punched card data was put on the magnetic tape off-line by the device called the controller which simply copied the card images to the tape. The data on the tape was then processed in connection with the CPU and output obtained on another megnetic tape This magnetic tape was then run via the controller to put the data on punched cards or the printer The technique of spooling is depicted in figure 69 The conversion via the controller did entail extra expenses but they were significantly offset by saving on the computer time

But even the magnetic tape peripherals operate on much lower speeds than those of the CPU Considerable CPU time was wasted even with spooling and also extra expenses and efforts were entailed in spooling. The third generation computers are capable of performing reading, processing and writing in an overlapped

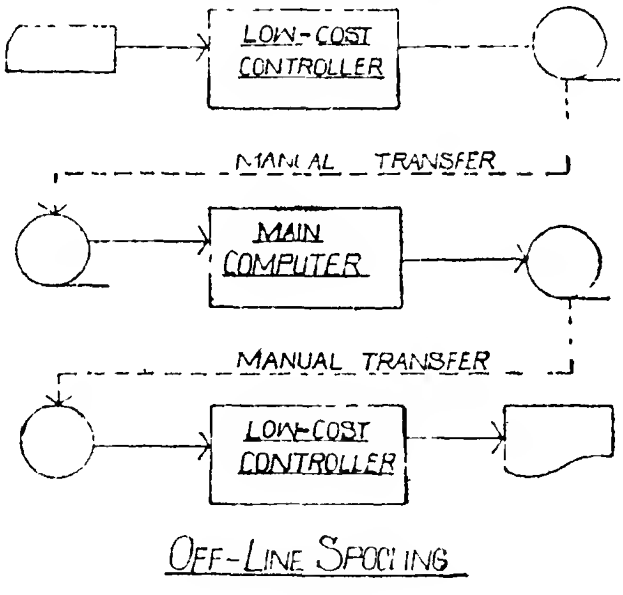


Fig 69

fashion as depicted in figure 68, i e, the input pheripherals, the CPU and the output peripherals can work simultaneously.

This is achieved by means of buffers which are high speed storage elements. In figure 68 it is to be seen that whilst record 2 is being read 1 is being processed Ordinarily record 2 connot be read in at the same time record 1 is being processed. The CPU cannot manage the two simultaneously The input buffer shown in figure 70 permits the simultaneity of the two operations, record 2 is read into the buffer whilst the CPU is engaged in the processing of record 1. In this case, the buffer should be large enough to hold two records. In another case, where speed

disparity between the input periperal and the CPU is greater the buffer storage would have to be larger The working of the output buffer may be interpreted in the same manner

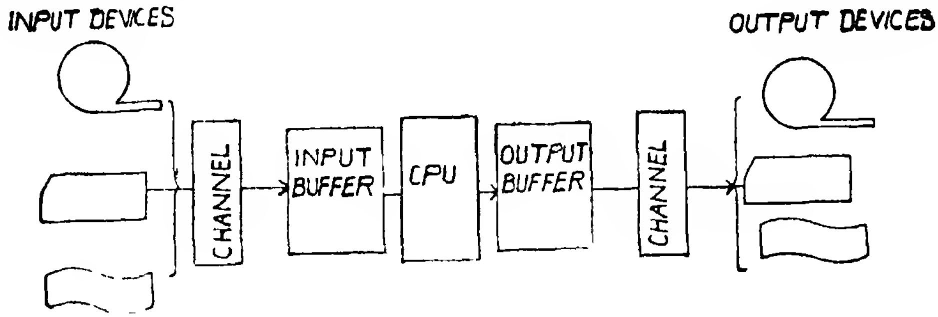


Fig 70

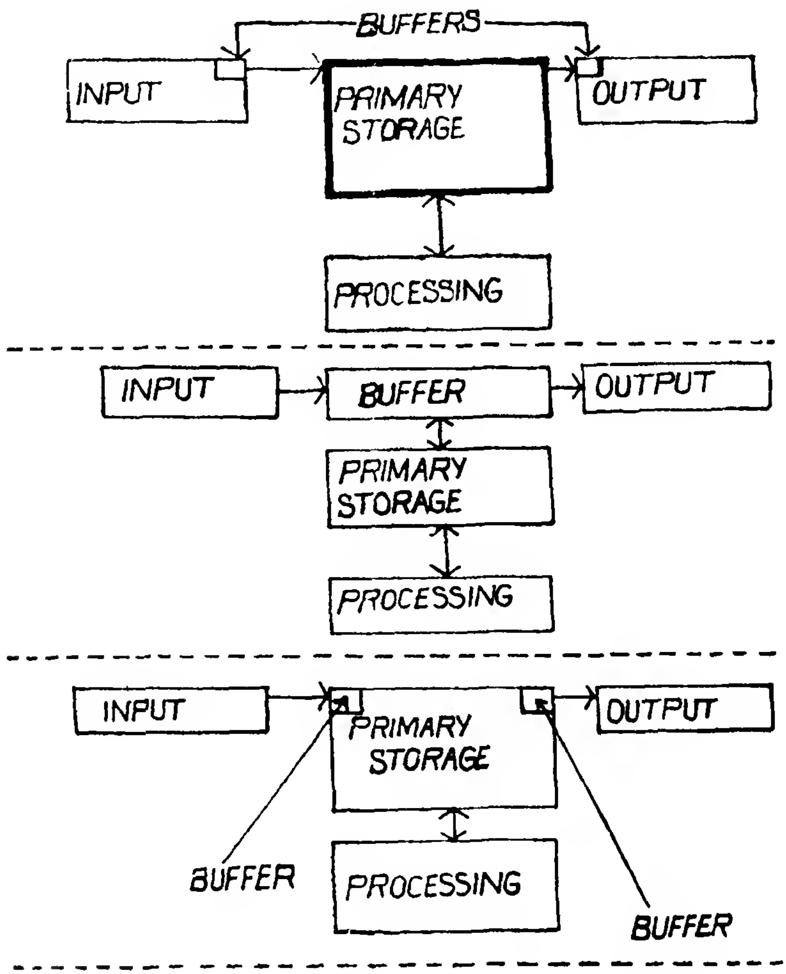


Fig. 71 A, B, C,

The buffers are connected to the CPU by means of channels which receive appropriate signals from the CPU and then proceed to operate independently and without supervision to bring in data and right information whilst the CPU is engaged in calculations

There exist three alternative ways of buffering as depicted in figure 71 A to 71 C In figure 71B the buffers are included neither in the CPU as figure 71C nor in the input/output units (as in figure 71 A) Usually, however, the buffers in the scheme of figure 72 are housed in what are known as the control units The control unit sits between the input/output device and the CPU acting with its buffering capability as an intermediary Aside from buffering the control unit performs another less obvious function of translation The card reader (without the decoder unit that is an alternative method of translation) would transmit the data to the CPU in the Hollerith's code, The paper tape reader would do so in BCD, the OCR reader would transmit it in its own reflected light codes, etc But the CPU would be working in one of the pure binary or the BCD version Thus there is a need to translate the data transmitted by the input unit to the CPU and the data transmitted by the CPU on to the output unit In order to save expenses and to simplify design of the computer the devices of converting all these codes is housed in what is known as a control unit that may also contain the buffers This control unit function is also known as the standard interface

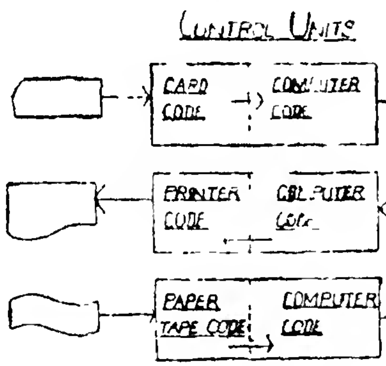


Fig. 72

***Data Base Concepts (in case of direct access media only)**

Under manual system, recording of various particulars are usually duplicated A customer's order, as an instance, bears the customer's name, item numbers, descriptions and quantities demanded that are copied in various files, viz sales, inventory, costing, etc Such duplication has more than one drawback , first it adds to paperwork , second, the same information recorded at different places is liable to be discrepant Another notable example in this regard would be the bills of material for which each department, not unusually, has its own version, an engineering bill, a purchasing bill, a costing bill, an assembly bill and an inventory bill The segregation of the same information has one advantatge though, this being that accessing the information is prompt Computer offers the possibility of pooling all the information in one place by integrating the various files by indexing and chaining In the past, there have been attempts to computerise the manual systems on

*More thorough treatment is given in Study VII.

the existing lines i e , different files for different applications These disregarded the capabilities of the computer in integration and after having advanced on these lines (independent files) integration was made still more difficult

Integration of files requires that each file must contain in its records the search argument (the key) in terms of which the other is either ordered or indexed. This is known cross indexing or chaining the files There is more of data integrity since all data is to be found at one place only Secondary storage (Master file storage media) space can be conserved, and during processing extracting and sorting is obviated to a great extent It should be obvious that the concept of cross-indexing is valid for direct access storage devices only Also, the access is slower, in view of complex cross-indexing. It is to be noted that the data base is a common property of all and belongs to no one particular functional department. The data base, therefore, weakens the traditional departmental jurisdictions. Formally,

Data base is a collection of data which are related in a meaningful way, and can be accessed in different logical orders but are stored once and only once

Consider figure 73 towards explanation of how this duplication is avoided in the three files (item Master, open purchase Order and Vendor Master File)

In col 4 of the item master file are the pointers leading to the open purchase order file, viz against item 1326, 820 gives the disc storage address where the particulars of the only open purchase order (no 104) against it are stored

In col 4B of the item master file are the pointers leading to the vendor Master File, viz for item 100, there are two regular vendors whose particulars are to be found in disc storage addresses starting from 2210 and 2230

In col 4 of the open purchase order file are the pointers leading to the Vendor Master File, viz the particulars of the vendor against whom the open P.O. 104 is pending are stored in the disc storage starting address 2230

With this background, consider the program for handling a receipt transactions against item 1326. The program retrieves its pa ticulars from disc storage starting address 100 into the CPU, updates the balances and looks for the pointer in the open P O file, i e , enters this file at address 820. The program updates the quantity received so far, quantity rejected so far and looks to col 4 in the open P.O file for the pointer 2230 in the Vendor Master File if the order has to be closed From disc storage address 2230 onwards the program retrieves the particulars of the vendor and updates his performance indices.

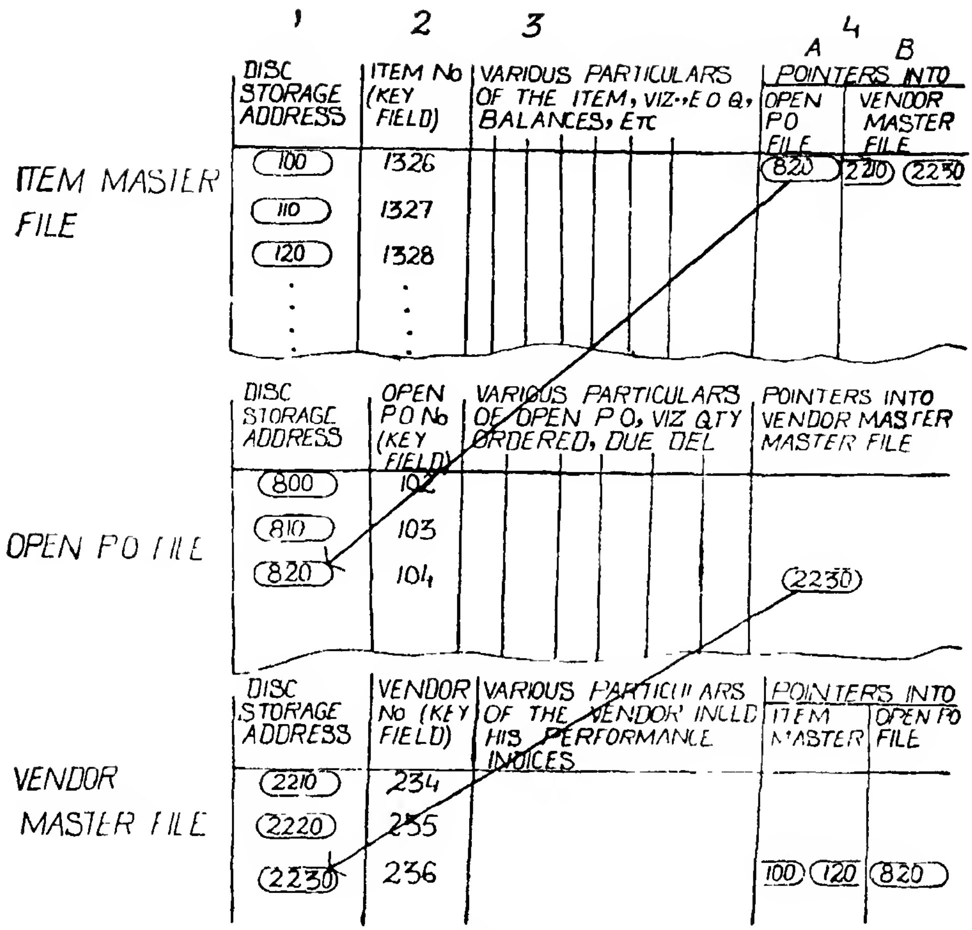


Fig. 73

Software

The word "software" has been coined to differentiate between (*i*) the computer equipment (i e , the hardware) and the means of controlling the equipment, the latter having been termed "software" Although in early days of the computer, only standard programs supplied by the computer manufacturers were called the software, the term, now-a-days, has a much wider meaning and is also inclusive of programs developed by the user or procured by him from an organisation dealing in software, and the associated policies and procedures

There is a definite trade off relationship between the hardware represented by the CPU and the software represented by the program Whatever is programmed can also be directly embedded in the computer circuitry For example, the computations of square root are ordinarily incorporated in the computer program They could also be alternatively provided for in the CPU circuitry so that with just one instruction the square root of any number is got But this alternative is highly expensive in terms of initial investment It can only be justified if the square roots were to be derived in an enormously large number, in some scientific application perhaps as an example As we shall see later only simple arithmetic/logic operations (like add, subtract, multiply, divide) are provided for in the CPU

circuitry so that these are performed in just one instruction each as they are all too frequent in both business and scientific applications. Neverthelesss, the trade-off relationship is of theoretical interest.

Before discussing the types of software, we shall discuss the various programming languages and phases of programming

Machine Language : In the early days of the computer, all programs were written in the machine codes. Each particular computer model has a machine language, which is based on the internal engineering architecture of the equipment, and which is developed and provided to user by the computer manufacturer. Simply a machine-level language is the language of the computer the only language the computer understands without translation If such a language is used in writing a program (say a user's payroll program), the programmer must be familiar with the specific machine-level language as well as with the details of the computer equipment to be used Programs written in machine-level language can only be used on the computer model for which that language is written. That is why this language is called machine-oriented

Writing programs in machine language is not only difficult for humans to do but is also subject to error This will be more clear below In attempt to facilitate human programming and to eliminate human errors, assembly language, which is very close to machine language was designed As we shall see later, the assembly languages assist the programmer in writing fairly error-free programs with far less labour and also as much suit the design of the machine on hand as its machine language. The procedural languages are completely oriented to ease up the programmer's task but disregard the machine design entirely.

The earlier machine languages employed the binary code i e , the instructions were codified in series of 0s and 1s in accordance with the bit pattern of the instuction to be performed. The binary code is now being abandoned in favour of the decimal code which is far more easy for the programmer and therefore, we shall not pursue the binary code any more

The computer manufacturer supplies a manual of the codes for the various operations which are unique to that computer

An example on some of the codes for a computer and use of these codes to write instructions in its machine languages follows :

Operation		*Code*
—Add the contents of two locations	10	00
—Add a constant to the contents of a location	10	01
—Subtract contents of one location from those of another	11	00
—Subtract a constant from the contents of a location	11	01
—Multiply the contents of two locations	12	00
—Multiply the contents of a location by a constant	12	01
—Divide the contents of a location by those of another	13	00
—Divide the contents of a location by a constant	13	01
—Transfer (copy) the contents of a location into another location	22	00
—Transfer (copy) partially the contents of a location into another	22	10
—Zeroise the contents of a location	23	00
—Compare the contents of two locations and branch if...	21	00
—Jump to such and such instructions	20	00
—Print the contents of a location etc. etc.	33	00

Either the programmer should refer continually to such a chart or, as it happens with practice, should memorise these codes Both of these are however, cumbersome and error-prone Below is given an actual instruction which can be seen as divided in 7 parts.

Instruction Number	Operation Code	XX	YYY	AAA	BBB	CCC
1	2	3	4	5	6	7
033	10	01	001	353	001	353

Explanation *Column 1* : Instruction number Computer executes the instructions sequentially except in so far there is a special instruction overruling the sequence After this, instruction 034 will be taken up for execution

Columns 2 3 (Operation) : As can be seen from the above chart 10 01 is the code for adding a constant to contents of a location.

Cols 57 to represent operand i e , location/data/instruction number upon which operations of cols. 2, 3 are performed.

Column 6 : It is the constant 001 that is to be added to the contents of locations 353.

Column 7 : The is result to be obtained back in 353 The length of the word (in a location) is 16 digits as shown below for location 353 whose contents at present are 455 The right most zero signifies that it is +455. Any other digit in the right-most position would make 455 negative.

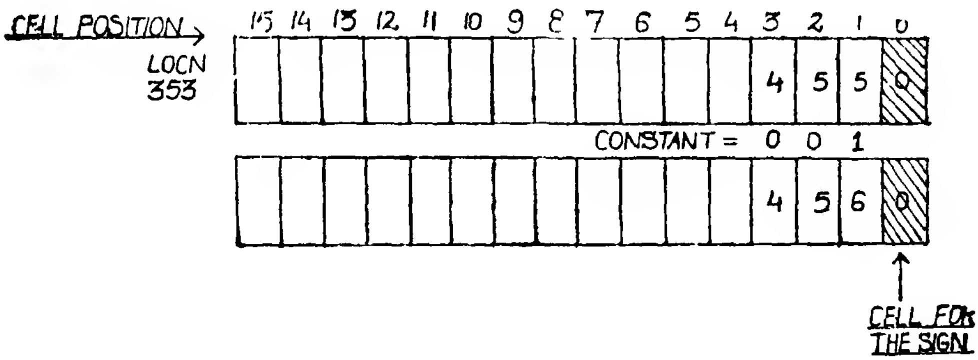


Fig. 74

Column 4 The constant C01 is to be added starting at position (1 to 3) of location 353 Suppose that location 353 is being used to count the number of master records that have been read in The programmer would have to memorise this fact too. This further contributes to the cumbersomeness and errorproneness of machine languages.

Assembler Languages are at the next level in improvement over the machine languages. They are also known as symbolic language because they employ symbols for both arithmetic and logical operations and even location addresses The above instruction (033) may be written as ADDC 001 COUNTER 001 COUNTER. **ADDC is add a constant. Such standard symbols for the various operations**

are supplied by the computer manufacturer who creates the assembly language for his computer COUNTER is the name given to location 353 by the programmer

Since mnemonic symbols are being used this eases up the programmer's task greatly *But* the computer follows its machine language only and the p ogram written in the assembler language has to be translated This is accomplished by a special program usually supplied by the computer manufacturer and known as the *assembler* The assembler simply translates each instruction in symbolic languages to its machine language equivalent. Thus there is a one for-one correspondence between the two languages. Another advantage possessed by the assembly language is that of flexibility Towards explanation of this, consider a program written in a machine language and consisting of 980 instructions If the programmer, upon an after thought, wishes to insert another couple of instructions he would have to alter the entire program Although it is possible to save a great deal on alterations by programming jugglery it is not only cumbersome but also liable to errors In a program written in an assembler language the extra instructions can be inserted wherever desired without any reprogramming, on the existing instructions Both the machine and assembler languages possess a definite advantage over the procedural languages discussed below Since these are machine oriented the programmer can write the programs that suit the capabilities of the computer at hand, i e , the pragram would occupy the minimum storage and take less processing time as compared to the one in a procedural language.

MACRO's

As mentioned earlier the first assembler languages had a one for one correspondence with the machine code It was noticed, however, that various sets or routines of instructions were used constantly and repetitively within programs and were common to most applications In particular, these were the sets of instructions to organise the transfer of data between peripheral devices and the core store, or to rearrange data within the store For example, three instructions, one after the other, might be needed to print out a number on a particular computer These three instructives, always in the same order, might be used over and over in the same program Rather than force the programmer to write out the set of these instructions every time, these routines were identified and written in a standard way by the manufacturer and maintained within the assembler program. The programmer, then, needs to write just one instruction whenever he wishes to use this routine These routines are called MACROS and the single instruction statement written by the programmer to invoke a MACRO is called the MACRO INSTRUCTION The MACROS save the programmer the time and trouble of writing and testing these routines over and again as they have been written by the computer manufacturer once and for all

Compiler Languages

Compiler languages are also known as high level languages or procedural languages They are procedure oriented (viz , a business application oriented language COBOL and a scientific application oriented language, FORTRAN) and employ plain English like and mathematical expressions. They are detached from the machine design and therefore the nomenclature 'High Level Languages Since they are procedure oriented and detached from the machine design an instruction of these languages may be equivalent to more than one instructions in a machine language. **An instruction in these languages is usually called a statement.**

A computation X+Y=Z would be written as below in FORTRAN (FORmula TRANslation) and COBOL (COmmon Business Oriented Language). This example is intended to bring out the similarity between the statements of these languages to plain English and mathematical expressions.

FORTRAN :	Z=X+Y	X, Y, Z designate
COBOL :	COMPUTE Z=X+Y	Storage locations.

Whereas each computer has its own machine language and assembly language devised by its manufacturer the compiler languages are universal

Since these languages employ plain English and mathematical expressions it is easy to learn these and write relatively error free programs This is further facilitated because of the fact that the programs written in them are much more compact than those in the low level (machine and assembly) languages But they have to be translated to the machine language for the computer on hand which is accomplished by an especial program known as the compiler written and provided by the comupter manufacturer Compiler usually occupies more storage space and requires more processing time than the assembler It however, also possesses the diagnostic capabilities. Thus they are cheaper than the low level languages in terms of learning and writing programs but this advantage is offset to quite an extent by more time on translation. Usually, therefore, an organisation would write frequently used programs in low level language and infrequently used programs in high level languages provided, of course, they are not constrained to favour one of them in view of available programmers being skilled in only that one

Besides FORTRAN and COBOL there are several more compiler languages.

Stages of programming

Program coding by means of various programming languages discussed earlier is just one of the several steps involved in writing computer programs which can be sub-divided into the following six phases broadly

(*i*) *Programming analysis* In this stage, the programmer ascertains for a particular application (e g , up-dating of the stock file) the outputs required (i e , the up-dated stock file, listing of the replenishment orders, stock movements report, stock valuation report, etc), the inputs available (i e , the stock master file, the receipt and issue transaction file) and the processing i e up-dating the physical balance, computiug stock value for various items, determination of the placement of the replenishment order, etc) The programmer then determines whether the proposed application *can* be or *should* be programmed at all. It is not unlikely that the proposal is shelved for modifications on technical grounds.

(*ii*) *Program design* In this stage, the programmer develops the general organisation of the program as it relates to the main functions to be performed. Out of several other tools available to him input, output and file layouts and flowcharts are quite useful at this stage These layouts, and flowcharts etc are provided to the programmer by the system designer as would be more clear later in the study III. The flowchart depicts the flow of data, documents etc. very clearly, what are the steps to be repeated or what are the alternatives or branches at a particular step is shown conspicuously. Such details may be difficult to bring out in descriptive language.

(*iii*) *Program Coding.* The logic of the program outlined in the flowcharts is converted into program statements or instructions at this stage For each language

N B : FORTRAN and COBOL are discussed in a little more detail at the end of Study II.

there are specific rules concerning format and syntax (vocabulary, punctuation and grammatical rules) available in the language manuals which the programmer has to follow strictly and pedantically). There are special sheets for writing the program instructions in each language. The format of these sheets facilitate writing error free programs Just as a mathematical problem can be solved in several ways, so is the case with writing a program Different programmers may write a program using different sets of instructions but each giving the same results In fact, there is a great scope for elegance in writing the programms but the limitations of time stipulated by management would not encourage the programmers to strive for such elegance In practice, therefore, the programers broadly pursue three objectives : simplicity, efficient utilisation of storage and least processing time It is highly desirable that the programs are simple to understand since a program written by one programmer is notoriously difficult for the other to understand. Since, the programs, upon implementation, may require frequent modification to suit the changing systems environment, there is a program maintenance gang employed on a permanent basis for modifying the programs and this gang is different from the ones who wrote the programs initially This, then should emphasise that programs are written as simple as possible to start with There is usually a trade off possible between the other two objectives : efficient utilisation of storage and least processing time

For each, instruction, then a punched card is prepared and deck of punched cards carrying the program instructions constitute the source program i e., the program in the source language which may be either an assembly language or a procedural language This program is then translated into the machine language of the computer on hand by an assember or a compiler both of which have diagnostic capabilities i e , they can point out several syntax errors like two labels used for the same location and invalid standard label, etc. The programmer, upon getting the print out of the assembly or compiler run, rectifies his program.

(iv) **Program debugging*. The assembly run or the compilation run can detect only few syntax errors. Considering that a program of average size would have thousands of instructions there is an ample scope for the programmers to make errors technically known as bugs. It is asserted that nobody can string together about 200 or more error free instructions Therefore, before putting the program into use there is a real necessity to debug it i e., to cleanse it of errors Towards this purpose, the programmers devise a set of transactions to test the various alternative branches in the program Since the branches in a program tend to proliferate a large number of transactions would have to be devised for a thorough test. The results of these tests on the master file are derived in long hand as per the logic of the program Then the file is up-dated by the transactions via the computer with the program to be debugged stored in it The results got from the computer are compared with the ones derived manually prior to computer processing. It is not at all unlikely that the results do not tally for the reasons mentioned above. The programmer then sits away from the computer and verifies his flowcharts and coding sheets in a hunt for the bugs. Since, debugging can be a tedious process computer manufacturers frequently provide facility for memory dump i e , print out of the data contents and instructions of the various CPU locations. Having had a first round of debugging a programmer would rectify his coding sheet and repunch the instruction card and go in for another assembly or compilation run. It is, however, to be noted that the identical results obtained manually and by computer processing do not guarantee that there are no bugs in the program since the program may not provide correct results on different set of transactions. Towards explanation of this consider the

*Further discussed in Appendix II, Study II.

past sales history of an item as 60, 62, 64, 66, 68, 70. The exponential smoothing model may be used here for sales forecasting and the forecasts derived manually in long hand according to appropriate formulas. The exponential smoothing forecasting program may then be used to derive the forecasts via the computer. Identical results here would be no guarantee that the program would yield correct results in another sales history of say, 232, 230, 228, 226 etc. This example, should bring out the fact that program debugging can indeed be a formidable task. In a survey conducted by the IBM it is estimated that program of some scope (trivial programs excluded) may require as many as 20 rounds of debugging before it is thoroughly cleansed of bugs

(*v*) *Program documentation.* Each program should be documented to diagnose any subsequent program errors, modifying or reconstructing a lost program. Program documentation including the following may be assembled

(i) The program specifications i.e , what the program is supposed to do.

(ii) The program description i e , input, output, and file lay out plans, flow charts etc

(iii) The test data employed in debugging the program. This could be highly useful later to the auditors.

(iv) The operation manual which details the operating instructions for the computer operator, viz , input the MLC when the yellow bulb is on.

(v) The maintenance documentation that is a listing of any subsequent amendments to the program.

(*vi*) *Program maintenance.* The requirements of business data processing applications are subject to continual change. This calls for modification of the various programs. There are usually a separate category of programmers called maintenance programmers who are entrusted with this task. Theirs is a difficult task of understanding and then revising the programs they did not write. This should bring out the necessity of writing programs in the first place that are simple to understand.

Types of Software : Having described the various computer languages and phases of programming we are now in a position to discuss the various types of software :

(a) Subroutines

(b) Utility programs

(c) Executive or Operating Systems

(d) Diagnostic Routines

(e) Assemblers

(f) Compilers

(g) Application Programs

} Supportive or System Software

(h) E D P. policies and procedures

(a) *Subroutine* : A subroutine is a subset of instructions that appears over and again in the program or finds applications in several programs. It is obviously economical to write it once and for all This would save time not only of writing it specifically or wherever it appears but also on debugging. In fact, it is not even necessary to repeat this subset over and again in the same program. This is sought to be depicted figuratively and explained below.

In two segments of the main program, a subroutine finds use. Instead of duplicating its instructions in the main program it is being used once only as shown in this diagram This has been made possible by the following scheme. To make the explanation easy we assume that this subroutine (S R), consists of deriving the value of 2^N, N being a variable i e , at one segment of a program it may be desired to compute 2^5 and at another 2^{18} etc Obviously, then, before any entry into SR by the main program the value of N has to be supplied by the latter to the former N, then, is a parameter of S R

Instruction 516 "Set N=5 in the entry instruction 901 of the S.R "

Instruction 517 "Enter into instruction 901 of the "S R "

As such the SR could compute $2^5=32$ upto instruction 908 but there is an important point that has not been clarified The SR must now return to control to the main program i e , after the execution of the SR instructions the main program instructions 518 should be taken up for execution We would avoid several complications here and roughly state that instruction 909, in plain English would read as "Go to the next instructions to the one from which the program entered the SR '. The program entered the SR at instruction 517 and the control would be returned at instruction 518 in the main program Likewise, 2^{18} would be computed in the second segment of the program

Subroutines may be incorporated in the main program in the assembly run or the compilation run, or may be recalled by the main program from the liabrary of subroutines in the backing storage. Both assembly/compilation run and the backing storage are further explained below.

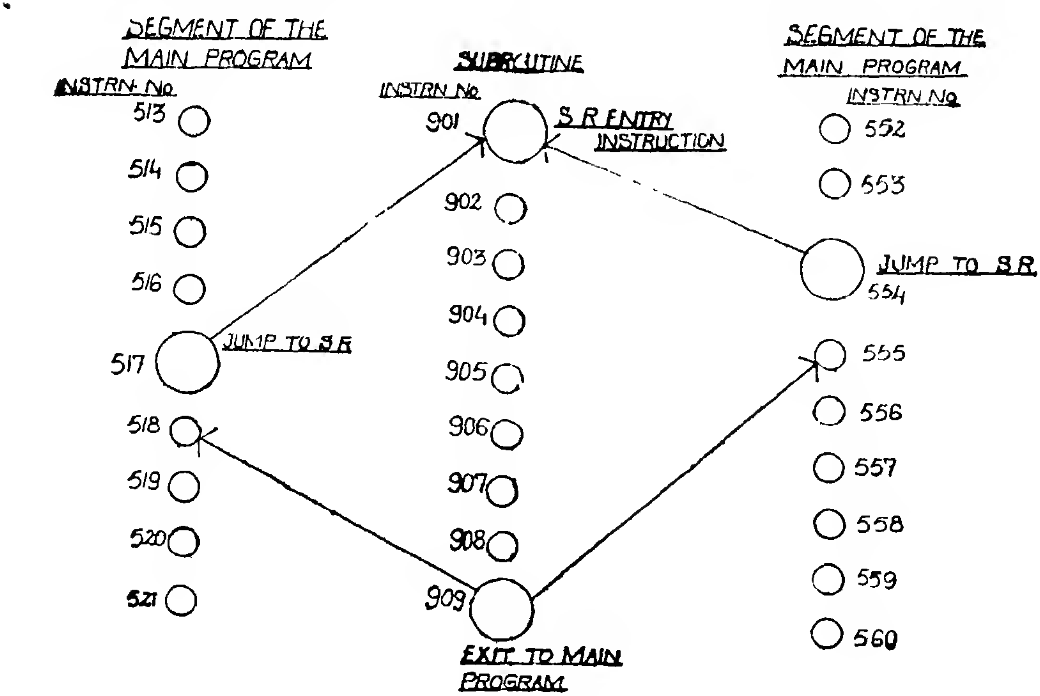


Fig. 75

Assembling the Program The punched card deck containing (i) the assembler, (ii) the standard subordinates (such as for sorting) supplied by the computer manufacturer, (iii) the subordinates written by the user and (iv) the main program are input to the CPU in the order prescribed by the computer manufacturer After having been read the assembler translates these and

(i) Prints each instruction in both the machine and the symbolic languages.

(ii) The card punch, upon instructions of the assembler program, punches one card for each printed instruction

(iii) Performs editing e g , invalid symbols, two symbols used for the same location or invalid instructions are indicated In other words the assembler (and for that matter also the compiler) possess diagnostic capabilities

After having amended the program deck for the syntax errors disclosed by the above run another run would be had, which would yield the final deck, provided, of course, there is no more syntax errors The output deck, constitutes the program to be input into the computer for routine processing but these may require several rounds of debugging prior to that.

The operation of compilation is similar to that of assembly as depicted in figure below.

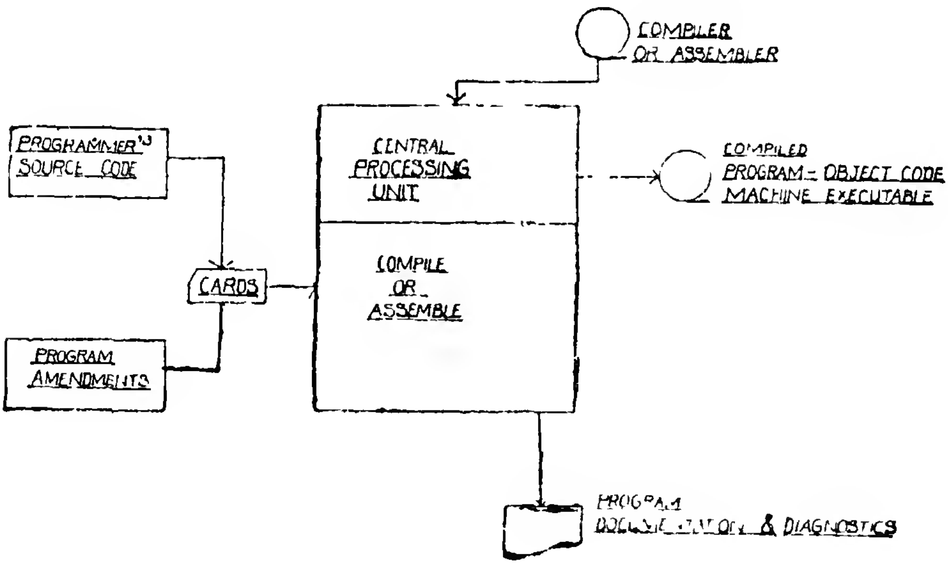


Fig. 76

**Modularity.* Modular programming refers to a technique in which the logical parts of a problem are divided into a series of individual routines so that each

*This is yet another objective of programming

may be programmed independently Consider, for example, a program to process payroll file periodically. It can be split into 11 modules at three levels as shown below

Process Employee
- —Process unmatched Employees
- —Validate Payroll Transactions
  - —Create current Weak Error Listing
- —Update Master Payroll file
- —Create payroll cheque
  - —Calculate—
    - —Calculate gross
    - —Calculate mandatory deductions
    - —Calculate voluntary deductions
- —Update Master Labour distribution File

Each programmer, out of several at disposal, can be entrusted with a module for writing and testing thereby speeding up the write up The group leader can then combine these and test the employee payroll file program

A great advantage of modularised programming is that any changes in an application, which are bound to occur now and then affect only one or a few modules which can be modified at a much lower cost than would have been the case for reprogramming in entirety.

To avoid any confusion the nuances amongst the MACROS, subroutine and modules must be brought out

MACROS are incorparated in the assemblers only by the computer manufacturer and they are subsets of instructions that are common in most programs

Subroutine are those subsets of instructions that are either repetitively used within a program or/and find application in several programs They are written once and for all by the computer manufacturer or software agencies or the user himself.

Modules arise by vivisection of a program into logical units just for the convenience of writing and debugging They may or may not appear over and again in the same program or different programs. In other words, a subroutine is a module also but a module may or may not be a subroutine Incidentally, more discussion on modular programming follows in Study III.

(b) *Utility Programs or Service Programs* are provided by the computer manufacturers to perform tasks that are common to all data processing installations. Some of them may either be programs in their own right or subroutines to be assembled/compiled in the application programs. The following tasks are performed by the utility programs.

(i) Sorting the data.

(ii) Editing the output data.

(iii) Converting data from one recording medium to another, viz., card to disc, tape to printer, etc.

(iv) Dumping of data to disc or tape.

(v) Tracing the operation of a program.

(c) *Operating or (Executive) Systems.* Considerable time is ordinarily wasted in computer set ups supervised by the operator. During compilation/assembly run time required to input the deck of (the main program and subroutines, etc.) punched cards or its tape equivalent is substantial and would naturally be pinching Likewise several application programs (viz, inventory control, A/c receivables etc) would be read into the CPU in turn everyday or so This too, eats into the working time of the computer. Thus time reduction can be had by maintaining all the programs (including utility programs, assemblers, compiler, etc) in the backing storage which may be of ferrite core or, more commonly, of magnetic drums but interlinked with the main memory. The required program can be recalled into the CPU far more quickly than in ordinary systems where computer is especially set up for each application run or assembly/compilation run etc.—saving considerable time on setups.

Besides, the operating systems work in multi-programming mode

Operating Systems are devised to optimise the man-machine capabilities. Programs are held permanently in the computer memory freeing thereby the operator from inputting a program for each application. The operating systems are also known as "executive systems", "control systems" and "monitor systems". Formally, an operating may by defined as an integrated system of programs which supervises the operation of the CPU, controls the input/output functions of the computer system, translates the programming languages into the machine languages and provides various support services. In a way, an E.D P system is an operating system in that it utilises control programs and assemblers and compliers. The operating systems are based on the concept of modularity. The operating systems are usually the creation of the computer manufactures who design these to suit the capabilities of the particular computer. It would be extremely difficult for the user to design, write and test operating system in view of the limited *brainwave* i e (systems analysts, programmers, etc) These operating systems have acquired unique individualities and have their own names viz GEORGE. It is a fact that the capabilities of the operating systems are far too numerous to be economically exploited by an average organisation. As a result, they tend to be white elephants A great deal of memory remains occupied by the sophisticated programs

The operating systems are characterised by the following :

(i) When presented with a job to be carried out, the operating system transfers the appropriate application program from backing storage into core store in accordance with the job description inserted by the computer operator via the console

(ii) A new program requiring compilation results in the relevant compiler being located from backing storage followed by compilation and notification of errors.

(iii) File storage is controlled so that the user can refer to his file by name, the operating system maintaining an index of user names, file names and file locations for this purpose.

(iv) A wide range of computer configurations can be used with the one operating system

(v) When the characteristics of the computer's job change from time to time the operating system is modified to suit a particular batch of jobs by means of operating parameters inserted by the computer operator

(vi) Temporary failure of a peripheral unit is accommodated by the operating system switching to other units, these are not necessarily of the same type Failure of a printer, for instance, is catered for by transferring the output to be printed on to a magnetic tape or disc storage pending the printer becoming operational once more

(vii) When trouble has occurred owing to machine breakdown, a postmortem analysis of the situation can sometimes be obtained

(viii) An accounting and budgeting system is inbuilt enabling the manager to estimate the separate user costs of the computer Similarly the usage of the computer is logged for all jobs and hence utilisation analyses are provided automatically.

(ix) Multi-access is possible

(d) *Diagnostic Routines* These programs are usually written and provided by the computer manufacturers They assist in program debugging. They usually trace the processing of the program being debugged The diagnostic routines are however also often treated as a category of the utility or service programs.

(e) (f) *Assemblers and compilers* are discussed in earlier sections.

(g) *Application Programs*, as self-evident, are the main programs for various applications, viz , inventory control, sales order processing, etc as more thoroughly discussed in Study II They may either be written by the company staff or acquired from the computer manufacturer or a consultancy organisation dealing in software as discussed in Study V

(h) *E D P policies and procedure* as discussed in Study IV.

Real-time Systems

At this stage it would be useful to give a practical turn to what we have studied thus far. There are two distinct modes of data processing. (i) Batch processing and (ii) On line Processing Realtime system are a subset of online processing. This is depicted in the following chart

Data Processing

Batch processing — On line processing

On line processing → Realtime Processing

Batch Processing. Most business applications consist of transaction processing. The high volume of input transaction occurring commonly in business systems necessitates batch processing A batch of transactions is accumulated, batch totals (as described below) compiled for it, the transactions transcribed on punch cards or other media, the punched cards (etc) sorted by the key field and processed against

the master file to update the latter and produce the desired outputs It is not necessary that only sequential media are used in batch processing Even magnetic discs may be used since it is possible to sequentially organise the files on them.

Batch Total consists of three totals· Record Count, Financial totals and Hash totals

Record Count is simply the number of transactions in the batch.

Financial total is the total of a financial data field over the batch.

Hash total is the total of such an odd data field as is not normally totalled over the entire batch For example, the total of the product stock codes in a batch of stock transactions is a hash total

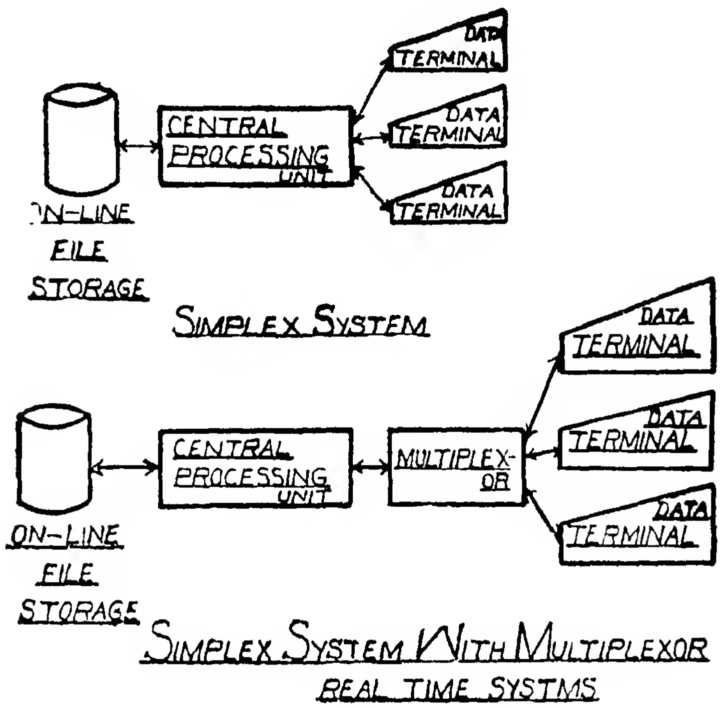
These totals are derived on an adding machine etc prior to processing Later, during, processing, the computer also derives them and prints them as summary information. The two sets of the totals are cross checked If they do not tally it means that either some transactions have accidently been overlooked or fraudulently removed or changed *The device of batch tools provides such a stringent control on errors and fraudulently motivated manipulations that batch processing is undertaken even on magnetic disc (which though offers all the attractions of online processing as discussed below)*

Online Processing refers to proeessing of individual transactions as they occur and from their point of origin as opposed to accumulating them into batches This is made possible by such direct access devices as magnetic disc and magnetic drum Aside from transaction processing and file updating inqui ies are also handled by the online processing systems Such inquiries may be as simple as a salesman wanting to know the availability of a stock item as desired by his customer They may also be complex, as, for example, the list of all customers whose accounts are more than 2 months past due, or the list of salesman in Calcutta whose total rupee volume of sales was highest for a particular month This requires advanced file organisation techniques (such as random access) discussed earlier

Online processing ensures that records are in updated status at any time whereas this is so with batch processing once, say, a week, but the fact remains that online processing is usually more costly Therefore, in choosing one of the processing both updatedness of records and costs have to be considered

Realtime Systems are basically online systems with one speciality in inquiry processing The response of the system to the inquiry itself is used to control the activity An online production data collection system that simply accepts input and utilises it for payroll and costing purposes could not be termed realtime , but if, on the other hand, the system was designed to provide immediate information to monitor the production process it is a realtime system Thus the response of a realtime system is one type of feedback control system The response time would naturally differ from one activity to another If, for example, it is the launching of a space vehicle the response time should be in split second whereas in business situations even a few minutes may be all right Online updating in real time usually leads to transactions as a by-product For example, the updating of stock records for a customer's order may lead to such transactions as despatch notes and purchase requisition or Works Orders Realtime systems offer accountants and others in business the chance to fulfil the potential of the computer as a tool of management.

Realtime systems usually operate in multi-programming and also *multi-processing* This increases both availability and reliability of the system. It is thus



Figs 77 and 78

easy to complement batch processing by one CPU with realtime operation by another; but they may also serve as stand by units to each other CPU's in realtime systems should also possess the capability of what is known as program interrupts These are temporary stoppages or halts in the execution of a program so that more urgent messages can be handled on priority.

Another specific characteristic of realtime systems is that terminals can be hooked to the CPU directly or via multiplexors as depicted in figures 77 and 78.

Data Terminals : (Serving as both input and output units)

These are of two types, Teleprinters and Cathode Ray Tubes (CRTS) The CRT may be likened to a typewriter attached to a television set (See figure 79) The teleprinter is simply typewriter like. The terminals can be placed at the desks of the various managers to input data into the CPU to receive answers to their inquiries The data is input via the typewriters. The answers are displayed on the television set in the case of CRT's and printed in the case of teleprinters The data being input may also be displayed in the case of CRT's and printed in the case of teleprinters for visual verification. Likewise. CRT's may be placed at the shop **floor for the workers to input the job ticket data directly annihilating thereby time-**

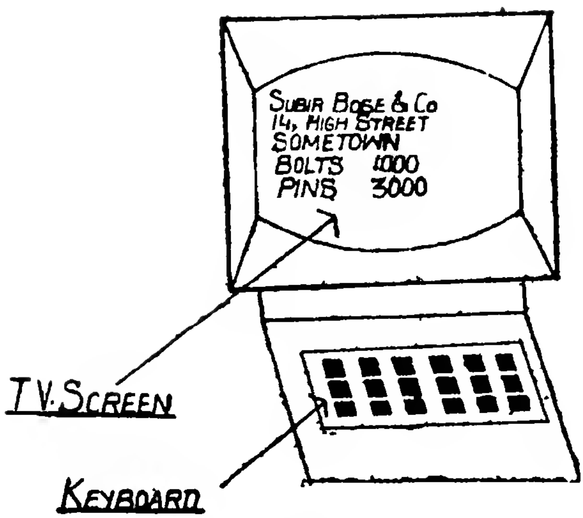


Fig 79

consuming and error-prone transcription process Also, a factory with a head-office keeping the computer and several warehouses scattered in the region may place terminals in the warehouses In this case the communication to the CPU at the head-office by the warehouses can be made via open-wire telephone lines The advantages/disadvantages of teleprinters/CRT's are given below

1 Teleprinters are much cheaper than CRT's
2 Teleprinters automatically produce a copy of all input/output Some CRT's are being marketed that produce a copy of what is displayed on the screen, but this feature makes the CRT's still costlier
3 It is possible with CRT's to display the format of the data in which it is to be entered. This makes it obviously quite easy for the operator to enter data.
4 It is easier to perform correction of errors with CRT's
5 CRT's are quicker than teleprinters.
6 Output speed of CRT's is higher than that of teleprinters.

It is to be particulary noted that the real-time system are usually valid at the operational control level. Real-time holds little prospects for management control Regarding strategic planning managements would be, as Dearden also concludes, inclined to assign such tasks to staff specialists Real-time systems are however being widely applied in process control applications which monitor physical processes such as petroleum refining Most business applications pertain to collection and utilisation of the accounting data Below are enumerated some of the leading areas for real-time applications

Sales order Processing

Real-time systems offer the possibility of a greatly prompt service to the customer His order can be processed within virtually no time. The finished goods inventory file may have data terminals throughout the sales territory So much that the CRT's can be made to display the invoice on a portable terminal for the salesman who is in the customer's plant and can input the particulars there and then ! The customer would get to know of the availabilities instantancously. All appropriate journal entries would be made immediately in the general ledger. The invoice can also be posted to the accounts receivable file instantaneously. The

selection of warehouse that is closest to the customer can also be made and a copy of the invoice could be displayed at the warehouse terminals. Besides, the following functions can be performed rapidly :

(i) The inquiries about the customer's account status can be answered by the salesman

(ii) Credit appraisal can be carried out on-line.

(iii) Sales analysis master file can be updated on-line This file provides a ready information to the sales manager regarding sales trends, etc

(iv) Inventory replenishment orders can be determined as a part of the sales order processing

Transportation and Travel Reservations. These activities pertain notably to hotel chains and airline reservations. Data terminals would be located at each reservation point so that customer reservations can be confirmed quickly The various accounting calculations can also be performed and the relevant files updated

Banking System. The data terminals would be provided at each teller window The following functions can be performed on real-time

1 Checking the balance against a customer's account.

2 Withdrawals and deposits can be posted in customer's account

This would obviously apply to distant branches as well

Retail Sales Credit checking and inventory control functions can be performed

Real-time manufacturing control system The production control, inventory control purchasing, receiving scheduling, shipping, quality control, responding to various inquiries and cost control can all be performed on real-time Efficiency, man-machines utilisation and service are enhanced by real-time, and responses to inquiries are quicker and control tighter.

Time Sharing In it, specific slices of time are allocated to different users in rotation However, time sharing systems are characterised by several users working with the computer via remote terminals The difference between the human speed and the computer speed is astounding This leaves the impression on the user that the computer is working for him exclusively, though the fact is that computer serves him only periodically The time sharing systems are the culmination of the man-machine interactive mode When the man is thinking over some response of the computer to his problem the computer is attending others and is back with him in time This amounts to *conversation* between man and machine.

Small Computers

Miniaturization of the computer equipment has been a fast development since when it started in the sixties and promises to continue even faster

Minicomputers are extremely compact, the central processing unit occupying an almost negligible amount of space due to the utilisation of monolithic integrated microcircuitry The effective volume of a mini-computer's CPU is only a fraction of that of a mainframe's The smallness, ruggedness and lower heat emission of a mini-computer enables it to be installed in a less protected environment, viz. there is no need of airconditioning The lower development costs combined with high sales and mass production result in mini-computers being considerably less expensive than the larger CPU's though this is not so fully true of their peripherals. The mainframe computers are usually sold with a large amount of software the cost of which is ultimately borne by the user, but the mini-computers are sold with minimal software though at a later date the user would have to expend on the development of the software around it The mini computers possess the modularity feature (Ref Study V) and, in fact, it is so easy to expand them as to replace the smaller circu it;

boards with bigger ones. Whereas the main-frame computers are usually under utilised, especially initially, this is not so with the mini-computers

Mini-computers also possess the following particular charcateristics which differentiate them from the main-frame computers.

1. The main memory consist of semi-conductor storage that is not only cheaper but also faster than the core memory.

2 The world length is 16 bits usually whereas that of the main-frame's is 32 bits or more. Thus longer digits (like 161489) cannot be handled with ease in programming.

3. Their capacity ranges from 4 to 32 K.

4 The instruction repertorie is much smaller than that of main-frame's therefore, more instructions would be needed for a problem than would be the case with the main-frame computer This means more processing time.

Visible Record Computer

It is a further refinement of the Accounting Machine (please refer your Advanced Accounting Study) and in fact, is half way in between the accounting machine and the computer It is in a compact single unit with core storage capacity ranging from ½ K to 10 K Its principal input and output media as well as the secondary storage media is the magnetic ledger card MLC) The MLC is an ordinary card (of different standard sizes) with a magnetic stripe bonded on the left side. The proper card portion affords printing of the data e g , stock record and, as far as visibility of the records goes, it is no different than manual ledger on kardex file cards But the magnetic stripe is capable of storing data just as a magnetic tape or disc The MLC's storage capacity varies depending upon the size and may store 10 to 80 words of data The mageneti̇c ledger card is input in the visible Record Computer through a hopper for printing on the card/or inputting and outputting the data into the magnetic stripes This magnetic stripe may be likened to a magnetic tape cut into pieces This retains the relative inexpensiveness of the magnetic tape but, since the MLC's are input manually, this also provides random access. There is no such restriction as to sequentialiy process the records. The disadvantage with the MLC's is that they have to be stored manually for input and, therefore, processing tends to be inherently slow. To some extent it has been successfully sought to speed up processing by pre-sorting of the MLC's and inputting them in magazines There are several advantages of the MLC media also which have been listed below The Visible Record Computer is sweeping the European market, especially for accounting and, since more recently, production and inventory control applications It is to be noted that most models are typical computers in that they also admit connection of all other input/output peripherals discussed in the context of main frame computers earlier and, possess a great deal of flexibility.

Advantages of Visible Record Computer It is relatively inexpensive since, as for storage capacity, it is a mini-computer. As a result, smaller organisations, which either cannot afford a large computer or find time sharing too expensive and not directly under control, are naturally falling in for a Visible Record computer that can be installed in their own premises under direct control (2) Owing to visibility of the records the querries by the customers/vendors and management can be mostly answered straightway as is the case with the manual systems without any processing in the visible Record computer whatever (3) In the main frame computers, the records are usually hidden, *i e*, invisible as they are stored in some magnetic media. This is a queer changeover from the manual systems and the staff take quite a bit of time to acclimatise themselves to the 'mysteries' of magnetic recording. The main-frame computers are often dreaded for this reason. (5) The

operation is exceedingly simple. All that the operator has to do is knocking console key board akin to an ordinary typewriter, put in or take out the MLC and derive guidance from control panel consisting of a few battery bulbs. If for example, light, 1, 2, 4 (out of total of 4) are lit this may imply that a wrong MLC has been put in The operator can, then, input the right one upon such a signal. (4) The visible record computer has remarkably infrequent breakdown. (6) They do not need any especial environment (e.g.) air-conditioned installations in case of the main-frame computers They are particularly intended for commercial applications. (7) Although the MLC is the principal input/output and secondary storage media the visible record computer admits connection of the conventional input/output peripherals. (8) It is an open secret that applications on many a main frame computers in business have been failures in the larger organisation because of the complexity. Such organisations are looking to the visible record computer for decentralisation of their various functions. viz Production control, and instead install a number of small compact visible record computers in their different units

Generation of Computers

Fourth Generation Information system : Fourth generation machines appeared in 1970 s, utilizing still newer electronic technology which enabled them to be even smaller and faster than those of the third generation Many new types of terminals and means of computer access were also developed at this time.

One of the major inventions which led to the fourth generation was the large scale Integrated circuit (LSI). The LSI is a small "chip" which contains thousands of small electronic components which function as a complete system. In effect an entire computer can be manufactured on a single chip of size less than ¼ inch square. A single chip may perform the functions of an entire computer, calculator or control device Research into future developments promise the manufacture of large computor systems, with enormous memory capacity, on small chips. This will reduce the cost and increase the speed of new systems still further.

Micro computers : Developed during 1970 s, the micro-computer is an even smaller version of a computers It is manufactured on a small chip of silicon mentioned above. Micro computers have many of the features and capabilities of the larger systems. The cost of micro computers has dropped substantially since their introduction. Many now sell a micro compnter for as low as $ 5 00 abroad. This reduction in cost will bring about a significant increase in the number of micro computers in use. The major application for micro computers lies in the field of industrial automotion, where they are used to monitor and control various manufacturing processes. Their low cost and light weight make it feasible to carry them on site or into a field or to package them with other portable equipment as part of larger system

Fifth Generation : In the 1980 s, the computers are being used to aid the management A new generation of managers has been educated in an environment that views the computer as an everyday tool. In addition, organizations are reaching some degree of sophistications relative to computers ; such organizations have talented computer professionals, data bases, and information systems. These ingredients are needed for an age that consider the computer as a management tool. Under such circumstances, the manager will be able to use the computer for the management functions (i e. planning, organizing, controlling and actuating) and actions (i e. decision-making and communicating).

We can anticipate that the computers of future will be even faster more compact, and more reliable than even before. Also, we predict significant simplified approaches to computer programming, that will include not only English like languages but also oral communications between human and the advanced computers.

APPENDIX 1

Unit Record Equipment (Syn. Punched Card Machines)

Unit Record Equipment comprises the following eight electromechanical machines which operate on *only punched card* files :—

1. Keypunch
2. Card Verifier
3. Card sorter

} These have been discussed in the context of the computer earlier

4. Reproducer
5. Interpreter
6. Collator
7. Calculator
8. Accounting Machine

} These will be discussed below

The nomenclature "Unit Record" arises because each punched card must carry one record unlike the computerised system in which one card may carry more or less than one record. Unit Record Equipment was used extensively in business situations prior to the advent of the computer ; but now-a-days, it is giving way to the computer. In fact, even the second generation computers had beaten it on cost benefit grounds.

Below, we discuss machines, 4 to 8 listed above.

4. *Reproducer* (*Syn. Reproducing Punch*) is used to copy all or part of a deck of punched cards into another deck of cards. It can reproduce in a straight forward way i.e., column by column duplication of a deck of cards. It can also reproduce with offset, shifting fields from one position on the original deck of cards to different position on the cards in the new deck. Finally, it can do gang punching which means punching of all, or a selected part of the data on the master card into all cards which follow it. These 3 modes of reproduction are shown on the next page by figs. 1 to 3.

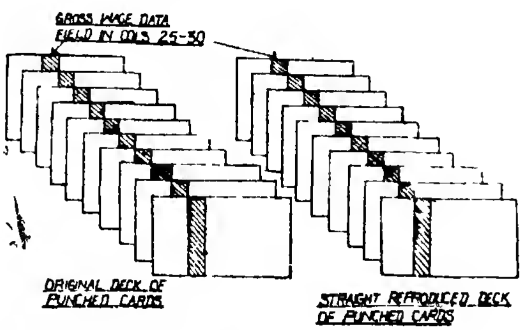


Fig. 1

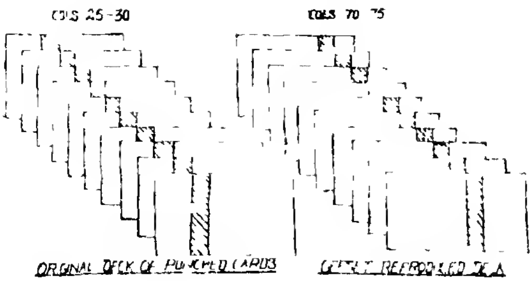


Fig 2

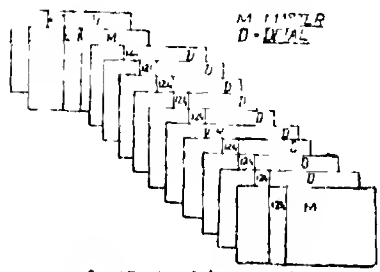


Fig 3

5. *Interpreter*

Interpreting is a means by which punched cards which do not contain printing at the top can be read, and the information read printed on the top of the card Depending upon the particular model of interpreter used, printing will take place on one or two lines throughout the body of the card Alphabetic or numerical information both can be printed by the interpreter.

It may be noted that some of the keypunch models not only punch the card but also do interpretation at their tops at the same time.

6. *Collator*

A collator is basically used to merge two different files of cards. It has two input feed mechanisms (Figure 4) and can drop cards into as many as 5 output hoppers.

Merging is the process by which two decks of cards arranged in some predetermined sequence, are combined to produce one large deck of cards in the same sequence. Fig 5 shows, for example, the merging of two files : Customer Order File and Customer Name/Address File.

Other capabilities of the collator are discussed below.

Select specific cards from a file Fig. 6 shows, for example, the selection of master cards from a file. In a way, selection is decollation.

7. *Calculator*

It is a machine that can do addition, substraction, multiplication and division. It can be put to 2 types of uses.

(i) Data calculated from a card is punched in same card, viz. net wage of a worker computed from his basic pay, allowances, deductions, etc. is encoded in the last 5 blank column of the same card.

(ii) Data calculated from a set of cards is punched on a summary card.

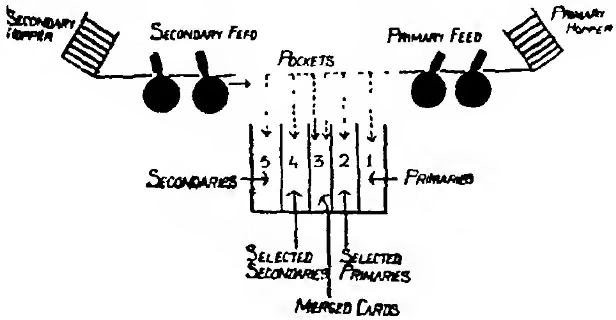


Fig. 4

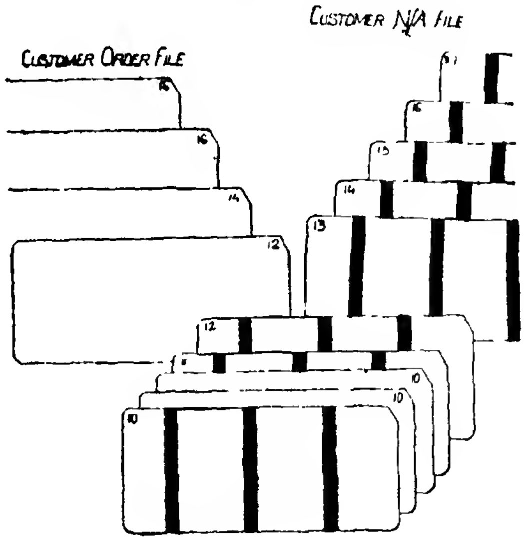


Fig. 5

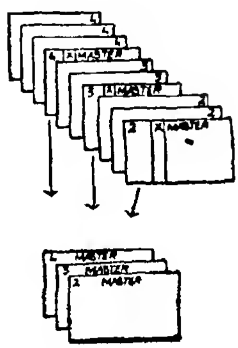


Fig. 6

Match merging : It means merging two files (fig. 7), selecting out all those cards that do not have a match in the other file.

Check the sequence of a deck of cards (fig. 8), routing any out of sequence cards to a reject hopper.

Simply select cards, from either of two files, which do not have a match in the other file (select all cheques for which there is no master file entry, for example).

8. *Accounting Machines* possess the following capabilities :—

1. Addition/substraction (in some models also multiplication/division) using a series of a counters.

2. Printing the data punched in a card, as well as printing the contents of its counters.

3. Punching the contents of its counters in a card through a reproducing punch ; and

4. Printing reports.

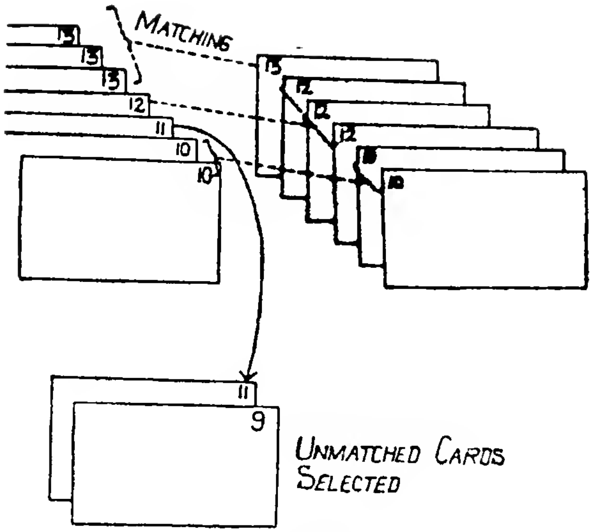

MATCHING
13
13
13
12
11
10
10
13
12
12
12
10
10
11
9
UNMATCHED CARDS SELECTED

Fig. 7

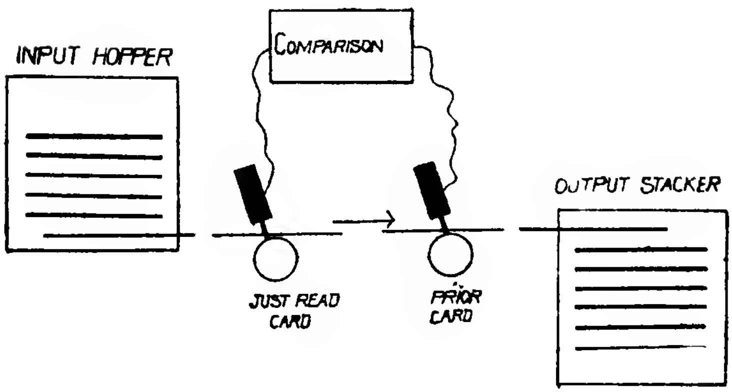

INPUT HOPPER
COMPARISON
JUST READ CARD
PRIOR CARD
OUTPUT STACKER

Fig 8

A typical Stock Application on Unit Record Equipment

(See fig. 9)

Step A. 1. Sales orders are assembled in batches.

A 2. A card is punched for each line on the Sales order. The deck of the punched cards is then verified, going the verified deck A 3.

A 4 The verified deck of punched cards is sorted by the stock item code on a card sorter. going the sorted depletions deck A 5.

A 6 From the sorted depletions deck fed into the accounting machine a detailed sales listing in the following specimen format is printed

Item code	*Qty. demanded*	
1141	14	
1141	13	
1141	27	(designate the summary)
1142	10	

A 8. Also produced by the reproducing punch connected to the accounting machines is a deck of punched cards, containing the total of all depletions for each stock items to a card, in proper sequence.

Steps B 1 to B 8 may be interpreted likewise for the receipt transactions

Steps 2. The depletion and addition deck (A 8 and B 8) are merged, giving the merged deck (3), on the collator.

Step 5. The master file (4) and the additions depletion deck (3), are match merged, into the combined file (7) the rejected cards thrown out as errors (6)

Step 11 The combined file (the master card followed by its summary depletion addition cards) is updated on the accounting machine, giving the updated master file (12) via the reproducing punch and the summary listing (13) in the specimen formate below :

Stock item No.	Description	Old balance	Additions	Depletions	New balance

Step 8. **In the step "the things are put back in the order", on a collator. From the combined file (7) the old master and transactions are obtained after decollation.**

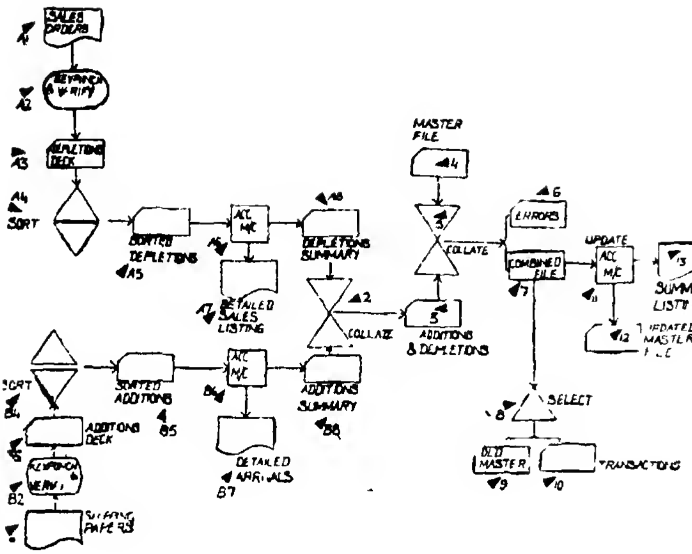


Fig. 9

Limitations of Unit Record Equipment

The unit record equipment has almost been completely overtaken by third generation computers The reasons should be clear from its limitations relative to electronic computers discussed below.

Punched card machines (or unit record equipment) have electronic computer as a serious contender and their limitations have to be appraised particularly in relation to it. Electronic computer is much faster, accurate, capable and versatile than punched card machines. Each successive generation of the electronic computer has brought about a tremendous decrease in cost (and size) permitting corresponding increase in capability in handling complexities. This has eroded whatever little cost advantage the unit record equipment had over the second generation computers. The principal limitations of the unit record equipment are stated below.

1. It is widely, and often wrongly, believed that computers are very expensive and a company should be large and prosperous to afford the luxury of computer processing One should remember that the hardware costs have been steadily falling over the last 20 years and the trend appears to continue at an acclerated rate. Further, a few hours of computer time can be hired at low cost for selected applications, properly conceived and implemented, the effort can be very profitable. A recent development is visible record computers, which have yet to find their way into India, are inexpensive and can be installed in house for smaller organisation also.

2. The most significant limitations of punched card processing are the lack of memory and rudimentary processing capability. There is virtually no memory except for the card currently being processed and the facility to take a few totals and sub-totals Hence it is virtually impossible for tasks involving some degree of computational complexity. This has limited the use of punched card machines merely to transaction processing (viz. such applications as inventory accounting or payroll).

4. Even at the transaction processing level, these machines have a serious limitation Each record has to be accommodated on one (and only one) punched card , therefore, applications involving longer records e g., customer billing entailing customer name and address) are hard to mechanise.

4. Data files on cards are bulky and therefore, difficult to store and handle. there being ever present hazard of dropping a pack of punched cards. The punched cards have to be run repeatedly through the punched card machine After a few passes they wear out and must be duplicated. Direct access is virtually impossible, thereby forbidding not only inquiry handling (except for manual access which is error prone) but also to make the most use of every piece of information and update a number of files in a stroke Since parts and components of these machines keep on moving, they soon wear out and give rise to mistakes not apparently detectable

5. Retrieval of data is another problem that we come across in the use of punch card machines Data are gathered in numerous batches and transaction cards are kept in bunches Though not impossible but it is definitely difficult to trace some old data from these cards There is also the attending possibility of these being misplaced in the process.

6. The 'cards grow in number with every transaction and in a short span of 'time may assume a volume that is difficult to handle Also without adequate control measures, keeping track of the cards will be impossible

7. Mechanical devices are inherently slower than the electronic components in the computer Typically, input and output speeds are much slower on unit record equipment and these limit the amount of work that can be done in a given time.

8. Programming is cumbersome and is done by inserting contacts in a plugboard or pannel.

The biggest problem in starting with unit record equipment and later moving on to a computer is that unit record processing embodies completely different concept from the computer Thus, the systems requirements with punched card equipment are very different from those with a computer Retention of these principles can be very wasteful and inefficient on the computer One of the most important principles in data processing is to make the best use of every piece of information. This is usually not practicable with unit record equipment, where systems are designed to produce specific reports Similarly, using a run to update a number of files is foreign to unit-record-trained personnel

It has been the experience in many installations that personnel trained on punched card equipment find it very difficult to adapt their thinking to computer techniques. This can lead to poorly designed computer systems The management also gets a restricted view of data processing and this colours their subsequent decisions, often crippling the data processing effort.

APPENDIX 2

DIRECT DATA ENTRY DEVICES

Key-To-Disk

Punched card as a media is now-a-days totally eliminated by the key-to-disk system The data from a source document is directly keyed to a magnetic storage A key-to-disk system comprises of several keying stations, a miniprocessor, magnetic unit and a supervisor's console A key station consists of a typewriter like keyboard with a panel or visual display unit Every key station is connected to the miniprocessor, and when input data is keyed, it is transferred to core storage in the miniprocessor For each key station, 40 (also 80, 128) characters of core storage in the miniprocessor are allocated.

Each character of the data that is keyed in is checked for validity, and errors are signalled by the miniprocessor to the keying station and the keyboard is locked The operator either corrects the error immediately or flags it for subsequent correction The panel or visual display units facilitate the correction of errors by displaying the erroneous data Sophisticated editing of input data can be performed at the time of keying by the use of the read-only program in the miniprocessor.

When the allocated area in the miniprocessor's storage (called buffer) for a key station is full, the keyed data is transferred to a designated area on a disk in the form of a record This is retrieved when the key station is in verify mode, and entered into the buffer storage, so that the same record can be compared when it is keyed for verification Any discrepancies during verification are displayed on the panel or visual display unit.

The records stored on a disk are subsequently transferred to a magnetic tape, after the verification and correction of records are completed The records are moved to an output buffer on a disk before they are written on a magnetic tape

The supervisor's console is used to monitor usage of the disk, the keyboard operator's performance and the status of input data Until this data is processed by the main computer system, the data on the disk is preserved While the key-to-disk systems have the advantage of ensuring accuracy of input, security and monitoring of progress, they suffer from a serious advantage, viz, all the key stations will be paralysed when there is a breakdown of the miniprocessor

Key-To-Diskette

In key-to-diskette systems, the data is keyed into a stand alone data station, and recorded on floppy disks A floppy disk (diskette) can hold about 150,000 characters in the form of 80 character records, or 240,000 characters in the form of 128 character records.

The data station consists of a keyboard and a visual display unit. The keyed data is held in a buffer before recording on a floppy disk.

The data recorded on a floppy disk can be verified by setting the keyboard in verify mode. It is displayed on the VDU in the form of 2 lines of 40 characters each and a third line indicating the format of the record. The data recorded on floppy disks is later transferred to magnetic tapes at a speed of 400 characters per second with the help of a data converter. The data converter can handle 20 diskettes at a time, transferring 300 records per minute The data converter can perform editing when data is transferred to a magnetic tape, and thereby guides the operator with error messages to initiate correction procedures.

The advantage of key-to-diskette systems is that they are stand-alone systems, in the sense they are self-contained, and a total break-down is avoided. The keying stations can be dispersed and located close to the sources of data The key-to-diskette systems allow decentralised recording of data and centralised conversion to a magnetic tape for processing by a mainframe computer.

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THE INSTITUTE OF CHARTERED ACCOUNTANTS OF INDIA

STUDY MATERIAL F S P (N) S A & DP-2 COMBINATION B

FINAL COURSE (N)

FLOWCHARTING

STUDY—II

Contents

FLOWCHARTING

Program Analysis

It was pointed out in Study I, the digital computer does not do any thinking and cannot make unplanned decisions Every step of the problem has to be taken care of by the program. A problem which can be solved by a digital computer need not be described by an exact mathematical equation, but it does need a certain set of rules that the computer can follow. If a problem needs intuition, guessing or is so badly defined that it is hard to put it into words, the computer cannot solve it.

A good deal of thought must be put into defining the problem and setting it for the computer in such a way that every possible alternative is taken care of. Thus the steps that comprise a *computational procedure* must be delineated before the procedure can be programmed for the computer and the computer procedure must be sufficiently detailed at each stage of a computation to permit the required calculations to be performed Also, computer procedures are designed to solve a whole class of similar problems. The procedure for adding two signed (i e positive or nagative) numbers a and b serves as an example :

1. If a and b have the same sign, go to step 5.
   (If a and b have different signs, continue with step 2)
2. Subtract the smaller magnitude from the larger magnitude.
   (continue with step 3)
3. Give the result the sign of the number with the larger magnitude
   (continue with step 4)
4. Stop
5. Add the magnitudes of the numbers a & b.
   (continue with step 6)
6. Give the result the sign of number a.
7. Stop

The procedure, in this case, is fairly detailed and would work for any two numbers a and b. For example, $(-5) + (1-4) = 9$, $16 + (-11) = 5$, $10 + 20 = 30$, and so forth.

Algorithm

A specific procedure of this type that exists as a finite list of instructions specifying a sequence of Operations and that gives the answer to any problem of a given type is called an *algorithm*. Computer programs are based on the concept of an algorithm.

Example-2 : Consider an algorithm used to generate a sequence of numbers as Fibonacci numbers.

1, 1, 2, 3, 5, 8, 13, 21, 24.........

If Fi denotes the ith Fibonacci number, then $F_1 = F_2 = 1$ and $F_i = F_{i-1} + F_{i-2}$, for all i greater than 2.

An algorithm for computing Fibonacci numbers that are less than 100 is given as follows :

1. Set N1 to 0. (This is not a Fibonacci number, and is used only to start the procedure)
2. Set N2 to 1 (This is first Fibonacci no)
3. Write down N2
4. Set N3 equal to N1+N2.
5. If N3 is greater than 100, then stop the calculations.
6. Write down N3.
7. Replace N1 by N2
8. Replace N2 by N3
9. Continue the calculations with step 4.

An algorithm exists for each computational problem that has a general solution. The solution may exist as a set of mathematical equations that must be evaluated or as a set of procedural steps that satisfy a pre-established procedure—such as the well-known procedure for calculating income-tax liability

Example-3 : Consider the Euclidean algorithm stated as follows :—
Given to positive integers a and b, find their common divisor.

The algorithm involves the construction of a descending sequence of numbers. The first is the larger of the two numbers, the second is the smaller, the remainder from dividing the first by the second, the fourth is the remainder from dividing the second by the the third; and so forth. The process ends when there is zero remainder. The greatest common divisor is the last divisor in the sequence. For example, the descending sequence of numbers for greatest common divisor of 44 and 28 is

44 28 16 12 4 0.

The last divisor is 4, which is the result. The algorithm can be summarized in the following list of instructions :

1. Write down a and b
2. If b is greater than a, exchange them.
3. Divide a by b giving the remainder.

4. If r is equal to zero, stop; b is the G.C.D.
5. Replace a by b; (that is, b→a)
6. Replace b by r ; (that is r→b)
7. Go to step 3.

From the above discussion, several characteristics of an *algorithm* can be given :

1. It consists of a finite no. of instructions, however, some instructions may be executed more than once and others may not be executed at all depending on the input data.
2. The instructions are precise.
3. The intructions are unambiguous.
4. The no of operations that are actually performed in solving a particular problem is not known beforehand, it depends on the input and is discovered only during the course of computation.

Flowcharts

For many applications, a simple list of the steps that comprise an algorithm is sufficient for stating the problem in a clear and unambiguous manner. However, when procedure is complex and different options exist, then a list of instructions is hard to follow. For describing a complex process a flow diagram (or flowchart) is the computer field. A *flowchart* is a diagram, prepared by the programmer, of the sequence of steps involved in solving a problem. It is like a blueprint, in that it shows the general plan, architecture, and essential details of the proposed structure. It is an essential tool for programming and it illustrates the strategy and thread of logic followed in the program. It allows the programmer to compare different approaches and alternatives on paper and often shows interrelationships that are not immediately apparent. A flowchart helps the programmer avoid fuzzy thinking and accidental omissions of intermediate steps.

Flowcharts can be divided into 4 categories as below and as such they may be likened to the geographical map with regard to the extend of detail :

1 Program flowcharts	(District map)
2. Run flowcharts	(State map)
3. System flowcharts	(National map)
4. System outline charts.	(Global map)

In this study we shall concern ourselves with Program flowcharts and Run flowcharts. The other two types of flowcharts are discussed in Study III,

Part I Program Flowcharts :

The program flowcharts are the most detailed and are concerned with the logical/arithmetic operations on data within the CPU and the flow of data between the CPU on the one hand and the input/output peripherals on the other. Prior to taking up an actual program flowcharts, we first discuss below a flowchart (Fig. 1) of the morning routine of an office employee to bring out concepts and the use of the following flowcharting symbols involved.

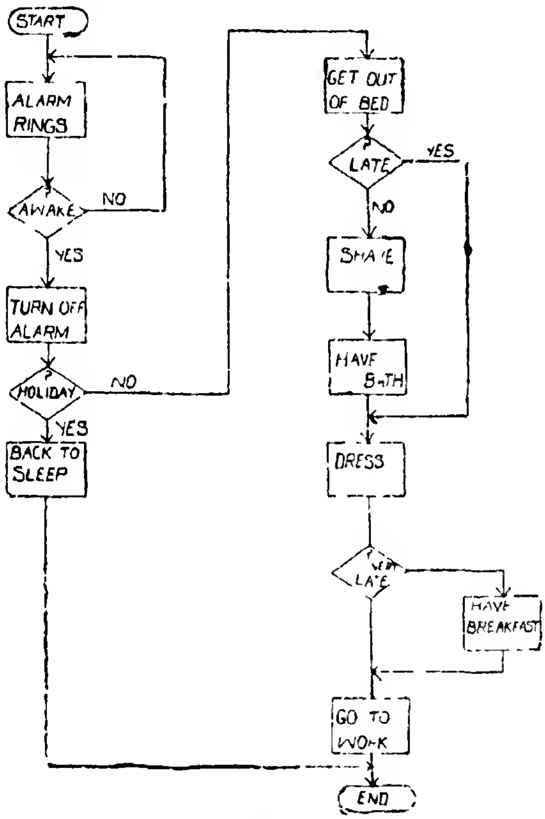


Fig. 1

Box

Fig. 2

The box is an action symbol. Such actions as "dress" etc. would be written within box. In the context of program flowcharting such actions would, for example, consist of the various arithmetic operations.

Diamond

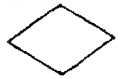


Fig. 3

The diamond is the symbol for posing a question and it leads to two branches with yes and no as answers to the question In program flowcharting, however, this is a comparison or conditional symbol. For example, if in an inventory control application, reorder level has reached the program instructions for placement of the replenishment order would be executed If it has not reached, a different path would be taken. This path may involve alternative set of instructions. Suppose that reorder level is placed in location number 536 and the stock level is placed in location number 637 the, necessary comparion may be symbolised in either of the following ways:

Fig. 4

This is t e symbol for start and end of a routine.

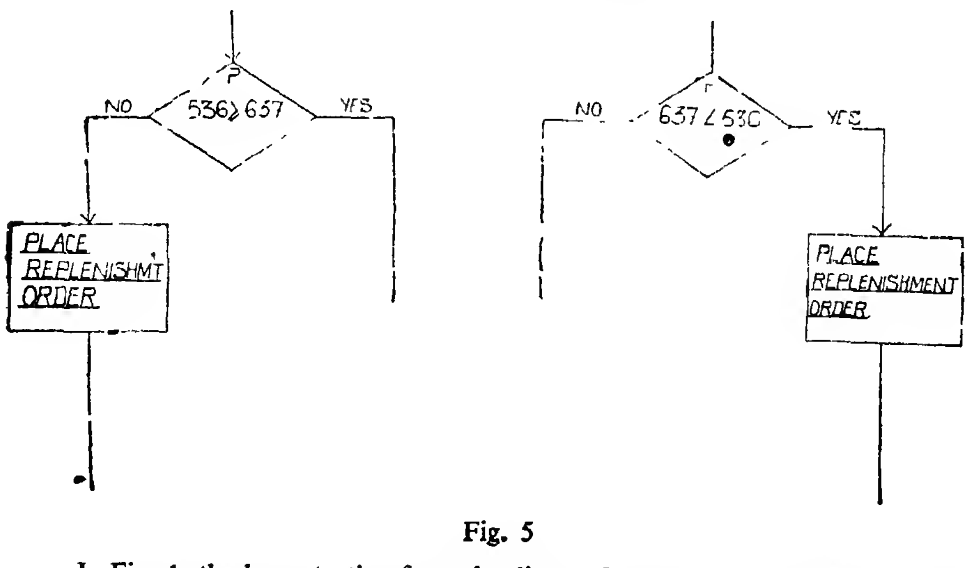


Fig. 5

In Fig. 1, the loop starting from the diamond symbol for "awakes" and enclosing the box for "alarm rings" should be of interest in view of its importance in program flowcharting It portrays the recurrence of the alarm until the person is awake. Later, in a program flowchart, we shall encounter a loop for computing 2^7.

Some other points of relevance to program flowcharting are well brought out by this flowchart. Obviously, different people have different approaches to the

routine Likewise, a problem may be flowcharted or programed differently by different programmers. For example, consider the problem of finding the value of the expression $2^7 \times |\underline{7}$. One programmer may compute $|\underline{7}$ first and then 2^7. Another programmer may do just the reverse. Still another programmer may work in the following fashion :

$$(2 \times 7) \times (2 \times 6) \times (2 \times 5) \times (2 \times 4). \ ...$$

Whereas in personal routines one is at complete liberty, in Program flowcharting, one of the objectives is to keep the program as simple as possible. Therefore, the third approach of computing $2^7 \times |\underline{7}$ in the fashion of

$$(2 \times 7) \times (2 \times 6) \times (2 \times 5)$$

is rather clumsy and, therefore, not sound

Another point concerns the level of detail in flowcharting. In the Box "dress"t we have implied dress including combing the hair But someone may want to use two boxes for dressing and combing. For example, the computation $2 \times 3 \times 7 = 42$ may be showm in just one box as such or in two boxes as $2 \times 3 = 6$ and $6 \times 7 = 42$. The most detailed program flowchart would have exactly one instruction for each symbol in the flowchart i.e , coding would be simple and straightforward but the flowchar being biggish would defeat its purpose of showing the flow of data at a glance.

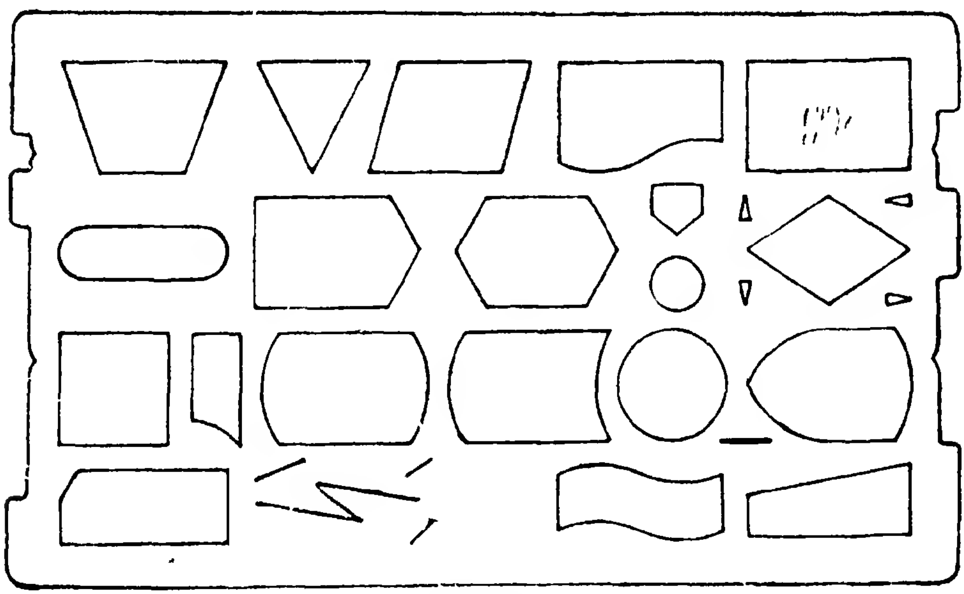


Fig. 6

Finally, abbreviations, annotation etc. of the various operations, comparisons etc are desirable and, in fact, necessary to save cluttering the flowchart with long plain English expressions. For example, in figure 4 "637<536" is the abbreviation for "Are the contents of location 637 less than those of location 536" ?

More flowcharting symbols are explained in Fig. 7. There are being marketed flowcharting stencils (figure 6) but neither they are readily available nor really

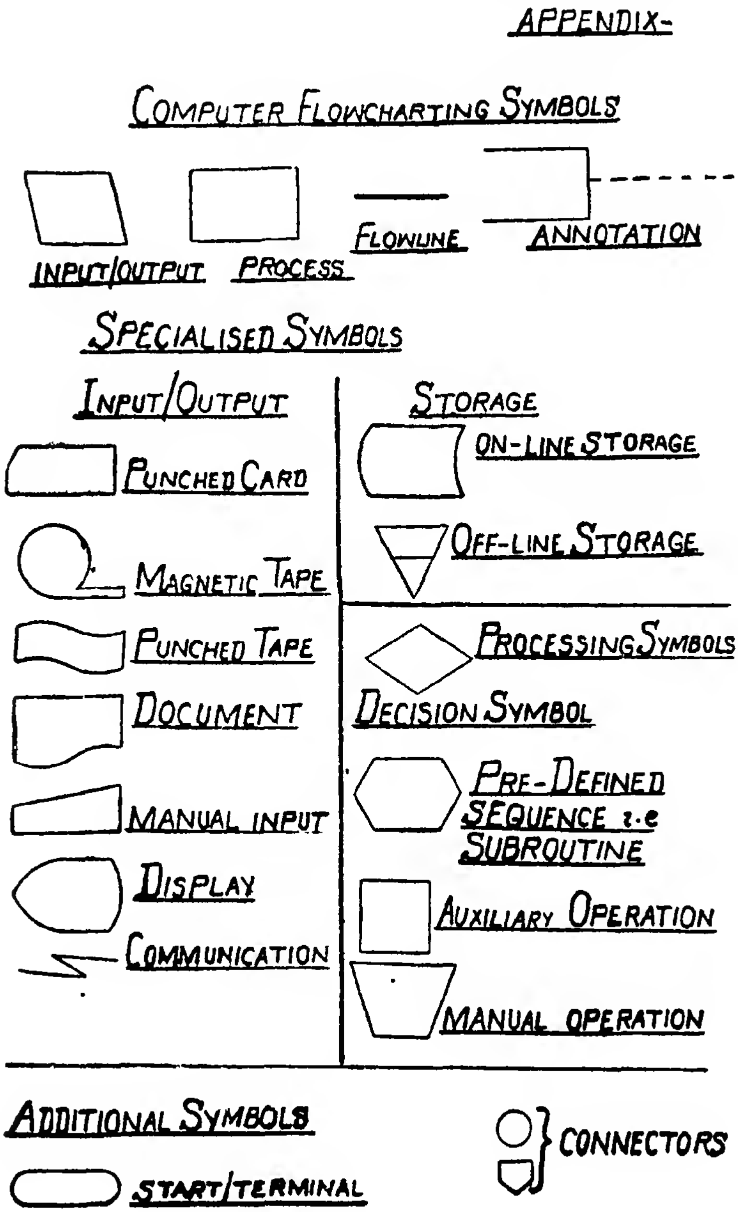
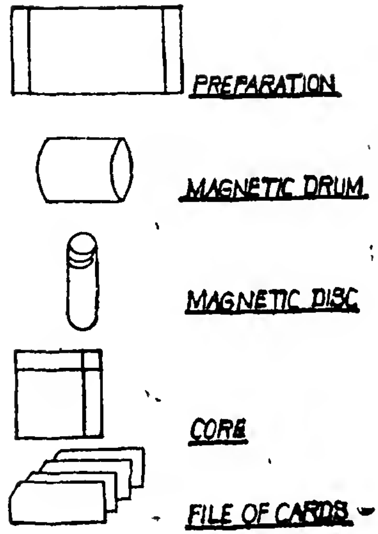


Fig. 7


PREPARATION
MAGNETIC DRUM
MAGNETIC DISC
CORE
FILE OF CARDS

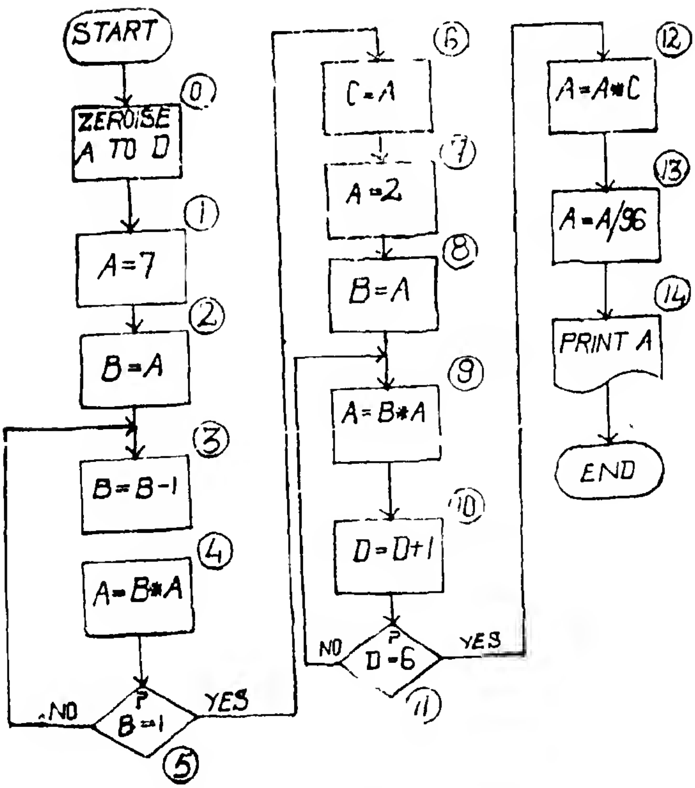

START
0
ZEROISE A TO D
1
A=7
2
B=A
3
B=B-1
4
A=B*A
5
P
B=1
NO
YES
6
C=A
7
A=2
8
B=A
9
A=B*A
10
D=D+1
11
P
D=6
NO
YES
12
A=A*C
13
A=A/96
14
PRINT A
END

Fig. 8

necessary for the student whom we would rather encourage to draw these symbols in free hand.

Let us now introduce a problem on program flowchart for computing and printing the result of $\frac{2^7 \times |\underline{7}}{96}$ The flowchart for this is shown in figure 8. The serial numbers in circles against the various symbols are not a part of the flowchart but are merely intended to facilitate the following explanation to the flowchart. At each serial number in this explanation, we have also given the contents of the four locations of the CPU, 001 to 004 which have been used to solve this problem. These 4 locations have been symbolised as A, B, C and D respectively.

The = sign should be interpreted as "becomes" and not 'equal to', e g. in step 3, B=B−1 means location B becomes one less than its original contents

Serial Number — *Contents of the CPU*

*Zeroise A to D (i e clear all working locations.

A	0	0	C
B	0	0	D

1. *Put 7 in location A*
   (This is with the view to start computation of $|\underline{7}$)

A	7	0	C
B	0	0	D

2. *Transfer the contents of location A to location B*
   (Transfer here implies copy)

A	7	0	C
B	7	0	D

3, *Subtract one from the contents of location B.*

A	7	0	C
B	6	0	D

*After start, it is highly desirable to give an instruction "clear all working locations". In this case it would ensure that locations A to D hold zeros.

4. *Multiply the contents of location A with those of location B and put the result in location 001.*

A	42	0	C
B	6	0	D

(It is to be carefully noted that 42 has come in location A, 7 having been automatically erased).

5. *If the contents of location B equal 1 go to the next step, otherwise go back, to step 3,*

This amount to looping alluded to earlier. The idea is to decrease the contents of location B by 1 successively, multiply these with those of A and store the intermediate results in A. Intermediate results in the CPU are shown below).

A	$(42\times5)=210$	0	C
B	5	0	D

A	$(210\times4)=840$	0	C
B	4	0	D

A	$(840\times3)=2520$	0	C
B	3	0	D

A	$(2520\times2)=5040$	0	C
B	2	0	D

A	$(5040\times1)=5040$	0	C
B	1	0	D

(Thus, at the end of step 5 we have $|\underline{7} = 5040$ in location A.)

6. *Transfer the contents of location A to location C.*

A	5040	5040	C
B	1	0	D

(It is to be noted that transfer really means copy i.e., we transfer the contents of location A to location C and also they are retained, and are not erased, in location A).

7. *Put 0002 in location A.*

A	2	5040	C
B	1	0	D

(This, of course, erased 5040 held earlier by location A. This has been done with the view to start the computations of 2^7).

8. *Transfer the contents of location A to location B.*

A	2	5040	C
B	2	0	D

9. *Multiply the contents of location A with those of location B and put the results in location A.*

A	4	5040	C
B	2	0	D

(Note that we have to multiply the contents of A 6 times to obtain the value of 2^7. We have done this once at this stage).

10. Add 1 to the contents of location D.

A	4	5040	C
B	2	1	D

(This is with the view to "remember" that the aforesaid multiplication has been carried out once. This is quite similar to men doing the counting on their finger tips as to how many times the multiplication has been done. In the loop to follow at the next step we shall go on incrementing the contents of location D by 1 until it equals 6. In the technical jargon, we have "set a counter" in location D to keep track of the number of times this multiplication has been carried out).

11. If the contents of location D equal 6 go to the next step, otherwise go back to step 9.

(This is the loop to carry out the multiplication six times. The intermediate results are ghown below).

A	$(4\times2)=8$	5040	C
B	2	2	D

(Multiplication 2 times)

A	$(8\times2)=16$	5040	C
B	2	3	D

(Multiplication 3 times)

A	$(16\times2)=32$	5040	C
B	2	4	D

(Multiplication 4 times)

A	$(32\times2)=64$	5040	C
B	2	5	D

(Multiplication 5 times)

A	(64×2)=128	5040	C
B	2	6	D

(Multiplication 6 times)

The final result 128 is retained in location A.

12. Multiply the contents of location A with those of location C and put the results in location A.

A	645120	5040	C
B	2	6	D

13. Divide the contents of location A with 96 and put the results back in A.

A	6720	5040	C
B	2	6	D

14. Print the contents of location A on the continuous stationary.

Several points emerge from the discussion on flowcharting and allied matters above.

(a) In the above explanation to the program flowchart, the unbracketed sentence against each serial number is instruction in plain English, viz., "Put 7 in location 001" is an instruction The serial number gives the instruction number. There are 15 instructions, the flowchart being detailed to the maximum extent i e. each symbol in it corresponds to one instruction. These plain English instructions are quite amenable to codification in the assembly language. The student must not confuse the expressions in the various symbols in this flowchart of Fig. 8 as the codified instructions in the assembly language. They are usually arbitrarily devised by the programmers.

The program flowchart is basically intended to facilitate encoding i e , writing the program instructions. And it is a function of the language in which coding is desired i.e., it may differ from one language to another. Sometimes, especially when the problem is small or simple, coding can be carried out directly without flowcharting; but, it is highly desirable to first draw the flowchart.

(b) The flowcharts, except for loops, should ordinarily proceed top to bottom and from left to right.

(c) As mentioned earlier different programmers may appoach the same problem in different ways. During the last few centuries in the history of engineering a design principle has emerged, "Simple engineering is the best engineering". This is as much applicable to program design i.e , flowcharts and systems design discussed in Study III.

(d) Another objective of programming, aside from its simplicity, is to use the minimum possible storage space. In this examble, we have used 4 CPU locations for computations, another 15 would be needed by the instructions i.e , a total of 19 location have been used. Considering that practical problems are much larger it is highly desirable to conserve the CPU storage space.

(e) The third objective of programming is to ensure that processing time is the least. In this regard, it may be pointed out that divisions, multiplications, subtractions, additions, transfers and comparisons take decreasing computer time in this order. However, an actual example on this objective is given in Study VI.

(f) These three objectives of programming are conflicting. For example, the programmer would get several opportunities where he can save on storage space by approaching a problem in rather a complex way and *vice versa* i.e. he may economise on computer time by using the storage space lavishly, etc. He has to reconcoile the three conflicting objectives If, for example, the storage space is at premium i e., the CPU is small and the program is likely to require slightly more or less space that it can make available the programmer may sacrifice the other two objectives of simplicity and least processing time and keep the emphasis on economising on storage space.

(g) Supposing that the fifteen instructions are stored in locations 005 to 019 the data in the start would be as shown on page 16.

Upon pressing the "Start" switch on the console these instructions would be executed one by one sequentially and the data in the first 4 locations would be processed as per explanation above. The intructions would stay the same location even after execution. *There is a kind of instruction that can modify other instructions as programmed during execution: but this type is not there in our instructions above.*

001	0	0	003
002	0	0	004
005	Instrn. 1	Instrn. 2	006
007	Instrn. 3	Instrn. 4	008
009	Instrn 5	nstrn. 6	010
011	Instrn 7	Instrn 8	012
013	Instrn. 9	Instrn. 10	014
015	Instrn. 11	Instrn. 12	016
017	Instrn. 13	Instrn. 14	018
019	Instrn. 15		020

Program flowcharting, to some extent, is a function of the language in which the program will be ultimately coded i.e. it will vary to some extent from one language to another Therefore, it also depends upon the instruction repertoire or mix of the computer on hand. Nevertheless the variations are minor. The following hypothetical instruction mix (say, for computer X) will be utilised throughout as a basis for drawing the flowcharts.

Logical/Arithmetic Operations

1. Addition

(i) Add the contents of two locations, say A and B, and put the results in either A or B, or any other location. An example follows :

C=A+B or C=B+A — means 'add the contents of two locations A and B, and put the results in location C. However, this interpretation is assembly language oriented The compiler language orientation would be "*C becomes* the sum of A and B". In this interpretation by '=' is meant 'becomes' and not "equal to", Also in this interpretation

C, A and B are treated as if they are variables. Most flowcharting in this study note is compiler language oriented. However, we shall use assembly language interpretations at places though rather sparingly.

A=A+B or A=B+A	A assumes the sum of the previous value of A and the value of B.
B=A+B or B=B+A	B assumes the sum of the value of A and the previous value of B.

(i) Add a constant to the contents of a location or the value of a variable.

C=A+13	C becomes the value of A plus 13
A=A+13	A becomes the previous value of A plus 13.

2 Subtraction

The student can interpret these on the lines of interpretations of the addition operations above.

B=A−B
C=B−A
A=A−B
A=B−A
B=A−14
A=A−14

3. Multiplication

The multiplication is best represented by asterisk in flowcharting so that it does not confuse with the widely used letter, X. The student can interpret the following operations himself.

C=A*B
C=B*A
A=A*B
A=B*A
A=A*7
B=A*7

4. Division

Two types of division can be carried out. Suppose for example, that we divide 7 by 4. In one type, we get 1.75. In the other type we just get the quotient 1 and the remainder 3 is consigned to a location reserved for remainder by the computer manufacture say, REM. We shall use the same format for the two types as below.

Type 1	*Type 2*	
Results in the location on L.H.S.	Only the quotient in the location on the L.H.S. The remainder is consigned to standard location symbolised by REM.	
		Remainder i
C=A/B	C=A/B	REM
C=B/A	C=B/A	REM
A=A/B	A=A/B	REM
A=B/A	A=B/A	REM
A=A/131	A=A/131	REM
B=A/131	B=A/131	REM

Though we are not using in the Study note any symbol for exponentiation (raising to power) the student may use * * e g. * * 3 means cube x.

5. Transfer

Transfer the contents of one location into another location. In other words the variable of the R.H.S. becomes (or assumes) the value of the variable on L H S Examples follow ·

A=B   If B were 13 and A were 7 or whatever, A would now become 13. The value of B remains 13 with this operation.

A=17   A, whatever its previous value, becomes 17.

Partial Transfer

It is possible with this type to transfer only some digits, (and not all for which operation type 5 above exists), from one location into another location. For example, if we want to transfer the middle two digits, 76 of location B below into A we can do so, anywhere in A.

B											4	3	2	1	0
											4	7	6	9	+

A											4	3	2	1	0
													7	6	+

We would not give the format of this type. We can state it in the flowchart in plain English "Transfer the contents of B (2nd and 3rd digit) into A in its 1st and 2nd cell".

All these operations 1 to 6 above are depicted in the flowchart in a box For example, A=A−14 would be depicted as below :

$$\boxed{A=A-14}$$

It may also be desired to designate a location or a variable by a suggestive symbol. Thus the step below means that we want to increment "COUNT" by 1.

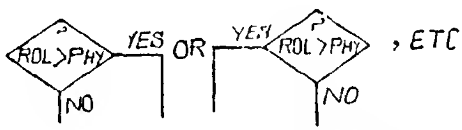
COUNT=COUNT+1	COUNT becomes the previous value of COUNT plus 1.

Desirably the length of symbols should not exceed six characters. Also they must start with an alphabet letter and never with a numeric or a special character. However, the following five characters could be of any kind. These symbols are devised by the person who draws the flowchart. *Alphabets must be in capitals.*

7. Comparison

In it, the value of two variables (i.e. contents of two locations) are compared and one action is taken if the answer to the comparison is "yes" and other action if the answer is 'no'. A comparison is always shown in a diamond as below in which ROL (the symbol for recorder for level) is compared with PHY (the symbol for the physical balance.) If ROL is greater than PHY we would place the replenishment order, otherwise not.

OR



The following types of comparisons are possible in most instruction repertoirs.

Variables on R.H.S.	*Constant on R.H.S.*	
A>B	A>13	Instead of constants
A<B	A<13	alphabetic character (s)
A=B	A=13	or special symbol (s)
A≠B	A≠13	can be had on R.H.S.

8. Print

The following types of print operation formats are available.

(i) Print (Material) at position. ... (literally)

e.g., Print "RAMU, 28" at 005.

We want to print RAMU, 28 which constitutes the material. The continuous stationery usually can accommodate 160 characters. Thus, there are 160 print positions from 001 to 160. In the example above, we want to start printing at position 005.

(ii) Print (Location or Variable) at position .. .. e.g. Print A at 010, with which we want to print the value of the variable (or the contents of location) A starting position 010.

9. Feed

This means raising the continuous stationery by 1 or more lines for printing the next line. The format is as in the examples below.

1 line C S. feed

3 line S.S. feed

We shall write other input/output instructions (viz. read a punched card) in plain English with some exceptions to be explained where the use is made.

EXAMPLES ON FLOW CHARTING

Numerous examples on flowcharting follow. The student should, however, familiarise himself with example on $\frac{27|7}{96}$ at this stage before proceeding with the following material.

Example 1. Draw the program flowchart for finding the sum of first 100 odd numbers.

Solution The flowchart is drawn as figure 9 and is explained step by step below. The step numbers are shown in the flowchart in circles and as such are not a part of the flowchart but only a referencing device.

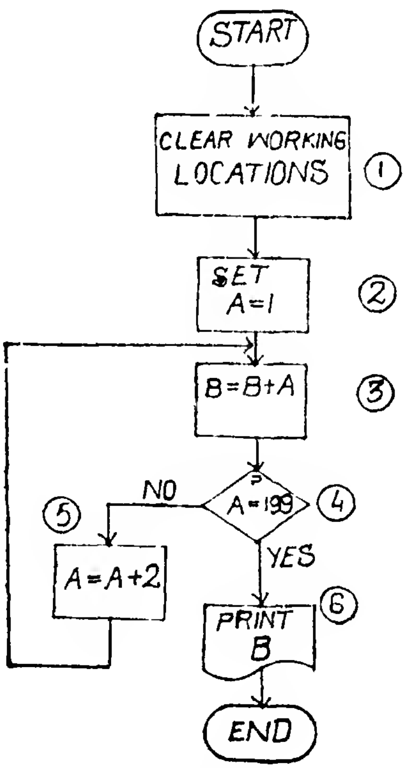


Fig. 9

Our purpose is to find the sum of the series 1,3,5,7,9........ (100 terms.) The student can verify that the 100th term would be 199. We propose to set A=1 and then go on incrementing it by 2 so that it holds the various terms of the series in turn. B is an accumulator in the sense that A is added to B whenever A is incremented. Thus B will hold

1

1+3=4

4+5=9

9+7=16, etc in turn.

Step 1. All working locations are set at zero. This is necessary because, if they are holding some data of the previous program that data is liable to corrupt the result of the flowchart.

Step 2. A is set at 1 so that subsequently by incrementing it successively by 2 we get the wanted odd terms · 1, 3, 5, 7, etc

Step 3 A is poured into B *i.e.* added to B B being 0 at the moment and A being 1, B becomes 0+1=1.

Step 4. In step 5 we shall increment A by 2 So that although at the moment A is 1, it will be made 3 in step 5, and so on Since we have to stop at the 100th terms which is 199 step 4 poses a question "Has A become 199?" if not go back to step 3 by forming a loop Thus A is repeatdly incremented in step 5 and added to B in step 3. In other words, B holds the cumulative sum upto the latest terms held in A.

When A has become 199 that means the necessary computations have been carried out so that in step 6 the result is printed.

Perhaps more suggestive symbols for A and B could be ODD and SUM respectively.

Example 2. Draw the flowchart for finding the value of $\underline{|K}$ where K represents an integre greater than one whose value will be read into the computer each time the program is run.

(In Example 8, we shall use this as a subroutine with K as the parameter)

The flowchart is drawn as figure 10 on page 22. It may be recalled that we drew a flowchart for computing factorial of 7 but here we intend to generalise for any value designated by K. Thus the number for which the factorial is needed is read into the CPU (via, say, a punched card) and this number is held in a location which is also designated as K. Thus K may be given any integral value, *viz* 7, 17, 20 etc. This is done in step 1.

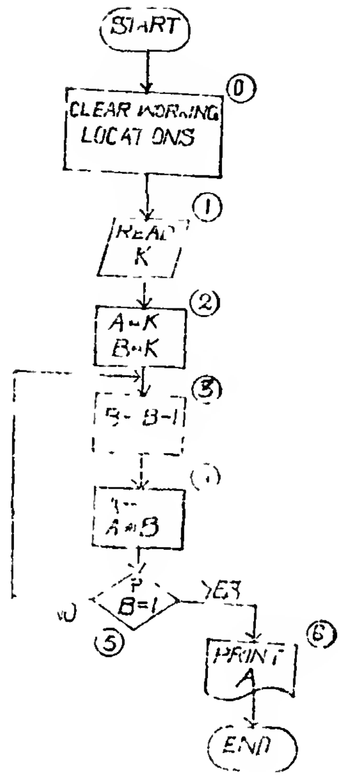


Fig. 10

Step 2. A and B are both equated to K. In the following steps we shall repeatedly decrement B by 1 and go on multiplying it with A successively so that A holds K (K—1), K (K—1) (K—2) etc. in turn and B becomes K, (K—1), (K—2), etc. in turn.

Step 3. As already stated above B is brought down K to K—1.

Step 4. A becomes the product of A and B *i e.* K (K—1).

Step 5. is a comparison step for looping. Obviously the factorial would have been computed when B, after having been successively decremented by 1, becomes 1. But since at the moment B has come down to K—1 and not 1 by looping we go back to step 2 by which B becomes (K—2) and A, in step 3, becomes K (K—1) (K—2); so on until A holds |K, which is printed in step 6.

Example 3. Draw the flowchart for finding the value of K^N where K and N are read into the computer each time the program is run, N has to be a +ve integer.

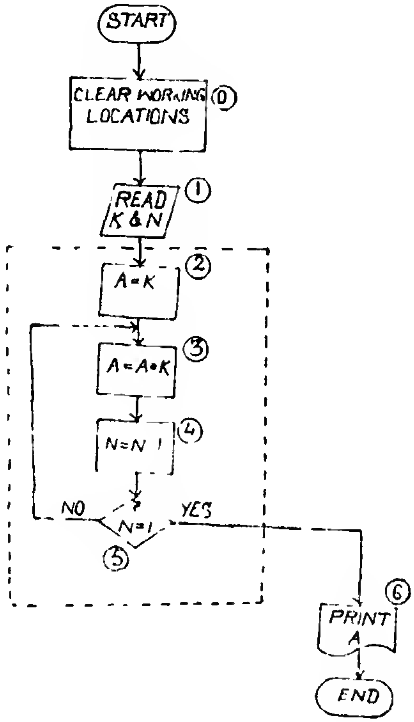


Fig. 11

Figure 11. (Please ignore dotted lines. They are referenced in a subsequent example).

Step 0. We zeroise all working locations.

Step 1. Values of K and N are read, say, via punched card.

Step 2. A is equated to K. We shall subsequently go on multiplying A by K successively via a loop so that A is made A^2, A^3, etc. in turn.

Step 3. A becomes AK *i e.* A^2 since K is equal to A.

Step 4. We have to carry out the multiplication of A by K, (N−1) times to get the value K^N. In this step, therfore, we decrease N by 1. Via the loop we shall continue to decrement it by 1 until it is brought down to 1.

Step 5. This is a comparison step where it is decided whether to continue with the loop or not When N comes down to 1 (in Step 4). A, which becomes K^N, is printed in step 6.

Example 4 Draw the flowchart which will calculate the sum of the first N multiples of an integer K. (For example, if $K=3$ and $N=10$), then calculate the sum of $(1\times3+2\times3+ \quad +10\times3)$. Make the flowchart completely general by reading an integer value for N and K each time this program is run.

(*See Figure 12*)

Step 0. The working locations A, B and C are cleared *i e.* zeroised to erase data sticking in from the previous program, if any.

Step 1. Parameters of the problem, K and N are read in via, say, a punched card.

Step 2. It is the intention to hold 1, 2, 3 etc , in turn, in A, therefore, A is incremented by 1

Step 3 In B is held the first term of the given series which is multiplied by K *i e.* $1\times K$ to start with.

Step 4. It is the intention to add the terms of the given series one by one in C; therefore, the first term, to start with, is accumulated in C.

Step 5 When A becomes equal to N we would print K, N, C as per step 6, but at the moment we form a loop back to step 2 so that A is made $1+1=2$ to prepare the 2nd term in the following steps.

Example 5. There are three quahtities; Q_1, Q_2 and Q_3. It is desired to obtain the highest of these in location H and lowest of these in location L.

(*See Figure 13*)

Step 1. The three quantities Q_1 Q_2 and Q_3 are read in via, say, a punched card.

Step 2 Any two quantities, say, Q_1 and Q_2 are compared. If Q_1 is greater than Q_2 we tentatively make $H=Q_1$ and $L=Q_2$ in Steps 3B and 4B : otherwise, in Steps 3A and 4A we make $H=Q_2$ and $L=Q_1$.

At Step 5 we are holding the higher of Q_1 and Q_2 in H and the lower of these in L. In Step 5 we see if Q_3 is greater than the higher of Q_1 and Q_2 in H. If it is so obviously in Step 8 H is made Q_3. If Q_3 is not greater than H we compare Q_3 with L in Step 6.

In Step 6, if $Q_3 < L$ we go to Step 7 and make $L=Q_3$, otherwise, the job has already been done prior to Step 5.

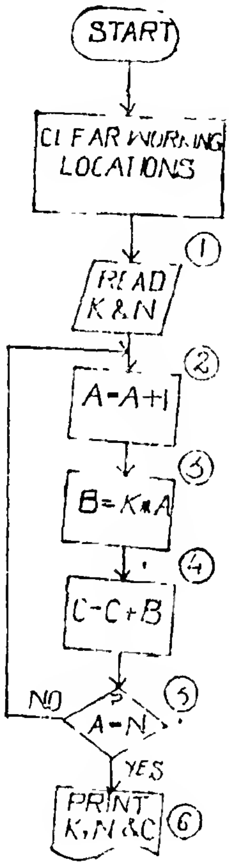


Fig. 12

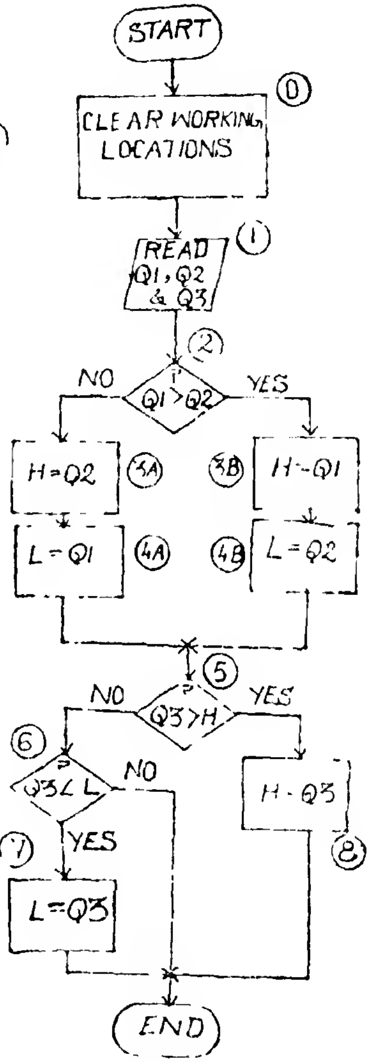


Fig. 13

Example 6. The square root of number can be computed by an iterativ procedure. The following computational steps are performed.

1. Select a first guess for the desired square root A reasonable value fc the first guess might be obtained by dividing the given number by 2.

2. Divide the given number by the assumed square root.

3. If the quotient and divisor are sufficiently close, then the desired square root has been obtained to a sufficient degree of accuracy and the computation ceases.

4 If the quotient and the divisor do not agree, then a new guess must be obtained for the square root and the procedure repeated. The new guess is obtained by calculating the arithmetic average of the most recent divisor and quotient. The computation then returns to step 2.

Say, N=the given number whose square root is desired.

D=the divisor.

Q=the quotient.

R=the desired square root.

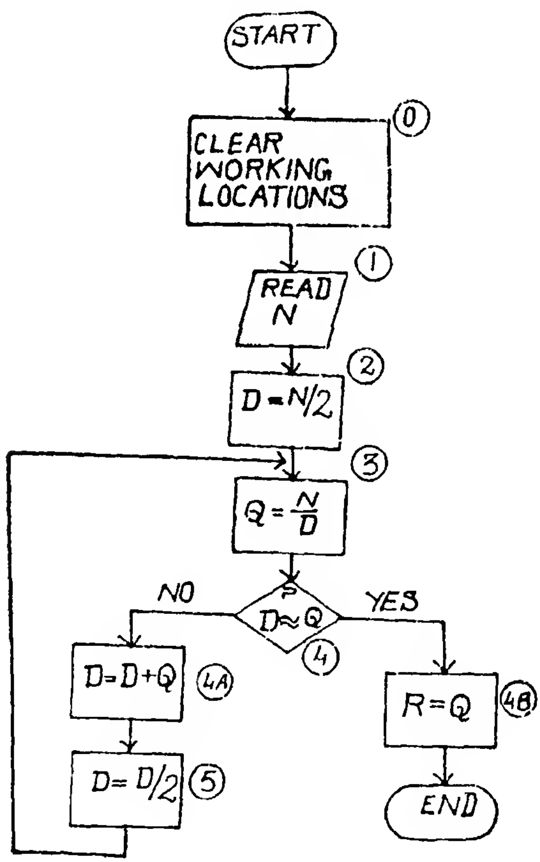


Fig. 14

[Let us now apply this method to a problem namely, computing the square root of 8. The computation will proceed in the following steps.

(a) $D_1=\frac{1}{2}(8)=4$ $\quad Q_1=8\div 4=2$

(b) $D_2=\frac{1}{2}(4+2)=3$ $\quad Q_2=8-3=2\ 6666$

(c) $D_3=\frac{1}{2}(3+2.6666)=2.83333$ $\quad Q_3=8\div 2\ 83333=2.82353$

(d) $D_4=\frac{1}{2}(2\ 83333+2.82352)$ $=2.82843$ $\quad Q_4=8\div 2.82843=2.82842$]

The (rather condensed) flowchart is given above in fig. 14.

Step 1. N, the number for which the square root is wanted.

Step 2. D is made half of N as the initial estimate of square root of N(=8 may be imagined).

Step 3. N(=8) is divided by the estimate D to get Q.

Step 4. If D is approximately equal Q we have computed the squre root which, however, is not the case as yet (since $D=\frac{N}{2}=4$ and $Q=\frac{N}{2}=\frac{8}{4}=2$ and $D\neq Q$); therefore, go to steps 5 and 6. In these to steps we find the average of D and Q and put it in D. This average $\left[=\frac{2+4}{2}=3\right]$ is taken in D as the new estimate of the square root and we loop back to step 3.

*In fact, there does not exist any instruction in any computer by which we can compare if two quantities are roughly equal. What therefore, would actually be done is to find the difference between D and Q and if it is $\leqslant$ prescribed difference, say 0.001 we accept them as equal.

Example 7. Draw the flowchart for deriving the sum of the squares of first 20 odd numbers.

The flowchart is shown in fig 15.

Step. 0. All working locations are zeroised.

Step. 1. In step 2 we employ the square subroutine (which is the set of steps enclosed in the dotted box in figure 11 for computing K), therefore, we set N=2 for ever in this program and K=1.

Step. 2. K^n i e. 1^2 is computed by the aforesaid subroutine (S.R.). A.S.R. is being depicted in the hexagonal symbol in program flowcharting.

Step 3. We accumulate the first term i.e. square of the first odd number, 1 in location C.

Step 4. The 20th odd number is 39, therefore, in this step we see if K has become 39 from 1 (by step 5).

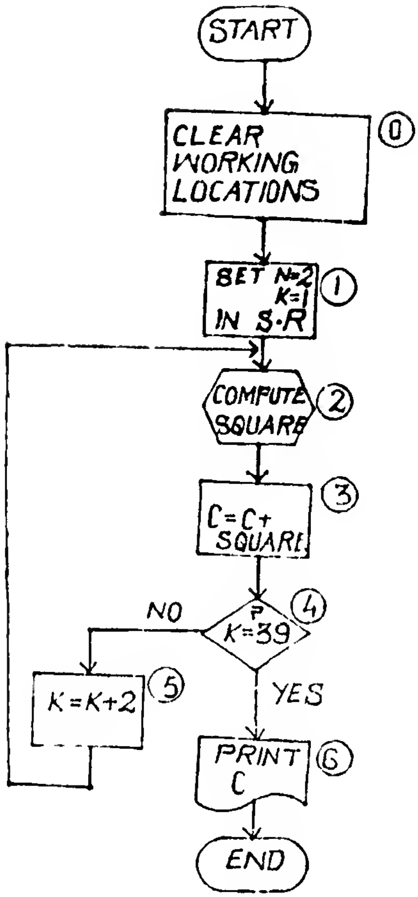


Fig. 15

Step 5. K is incremented by 2 i.e. it becomes 1+2=3.

Note: The problem can be solved without using sub-soutine also as depicted in Fig 15-A on Page 29.

Example 8 The sine of X can be approximately calculated by summing up the first 100 terms of the infinite series

$$\text{Sin } X = X - \frac{X^3}{\underline{|3}} + \frac{X^5}{\underline{|5}} - \frac{X^7}{\underline{|7}} + \ldots\ldots$$

Draw the program flowchart in which the value of X in radians will be read in and then the sine is computed.

(*See Figure 16*)

Step 0. Clear all working locations.

Step 1. The value of X in radians is read in via, say, a punched card.

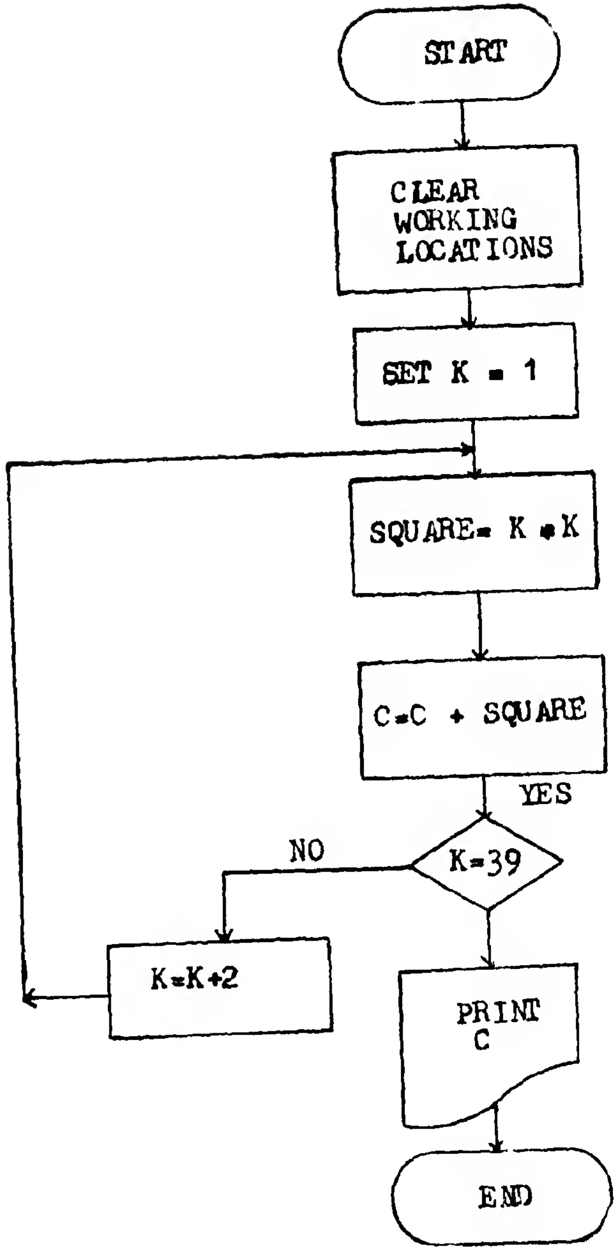

START
CLEAR
WORKING
LOCATIONS
SET K = 1
SQUARE= K * K
C=C + SQUARE
YES
NO
K=39
K=K+2
PRINT
C
END

Fig. 15—A

Step 2 In step 4, we shall use the power S. R. for computing K^N In this step 2, we set A=1 and in step 3, we set K=X and N=A (i e.=1).

Step 4. K^N (=K^A) is computed by the S. R. (A having been at present set at 1 in step 2)

Step 5. In step 6 we compute the factorial of a A (which is 1 at the moment). Since in fig 10 we used the sympol K we set K=A in the S. R. of step 5. A being 1 at the moment

Step 6. $\underline{|A}$ i e. $\underline{|1}$ is computed. It constitutes the denominator of the first term, $\frac{X^1}{\underline{|1}}$,

Step 7 We divide X^A (i.e. X^1) by $\underline{|A}$ i.e. $\underline{|1}$ and put in location term TERM

Step 8. We count the number of terms in location COUNT which becomes 1 by incrementing it by 1.

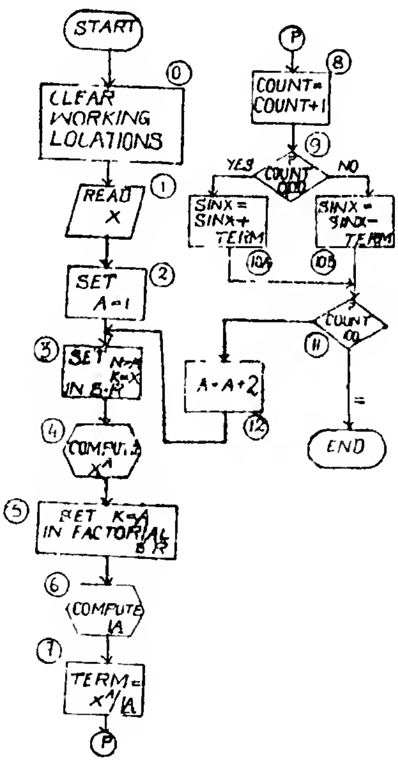


Fig. 16

Step 9. We see if COUNT is odd or even. In the given series (on R.H.S.) it is to be seen that odd terms are preceded by plus sign and even terms by minus sign.

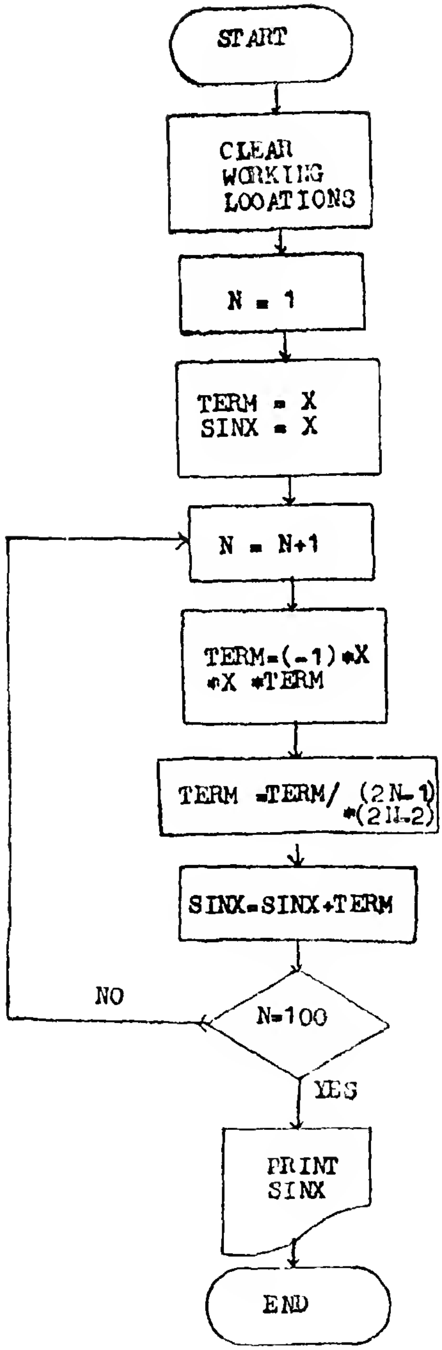

START
CLEAR WORKING LOCATIONS
N = 1
TERM = X
SINX = X
N = N+1
TERM=(-1)*X
*X *TERM
TERM =TERM/ (2N-1)
*(2N-2)
SINX=SINX+TERM
NO
N=100
YES
PRINT SINX
END

Fig. 16—A

Step 10 A and B. If count is odd we accumulate it in location SINX otherwise we subtract it from SINX. Thus SINX holds the sum of R.H.S. upto st, 2nd, 3rd . , etc term in turn

Step 11 It is ascertained if the 100 terms have been processed. If so we end, otherwise, in step 12 we increment *A* by 2 (e.g. it becomes 3 from 1) and loop back to step 3 process for the second term, $\frac{X^3}{\lfloor 3}$.

Note: Alternatively, the problem can be solved as shown in Fig 16 A

Example 9 Draw the program flowchart for computing the annual acquisition, inventory carrying and total costs for lot sizes of 100, 200 ..2400. The various variables of interest are supposed to be there in the locations symbolized below.

REQ	Annual requirements of the item
ACQ	Procurement cost/order
COST	Cost per unit
RATE	Inventory-carrying rate, I.

The flowchart is drawn in fig 17. The following symbols represent the working locations that are put to use by this flowchart

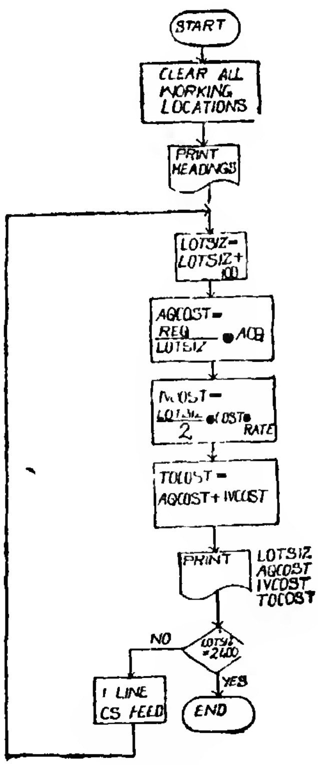


Fig. 17

LOTSIZ	Lot size
IVCOST	Annual inventory-carrying cost
AQCOST	Annual acquisition cost
TOCOST	Annual total cost

Example 10. Draw the flowchart for finding the amount of an annuity of Rs A in N years. Rate of interest = $r\%$. $R=(1+r)$. This amount is given by the following series : $A+AR+AR^2+\ldots AR^{N-1}$.

The flowchart is drawn in figure 18. The following symbols are employed.

TERM	To hold A, AR^2, etc. (i e. the various terms) in turn.
SUM	In it is accumulated the sum of term.
COUNT	Counter to count the number of terms accumulated.

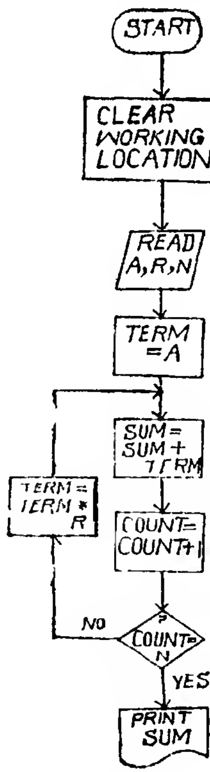


Fig. 18

Example 11. (On Computing Customs Duties): Assume that imported goods from foreign countries are classified into 4 categories for the purpose of levying customs duty. The duty rate for each category is as follows :

Class No.	*Class of Goods*	*Customs duty (%) K, on Values of Goods V*
1	Foods, beverages	10
2	Clothing, footware	15
3	Heavy machinery	17½
4	Luxury items	40

Draw the flowchart for computing the appropriate customs duty.

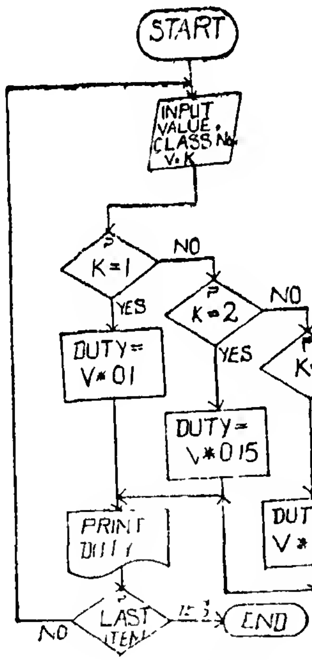


Fig. 19

Example 12. The problem is to compute, for a series of transactions, the gross sales (G) : the quantity discounts, (D), if any ; and the net sales (N). The raw data to be supplied in the program includes the quantity sold (Q) and unit price (P) The quantity discount schedule is as follows :

If quantity sold is :	*The discount rate would be ;*
less than 100 units	none
100 to less than 200	10%
200 and over	20%

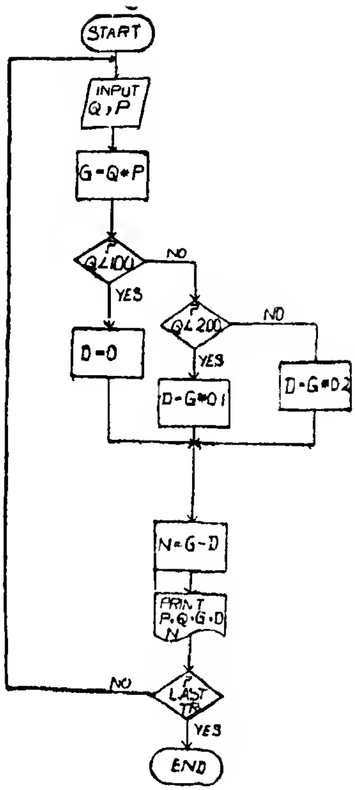


Fig. 20

Example 13. Given the following set of data :

Account No.	*Age of customer*	*Sex*	*Unpaid Balance*
13466	28	M	Rs. 145.23
43156	20	F	49.50
33215	45	F	89.24
44178	19	M	115 23
56723	28	F	75 95
47892	33	F	25.78
24567	19	M	54.75
56783	55	M	24.78
43579	39	F	67.85
56782	30	M	150 97
79234	18	F	39.95
63423	29	F	69.95

Draw the flowchart to compute and print out the following :

Average	*Unpaid Balance*	
Customer Age	Males	Females
Under 20	Rs. ×××.××	Rs. ×××.××
20 to Under 30	×××	×××
30 to Under 40	×××	×××
40 and over	×××	×××

The program flowchart is given in Fig 21 M1 to M4 accumulate balances for the 4 age groups of male customers and likewise F1 to F4 for the female customers, Age is symbolised by A, balance by B and Sex is codified as M or F. *The last record is dummy,*

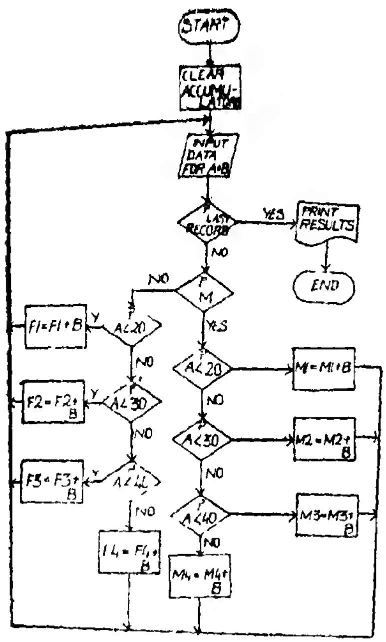


Fig 21

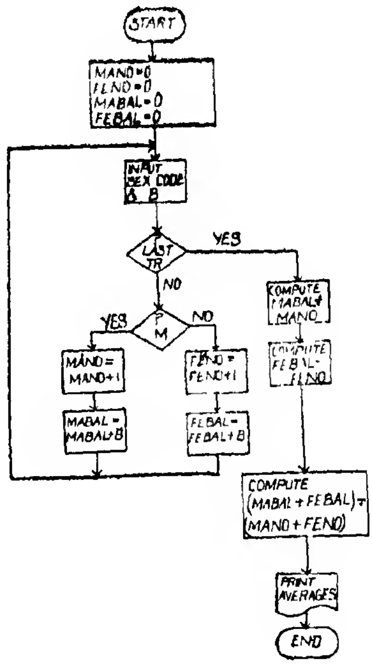


Fig. 22

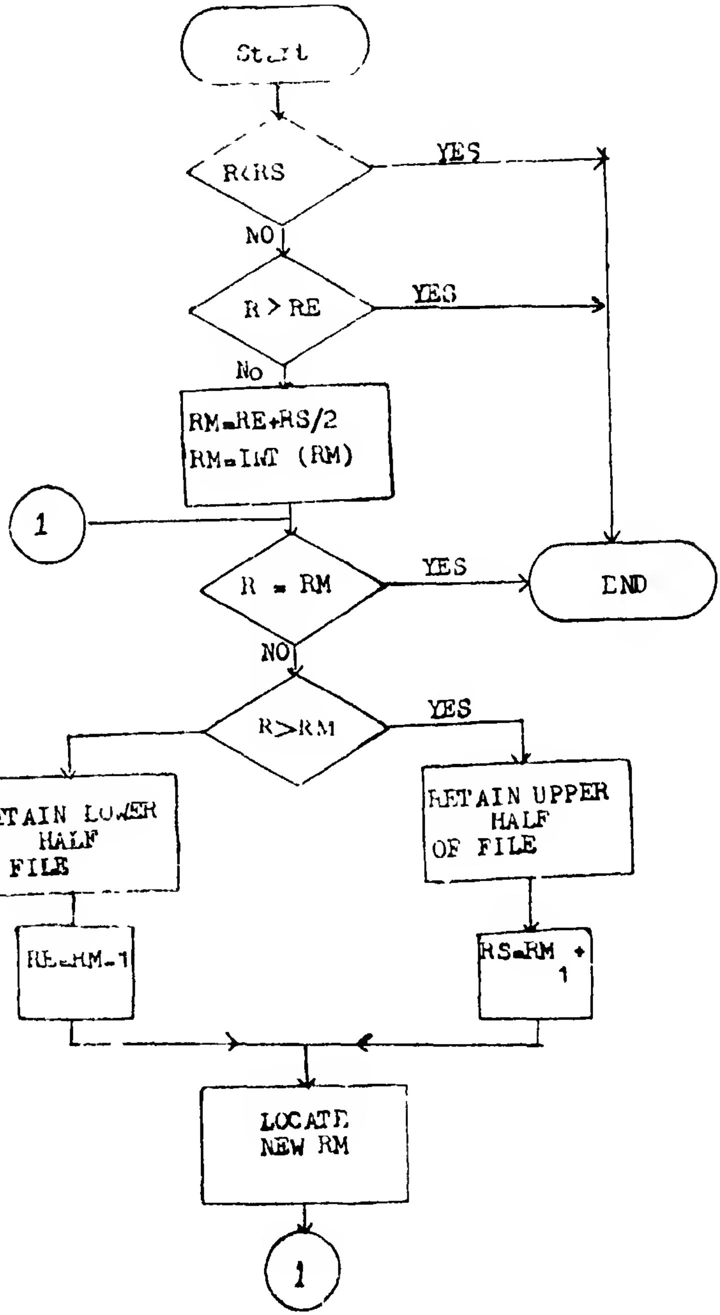
Example 14. Using the data of the previous example, draw a flowchart for computing and printing out the following statistics :—

Sex	*Average United Balance*
Male	Rs. ×××.××
Female	×××.××
Overall	×××.××

The program flowchart is shown in Fig 22 above. The following Symbols are used.

MANO	Counter for males
FENO	Counter for females
MABAL	Sum for male balances
FEBAL	Sum for female balances

Example 15. Flowchart for Binary Search, (Re. : Magnetic Disc. Study I)



R = Desired Record
RE = Record at the end of file
RM = Record in middle of file
RS = Record at start of file

Fig. 23

Modification/Initialisation Instructions

These instructions can change the value of a *variable location number* in an existing instruction during the program execution process. The initialisation instruction can set or reset the value of this variable to any desired number. The modification instruction can increment/decrement this variable during the loop execution by any constant (viz. 1,2,13).

Example 16 Marks on each student (in a class) in 12 papers are encoded on a punched card which are read (one card by one) into the CPU locations MARKS 001 to MARKS 012 You are required to draw the flowchart for computing and printing the average marks of each student.

Solution MARKS 001, MARKS 002. MARKS 012, are holding the marks in 12 papers of a student We propose to accumulate them as ACCUM. This could be accomplished in 12 instructions as below.

ACCUM=ACCUM+MARKS 001
ACCUM=ACCUM+MARKS 002, etc. to—,
ACCUM=ACCUM+MARKS 012.

But we do not do this way. We shall adopt a cleverer approach which is made possible by the facility of what is known as the "modifying" instructions in the instruction repertoire.

It is to be seen that the 12 instructions above can be generalised as

ACCUM=ACCUM+MARKS (X) .. (A)

We have to start with X=1 and then go on incrementing it by one to generate the above 12 instructions. This we do as below.

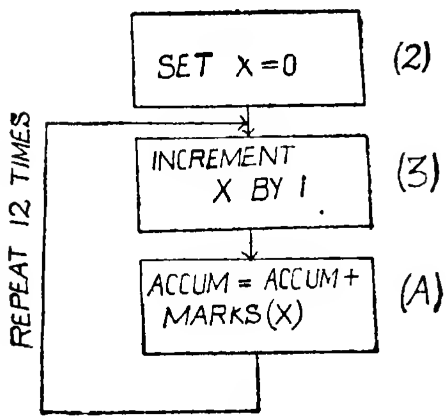


Fig. 24

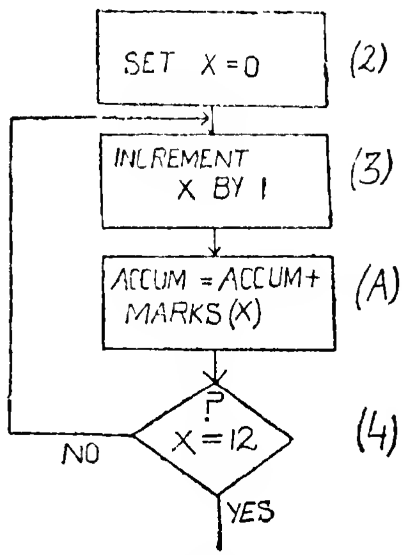


Fig 25

Set X=0 in (A) ...(2

Increment X by 1 in (A) ...(3

ACCUM=ACCUM+MARKS (X) ...(A

By step (2) we made X in step (A)=0 and by step (3) we incremented it b 1 so that MARKS (X) in A has been made MARKS (1) which is the same thing a MARKS 001 If we repeat steps (3) and (A) 12 times as per the left flowchart se ment above, we, in effect, will have performed the aforesaid 12 instructions.

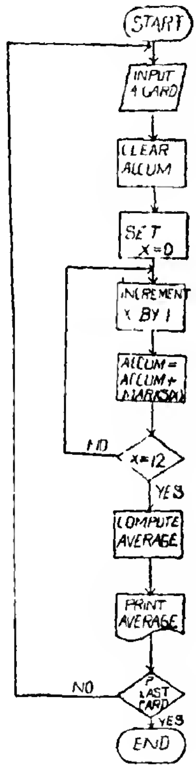


Fig. 26

But, do we repeat this loop 12 times ? This is accomplished by incl the comparison step (4) as per R.H.S. segment above. In thist step, we pos question if X has become 12.

The completed flowchart is shown in Fig 26.

Step (3) in Fig. 25 corresponds to what is known as the mo instruction since it modifies the instruction corresponding to step (A). The st is a sort of initialisation step or instruction since it sets the value of the variab 0 for each student's 12 papers.

Modification of the 'Comparison' Step.

Example 17. Prices for ten commodities in the current year are designated by J(X), X varying from 1 to 10. Likewise their last year's prices are designated by K (Y) Y varying from 1 to 10. Draw the flowchart for finding the number, N of commodities of which prices have increased,

The flowchart is drawn in figure 27.

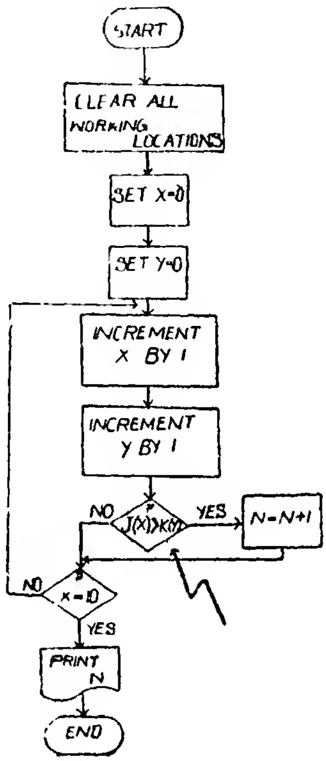


Fig. 27

The *crooked arrow shows the comparison step that is initialised and modified for looping. The following is the comprehensive list of comparisons of this type that are valid.

$$J\,(X) > K\,(Y)$$
$$J\,(X) = K\,(Y)$$
$$J\,(X) \neq K\,(Y)$$
$$J\,(X) < K\,(Y)$$

Example 18. Prices of a commodity in ten major cities are designated by J (X), X varying from 1 to 10. The price prevailing in the capital is designated by C. Find the number of cities having the price less than that in the capital.

*Throughout this study note, we are using crooked arrow to emphasise a step of major interest. As such, it is not a part of the flowchart.

The flowchart is drawn in figure 28 with the crooked arrow showing the comparison step that is initialised and modified for looping. The following is the comprehensive list of this type of comparisons possible in flowcharting

$$J\ (X) < C$$
$$J\ (X) = C$$
$$J\ (X) > C$$
$$J\ (X) \neq C$$

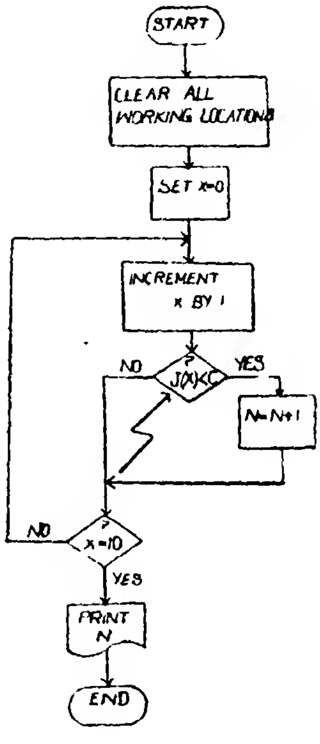


Fig. 28

Example 19. There are 1000 students in for an examination. The roll number of a student, his name and marks obtained by him in the 10 papers are encoded in a punched card. Cards are input into the CPU one by one for finding for each students the number of papers, N in which he scored distinction by obtaining 75 out of 100 or more marks. Name is held by NAME, Roll number by ROLLNO and marks by MARKS (X), X=1,2,3,.... .10.

The flowchart is shown in figure 29. The crooked arrow shows the comparison step of major interest. This comparison involves a constant 75. The following is the comprehensive list of this type of comparisons valid in flowcharting.

J (X)<75 (On R.H.S. alphabetics of special symbols
J (X)=75 can also be had).
J (X)≠75
J (X)>75

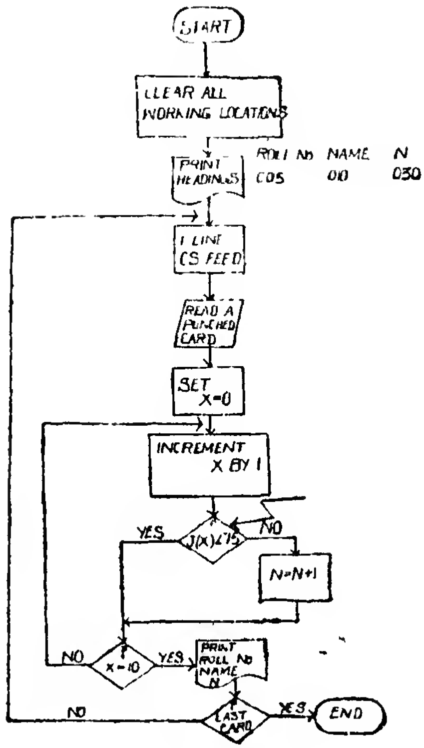


Fig. 29

Modification of the "Transfer Step".

Example 20. Assign a value of 37 to each of the array, J (X), X=1,2...10.

The crooked arrow in the flowchart of fig. 30 shows the "transfer" step of interest wherin 37 is put in each of the 10 locations designated in general by J (X).

Example 21. Transfer the contents of locations J (X), X=2, 4, 6, 8, 10...... 20 to K (Y), Y=1, 2, 3......10.

The flowchart is shown in figure 31. The crooked arrow shows the step of major interest wherein K (Y)'s are successively equated to J (X)'s.

Example 22. Transfer the contents of location J (0) to each of the following 10 locations.

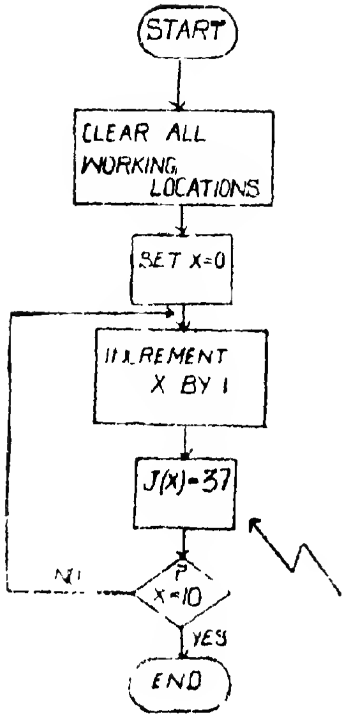


Fig. 30

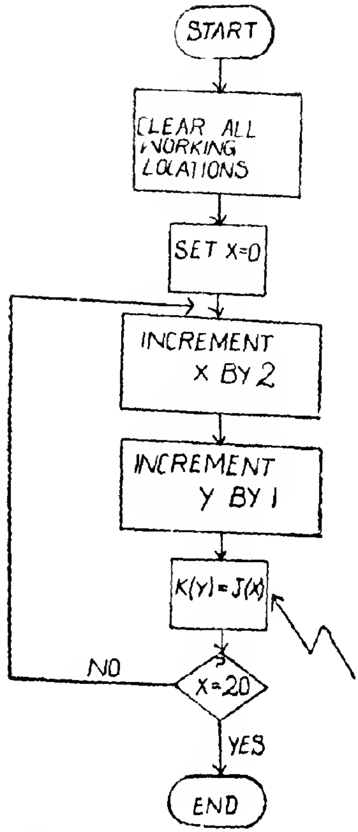


Fig. 31

The flowchart is shown in figure 32 for transferring the content of J (0) tc each of J (1), J (2), J (3)......J (10). The crooked arrow shows the step of majoı interest.

Modification of Arithmetic Steps

Example 23 It is desired to add contents of 10 locations J (X), X=4, ; 10......31 and K (Y), Y=1, 2, 3......10 on a one to one basis and put the results i R (Z), Z=2, 4, 6.. .. 20.

The flowchart is shown in figure 33. The step of major interest has a croo ed arrow to it. The following is the comprehensive list of such types of steps.

R (Z) = J (X) + K (Y)	Such types are
R (Z) = J (X) − K (Y)	also valid;
R (Z) = J (X) * K (Y)	R (Z) = R (Z) + J (X)
R (Z) = J (X) / K (Y)	R (Z) = R (Z) / J (X), etc.

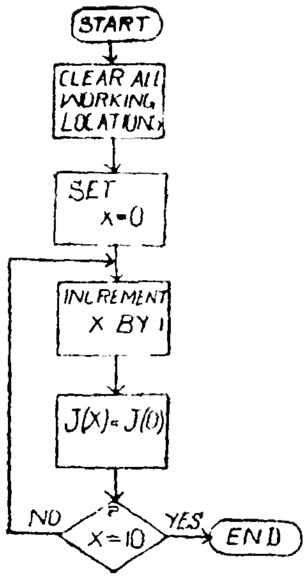


Fig. 32

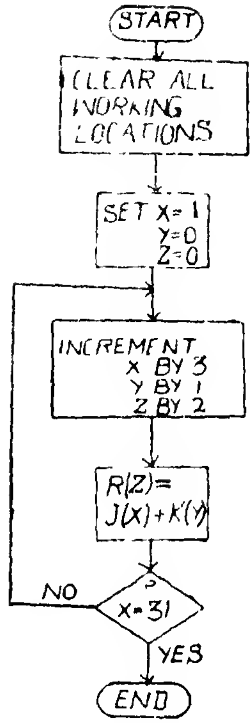


Fig. 33

More Examples on Modification of Arithmetic operations.

Example 24. Multiply R (Z), Z=2,4,6......20 and K (Y), Y = 3, 6, 9...30 on a one to one basis and put the results in J (X), X = 19, 18, 17......10.

The flowchart is shown in figure 34. It is the intention here to bring out the fact that *decrementing* of L.H S J (X) (in general even one or both designations R.H.S), is also valid by one (in particular and in general any integer).

Exampale 25. Add Rs. 45 (a constant) to the wages of 10 persons designated by J (X), X = 1, 2..... 10.

The flowchart is drawn in figure 35. The crooked arrow shows the step for adding a constant to the contents of a location. Other permissible steps of this type are as below :

J (K) = J (X) − 45
J (X) = J (X) * 45
J (X) = J (X) / 45
K (Y) = J (X) + 45, etc.

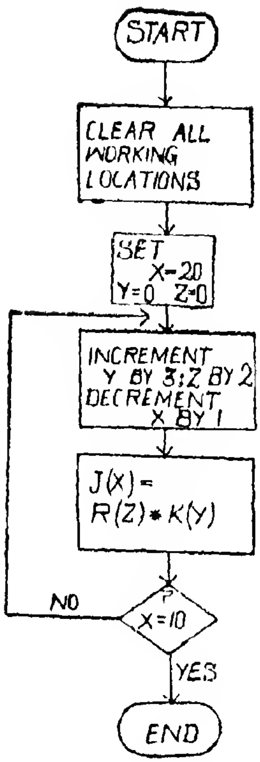


Fig 34

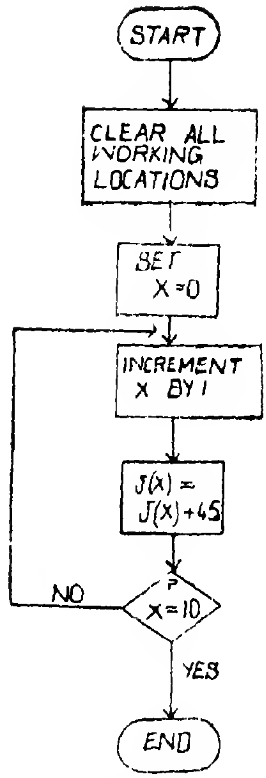


Fig 35

Example 26. Point 6 'P's in the pattern below.

012 (print position)
P
P
P
P
P
P
007 (print position)

The flowchart is drawn in figure 36 The print instruction has the following two formats. Printing is done in one line by an instruction.

1 Print (Given material as 'P' here) starting at print position (001 to 160) We want to print 'P' at starting print position 012; therefore, we give the instruction "Print 'P' at Y". Y is the print position on the continuous stationery which usually can accommodate 160 characters in one line. We want to print 'P' at the Y th position. In the given pattern 'P' in the first line is to be printed at position 12

therefore, at 13 and then decremented by 1. This is followed by "1 line CS fee which means programming to raise the continuous stationery by one line so that is set for printing the 2nd 'P' in the 2nd line. By means of the loop Y is decrement from 012 one by one so that 'P's are printed at positions 012, 011, 010, 009, 008, C in successive lines,

2. Print (Contents of a location) at starting print position (001 to 160).

This type is illustrated in the following example.

Example 27. 64 locations J (X), X = 1,2..... 64 hold 64 3-digit quantiti It is required to draw the flowchart for printing these in an 8×8 matrix as below :

412 331 602 400 405 403 408 421
424 425 423 422 421 420 419 426
⋮
531 310 410 212 111 402 124 429

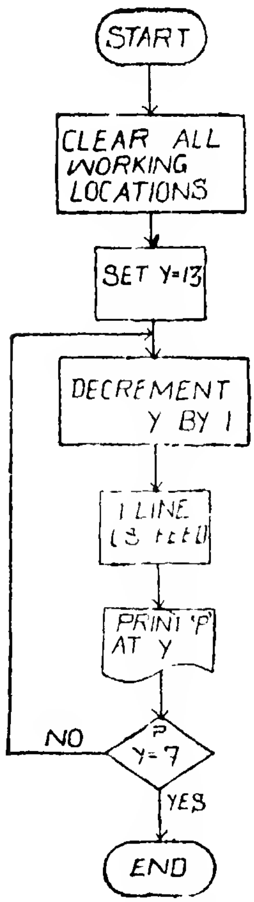


Fig. 36

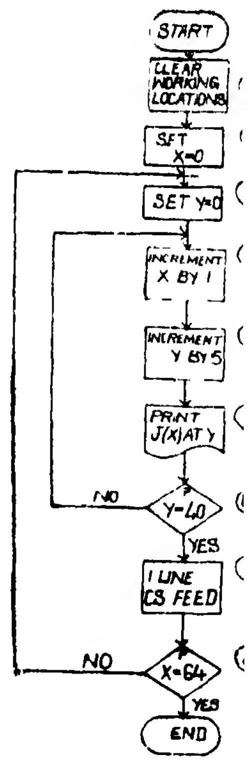


Fig. 37

The flowchart is drawn in figure 37. The first row figures in the above matrix are the contents of J (1) to J (8). The second row figures in the above matrix are the contents of J (9) to J (16). Moving this way the figures of the last row are the contents of J (57) to J (64).

The first column is printed at start position Y = 005, the 2nd column at Y = 010, and so on, so that the 8th column has the print position, Y = 040. Thus in the flowchart when Y becomes 040 it is a signal that the printing of a line is over; therefore, the continuous stationery is raised by a line, Y reset at 005 for commencing the printing of the next line.

We are giving an increment of 5 to Y which is the minimum necessary. It could be more but it should not be less because the three digits and the sign (for, say debt / credit as + or −) would require 4 print positions and the fifth position would be left blank as a gap between two neighbouring quantities.

Example 28. Q (X), holds 9 quantities Q1, Q2, ...Q9. Obtain the highest quantity in location H and the lowest quantity in location L.

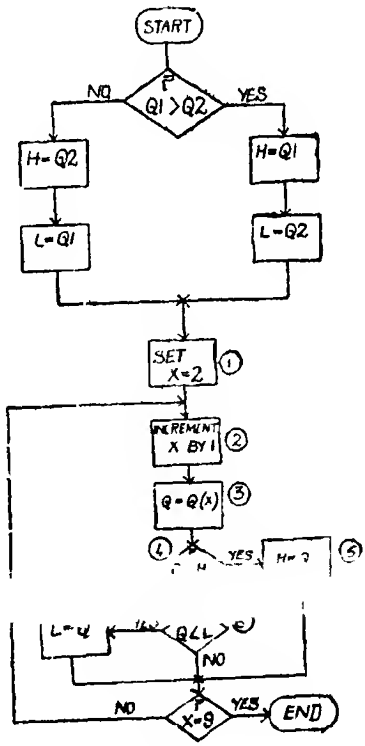


Fig 38

The flowchart is shown figure 38. In addition to the given symbols anoth symbol Q [which is the same as Q (0)] is used to hold Q 3, 24, .. Q 9 in turn.

We start in the manner of example 5 on page 24. Having put that way th higher of Q1 and Q2 in H and the lower in L tentatively, from step 1 onwards, w want to compare Q3, Q4 ... Q9 in turn with H and L. In fact, steps 4 to 7 ai similar to the later part of the flowchart of example 5 on page 24.

In steps 1 and 2 we prepare X = 3 for step 3 which now reads "Q = Q (3)' What we have done is that we have put the contents of Q(3) in Q and in steps 4 t 7 we work on Q instead of Q (X). Why are we reluctant to work with Q (X) straigl in steps 4 to 7 ? well we could do so and we would get the wanted results. But b working with Q (X) directly in step 4 to 7 these steps would read as below on th L.H S. 'rather than on the R.H S. as in the flowchart.

Step 4	Q (X)>H		Q>H
Step 5	H = Q(X)	rather	H=Q
Step 6	Q (X)<L	than	Q<L in the flowchai
Step 7	L = Q (X)		L = Q

So if we work with Q(X) straight in steps 4 to 7 we shall have to set X = in each of these steps. But by having equated Q with Q(X) we can work with (and we do not have any problem of setting X in steps 4 to 7.

Example 29 In locations J (X), X = 1, 2..... 200 are held 200 quantitie: Draw flowchart for finding the ratio of the total number of quantities indivisible b 10 to that of divisible by 10.

The flowchart is shown in figure 39. The following symbols are used in i

NONTEN Total number of items not divisible by 10

NO

TENNER Total number of items divisible by 10

RATIO Ratio NONTEN/TENNER.

J (X), X = 1, 2.........200

This used to hold the last digit of a quantity.

Partial transfer (as we have done in this flowchart) of one or more consecutiv digits from one location into another location is valid.

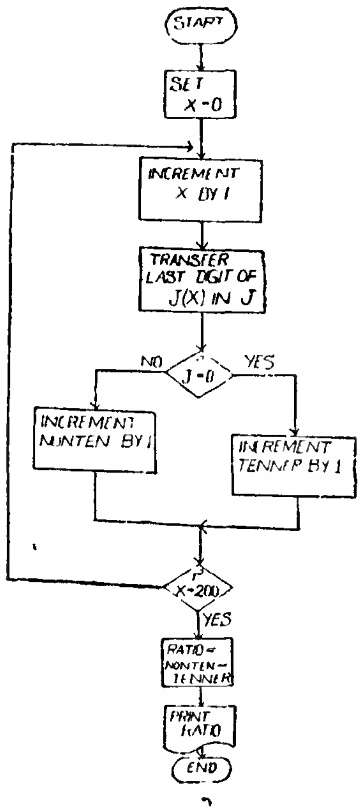


Fig. 39

Example 30. It is required to compute the geometric mean of six past prices for each commodity in an inventory of 50 commodities, the 6 prices having been encoded in a punched card which are read into the CPU in locations designated by VALUE 001 to Value 006. Draw the flowchart. Use would be made of a S R. to compute the 1/6 powers.

Here again in Fig 40, modification instruction is put to use, which is always the case whenever an array or a list of variables are to be processed similarly.

Example 31. It is desired to sort 5 quantities in a list held in the CPU locations symbolised by LIST 001 to LIST 005. Draw the program flowchart.

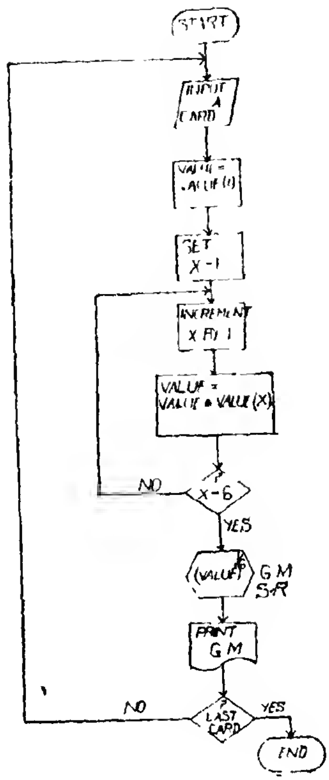


Fig. 40

Switching or Exchange Method of Sorting Within the CPU

The LIST X to be sorted is assumed to be as follows :

LIST X (1)
LIST X (2)
LIST X (3)
LIST X (4)
LIST X (5)

The logic of method depicted in the flowchart in fig. 41 may be summarised as follows —

1. Check the first pair of values in the list X-that is, compare X (1) and X (2). If they are in the right order, X (1)$\leqslant$X(2), leave them alone and proceed to check

the next pair of values. If however X (1)>X(2) they are in the wrong order and need to be switched i.e. exchanged before proceeding to a check of next pair, X (2) and X (3).

2. After one pass through comparing each neighbouring pair of the values of X, it is necessary to go through another pass to ensure that each pair of values is now in the right order (that is if no switching occurs during the pass) another pass is still required to ensure that a sorted list is being achieved.

For illustration of how the sorting logic work's the LIST X is assumed to be as follows before sorting begins :

LIST X (1) = 1
LIST X (2) = 5
LIST X (3) = −2
LIST X (4) = 7
LIST X (5) = 4

The values of the elements of X during the first pass through the list are summarised below :

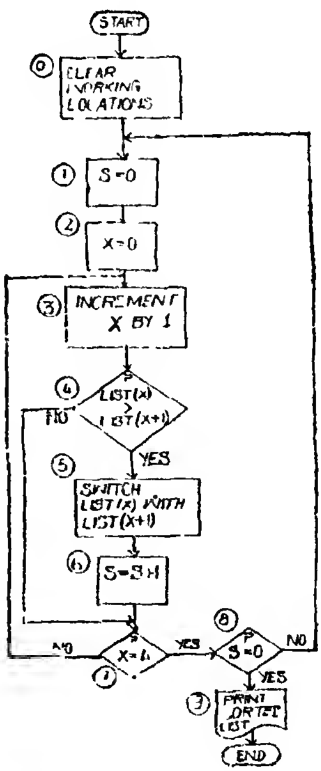
LIST	VALUE OF X DURING 1ST PASS				AFTER I LOOP
	I=1	I=2	I=3	I=4	
X(1) =	1	1	1	1	1
X(2) =	5	5	-2	-2	-2
X(3) =	-2	-2	5	5	5
X(4) =	7	7	7	7	4
X(5) =	4	4	4	4	7
? SWITCHING	NO	YES	NO	YES	
VALUE OF S	0	1	1	2	2

The value of I only goes from 1 to 4, since when I = 4 the last pair of values of X, X (4) and X (5), will be compared.

As demonstrated above, after one pass through the list, the value in LIST X are not all in the right order since two switches occurred during the pass. Hence a second pass through the list is required. The value of the elements of X during the second pass are as follows.

LIST	VALUE OF X DURING 2ND PASS				AFTER I LOOP
	I=1	I=2	I=3	I=4	
X(1) =	1	−2	−2	−2	−2
X(2) =	−2	1	1	1	1
X(3) =	5	5	5	4	4
X(4) =	4	4	4	5	5
X(5)	7	7	7	7	7
SWITCHING ?	YES	NO	YES	NO	
VALUE OF S	1	1	2	2	2

In the third pass no switching would be there i.e. $S = 0$, meaning that the list is sorted.



The flowchart is drawn above and explained below.

Step 1. The switch counter is set equal to zero.

Step 2. This step initialises the loop to follow.

In steps 4 and 5, we are using LIST (X) as the general symbol for the five locations :

LIST (1), LIST (2), LIST (3), LIST (4) and LIST (5) holding the five numbers to be sorted in the ascending order. Naturally, therefore, if LIST (X) is made LIST (0) by setting X = 0 in step 2 under explanation LIST (X+1) would mean LIST (1). Thus having set X = 0 in steps 4 and 5, LIST (X) means LIST (0) and LIST (X+1). means LIST (1). Surely LIST (0) is not one of the five locations holding the five numbers. This is set right in step 3.

Step 3 X is incremented by 1 in the following steps, 4 and 5. This makes LIST (1) of LIST (X) and LIST (2) of LIST (X + 1).

Step 4, LIST (1) is compared with (2).

Since we know LIST (1) = 1 and LIST (2) = 5 are in the right (ascending) order no switching is needed, therefore, steps 5 and 6 are bypassed.

Step 7. Since X=1 and not 4 therefore, the program flowchart loops back to step 4.

Step 3. X is incremented by 1 so that step 4 reads "Is LIST (2)>LIST (3)?" Since we know LIST (2) = 5 and LIST (3) = −2 and as such the answer to the question of step 4 is in affirmation We, therefore, proceed with step 5 and switch the contents of the two locations In step 6, the switch counter is incremented by 1 to count that one switching has taken place.

In this manner, the loop is executed 4 times (X = 4 in step 7) and then we take up step 8 which poses the question, "Is S, switch conuter = 0?" We know S≠0. therefore, the flowchart loops back to step 1 for the 2nd pass.

Note 1 : If it were required to draw the flowchart for sorting those quantities in the *descedıng order* the above flowchart with step 4 modified as below would serve the purpose.

LIST (X)<LIST (X+1)

Ref to example 31 (B)

Note 2·· In the above flowchart, we have condensed the printing step 9. Supposing the list were to be printed in the format below expand step 9 as an exercise.

—2 (Start print position 035
1
4
5
7

Example 31 B. Write a computer programming flow chart to arrange 20 members in descending order.

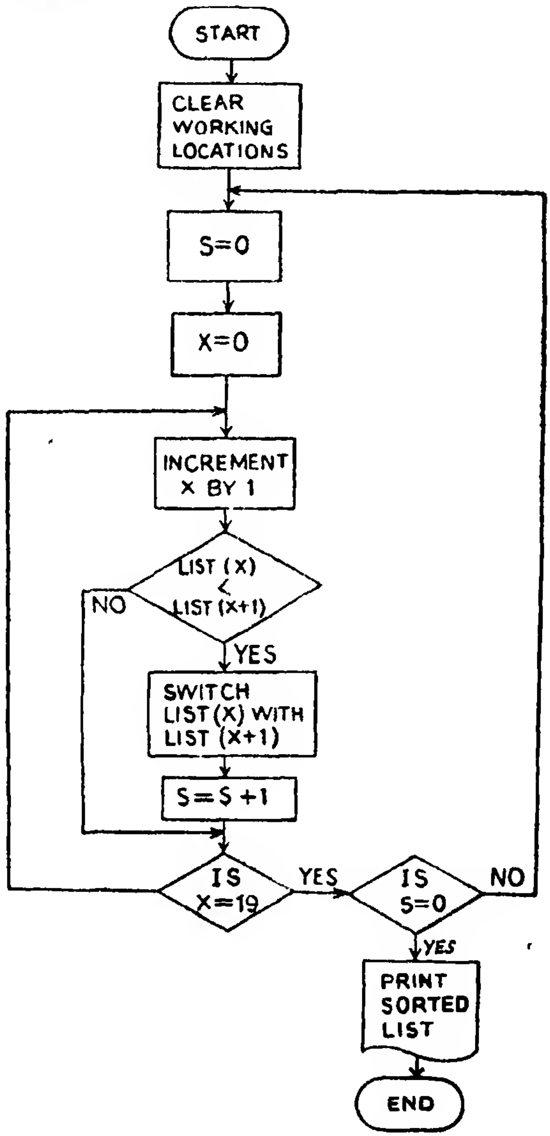


Fig 41-A

Exercises Set 1

1. Draw flowcharts, one each for summing up the following series

$1A+2A^2+3A^3+4A^4+\ldots\ldots$ N terms

$\frac{1}{2.3}+\frac{2}{3.4}+\frac{3}{4.5}+\frac{4}{5.6}+\ldots\ldots$ N terms

$\frac{1}{\underline{|2}}+\frac{2}{\underline{|3}}+\frac{3}{\underline{|4}}+\frac{4}{\underline{|5}}+\ldots\ldots$ N terms

$\frac{1.3}{2.4.5}+\frac{2.4}{3.5.6}+\frac{3.5}{4.6.7}+\ldots\ldots$ N terms

$1A-2A^2+3A^3-4A^4+\ldots\ldots$ N terms

2. Draw a flowchart for finding the 16th root of a number.

3. Draw a flowchart for computing and printing the simple interest for 10, 11, 12, 13 and 14 years at the rate of 3% per annum on an investment of Rs. 10,000.

Miscellaneous Solved Examples

(After having gone through these the student may want to redraw these by closing the study paper)

Example 32. Salaries of 100 persons are designated by J (S), S=1,2,3 .100. Draw flowchart for finding%age of the following salary ranges.

<Rs. 1500 (per month)

1500 to 3000

>3000

The flowchart is drawn in figure 42.

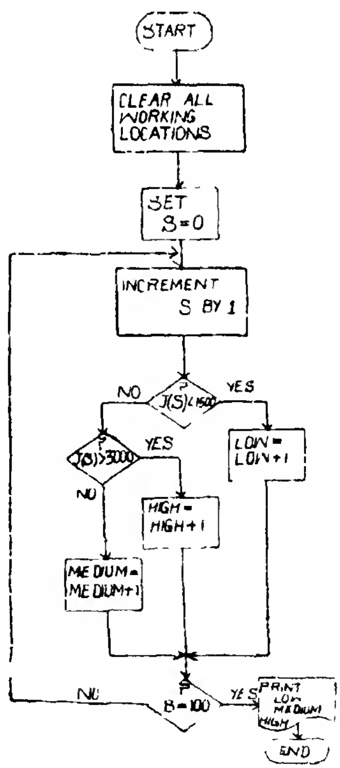


Fig. 42

Example 33 : The commission rates for salesman of Acme Company are given in Table 1 Salesmen in different titles have different quotas and their commission rates depend on whether or not they meet their quota. The quotas are : for junior salesman, none, for associates, an average monthly sales amount of Rs. 40,000, for seniors, an average monthly sales amount of Rs. 1,00,000. For each salesman, his number, class and the sales made by him during the last twelve months are available.

Draw a flowchart suitable for preparing a program in any higher-level language to do the following :—

Read in the details pertaining to each salesman; calculate his average monthly sales, compare it with the quota fixed for him; calculate the commision for each of the twelve months' sales based on commision rates given in Table. Report as output his number, class, his monthly sales value and the corresponding commisson and finally his total sales amount and the total commission due to him.

Table 1—Commision Details

Title	Commission if quota is not met	Commission if quota is met
Junior (Class 3)	5 per cent of sales amount on all sales.	5 per cent of sales amount on all sales.
Associate (Class 2)	5 per cent if monthly sales amount is less than the quota amount, 10% if monthly sales amount is greater than or equal to the quota.	10 per cent if the monthly sales amount is less than the quota amount; 15% if it is greater than or equal to quota.
Senior (Class 1)	10% if monthly sales amount is less than quota amount, 15% if greater than or equal to the quota.	15% if monthly sales amount is less than quota amount; 20% if it is greater than or equal to quota.

Solution : See Fig 43 on Page 57 & 58.

Example 34 : Acme India is engaged in selling of electrical appliances to different categories of customers. In order to promote its sales, various types of discounts are offered to various customers. The present policy is as follows :

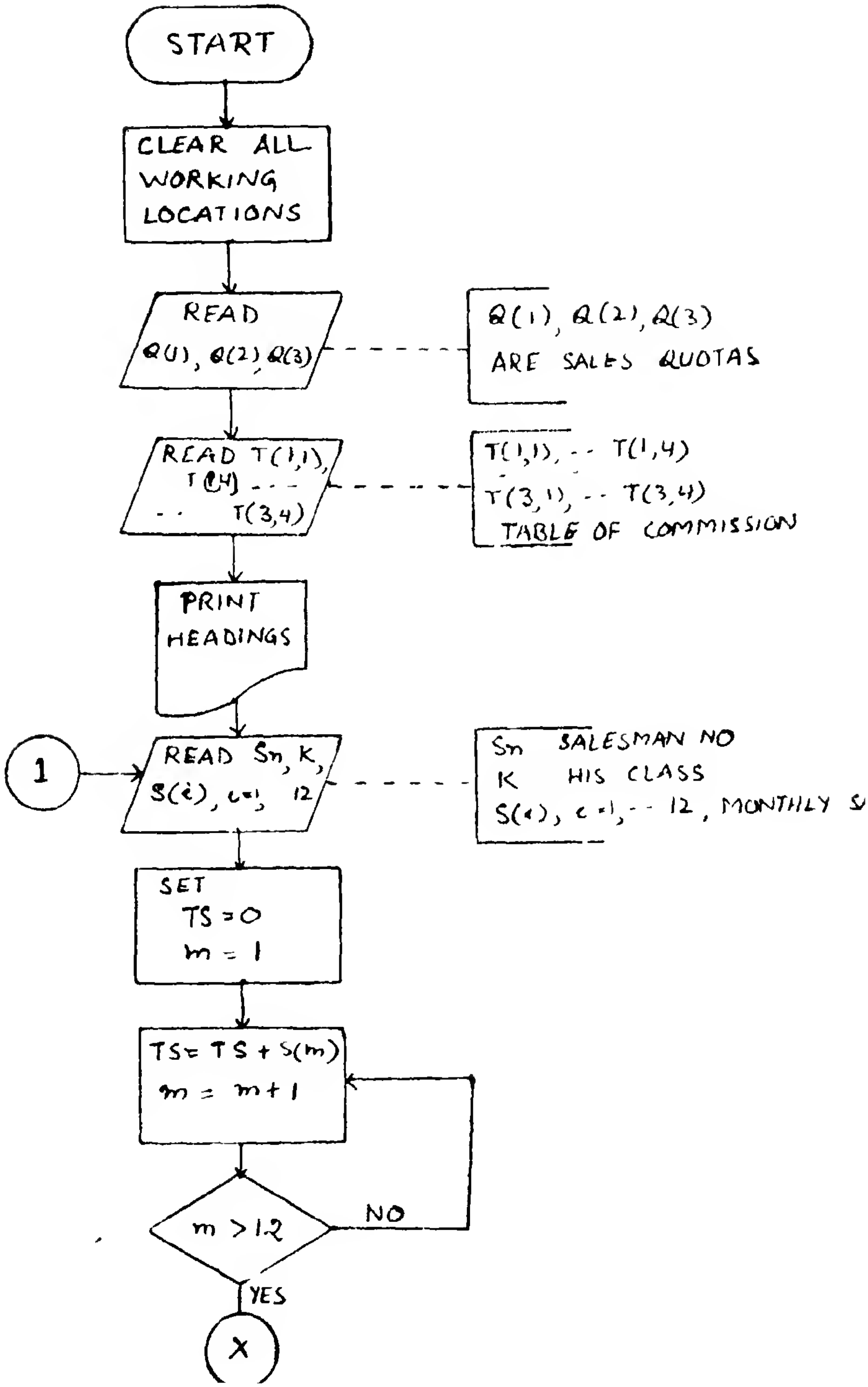
(i) On cooking range, a discount of 10% is allowed to wholesaler and 7% to retailers if the value of the order exceeds Rs. 5000. The discount rates are 12% and 9½%, if the value of the order is Rs. 10,000 and above

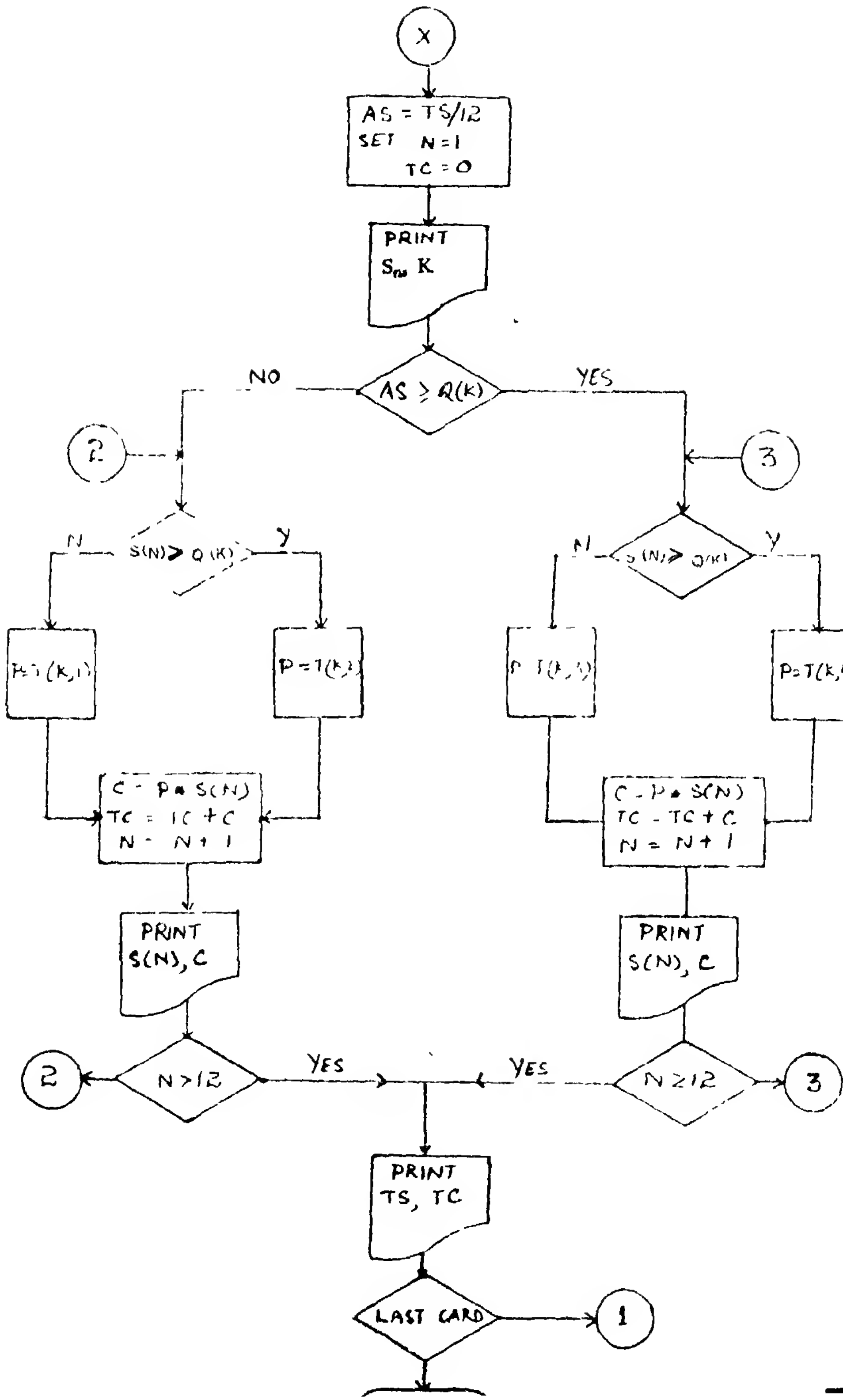
(ii) A discount of 12% is allowed on washing machine irrespective of the class of customer and value of the order.

(iii) On decorative items, wholesalers are allowed a discount of 20% provided the value of the order is Rs. 10,000 and above. Retailers are allowed a discount of 10% irrespective of the value of the order.

Draw a program flowchart for the above procedure.

Solution : **See Fig 44 on Page 59 & 60.**


START
CLEAR ALL WORKING LOCATIONS
READ Q(1), Q(2), Q(3)
Q(1), Q(2), Q(3) ARE SALES QUOTAS
READ T(1,1), T(1,4) ... T(3,4)
T(1,1), -- T(1,4)
T(3,1), -- T(3,4)
TABLE OF COMMISSION
PRINT HEADINGS
1
READ Sn, K, S(i), i=1, 12
Sn SALESMAN NO
K HIS CLASS
S(i), i=1, -- 12, MONTHLY S
SET TS = 0 m = 1
TS = TS + S(m)
m = m + 1
m > 12
NO
YES
X


X
AS = TS/12
SET N=1
TC = 0
PRINT
S, K
AS ≥ Q(K)
NO
YES
2
3
S(N) ≥ Q(K)
N
Y
S(N) ≥ Q(K)
N
Y
P=T(K,1)
P=T(K,2)
P=T(K,3)
P=T(K,4)
C = P * S(N)
TC = TC + C
N = N + 1
C = P * S(N)
TC = TC + C
N = N + 1
PRINT
S(N), C
PRINT
S(N), C
2
N > 12
YES
YES
N ≥ 12
3
PRINT
TS, TC
LAST CARD
1

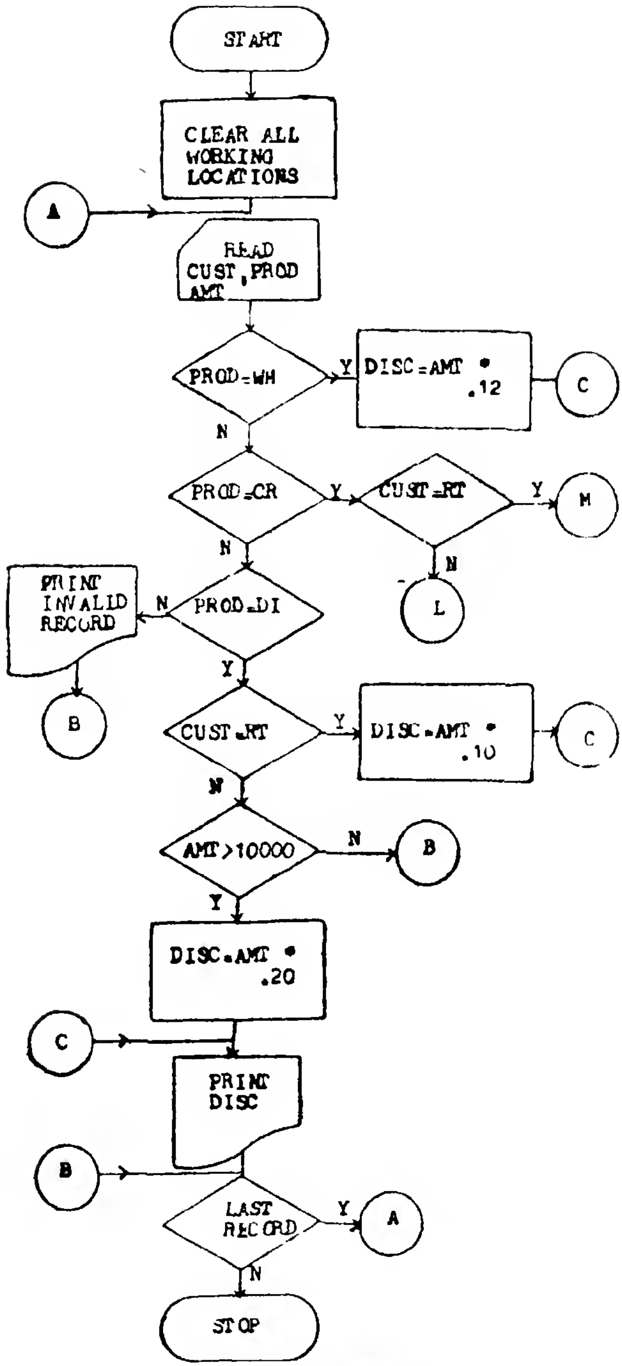
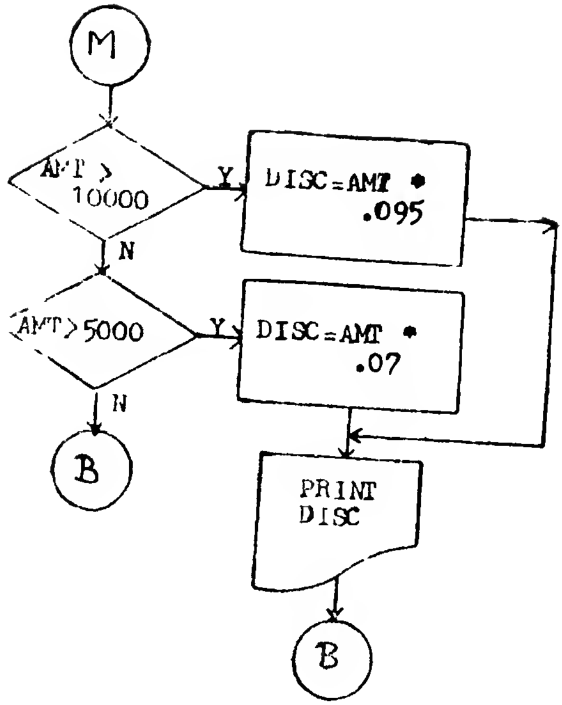
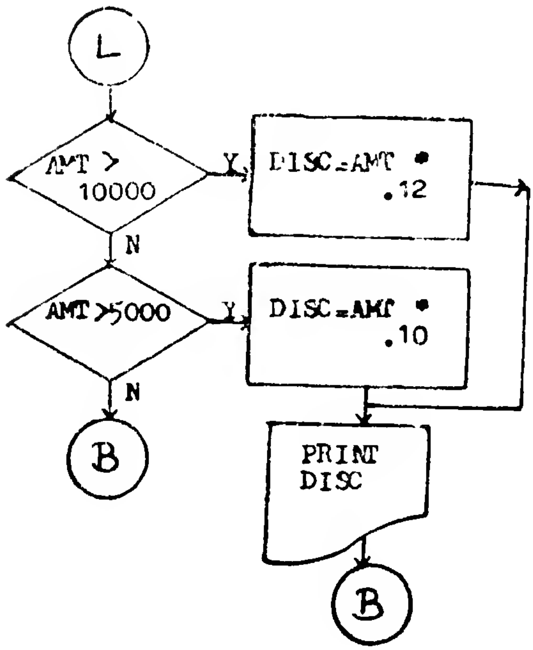


Fig. 44

(CNTD 60.

Continued—


M
AMT > 10000
Y
DISC=AMT * .095
N
AMT > 5000
Y
DISC=AMT * .07
N
B
PRINT DISC
B


L
AMT > 10000
Y
DISC=AMT * .12
N
AMT >5000
Y
DISC=AMT * .10
N
B
PRINT DISC
B

Examples 35 to 37 should be done after having gone through OR study V

Example 35. Referring to the first solved example in Replacement Theory in OR study V draw the flowchart for finding the optimal replacement interval and the least cost. The following symbols may be used.

C Initial Capital Expenditure

S Salvage Value.

CAPT C−S(fixed for all periods)

Printout is to be had in the format below. Also given are the necessary symbols.

Period	C−S	Running Cost	Cumulative Running Cost	Total Cost	Average Cost
Symbol N	Symbol CAPT	Symbol R (X) X=1,2 ..T	Symbol CUMR	Symbol TOCOST	Symbol AVCOST X=1,2.. T
⋮					
Last period N=T					

The flowchart is drawn in figure 45. Stepwise explanation follows :

Step 4 X of R (X) is set at zero. This initialises the loop from step 5 through step 13.

In this loop, in step 8, R (X) are accumulated in CUMR one by one Total Cost is computed by adding C-S to CUMR in step 9, average cost is computed in step 10 by dividing total cost by N and the following are printed in step 11 in a line as above.

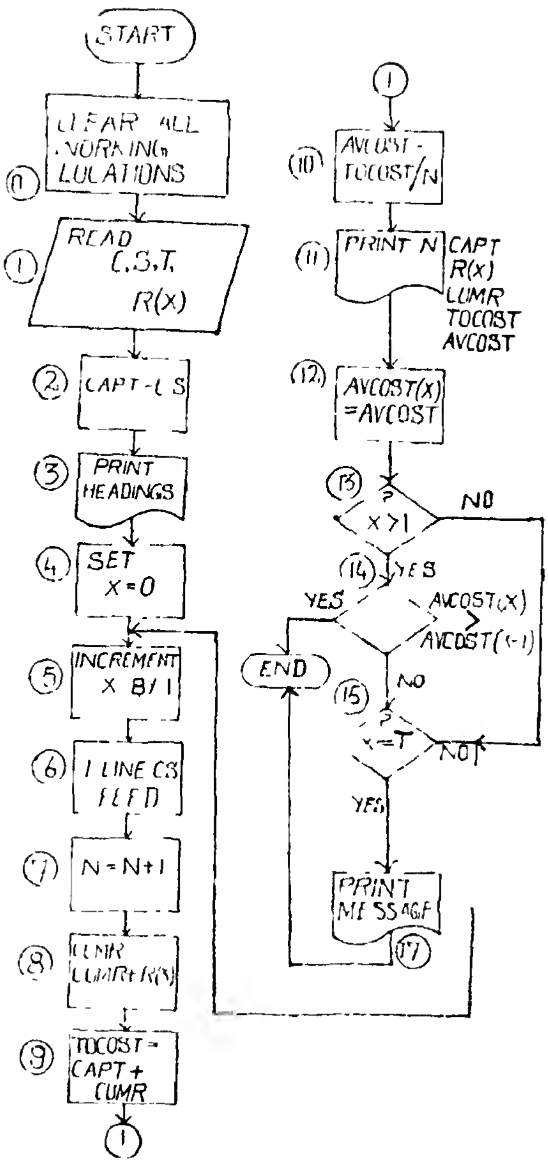


Fig 45

Example 36. Draw the program flowchart for finding the optimal replacement interval for example 8 of the replacement theory, study V of OR Rate of interest, r is given,

An excerpt of the solution of this example for Machine A is given below. Symbols used in the flowchart below are also given at the bottom of the excerpt. $R=1+r$

Year	Running Cost	V^{n-1}	Col. 2× Col 3	Σ col. 4	Total Cost	$\frac{V^{n-1}}{V-1} = \Sigma V^{n-1}$	Col 2× Col 7 e g. 2nd fig of Col 2× 1st fig. of Col 7
1	2	3	4	5	6	7	8
1	800	1.0000	800	800	5800	1.0000	800
2	*800	0.9091	727	1527	6527	1.9091	1528
3	800	0 8264	661	2188	7188	2 7355	2189
X=1, 2 .T	R (X)	F	F*R (X)		TOCOST (X)	CUMF	COL 8

COL is Accumulated in TOCOST

Step 1. Initial Cost, Salvage Value, S, the total years for which data is given T. $R=1+r$ and R (X), the running costs are read via, say, a punched card.

Step 2. Let TOCOST=C−S

Step 3. Set F=1 and X=0. F would be made $\frac{1}{R}, \frac{1}{R^2}$.... in turn in the loop to follow.

Step 4. X is incremented by 1 in steps 4 to 8 i.e., in the loop whenever it is used.

Step 5. TOCOST (X) i e each figure of Col. 4 is computed by discounting R (X) with the discount factor F. F is 1 to start with. In step 10 it would be made $\frac{1}{R}, \frac{1}{R^2}$ etc. in turn.

Step 6. Col. 6 figures are obtained in TOCOST which accumulates Col. 4.

Step 7. $\frac{V^{n-1}}{V-1} = \sum_{i=0}^{i=n-1} V^i$, therefore, it is obtained (i.e. figures of Col. 7) by accumulative F's in CUMF.

Step 8. COL 8 (i e. col 8 figures) are derived by multiplying CUMF with R(X+1) i.e. running cost of the following row.

Step 9. COL 8 is compared with TOCOST.

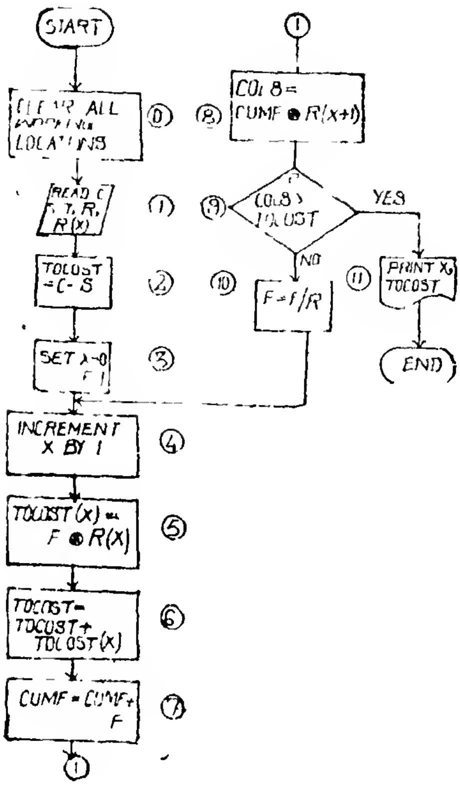


Fig. 46

Example 37. Draw flowchart for finding the optional replacement interval for group replacement (for items that fail suddenly) with reference to example 9 on Replacement Theory in OR Study V)

The flowchart is drawn in figure 47. The following symbols have been used.

N	Number of bulbs in the installation.
GRCOST	Cost of replacing a bulb in a group.
INCOST	Cost of replacing an individual bulb.
P (X)	Probability of failure (Not to be confused with the given "probability of failure to date" in example 9, OR Study V) in X-th week, X=1,2......

The above are read in :

T	Replacement Interval, T will assume values 1,2......
N (Y)	No. of bulbs individually replaced in period Y=1,2,3,......
TERM	$N_0 p_1$, or N_0 p_2, etc. In general, it is N (Y)* P (X). It is to be noted that X+Y=T.

TOCOST Total cost. It holds the total costs for 1 week, 2 weeks, 3 weeks, etc. in turn.

AVCOT Average cost for replacement interval T.

Step-wise Explanation :

Step O. Clear all working locations.

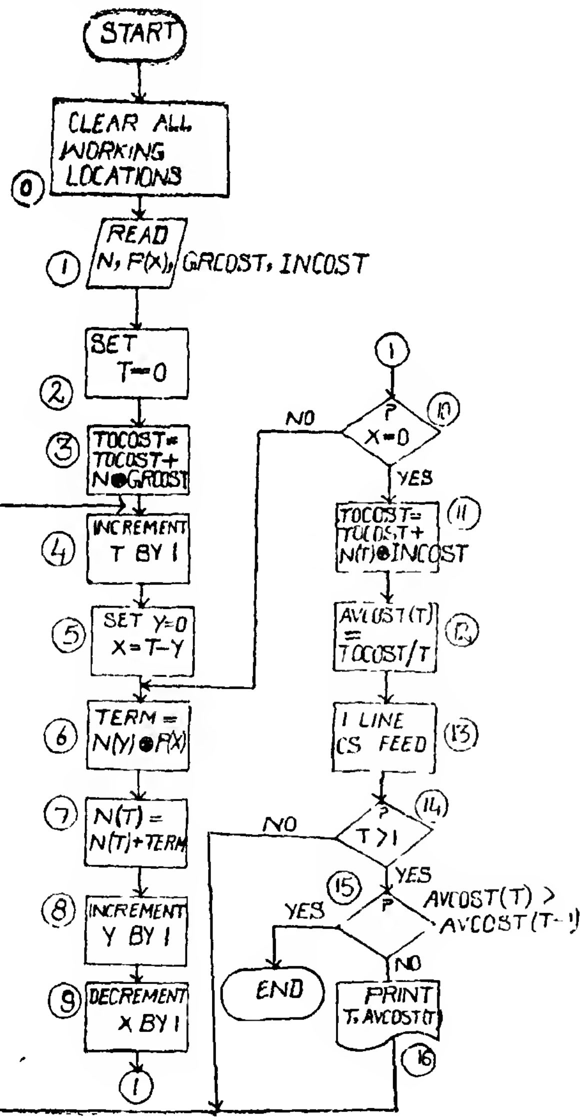


Fig. 47.

Step 1. N, GRCOST and INCOST are read in via, say, a punched card. Also read in are all P(X). To tie P(X) to the solved example 9 of OR Study V

$$P(1) = 0\ 05$$

$$P(2) = 0.08$$

$$P(3) = 0.12, \text{ etc. } i.e. \text{ they are pre-computed.}$$

Step 2. T is set at 0 for the outer loop from steps 4 through step 14.

Step 3. Total cost is set at the periodic group replacement of N bulbs at $1000 \times 0\ 60$.

Step 4. T is incremented by 1 i e to start with it is 1 weekly replacement policy. Thus step 4 through step 15 constitute the replacement policy loop.

Step 5. $\left.\begin{array}{l} Y = 0 \\ X = T-Y \end{array}\right\}$ for the inner loop from steps 6 through Step 10. In this loop we shall compute

$$N_1 = N_0 p_1$$

$$N_2 = N_0 p_1 + N_1 p_1$$

$$N_3 = N_0 p_3 + N_1 p_2 + N_2 p_1, \text{ ete, in turn.}$$

Step 6. TERM = N (Y)*P(X)

Since to start with T = 1, Y = 0, X = T−Y+1;

TERM = N(0)*P(1) i.e. $N_0\, p_1$

Step 7. We compute N(T) i e. N(1) to start with as the sum of previous value of N (1) which is zero and TERM which is $N_0 p_1$.

Step 8. & 9. Y becomes 1 and X becomes 0.

Step 10. For N_1 only one term; therefore X = 0 makes it an exit from the loop.

Step 11. The inner loop yields the values of N (1), N (2), etc. in turn i e. the number of individual bulbs to be replaced in each week chronologically. To TOCOST is added the cost of individual weekly replacements.

Step 12 Average costs are computed and stored in AVCOST (T) i.e.

AVCOST (1), AVCOST (2), etc.

AVCOST (0) = 0, made in step 0.

Step 15. AVCOST (T) is compared with AVCOST (T−1). If it is more it means AVCOST (T−1) was optimal with (T−1) as the optimal replacement period; AVCOST (0) the latter being zeroised in step 0 it is an exception to step 15.

Step 16. The interval T and the corresponding average cost are printed.

Example 38. There are 1500 students for an examination. Each student appears in 12 papers. Pass marks for each paper are 40%. His roll number and marks obtained by him in each paper are transcribed on a punched card, thereby giving a deck of 1500 punched cards. It is desired to draw the program flowchart for printing the following report. If a student scores 75% or more marks in a paper the marks would be followed by 'D' in printing to designate distinction in that paper. Provided student has failed in not more than 1 paper he would be exempted in each paper where he obtains 60% or more marks and this would be designated by printing 'E'. However, a student failing in one or more papers fails.

	Marks in Paper No.												
Roll No.	1	2	3	4	5	6	7	8	9	10	11	12	Total marks or 'FAIL'
Print Position 020	025	030	035	040	045	050	055	060	065	070	075	080	090
No. of Exemptions													
No of Distinctions													

No. of students passing all papers......

No. of failed students......

The flowchart is drawn on p. 69 to 71. The following symbols have been devised and put to use in it.

ROLLNO Roll Number of a student

M (X), X=1, 2 .. .12	Marks obtained by a student in paper No. X=/1,2...12
FAIL	Counter for accumulating the number of papers in which a student fails.
TOPASS	Total number of pass students
TOFAIL	Total number of fail students
TOTAL	Total marks obtained by a student in the 12 papers are accumulated in it.
MARKS i e. 12 locations	It holds the marks obtained by a student in paper no. 1, 2 12, one by one, in turn.
E(X), X = 1, 2 . 12	Each of these accumulates the number of students exempted in paper no X, X = 1,2,......12.
D(X), X = 1, 2 . 12 i e. 12 locations	Each of these accumulates the number of students getting distinction in paper no. X. X = 1, 2. . 12,

Step-wise Explanation

Step 0 All working locations are cleared. This ensures erasure of any data in these of a previous program.

Step 1. The headings (ROLLNO , MARKS in each paper etc) are printed. This is followed by raising of the continuous stationery by one line in step 2.

Step 3. The roll number of a student and marks obtained by him in each of the 12 papers are read into the CPU from a punched card.

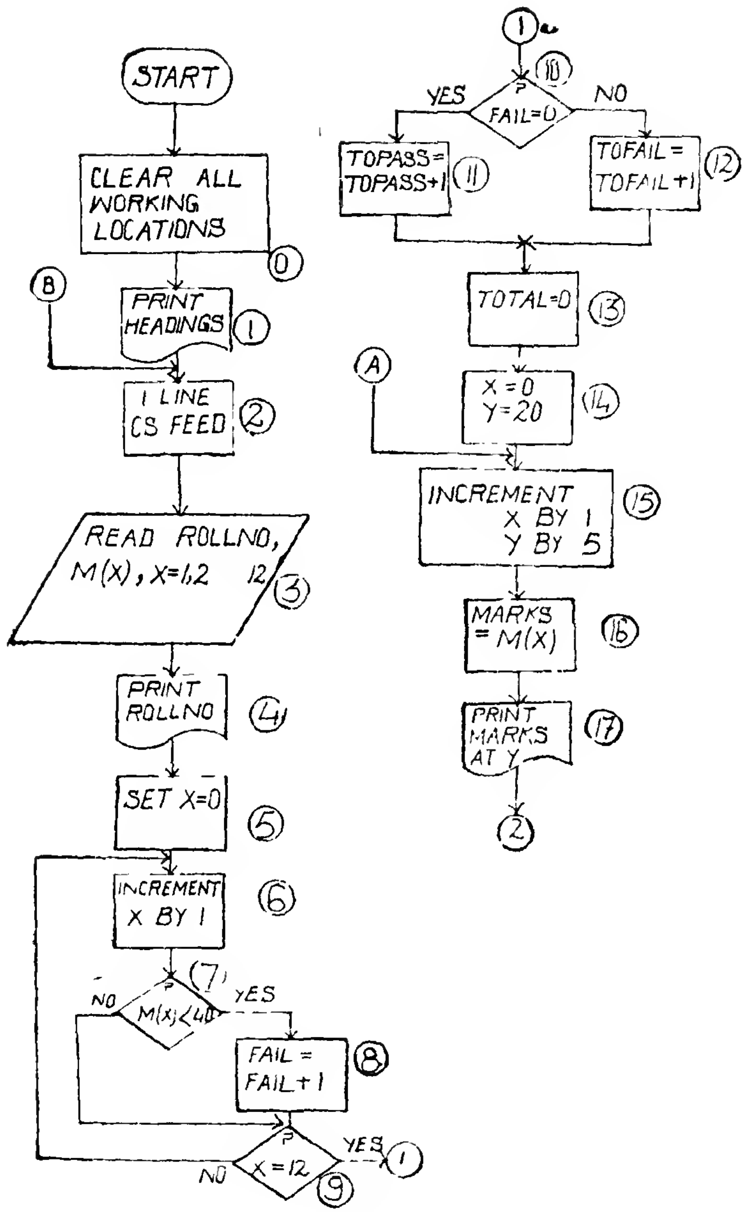
Step 4. Roll Number is printed at print position 025.

Step 5. X representing the paper number is set at 0 as initialisation of the following loop.

Step 6. X is incremented by 1 in the following loop so that M (X) becomes M (1) in it.

Step 7 It is ascertained if M (X) is less than 40 or not i.e. if the student has failed in this paper or not.

Step 8. If he has failed (i.e., scored less than 40 marks). FAIL is incremented by 1.

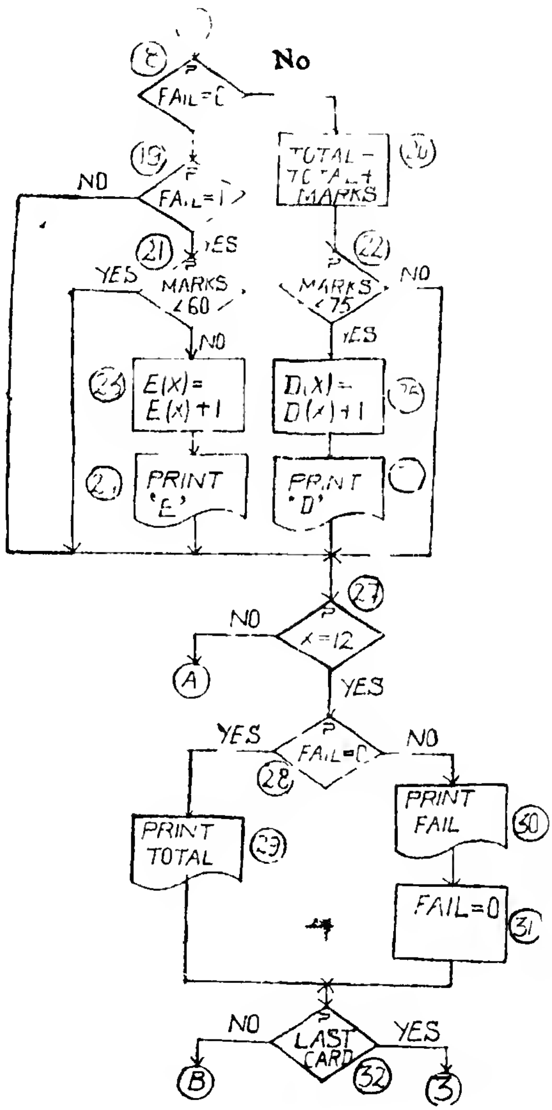


(*Continued*)

Step 9. It is ascertained if X has become 12 i.e. if all the 12 pepers have been checked for pass or fail.

Step 10, 11, 12. The number of students passing or failing is incremented by 1 depending upon if FAIL = 0 or not.

Step 13. TOTAL (used for accumulating the marks of the student in the 12 papers) is zeroised as initialisation of the following loop.



(*Continued*)

(N.B. Before step 32 another comparison step to ascertain if it is the end of the page may also be included).

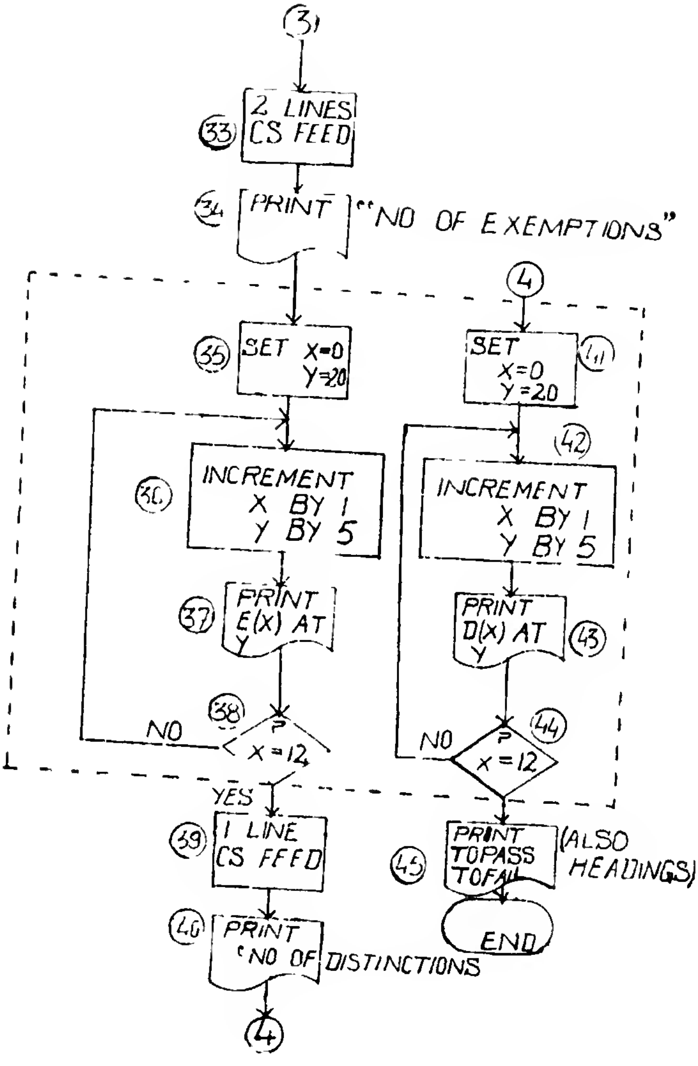

3
33
2 LINES
CS FEED
34
PRINT
"NO OF EXEMPTIONS"
4
35
SET X=0
Y=20
INCREMENT
X BY 1
Y BY 5
37
PRINT
E(X) AT
Y
38
NO
?
X=12
YES
39
1 LINE
CS FEED
40
PRINT
'NO OF DISTINCTIONS
4
41
SET
X=0
Y=20
42
INCREMENT
X BY 1
Y BY 5
PRINT
D(X) AT
Y
43
44
NO
?
X=12
45
PRINT
TOPASS
TOFAIL
(ALSO
HEADINGS)
END

Fig. 48

Step 14. X, and Y (representing the print position which for mark in the first paper is 025) are set at 0 and 020 respectively as initilisation of the following loop.

Step 16. Marks in paper no X are transferred to MARKS.

Step 17. Marks in a paper are printed at position Y.

Step 18 to 26 Total marks of a pass student are accumulated in TOTAL.

For a failed student E(X) (used for accumulating the number of students exempted in paper X) is incremented by 1 if he has not scored less than 60% marks n paper X then E is printed

Likewise a pass student scoring not less than 75% marks in paper X is awarded distinction in it by printing D Also D(X) is incremented by 1,

Step 27. If all the 12 papers have not been processed the flowchart loops back to step 15.

Step 28. It is ascertained if a student has failed in no paper or not.

Step 29 If he has passed in all his total marks are printed at print position 90.

Otherwise, in step 30, "FAIL" is printed and FAIL is subsequently zeroised in step 31.

Step 32 If all 1500 cards have not been processed the flowchart loops back to step 2.

Step 33. The continuous stationery is raised by 2 lines to print "NO OF EXEMPTIONS" in step 34.

Step 35 to 39. The total number of students exemped in each of 12 papers is printed.

Step 41 to 44. The total number of students getting distinctien in each of 12 papers is printed. [Since, the two segments in the dotted portion here are almost identical a subroutine could be used for these].

Step 55 (condensed). Rest is summary data that is printed.

Example 39. Write a program flowchart to process data in the format shown below ,

Employee No.	Job classification code	No. of years employed	Employee's current annual salary

Punched Card

Each employed is to receive a% age salary increase based on the job classification and the number of years employed. The following table shows the % age increase for the different job classifications and years of employment.

Years Employed	Job Class 1	2	3	4	5	
0–1	5%	5%	5%	5%	5%	J(1) to J(5)
2–5	10%	10%	15%	15%	15%	J(6) to J(10)
6–10	15%	15%	15%	15%	20%	J(11) to J(15)
11–20	20%	25%	25%	25%	25%	J(16) to J(20)
21–99	28%	28%	28%	28%	30%	J(21) to J(25)

The Problem is to determine for each employee the rupee amount of the increase and print on one line the employee number, the old salary, the increase and the salary after the increase. At the end of the report print the tolal of all increases and the total of all new salaries.

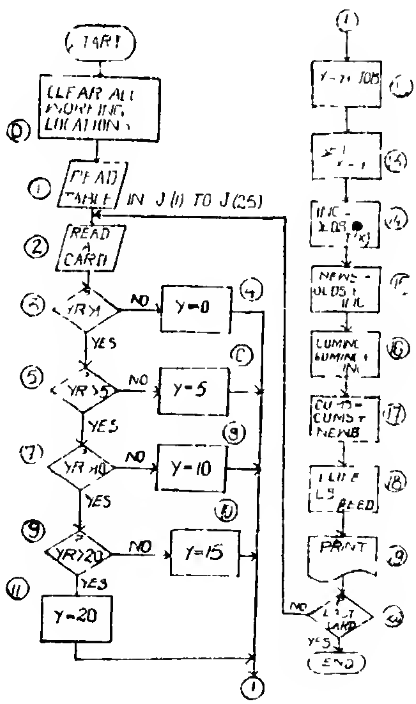


Fig. 49

Solution : The flowchart is drawn in the figure 49 above The following symbols have been employed.

J(1) to J(25) hold the table of% age increase. J(1) to J(5) are meant for the 1st row, J(6) to 10 are meant for the 2nd row and so on.

YR	Years Employed
Job	Job Class (1, 2, 3, 4, or 5)
Y	Working location tp determine location holding the % age from the given table on the basis of the years employed and Job Class.
INC	Increase in Salary
OLDS	Old Salary
NEWS	New Salary
CUMINC	Cumulative Increase in salary.
CUMS	Cumulative new salary

Explanation. First, we shall explain how Y is assigned a value. Suppose there is an employee with 7 years of service and Job class 4. Looking along the years of service across the% age table this employee's %age is to be found in the third row i e. in one of the locations J(11) to J(15). Thus, if we first set Y=J(10) and then make it J(13) in view of his Job Class 3 we would have got J(13) as the address of the location that contains the % age applicable to this employee. Steps 3 to 11 set Y=0, 5, 10, 15, 20 for the 1st, 2nd, 3rd, 4th and 5th rows respectively. In Step 12 Y is incremented by the Job Class.

In Step 13 we set X=Y for step 14. In step 14 we compute the increase in salary by multiplying the old salary with J(Y), the % age increase.

Example 40. A Comprehensive Flowchart for Processing Sequential under Batch Processing Mode. (Fig. 50)

To explain the various steps let us suppose that the following are the master file and transaction file record keys.

Master File	1 2 4 6 9 11 17 20 21 22 29*
Transaction File	1 2 5 6 7 9 11*
	*means end of the file.

Also, please note that there are gaps in the keys of the master file. Keys 5 and 7 in the transaction file represent new records created to be put into the master file appropriately in sequence.

Please have a look at the definition of the various symbols used at the top of the flowchart

Step 1. A master record is read Key=1

2. A transaction record is read Key=1

3. Is transaction Key*?

No, it is 1 ; therefore, step 4

4. Transaction Key Versus Master Key They are equal therefore, Step 5.

5. Master record is updated. Also accumulated are batch totals from the transaction record. Back to Step 2 Note that updated MR is not put on the updated master file

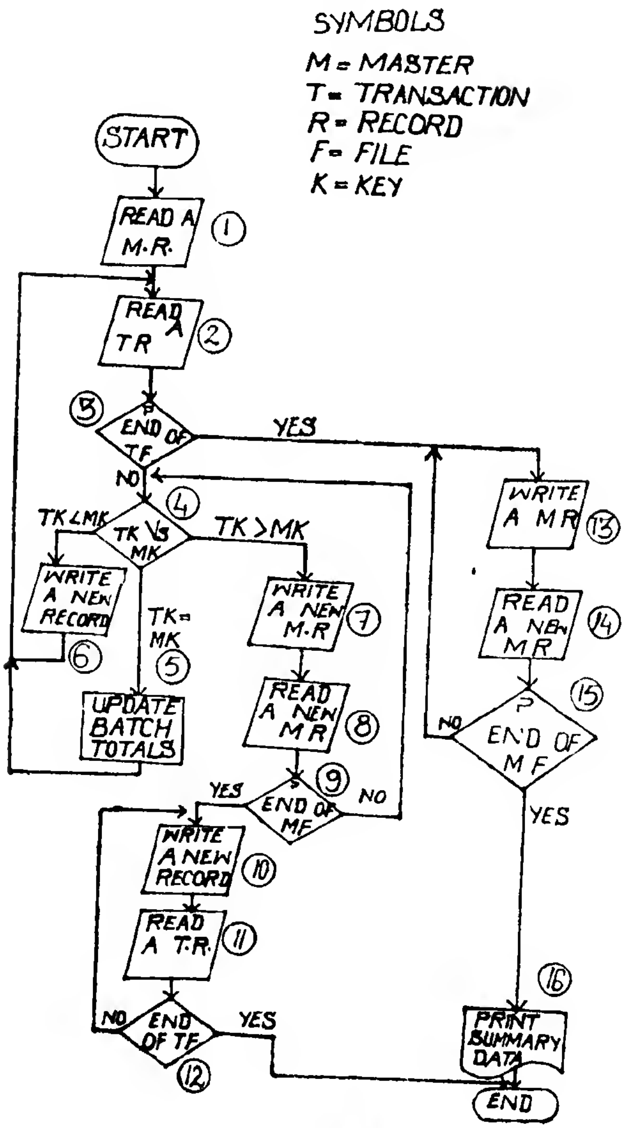


Fig. 50

2. Read a transaction record Key=2

3. Is transaction Key*?
   No, it is , therefore, step 4.

4. Transaction Key (2) is greater than
   Master key (1), therefore, Step 7.

7 Updated master record 1 is written on the
  output (C/F) magnetic tape,

8. Next Master record is read Key=2

9 Is master Key*? No, it is not,
  therefore, Step 4.

4. Transaction Key (2)= Master Key (2);
   therefore, Step 5.

5. Update master record 2. Also accumulate
   batch totals for transaction 2 Back to
   Step 2.

2. Next transaction record is read Key=5

3. Is transaction key*? No, it is
   not ; therefore, to Step 4.

4 Tran saction Key (5)<Master Key (2) ;
  therefore, Step 7.

7. Updated master record 2 is written onto C/F tape.

8. Next master record is read Key=6

9. Is master Key*? No, it is 6 ;
   therefore, back to Step 4.

4. Transaction Key (5)<master Key (6) ,
   therefore, Step 6.

6. New master record is creatsd by
   copying transaction record 5 on to CF
   tape. Back to Step 2.

The student may proceed on this way until both the asterisks have been reached one by one. Likewise, he may "dry process" (just as we did above) the following, master and transaction files.

Master File	1, 2, 3, 4, 6*
Transaction File	1, 4, 7, 8, 9, 10*

Here new records 7, 8, 9, 10 have to be put at the end of the C/F tape.

Dry Run and Debugging the Program

In study I, we stated that any program of some scope even written with great care is likely to contain some mistakes known as bugs in the technical jargon. There, therefore, is a need to remove these mistakes or debug the program. Debugging should start with the review of the flowchart through review of the program code and finally testing the program with fictitious data in one or more computer setups.

Review of the flowchart also carried out by means of fictitious data and as such is known as the *dry run* since no computer setup is involved. We shall take up the flowchart of example 5 to elucidate the means to carry out the dry run. This flowchart is concerned with picking up the highest and the lowest of three quantities: Q1, Q2 ond Q3 putting them in locations designated by H and L respectively.

The flowchart of fig 13 is reproduced below in fig A except for the deliberate mistakes indicated by the crooked arrows in this fig (A). "Yes" and "No" have been interchanged, i e bugs have been deliberately introducted. Now let us see how these bugs are detcted by means of the dry run.

We shall try three sets of values for Q 1, Q 2 and Q 3 as below:

	Q 1	Q 2	Q 3	
Set 1	6	2	14	Q3>Q 1>Q2
Set 2	3	7	15	Q3>Q 2>Q1
Set 3	2	4	3	Q2>Q 3>Q1

In Fig. A the data of the Ist set have been 'flown' in the flowchart and it flows across the dotted lines in the flowchart. Ultimately, we end up with 14 in H and 2 in L. This is correct since we can see for ourselves that in the first set 14 is the highest and 2 is the lowest.

In Fig. B on 87 the data of the 2nd set is flows and it flown across the dotted lines in this flowchart. Again, we end up with the correct result, 15 as the highest and 3 as the lowest in H and L respectively.

In Fig. C on p 79 the data of the 3rd set is flown Here we end up with 4 in H and 3 in L which is wrong since we can see that 2 is the lowest in the 3rd set where as we are getting 3 as the lowest. This arouses our suspicion and we would carefully scrutinise the lower portion of the flowchart until we detect the bugs.

Following this up we shall rectify the flowchart and the program code.

N. B : Dry run is synonymous with desk checking.

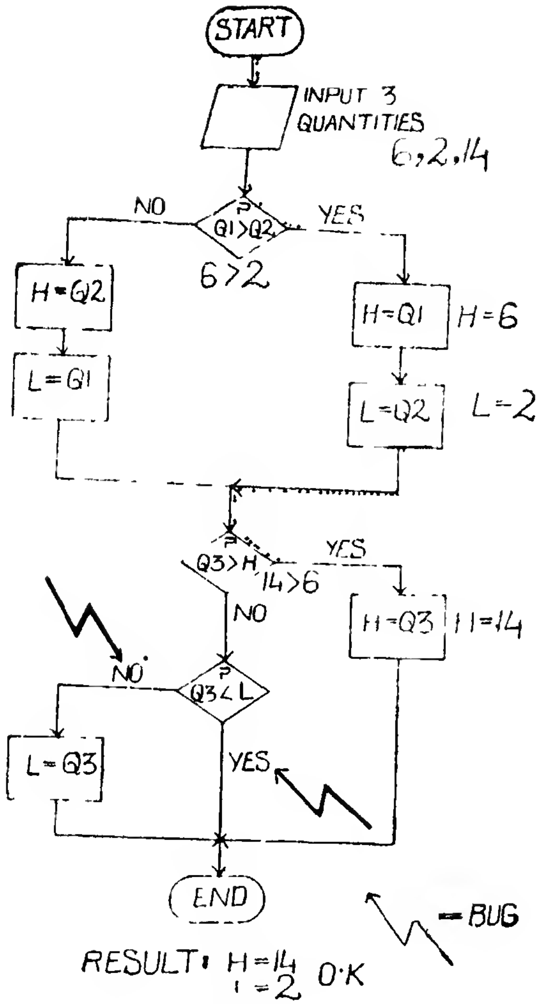


Fig. A

[By means of such dry runs student may want to verify the flowchart for the exercises he draws].

In larger flowchart with many more branches we shall not be content with a mere dry run. We shall actually set up the computer with the given program loaded in its memory, input the test data, and compare the results output by the computer with the ones computed in longhand. The task of debugging is formidable indeed.

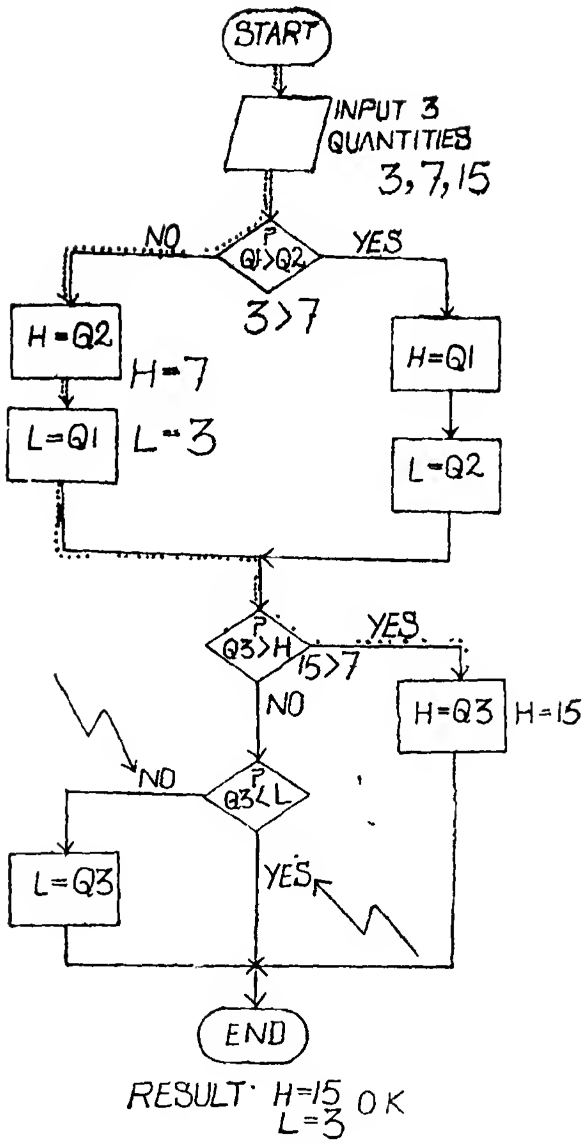


Fig. B

In complex programs there may be tens of thousands of different paths through the program It simply is not practical (and may not be even possible). to trace through all the different paths during testing. Boehm determined for the rather simple looking program flowchart that the number of the different paths is as astoundingly high as 10^{20}. He further observed, if we could some how check out

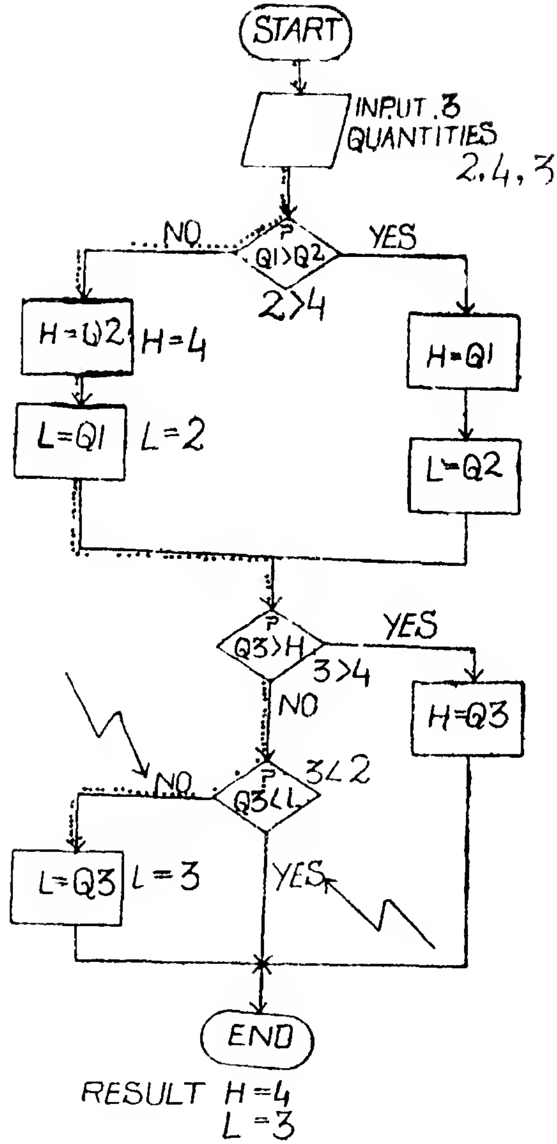
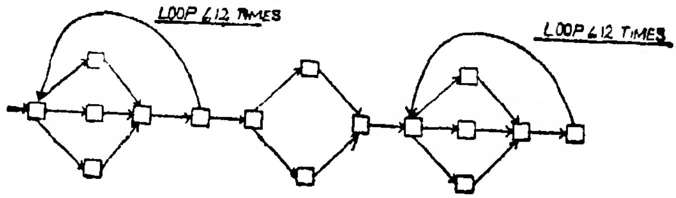


Fig. C

one path per nanosecond, and if we had started our testing in the year, 1. A. D., we would only be about half done at the present time !

It is to be noted, however, that removal of the syntax errors diagnosed by the compiler is not the part of the debugging procedure. The programmer compiles the test data deck which should contain (1) typical data which will test the generally

used program paths; (2) unusual but valid data which will test the program paths used to handle exceptions ; and (3) incorrect, incomplete, or in appropriate data which will test the error handling capabilities of the program.



The programmerr, after the dry runs loads the computer with the program to be tested, inputs the test data and obtains the output results which he compares with the results derived by him in long hand prior to processing If the program does not pass the test, i. e , the results do not tally. The programmer may do the following :

1. Trace through the program, a step at a time at the computer console ; but this facility is, usually, available with smaller and mini computers only.

2. Call for a trace program run. The trace program prints the results of execution of each instruction in detail. It is thus comparable to console checking. However, less machine time is required.

3. Call for a storage dump when the program hangs up (i. e computer hums) i.e. obtain a printout of the contents of the primary storage at the time of the hangup. The programmer can then study this listing for possible clues to the cause of the programming errors,

However more bugs may come to notice upon parallel running, which is done upon program implementation with live data.

Exercise, Set II

(The student must do these. Mere understanding our examples is not enough)

1. Wages of 500 workers are hold in J(w), w=1,2,...500. Draw the pragram flowchart for compiling the frequency distribution and printing it in the following formats.

Class Intrreval	*Frequency*
< 300	
$300 < 400$	
$400 < 500$	
$500 < 600$	
$600 < 700$	
$700 < 800$	
$800 < 900$	
> 900	

2. Out of an array of number J(N), N=1,2...100, 5 numbers are known to be zeros You have to draw the program flowcharts for squeezing the zeros out i e rearrange the 95 non-zeros number in location J(N), N=1, 2.. 95.

Also, extend the flowchart for printing these 95 numbers in a 19×5 matrix: assume that a number is at most of six digits.

3. J (E) E=1, 2. .. 42 contains 42 quantities of 7×6 matrix. Draw the program flowchart for printing its transpose, assume that a number is at most of 5 digits.

4. Names of eleven cricket playears are held in J,(X), X=1, 3, 5..... 21 and their respective batting averages in J(X), X=2, 4, . .. 22. You are to arrange the eleven players in the descending order of their batting averages

5. Draw the program flowchart for summing up 1,11,111,1111, (10 terms)

6. J(X), X=1,2 200 designate numbers. Draw the program flowchart for computing the following.

   1. The % ages of negative, zero and positive numbers.

   2. The sum of the-ve and+ve numbers separately.

   3. The sum of the absolute numbers.

   4. The sum of squares of all the numbers.

7. Draw the program flowchart far printing the following pattern on the continuous stationery.

```
M               M
 M             M
  M           M
   M         M
    M       M
     M     M
      M   M
       M M
        M
```

8. Referring to OR Study II, for the Newspaper Boy's problem draw the program flowchart for finding the expected payoff of each strategy and picking up the best strategy.

9. Assume that you opened a savings account with a local bank on 1-1-1980. The annual interest rate in 5.75%. Interest is compouned at the end of each month Asuming that your initial deposite is×rupee draw a flowchart to print out the balance of your account at the end of each month for two years.

10. Write a program flowchart to compute the mean and S. D. of Numbers denoted by J(X), X=1,2.. ...N.

11. The median of a list of numbers is defined to be item in the list that has as many items greater than it as less than it, if there is an odd number of items in the list the median is the average of the two middle items Draw a Program flowchart to read a list of numbers and print the median

12 A+ve integer is called "perfect" if it equals the sum of its proper divisors. For example, the number 6 and 28 are perfect because,

(Hints : Sort the list first)

$$6=1+2+3$$

$$28=1+2+4+7+14$$

Write the flowchart to decide whether a +ve integer is perfect.

Decision Table and Flowcharting

Decision Table

Introduction :

A decision table is a table which may accompany a flowchart, defining the possible contingencies that may be considered within the program and the appropriate course of action for each contingency.

Decision tables are necessitated by the fact that branches of the flowchart multiply at each diamond (comparison symbol) and may easily run into scores and even hundreds. If, therefore, the programmer attempts to draw a flowchart directly he is liable to miss some of the branches.

A decision table is divided into four parts :

(1) Condition Stub (which comprehensively lists the comparisons or conditions) ; (2) Action Stub which comprehensively lists the actions to be taken along the various program branches : (3) Condition entries (which lists, in its various columns the possible permutations of answers to the questions in the condition stub) and (4) Action entries (which lists, in its columns corresponding to the condition

entries the actions contingent upon the set of answers to questions of that column). A decision table is given below as an example :—

Part		Granting Credit Facility	R1	R2	R3	Part
Part 1	C1	Credit limit okay	Y	N	N	Part 3
	C2	Pay experience Favourable	—	Y	N	
Part 2	A1	Allow Credit Facility	X	X		Part 4
	A2	Reject Order			X	

There are two conditions : C_1 and C_2 in this table and two actions : A_1 and A_2. According to R_1 (a set of rules) if there is a "yes" to C_1 and C_2 is to be by passed, Action A_1 will be taken that is "Allow credit facility". Under R_3, Nos to both C_1 and C_2 requires action A_2 to be taken.

With this example, we given below the components of the decision table in more detail.

(a) *Condition Statement* : Statement which introduce one or more conditions (i. e. factors to consider in making a decision).

(b) *Condition Entries* : Entries that complete condition statements.

(c) *Action Statements* : Statement which introduce one or more action (i.e. steps to be taken when a certain combination of conditions exist).

(d) *Action Entries* : Entries that complete the action statements.

(e) *Rules* : Unique combinations of conditions and action to be taken under those conditions.

(f) *Header* : Title identifying the table.

(g) *Rule Identifiers* : Code (R1,R2,R3) uniquely identifying each rule within a table.

(h) *Condition Identifiers* : Codes (C1, C2, C3....) uniquely identifying each condition statements/entry.

(i) *Action Identifiers* : Codes (A1, A2, & A3......) uniquely indentifying each action statement/entry.

These items are contained within the body of the table which is divided into four major sections by double or heavy vertical and horizontal lines as in the table above.

Types of Decision Table : There are three types of decision table :

Limited Entry Tables: In a limited entry table the condition and action statements are complete. The condition and action entries merely define whether or not a condition exist or an action should be taken. The symbols used in the condition entries are:

Y . Yes, the condition exists

N : No, the condition does not exits.

—: Irrelevant, the condition does not apply, or it (or blank) makes no difference whether the condition exists or not.

The symbols used in the action entries are:

X : Execute the action specified by the (or blank) action statement.

—: Do not execute the action specified by the (or blank) action statement.

Extended Entry Table

The condition and action statements in an extended entry table are not complete, but are completed by the condition and action entries.

Example :

	Granting Credit Facility	R1	R2	R3
C1	Credit Limit	OK	Not OK	Not OK
C2	Pay Experience	— —	Favourable	Unfavourable
A1	Credit Facility	Allow	Allow	— —
A2	Credit Action	— —	— —	Reject Order

Mixed Entry Table

The third type of decision table is the mixed entry form which combines both the limited and extended entry forms. While the limited and extended entry forms can be mixed within a table, only one from may be used wrthin a condition statement/entry or an action statement/entry.

Example 1

Granting Credit Facility		R1	R2	R3
C1	Credit Limit Okay	Y	N	N
C2	Pay Experience	—	Favourable	Unfavourable
A1	Credit Facility	Allow	Allow	
A2	Reject Order			X

A systematic approach to the development of a limited entry decision table is presented below :

Steps in preparing a limited entry decision table

1. List conditions and actions.
2. Combine conditions which describe the only two possibilies of a single condition. In other words delete conditions which can be derived from the responses of the other conditions.
3. Make yes or on (Y or N) responses and mark actions to be taken for each rule with X.
4. Combine redundant rules to simplify table.
5. Check for completeness.

An example will be ueed to explain and illustrate the procedures.

Example 2

A shop owner allows credit facility to his customers if they satisfy any one of the following conditions :

1. Holding the present job for more than 3 years and residing in the same place for more than 5 years.
2. Monthly Salary exceeds Rs. 1500/- and holding the present job for more than 3 years.
3. Residing in the same place for more than 5 years and monthly salary exceeds Rs. 1500/-

The facility is rejetcd for all other customers.

Step 1 is to write down all of the conditions and actions :

Conditions involved in the problems are :

1. Holding the present job for more than 3 years.
2. Holding the present job for 3 years or less than 3 years.
3. Monthly salary exceeds Rs. 1500/—.
4. Monthly salary is Rs. 1500/- or less than Rs. 1500/—.
5. Residing in the same place for more than 5 years.
6. Residing in the same place for 5 yeers or less than 5 years.

Actions involved in the problem are :

1. Allow credit facility.
2. Reject credit facility.

Step 2 is to combine conditions which only describe the two possibilities of a single condition :

"Holding the present job for more than 3 years" and "Holding the present job for 3 years or less than 3 years" can be combined, A single condition "holding the present job for more than 3 years" can represent both because a "No" answer means holding the present job for 3 years or less than 3 years. The same reasoning allows the combination of 3 & 4 and also 5 & 6.

There are thus only three conditions :—

1. Holding the present job for more than 3 years.
2. Monthly salary exceeds Rs. 1500/-.
3. Residing in the same place for more than 5 years.

Step 3 is to prepre the Yes and No responses using Y and N for all possible combinations of conditions and for each set of conditions make the actions to be taken with an X.

No. of rules $= 2^{\text{no of conditions}}$

In the exampie there are three conditions, so there will be 2^3 or 8 rules. The Y's and N's can be inserted in any order, but a systematic method will reduce the effort of filling in the table and reduce the chance of error. Start with the bottom row of the condition entries and fill in the row starting with Y and then alternating between N and Y. The row above this is filled in by writing two Y's two N's, two N's etc. The third row from the bottom uses sets of four Y's and four N's. This doubling of the sets of Y's and N's continues until the table is complete. Then analyse each rule and fil' in the action entries. The figure below shows the completed table at this stage.

	Allowing Credit Facility	R1	R2	R3	R4	R5	R6	R7	R8
C1	Hoeding the present job for more than 3 years	Y	Y	Y	Y	N	N	N	N
C2	Monthly salary exceeds Rs 1500/-	Y	Y	N	N	Y	Y	N	N
C3	Residing in the same place for more than 5 years	Y	N	Y	N	Y	N	Y	N
A1	Allow credit facility	X	X			X			
A2	Reject credit facility				X		X	X	X

Step 4 is to combine rules where there are redundancies :

Two rules can be combined into a single rule if :—

(i) all of the conditions except one have the same Y or N (or—) condition entries and

(ii) the actions are the same for both.

(Rule with impossible combination of condition entries can be combined with any other rule if

(iii) all of the conditions except one have the same Y or N (or—) condition entries (See Example 2)

Combine the two rules into one and replace the condition entry of Y and N with a dash (—), which means the condition does not affect the actions to be taken. Using this procedure rule R1 and R2 can be combined. In other words, if holding the present job for more than 3 years and monthly salary exceeds Rs. 1500/— then the credit facility is allowed without regard to the third condition viz. residing in the same place for more than 5 years. Rule R7 & R8 (or R4 &R8) can be combined. The resulting table with redundancies removed is shown below :

	Allowing Credit Facility	R1	R2	R3	R4	R5	R6
C1	Holding the present job for more than 3 years	Y	Y	Y	N	N	N
C2	Monthly salary exceeds Rs. 1500/-	Y	N	N	Y	Y	N
C3	Residing in the same place for more than 5 years	—	Y	N	Y	N	—
A1	Allow credit facility	X	X		X		
A2	Reject credit facility			X		X	X

Step 5 is check for completeness of the rules :

(1) Count number of dashes in the condition entries for each rule. The number of rules "represented" by each rule are 2^m where m is the number of dashes. Where there are no dashes, the number represented is 2^0 or 1. A single dash means 2 rules have been combined etc.

(2) Sum the number of rules represented by the different rules as computed above.

(3) Compare the number of rules represented by the reduced table with the number to be accounted for, which is 2^n (n no. of conditions). If they are equal (and all othre features are correct), the table is complete.

In the example, rules R1 & R6 have one dash and rules R2, R3, R4 and R5 have no dashes. The sum of the rules represented by the rules in reduced tables is 2 1 1 1 1 2 which is equal to 2^3 or 8.

Therefore the reduced table is complete.

Example 2. Select the largest of three distinct numbers A,B,C

Step 1. Conditions involved in the problem are :

1. A>B
2. A>C
3. B>A
4. B>C
5. C>A
6. C>B

Actions involved in the problem are :

1. A is largest
2. B is largest
3. C is largest

Step 2. Conditions 1 & 3 can be combined

Conditions 2 &5 can be combined

Conditions 4 & 5 can be combined

Therefore there are only three conditions :

1. A>B
2. A>C
3. B>C

Step 3 No. of rules $=2^n$ conditions

$=2^3=8$

	Select Largest	R1	R2	R3	R4	R5	R6	R7	R8
C1	A>B	Y	Y	Y	Y	N	N	N	N
C2	A>C	Y	Y	N	N	Y	Y	N	N
C3	B>C	Y	N	Y	N	Y	N	Y	N
A1	A is largest	X	X						
A2	B is largest					X		X	
A3	C is largest				X				X

*R3 & R6 contain impossible combination of condition entries.

Step 4. R1 & R2 can be combined

R3 & R4 can be combined

R5 & R7 can be combined

R6 & R8 can be combined

Select Largest		R1	R2	R3	R4
C1	A>B	Y	Y	N	N
C2	A>C	Y	N	—	—
C3	B>C	—	—	Y	N
A1	A is largest	X			
A2	B is largest			X	
A3	C is largest		X		X

Step 5. All the rules in the reduced table have one dash. Therefore the sum of the rules represented by rules in the reduced table is $2^1+2^1+2^1+2^1$ which is equal to 2^3 or 8. No of conditions is 3 and therefore the no. of rules to be accounted for is 2^3 or 8, Therefore the reduced table is complete.

If problem has many conditions, the decision table may become quite large and difficult to follow. Since the objective of the table is to show the logic of the procedure as clearly and as simply as possible, a large, complex table should be avoided. In most cases a large problem with many conditions can be subdivided into two or more tables. One or more of the actions of the first table will specify that the user should proceed to another table to complete the logic. An example will be used to illustrate this use of more than one table.

Example 3. A sales organisation is seeking to hire some salesmen and saleswomen having special characteristics. They need only unmarried personnel between the age of 18 and 30. If male they want the salesman to be over 5½ ft in height but less than 75 kg. in weight and not bald If female, the saleswoman is to be less than 5½ ft. in height and less than 55 kg. in weight and is to have sholder-length hair. This problem has nine conditions, which would mean a table with $2^9=522$ rules before reduction. But the problem fits logically into three parts—the overall criteria, male criteria, and female criteria. This suggests that threee decision tables should be used—initial screening, male selection and female selection. The result of this use of three tables is shown below. All tables have redundancies removed.

INITIAL SCREENING		R1	R2	R3	R4
C1	Unmarried	Y	Y	Y	N
C2	Age between 18 & 30	Y	Y	N	—
C3	Male	Y	N	—	—
A1	Go to male selection table	X			
A2	Go to female selection table		X		
A3	Reject			X	X

MALE SELECTION		R1	R2	R3	R4
C1	Over 5½ ft. in height	Y	Y	Y	N
C2	Less than 75 kg. in weight	Y	Y	N	—
C3	Not bald	Y	N	—	—
A1	Hire	X			
A2	Reject		X	X	X

FEMALE SELECTION		R1	R2	R3	R4
C1	Under 5½ ft. in height	Y	Y	Y	N
C2	Less than 65 kg. in weight	Y	Y	N	—
C3	Shoulder-length hair	Y	N	—	—
A1	Hire	X			
A2	Reject		X	X	X

As a reader develops some skill, he may be able to arrive more directly at the final table. However, the beginner should proceed carefully.

EXERCISES :

(1) Complete and simplify the following decision table.

	Reservation Procedure	R 1	R 2	R 3	R 4	R 5	R R 6	U R 7	L R 8	E R 9	S R 10	R 11	R 12	R 13	R 14	R 15	R 16
C1	Request for I Class																
C2	Requested Space available																
C3	Alternate Class acceptable																
C4	Alternate Class available																
A1	Make I Class reservation																
A2	Make tourist reservation																
A3	Name on I Class wait list																
A4	Name on tourist wait list																

(2) Analyes the completeness of the following decision table :

TABLE X		R1	R2	R3	R4	R5
C1	Condition A	Y	N	N	N	N
C2	Condition B	Y	Y	N	N	N
C3	Condition C	—	N	—	Y	N
C4	Condition D	—	—	—	N	N
A1		X		X	X	
A2			X			X

(3) Prepare decision table for each of the following :

(a) A cheque is presented at a bank. The cashier has to decide what to do. The rules state that "on presentation of a cheque the cashier is required to ensure that there are sufficient funds in the account to meet the amount specified and to check that there exist no reasons why the cheque should not be honoured. Those cheques accepted and which are not outstation are not charged a handling fee, otherwise a charge at standard rates will be made".

(b) A University has the following criteria for deciding whether or not to admit a student for its graduate school :

Admit a student who has undergraduate grades of B or better, has test scores on the admission test of over 550 and has a grade average of B or better for the past two years. Also, admit if overall grade average is less than B but the last two years average is B or better and the test score is over 550. Admit on probation if the overall and 2 years grade averages are B or better and test scores is 550 or less. Admit on probation if overall grades are B or better and test score is above 550 but last 2 years are below B. Also, admit on probation if overall grades are less than B average and test score is 550 or less, but grades for past two years are B or better. Refuse to admit all others

Flowchart for a Decision Table

Example. Below is given the decision table for the wage calculation in an organisation. Gross Pay (G. P.) is derived from the Guaranteed Minimum (G.M.) as follows :

G P=1.05 GM when quantity produced, $Q > 100$
=1 15 GM when $Q > 120$
=1 25 GM when $Q > 130$

Also awarded is the quality bonus if a certain level of quality has been attained by the worker. However, in case $Q > 130$ and the worker also attains the aforesaid quality level his gross pay GP = 1 25 GM+Quality bonus is subject to an overall maximum check which is performed by means of subroutine, SR 2 with the details of which we would not concern ourselves for the limited purpose ahead In the programme also incorporated is SR 3 which validates the wage no, in a transaction i.e., it checks if the wage no, is correct. Here again we would not be concerned with the details of this subroutine which are beyond the scope of this discussion. The exit from the programme is made to a subroutine, SR 4 the details of which we ignore. The description has been captured in the first two parts of the decision table below.

Part I contains all the possible question, 5 in number.

Part II list all the possible actions,

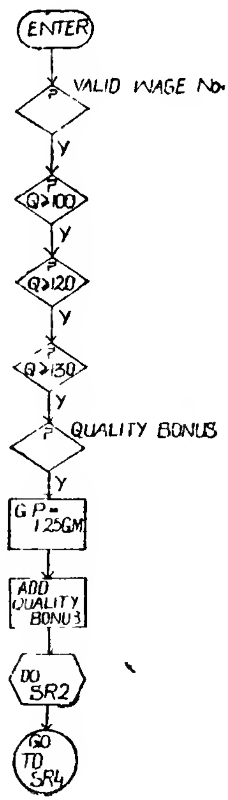
Part III lists the 9 feasible sets of answers. For example the first set has 'yes' to all the 5 questions and the last set has 'No' to the first question and it bypasses the other questions which is noted by dots in its column.

Part IV indicates, by means of crosses (×) the actions to be taken for each set of condition entries. For example, under the set of answers, 1 (all yes) there are four actions to be taken as noted by 4 crosses in the action entry column below it.

The systems analyst/programmer will first compile this decision table and therefrom draw the flowchart because he can set out the table without any likelihood of ignoring an answer set In this section, our, endeavour is to explain how the flowchart is drawn from a given table We shall take this table as an example.

N. B., Often the flowchart for a decision table (when it is small) can be drawn by common sense by comprehending its import.

	Conditions	Rules 1	2	3	4	5	6	7	8	9	
Part I Questions	Valid wage no ?	Y	Y	Y	Y	Y	Y	Y	Y	N	Part III Sets of Answer
	Qty. produced > 100 ?	Y	Y	Y	Y	Y	N	N	N	.	
	Qty produced > 120 ?	Y	Y	Y	Y	Y	N	.	.	.	
	Qty. produced > 130 ?	Y	Y	N	N	.	.	.	.	.	
	Quality bonus ?	Y	N	Y	N	Y	N	Y	N	.	
Part II Actions	Gross pay = GP	.	.	.	.	.	.	X	X	.	Part IV Sets of Actions
	GP = 1.05 GM	.	.	.	.	X	X	.	.	.	
	GP = 1.15 GM	.	.	X	X	.	.	.	.	.	
	GP – 1.25 GM	X	X	.	.	.	.	.	.	.	
	Add quality bonus	X	.	X	.	X	.	.	.	.	
	do max check SR 2	X	.	.	.	.	.	X	.	.	
	do invalid wage no S R 3	.	.	.	.	.	.	.	.	X	
	go to this table	.	.	.	.	.	.	.	.	X	
	do deductions calculations SR 4	X	X	X	X	X	X	X	X	.	



In fig. E we draw the segment of the flowchart for answers and actions of column 1 in the table.

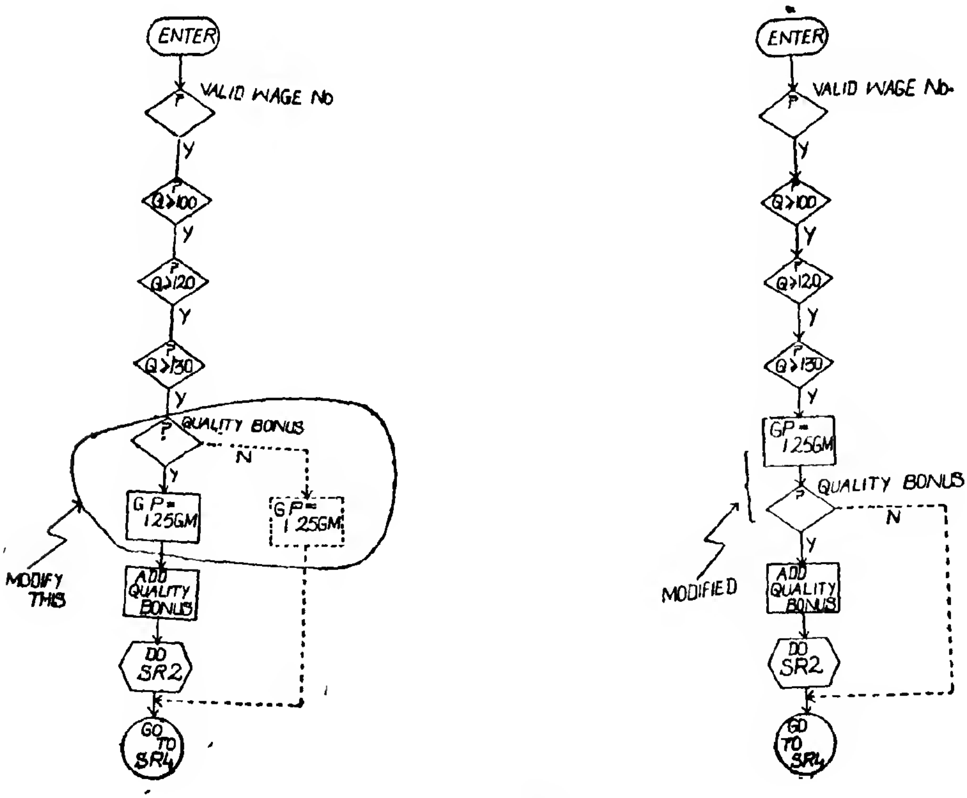
In fig F we endeavour to superimpose the segment of column 2 (shown in dotted line in fig. F) on fig. E But we see for the question "quality bonus" ? that for both 'yes' and 'no' to it we find

$$GP=1.25 \text{ G.M.}$$

Obviously then we should first compute

$$GP=1.25 \text{ G.M.}$$

and then pose this question i.e. fig. F needs modification which has been done in fig. G.



Figs. F and G

In fig. H, we have superimposed the segment for col. 3 (in crosses) onto that of fig. G and we notice that the question "quality bonus ?" has to be posed once again.

In this manner, we continue column-wise superimpositon of segments until we end up with final flowchart as in fig I below.

This shows that the flowchart is drawn by trial and error from the given table and quite a few erasures and rework would be involved. Also when the final flowchart has been drawn it can be verified against the give decision table.

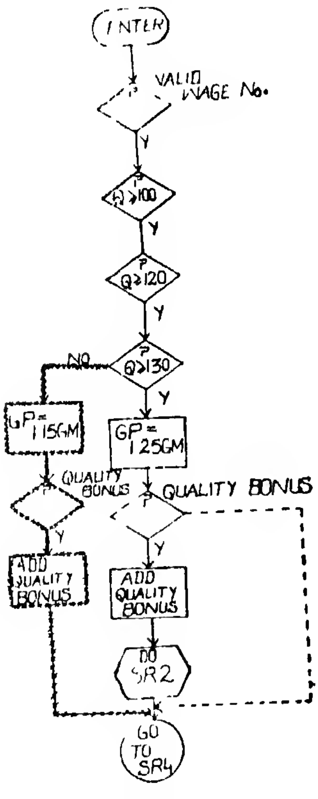


Fig. H

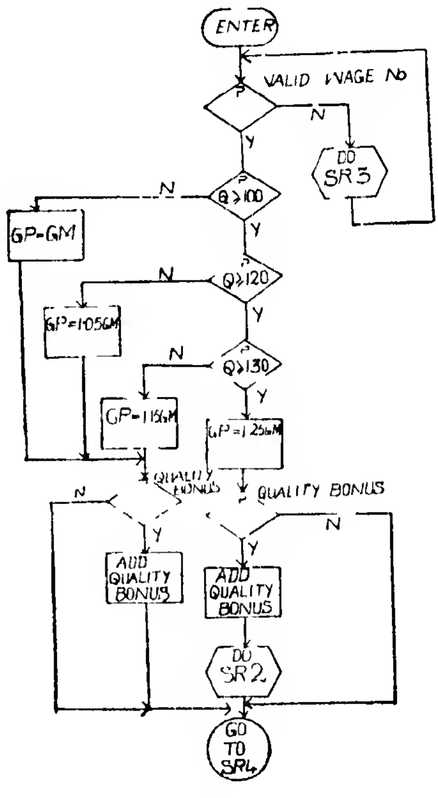


Fig. I

Advantages of Decision Tables :

(i) It is possible to check that all test combinations have been considered.

(ii) Alternatives are shown side by side to facilitate analysis of combinations.

(iii) The tables show cause and effect relationships.

(iv) They use standardised format.

(v) Semi standardised format.

(vi) Typists can copy tables with virtually no qeestion or problems.

(vii) Complex tables can easily be split into simpler tables.

(vii) They are easily understood by layman.

(ix) Table users are not required to possess Computer knowledge.

(x) Under certain cenditions, a program may be written directly from the decision table.

PART II RUN FLOWCHARTING

Several business applications (viz Inventory Control) are discussed in this part by means of run flowcharts These, however, cannot be grafted on a given organisation i e, they are just hypothetical in nature and are given here to explain flowcharting. All the steps of systems analysis and design discussed in detail in Study III have to be gone through for arriving at flowchart for the various applications for a given organisation in practice.

The transactions in these applications are punched on cards processed periodically against the master files to produce the desired reports, summaries projections etc. in a computer step (also known as the computer run). The Transactions may have to be sorted before the file up dating run. If these transactions are on punched cards these would be sorted by the key in which the master file has been arranged on a card sorter off line. If, however, the cards are on a magnetic tape they can be sorted on line in several passes in what known as a sorting run.

The punched cards have been used for the transaction files and the magnetic tape has been used for the master files throughout the study note merely for illustration, Alternative media, however, can be employed. Which media actually to employ in a given situation is again a system analysis and design matter and is adequatly by discussed in study III.

It is to be carefully noted that all the data fields appearing on the output reports, summaries, projections etc. are after all either directly picked up from the transactions/master files or derived, after some arithmetic operations, from these. If therefore the layouts of the outputs are well-defined and the input layouts designed to suit the former writing of the program for a particular up-dating run becomes simple and can be entrusted to the programmers. Incidently, we can also assert if the student is clear about layouts of outputs and of transactions/master files for an application the run flowcharts should flow out of his hands naturally. Besides, in the program have also to be incrporated some check and controls to serve as

what we call, judgement. For Example, in a manual system if a clerk finds the price of a washer as Rs. 50 instead of 50 P., he would be suspicious, run around, get the correct price and amend the records accordingly. The computer does not possess any such judgement and it would accept Rs. 50 as the correct price for a washer and even pass the invoices sent by the supplier. Therefore there is a need to build in checks and controls in the pragram so that, such errors or omissions can be detected and brought to the notice of the computer users Invariably, in an up-dating run one of the outputs is an error list and summary information which contains such errors and the batch totals computed by the computer. These totals are then compared with the ones derived manually prior to processing as discussed in study I, and the errors detected via the checks and controls are scrutinised by the user departments and ultimately rectified and re-submitted for computer processing.

Another point which needs some explanation at his stage is the term, integration. Integration of sub-systums implies use of the same transaction copies and files.

There are several gradations of integration. In a rudimentary way, it can be had even in manual systems. For example, a company has a simple integrated procedure for processing goods received notes. In this procedure, the receiving organisation puts a stamp on the back of the advice note received along with the goods from the suppliers. This stamp carries all the usual particulars to be found on a goods receipts note. This single copy of the advice note then travels from the receiving section through the inspection, stock control, purchasing to invoicing section etc. All these sections update their files with this single copy and make the appropriate noting on it. This is an example of the integration of various departments i.e., all of them use the same copy of the transaction in contrast to un-integrated systems where the goods receipt note is prepared in several copies and each department uses its own copy. In all the applications discussed in the study note, this concept of multiple use of the same transactions is to be found. The transactions may be put on punched cards or magnetic tape, etc. for computer processing but they are never multiplied in several copies. Integration at a higher level also implies fusing of the various files into a data base and is discussed in more detail in study III.

1. Sales

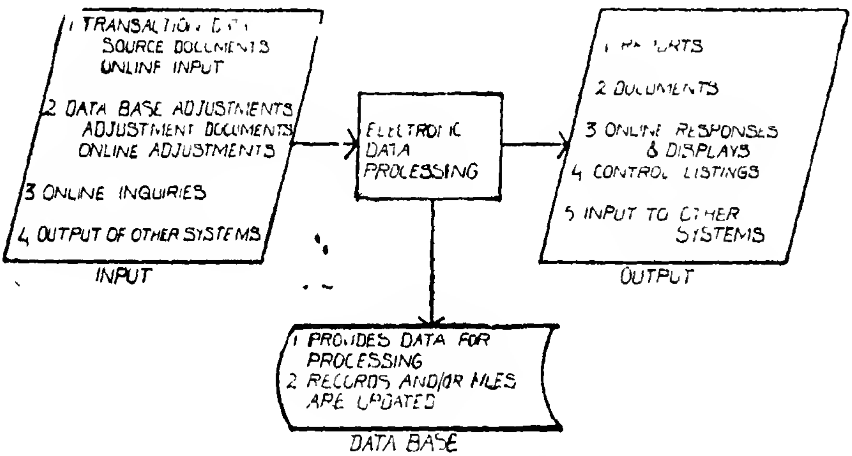
A computer-based sales order processing system characterised by the following possibilities of integration:—

1. Sales accounting and sales analysis are integrated with accounts receivable operation and records.
2. Credit functions can be integrated as far as possible into the overall flow of automated processing.
3. The sales application is closely related to inventory control processing.

Integration of credit functions is based on the assumption that credit review contains elements that are capable of being handled by a computerised system. This assumption is valid if credit review involves the application of set of criteria in a systematic way without any resort to subjective considerations. Nevertheless, even if the credit review cannot be completely formalised and automated the bulk of routine computations can still be performed by the computer. The rejections would contain the special cases that would be reviewed subjectively later.

General Form of Business Applications

The chart of figure below shows the general form of business applications. It emphasises that all the outputs are prepared from the inputs (transaction and master files).



GENERAL FORM OF BUSINESS COMPUTER APPLICATIONS

The flowchart in figure 1 to 3 depict a typical sales order processing system

1.1 Input preparation

The sales department prepares the sales order in duplicate upon receipt of the customer's purchase order or telephonic information from a salesman or the customer after satisfying themselves that the customer's account is not delinquent. One of the outputs of a computer run to be discussed subsequently is the list of delinquent accounts which the sales department consults to establish credit worthiness of the customer. One of the copies of the sales order is sent to the customer as an acknowledgement of the receipt of his purchase order. The other copy is sent to the shipping department as an authorisation to ship the goods to the customer. The format of the sales order should be so designed that it facilitates error-free and quick entry of data from the customer's purchase order as also transcription of data from it on to punched cards subsequently. The sales department enters the quantity shipped and quantity back ordered against each line on the order, i e., for each stock item on order, the price having already been entered by the sales department. The

shipping department assembles the sales order in daily batches and compiles the following batch totals for it on an adding machine, perhaps : record count, financial total of the values of items shipped and hash totals of quantities shipped, quantities back-ordered, customer's account numbers and stockitem numbers The batch of the sales orders together with the control slip bearing these batch totals is then forwarded to the data processing department for transcription on the punched cards.

Likewise, the mailroom assembles the daily batch of remittance advices received from the customers and compiles the following batch totals for it : record count, financial total of the amounts remitted and the hash total of the customer account numbers. The batch together with the control slip bearing these totals is forwarded to the data preparation section

We are dealing with only two types of transactions for this application for simplification of illustration though in practice there would also be the following inputs :

(i) Addition of new records to the file

(ii) Credits for sales returns and allowance

(iii) Account right-offs

(iv) Changes of addresses and other routine adjustment and corrections.

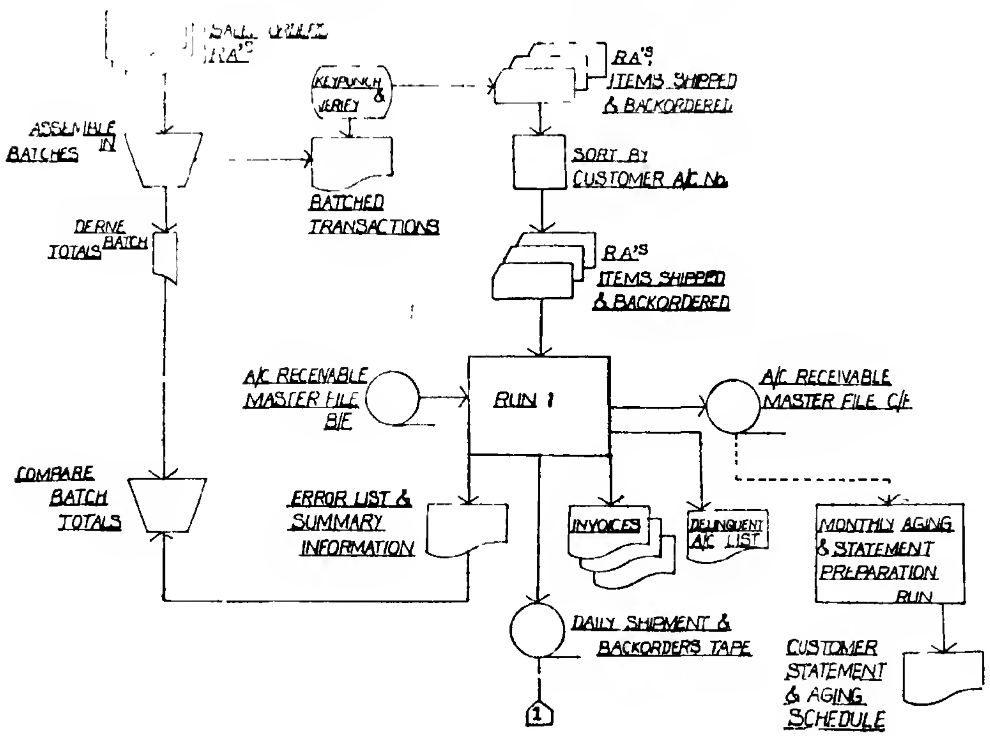


Fig. 1

The data preparation section of the data processing department prepares a punched card for each item back-ordered, each item shipped and each remittance advice on the key punch The deck of punched cards is then given to the verifier operator who verifies the cards by re-entering the transaction data via the verifier keyboard For economies in verification time it may, however, be desired that just the critical data fileds on the punched cards are verified.

The deck of punched and verified cards is then sorted off-line on a card sorter with the customer account number as the key.

The sorted deck of punched cards constitutes the transaction file which is used to update the accounts receivable master file by the account receivables update program in computer run. In this run, customer accounts are updated for the sales. Typically, the accounts receivable master file contains the following fields:

(i) Customer account number (control field).

(ii) Customer name and address

(iii) Credit rating.

(iv) Credit limit.

(v) Balance due as of last monthly statemet.

The following particulars of each transaction since then :

(i) Transaction type code.

(ii) Documents number.

(iii) Date.

(iv) Amount.

(v) Current balance.

Incorporated in the programme are also the various control cheks discussed in detail in study III which check the picture, limits etc. of the various data fields. The ouputs of this run are as follows:—

(i) The updated A/c receivable master file

(ii) Delinquent accounts list which contains the particulars of those customers who have crossed either the credit limit or the credit rating assigned to them.

(iii) Invoices in as many copies as desired. It has to be noted that invoice would be prepared for only the items shipped. A specimen of the invoice is given below :—

XYZ Manufacturing Company

15, High Street, Sometown Tx............Tel..............Invoice No..............

INVOICE

Customer Order No.	Date	Salesman Code	Cost Acct No.
....	......	....	....

Sold to	Ship to
ABC Mfg. Co.	Same
13. Nehru Road	
Allahabad	

Shipper	Date Shipped	Invoice Date	Terms of Sale
....	....	....	....

Item Code	Description	Qty. ordered	Qty. Back-Ordered	Qty. shipped	Unit Price	Total Price

(iv) *Daily Shipment and back-orders tape.* The punched cards data is put on to magnetic tape for a subsequent speedier resorting by stock item number as well as speedier pocessing against inventory file the discussion on which would follow.

(v) *Error list and summary information.* This contains the rejects i.e., those transactions which could not pass the control checks. These are ultimately investigated by the user department, rectified and re-submitted to the data processing department for re-processing. The

summary information consists of all the batch totals derived by the computer. These batch totals are compared with the ones derived manually prior to processing and entered into control slips travelling with the batches of the transactions.

The accounts receivable file is also processed every month to produce the customer statements and aging schedules a specimen of which is shown below.

Statement
XYZ Manufacturing Company

15, High Street
Sometown

To

ABC Limited — Date — Account No
13, Nehru Road
Allahabad

Date	Invoice Number	Charges	Credits	
				Previous Balance .... Current Account .... Total Amount

Past Due Amounts
Over 30 days. ......
Over 60 days. ....
Over 90 days.........

1.3. Inventory processing

The daily shipment and back-orders tape obtained as an output of a run in the previous flowchart is used to update the finished goods master file according to the concept of integration discussed at the outset of this part. Since however, the finished goods master file is sequenced by the product stock number as the key, the daily shipment and back-orders tape is also sorted in this order. In the daily finished goods update run, sorted daily shipment and back-orders tape constituter the transaction file and by updating finished goods master file produces the following output :

(i) *Finished goods master file* is updated for the various stock balances in it, Back order-sub-records may also be added

Back-orders and replenishment orders list The items for which the stock balance has fallen below the recorder level would find place in the replenishment orders list 'Backorder' means a consumer's order for an item against which goods cannot be supplied now but would be supplied as and when they are received from the works

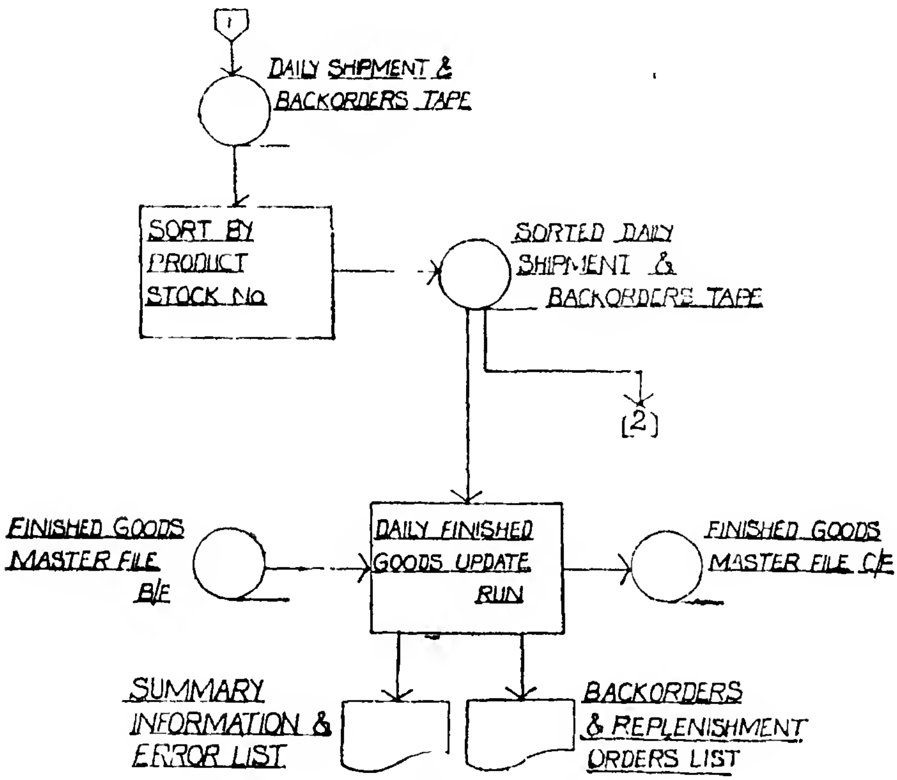


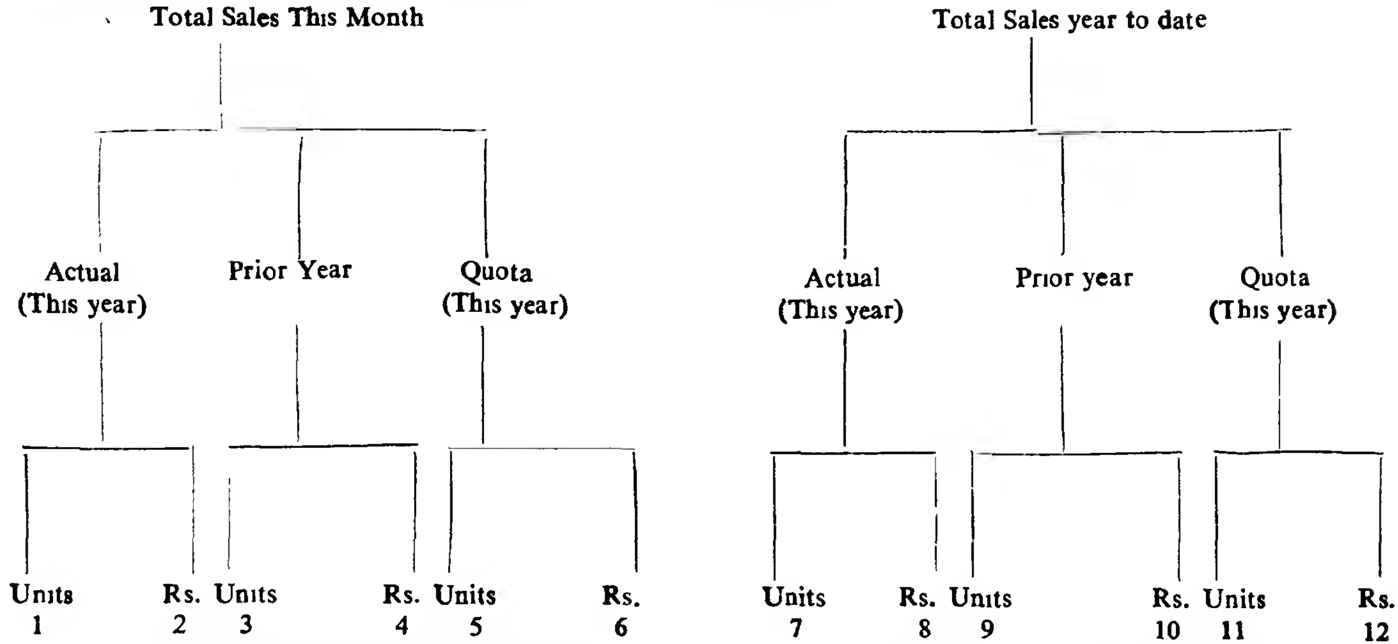
Fig. 2

(ii) *Summary information and error list.* The summary information consists of the vaious batch totals derived by the computer. These will ultimately be compared with their counterparts derived manually prior to computer processing. The error list would contain erroneous transaction which could not pass the various checks built in the computer programme.

The sorted daily shipment and back orders tape, in consonance with the concept of integration, is used in the following flowchart for sales analysis runs.

1.4. Sale analysis runs

The sales analysis are made by updating the sales summary Master File by the daily shipment and back orders tape. The sales summary file consists of the following data fields :—



This set of 12 data fields appears against each combination of a product, a customer and a salesman in the sales summary master file maintained on magnetic tape. For example, these 12 data fields would be there for Ramu (a salesman) who is selling refrigerators to P (caustomer). Likewise, these 12 data fields would be there again for Ramu who is selling refrigerators again to Q (another customer), and so on. In other words, for each combination of a salesman, a product and a customer this set of 12 data fields would be there.

To make the layout of the sales summary master file more clear let us take up the simple example of three products : Refrigerators, Air Coolers and Exhaust Fans; three customers : P, Q and R and three Salesmen . X, Y and Z.

The layout of the file sequenced by this product stock number as the key is pictured below :

Refrigerators	* PX	PY	PZ	QX	QY	QZ	RX	RY	RZ	Air Coolers	PX	PY	PZ	QX
	1-12	1-12	1-12	1-12	1-12	1-12	1.12	1-12	1-12		1-12	1-12	1-12	1-12

(Contd.)

*PX 1-12] Stands for 1 to 12 data fields of the chart above for refrigerators being sold to the costomer P by the Salesman X.

The Layout of the same file sequenced by the customer number is pictured below :

PX Ref	*PY Re	PZ Ref	PX Air	PY Air	PZ Air	PX Exh	PY Exh	PZ Exh	QX Ref
1-12	1-12	1-12	1-12	1-12	1-12	1-12	1-12	1-12	1-12

(Contd.)

*PY Ref 1-12] stands for the 1-12 data field for refrigerators being sold to the customer P by salesman Y.

The layout of the same file sequenced by the salesman code is pictured below :

PX Ref	QX Ref	RX Ref	*PX Air	QX Air	RX Air	PX Exh	QX Exh	RX Exh	PX Ref
1-12	1-12	1-12	1-12	1-12	1-12	1-12	1-12	1-12	1-12

(Contd)

*PX Air 1-12] Stands for the 1-12 data fields for the Air Coolers being sold to the customer P by the salesman Y.

(Note : In an actual situation several of these sub-records would be missing e.g., if the customer P does not buy air coolers the air coolers sub-records against this customer would not be there).

It is assumed, however, that to start with the sales summary master file is sequenced by the product stock number and, therefore, it is straightway upiated in run 1 by the daily shipment and back-orders tape. As an output, the sales analysis report by product stock number is obtained.

The sales summary master file is then sorted by the salesman number as the key and in run 2 a printout of the sales analysis by salrsman is obtained.

The sale summary master file is finally sorted by the customer number as the key and in run 3 a printout of the sales analysis by customer is obtained.

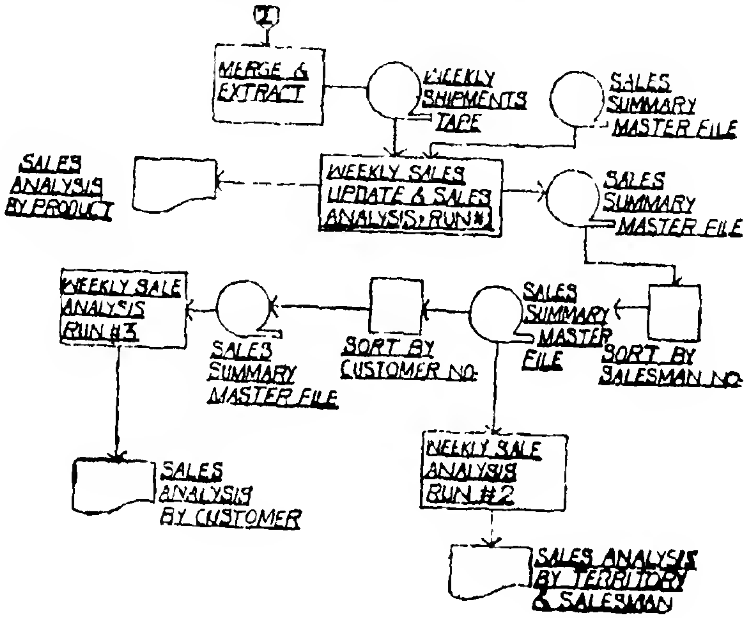


Fig. 3

SALES ANALYSIS BY SALESMAN

Period ending : August' 79

Salesman	Period	Sales		% Change	Quota	% Variance from quota
		Actual	Prior year			
Panikar	This month Year to date	Rs. 7,500 60,000	Rs. 8,000 75,000	−6.25% −20%	8,000 80,000	−6.25% −25%
Natrajan						
⋮						

2. Payroll

The basic purpose of the payroll system is to produce payslips and pay-cheques for the employees every month. Towards this purpose, the employees payroll file is run against their attendance cards, both being sequenced by the employee number. The attendance sheet of each employee may include the following data fields :

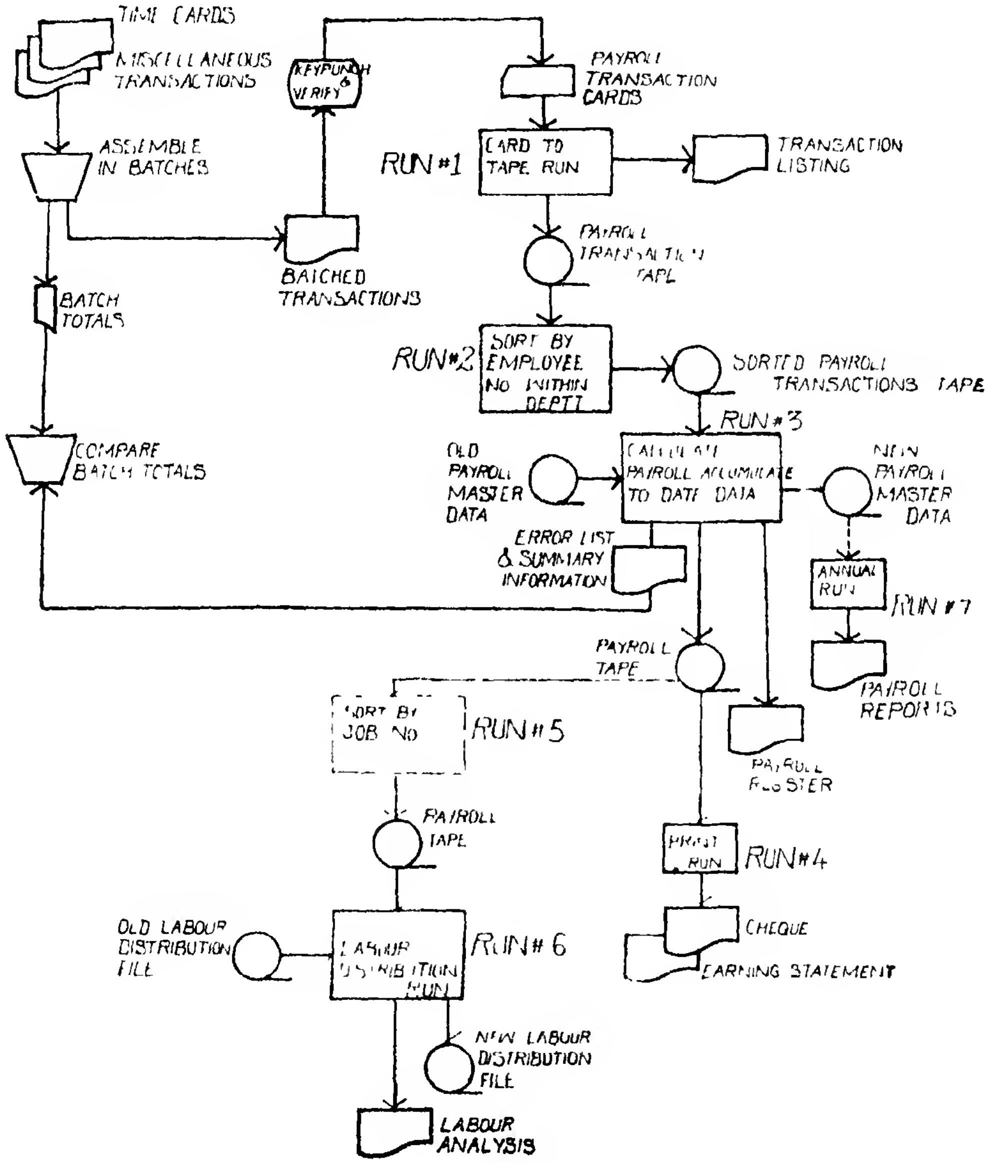


Fig. 4

Transaction code	1 Characters
Employee No.	6 Ch.
Deptt. No.	2 Ch.
Job No.	6 Ch.
No. of days worked	2 Ch
Special Deductions (Loan, etc.)	6 Ch.

Since each attendance records carries 6+2+6+2+6=22 characters it would be desired to put three records in a punched card.

Besides the attendance records the transaction would also include additions, deletions and amendments to the payroll file originating from the payroll section. These are also laid out on the punched cards.

In run # 1, the payroll transaction punched cards are put on magnetic tape for subsequent speedier processing. Also printed out is the transaction listing for control and audit purposes.

In run # 2, the payroll transaction file is sorted by the employee number within department number.

In run # 3, the sorted transaction file is used to update the payroll master file on magnetic tape. The latter has the followings typical fields.

Employee Number
Employee Name
Designation
Department Number
Job No.
Marital Status
Basic Pay
Type of cost code
Amount of authorised deductions (several fields)
Year-to date earning total (several fields)

The type of cost of code in the account number indicates the nature of the account to which the employees' gross pay is charged—direct labour, indirect labour. etc.

Gross and net pay are calulated in this run· year-to date and period-to date data are accumulated and written on the payroll master file, the data necessary f. r printing the payslips and the paycheques is put on a magnetic tape constituting another output of this run. Also,printed are the Error List and Summary Information and the payroll register.

In run # 4, the payroll tape is used to produce payslips and paycheques.

In run # 5, the payroll tape is sorted by job number, which in run # 6 is used to update the labour distribution file with a specimen format given below ;

Labour Distribution File on Magnetic Tape

Job No.	Trade 01	Value	Trade 02	Value		Trade 40	Trade
6, Ch	2 Ch.	6 Ch.	2 Ch.	6 Ch.	.....	2 Ch.	6 Ch.

The cumulative valued of various trades *as at the end of the last month are* updated to *as at the end of this month* by means of the payroll tape.

The labour Analysis is printed for each job as another output of this run.

In run # 7 (which is undertaken annually), the payroll master file is used to produce annual payroll reports.

Accounts payable Application (Fig. 5)

1. The purchasing department receives the invoices from the vendors. After verifying and editing each of these it prepares a disbursement voucher for it by assigning the voucher number serially and the vender A/c number. A specimen disbursement voucher is shown below :

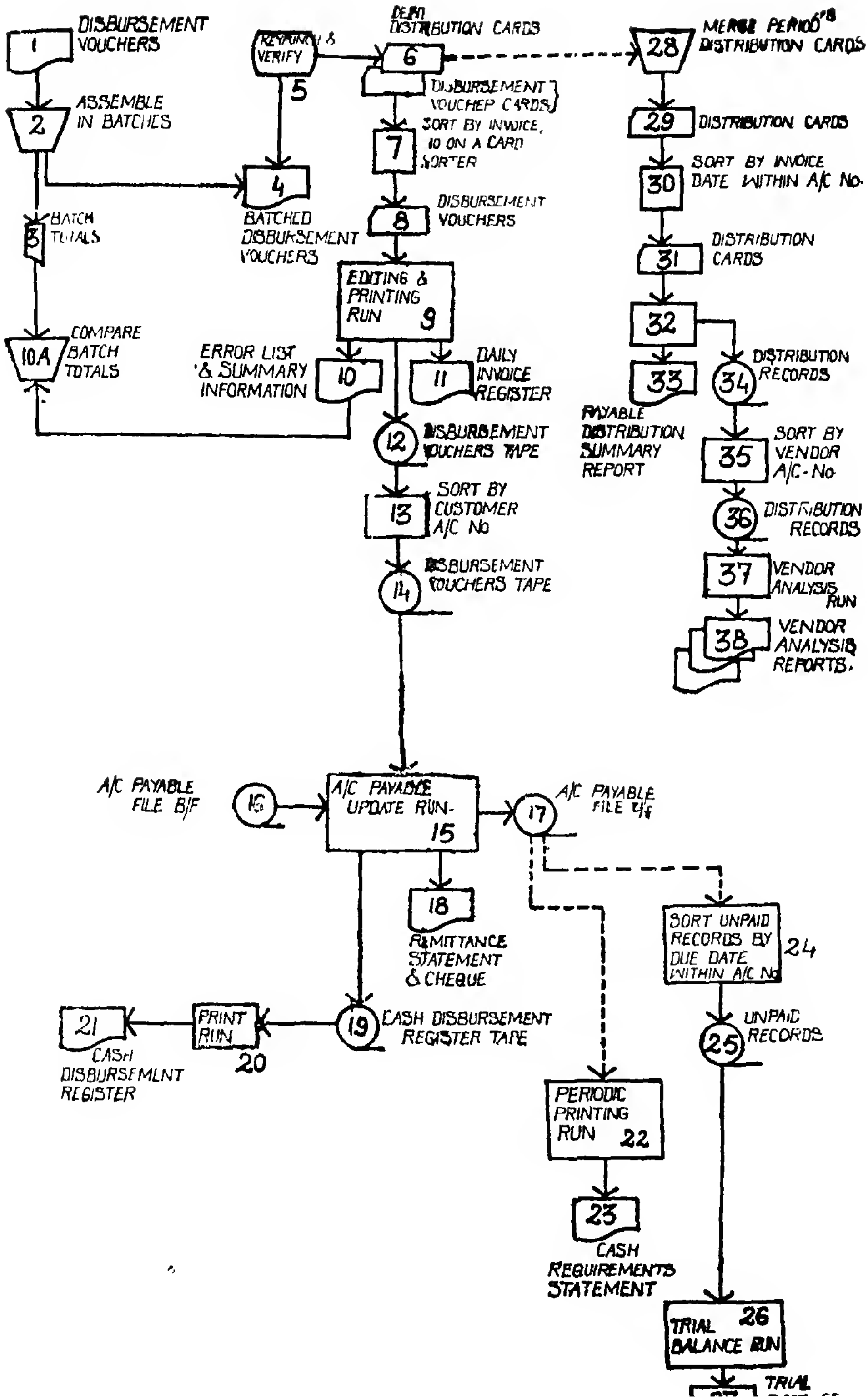
ACCOUNTS PAYABLE VOUCHER

Vendor Name : Kailash Hardware Stores
14, Laxman Road, SURAT

Vendor A/c No.	Invoice Date	Invoice Number
1321	16-7-79	2430
Voucher No.	**Due Date**	**General Account**
627	26-7-79	13-47-81

Approved by	Invoice Amount	96.8.30
	Discount	1936
(Signatures)	Net Amount	949.04

Account No.	Sub-code	Amount
326	185	339.84
326	114	101.06
326	227	527.40



1 DISBURSEMENT VOUCHERS
2 ASSEMBLE IN BATCHES
3 BATCH TOTALS
4 BATCHED DISBURSEMENT VOUCHERS
VERIFY 5
6 DISTRIBUTION CARDS
DISBURSEMENT VOUCHER CARDS
7 SORT BY INVOICE, 10 ON A CARD SORTER
8 DISBURSEMENT VOUCHERS
EDITING & PRINTING RUN 9
10A COMPARE BATCH TOTALS
ERROR LIST & SUMMARY INFORMATION 10
11 DAILY INVOICE REGISTER
12 DISBURSEMENT VOUCHERS TAPE
13 SORT BY CUSTOMER A/C No
14 DISBURSEMENT VOUCHERS TAPE
28 DISTRIBUTION CARDS
29 DISTRIBUTION CARDS
30 SORT BY INVOICE DATE WITHIN A/C No.
31 DISTRIBUTION CARDS
32
33 PAYABLE DISTRIBUTION SUMMARY REPORT
34 DISTRIBUTION RECORDS
35 SORT BY VENDOR A/C-No
36 DISTRIBUTION RECORDS
37 VENDOR ANALYSIS RUN
38 VENDOR ANALYSIS REPORTS.
A/C PAYABLE FILE B/F 16
A/C PAYABLE UPDATE RUN 15
17 A/C PAYABLE FILE C/F
18 REMITTANCE STATEMENT & CHEQUE
19 CASH DISBURSEMENT REGISTER TAPE
PRINT RUN 20
21 CASH DISBURSEMENT REGISTER
SORT UNPAID RECORDS BY DUE DATE WITHIN A/C No 24
25 UNPAID RECORDS
PERIODIC PRINTING RUN 22
23 CASH REQUIREMENTS STATEMENT
TRIAL BALANCE RUN 26
TRIAL

2. *The disbursement vouchers for the day are assembled in a batch.*
3. *The following batch totals are derived on an adding machine.*
   (a) Record Count i.e. the number of disbursement vouchers in the batch.
   (b) Hash totals of the vendor A/c numbers.
   (c) Financial total of the net amount data field.
4. The batch of the disbursement vouchers headed by the control totals slip is forwarded to the computer data processing department.
5. In the keypunching section, for each disbursement voucher a disbursement voucher punched card is prepared on the keypunch as also as many debit distribution cards as the number of item lines are prepared on the keypunch. For visual differentiation, the disbursement voucher and the debit distribution punched cards are of different colours. The data fields of the two sets of cards given below:

Disbursement Voucher Cards	*Debit Distribution Cards*
Entry Date	Entry Date
Invoice Date	Invoice Date
Invoice Number	Invoice Number
Vendor Number	Vendor Number
Vendor Invoice Number	Vendor Invoice Number
Gross Amount of Invoice	Department charged
Discount, if any	Invoice or Item Amount
Net Amount of Invoice	

   The punched cards verified on the card verifier. The 2 decks of the two sets of punched and verified cards are shown in symbol 6.

7. The disbursement voucher cards are sorted on a card sorter by the voucher number as the key.
8. The sorted deck of disbursement voucher cards.
9. *Editing and Printing Run.* Various checks are applied to the disbursement voucher transactions. Those transactions that fail in these are printed in the error list (10) on which are also printed the batch totals as the summary information as derived by the program in this run. The daily invoice register in the following format is produced as another output. The disbursement voucher data is put into magnetic tape (12) for subsequent speedier processing.

BHARAT STEEL APPLIANCES COMPANY

Daily Invoice Register

Entry Date	Invoice Date	Vendor Invoice No.	Vendor No.	Our Voucher	Entry Code	A/c No. / Gen. Sub	Deptt. Charged	Item No.	Due Date	Quantity / Discount	Item Amount / Invoice Total	Net Payable

10A. Error List and Summary *Information is compared with the batch* totals derived on an adding machine prior to computer processing. Also the erroneous transactions are investigated in the purchasing department. These would be resubmitted for processing after rectification.

13. The disbursement voucher tape (12) is sorted by the vendor A/c no. in this run and gives the sorted disbursement voucher tape (14) as its sole output.

15. *A/c Payable Update Run.* The A/c Payable file (16), typically containing the following data fields is updated in this run by this disbursement vouchers tape (15).

A/c Payable File (16).

Vendor A/c No. (Control field)

Vendor name and address

Vouchers Payables

Voucher number

Vendor's Invoice number

Date of payment due

Invoice Amount

Discount

Total Amount Payables.

The program scans each vendor record for those earlier disbursement vouchers for which payment is due today, prints cheques and remmittance statements for them, and deletes them from A/c Payable File (16). The new disbursement vouchers are added to their respective vendor records. This run yields the following outputs :

A/c Payable File, C/F (17)

Remmittance Statement & Cheques (18)

(Specimen format shown below)

Cash Disbursement register Tape (19).

20. In this computer run, the cash disbursement register (21) is obtained as the output. Its specimen format is also shown below :

BHARAT STEEL APPLIANCES CO.

Accounts Payable

Route Cash Disbursement Register Date

Vendor	Cheque No.				A/c Payable	Credit	
	Vendor No.	Day	Month		Date	Discount	Cash

BHARAT STEEL APPLIANCES CO.

Statement of Remittance

Cheque No.	Invoice Date	Vendor's Invoice No	Code	Invoice Amount	Discount	Net Amount

22. In this periodic (weekly, perhaps) run, the A/c payable file is used to produce the Cash Requirements of which the specimen format is shown below :

BHARAT STEEL APPLIANCES CO.

Accounts Payable

Cash Requirements Statement

Route to Mr. Subir Bose, Deptt. 101 **Date**

Vendor	Vendor No.	Due Date	Invoice Amount	Discount	Cheque Amount
Omega General Suppliers	1421	16/4	773.30	15.47	757.83
Peco Engg. Co.	1426	16/4	1620.18	32.42	158[illegible].78

24. In this run which is had at the end of each accounting period, the unpaid records on the A/c Payable file are extracted, sorted and put on a tape as the unpaid file (25).

26. From the unpaid records file (25) the trial balance (27) is derived in this run and printed out in the specimen format shown below.

BHARAT STEEL APPLIANCES CO.

Trial Balance

Open Items—Payable

Closing Date

Vendor	Vendor A/c No.	Invoice Date	Due Date	A/c Payable	Dis-counts	Net Payables
				Rs.	Rs.	Rs.

28. At the end of the A/c period all the distribution cards are merged in one deck (29).
30. The distribution cards are sorted by the invoice data within the A/c number on a card sorter to obtain the sorted deck of distribution cards (31).
32. In this computer run, the A/c payable Distribution Summary is printed under the following format :

BHARAT STEEL APPLIANCES CO.
Accounts Payable Distribution Summary

Date

Entry Code	Invoice Date	Vendor Name	Vendor A/c No.	Our Voucher No.	A/c No. Gen.	A/c No. Sub	Deptt. charged	Due Date	Qty.	Inv. Amt.	Gen. Led.

35. In this run, the distribution records (34) for the period are sorted by vendor A/c no.
37. The sorted distribution record tape (36) is used in this run to produce the various vendor analysis reports (38) of which a specimen format is given below.

BHARAT STEEL APPLIANCES CO.
Summary of Purchases

Account Name	Account Gen.	Account Sub.	Sub account total	Gen Account total
Raw Materials Brass Forgings Castings Rolled Stock Baked Parts Lumber				

INVENTORY CONTROL

Example : Company 'C' has mechanised its stocks and inventory system.

The following are the inputs for a month

(*a*) *Receipts with rate*

(*b*) *Issues.*

The following reports are to be prepared .—

(*a*) *Ledger*

(*b*) *Department-wise consumption report*

(*c*) *Excess Stock*

(*d*) *Under Stock*

(*e*) *Slow-moving items.*

(*f*) *Items-to be ordered tape.*

Draw a run flowchart and explain the computer processing.

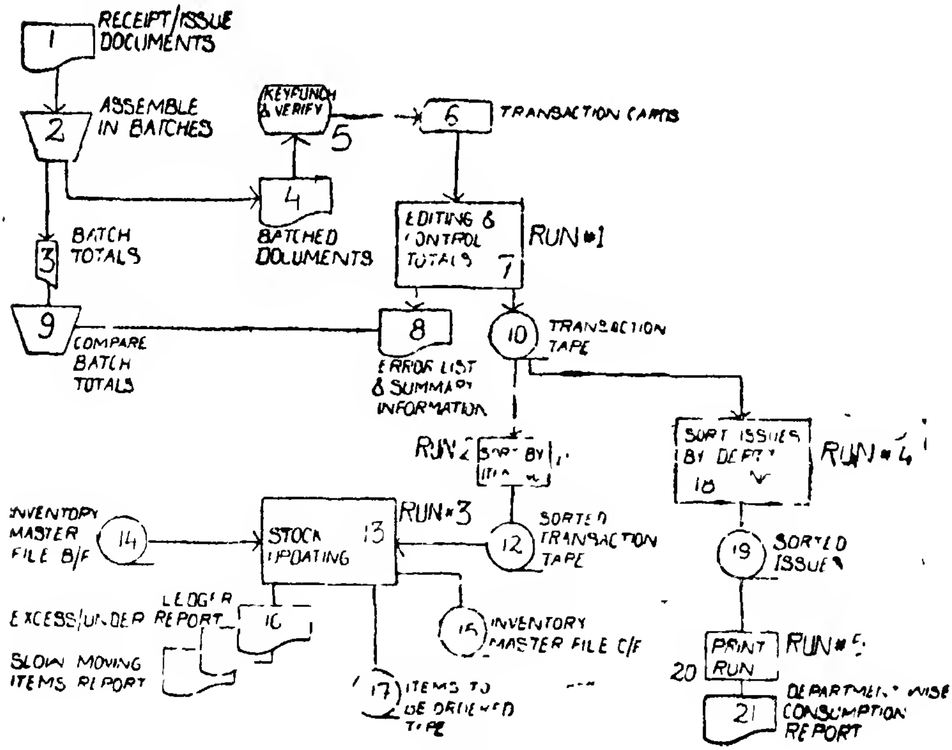


Fig. 6

Run Flowchart

This application on hand requires 5 runs. Each run has its own program and is performed in a computer setup after loading this program in the computer memory. The flowchart depicting the inputs to and outputs from the various individual runs for an application is known as the run flowchart or the block diagram. The run flowchart for the application on hand is shown in fig. 6 The symbols in it are numbered from 1 to 21 to facilitate the following explanation.

1. The receipt and issue notes are prepared in the stores depatment as and when these transactions arise. It is desirable that the format of such source documents are so designed that they facilitate transcription of data from them into punched cards or other computer media. For example the maximum length of each data field in them may be emphasised by putting as many boxes for it If, for example, the quantity received can not exceed 5 numerics 5 boxes could be placed and the actual quantity received may be entered in these right justified An incorrect 6 digit quantity would then have little chance for entry. Also the receipt and issue notes may have different colours The notes in a pad should be serially numbered to keep track of each of these.
2. The receipt/issue notes for the day are assembled in batch.
3. The following control totals for the batch are derived on an adding machine, perhaps :—

   (i) Record Count that is the number of the notes in the batch.

   (ii) Hash totals of the item numbers, quantities received and issued.
4. The batch of the receipt/issue notes alongwith the control slip bearing the batch totals is transmitted by the stores department to the data preparation section.
5. A card is punched for each transaction. A specimen layout of the transaction card is given below.

Transaction Code 1 Ch	Item Code 5 Ch.	Date of transaction 6 Ch.	Voucher No. 5 Ch.	Qty. received/issued 6 Ch.	Deptt. No. 2 Ch.	(It can be seen that one punched card can accommodate 3 receipt/issue transactions).

The punched cards are also verified on the card verifier by an operator who is different from the keypunch operator and who uses the same source documents (goods receipt notes and issue notes)

6. The deck of transaction cards which is used as the input to run 1.
7. In this run, editing is performed on the transaction cards. A couple of the possible checks that can be performed are;

   (i) Check digit on the item code, and

   (ii) Transaction code check. (See Study III for details)

Those control totals that were derived manually prior to processing are now derived by the program. The Error List and Summary Information (8) lists the erroneous transactions and also the control totals which are checked manually (9) with their opposite numbers derived manually (3) prior to processing. If they do not tally investigations for the accidental/fraudulently motivated reasons would be under taken also, the Error List would be forwarded to the stores department to scrutinise the erroneous transactions, rectify and resubmit there for reprocessing.

10. The transactions are output on a magnetic tape for subsequent speedier processing
11. In this run, the transaction tape (10) is sorted by the item numbers as the key to give the sorted transaction tape (12).
13. In this run, the sorted transaction tape is used to update the inventory master file on magnetic tape the layout of which is given below.

....	Item code	Description	Units	Units Price	Physical balance	E. O. Q	ROL	Max. Level	Min. Level	Date of last transaction	: ... :

The following are the outputs of this run.

Inventory Master file C/F (15)

Items to be ordered tape (17) containing particulars of those items for which the stock level has fallen below the reorder. This tape may be used in another routine to produce purchase requisitions or orders, but that is beyond the scope of this application.

Printouts of the stock ledger, Excess/Under Report and Slow moving items report (16) under the following formats

STORES LEDGER

Month *Year* *Part No.*
Description *Order Point*
Batch Qty. *Min Level*
Max Level

Date	Voucher No.	In	Out	Balance		Qty.	
				Qty.	Value	Excess	Under

(Excess Qty. is stock above the max. level and Under Qty. is the stock below the min. level).

Report on slow Moving Items Month. .						
Part No.	Balance Qty /Value No. Movements Since					
	6 months		1 year		2 year	
	Qty	Value	Qty.	Value	Qty.	Value

18. The transaction tapes for the period are merged, recipts ignored and sorted by the department number in run 4 to give the sorted issues tape (19).

20. In this run the sorted issue tape (19) is used to produce the printout of the departmental consumption report (21) the forma of which is given below.

Department-wise Consumption Report		
Deptt. No.		Month
Part No.	Consumption	
	Qty.	Value

Purchase Application

(The flowchart of fig. below may be taken in continuation to the one of fig. 6, p 120).

23 In this run, the open purchase order file (22) is sorted by the vendor A/c number.

27. In this run, the items to be ordered tape is also sorted by the vendor A/c number.

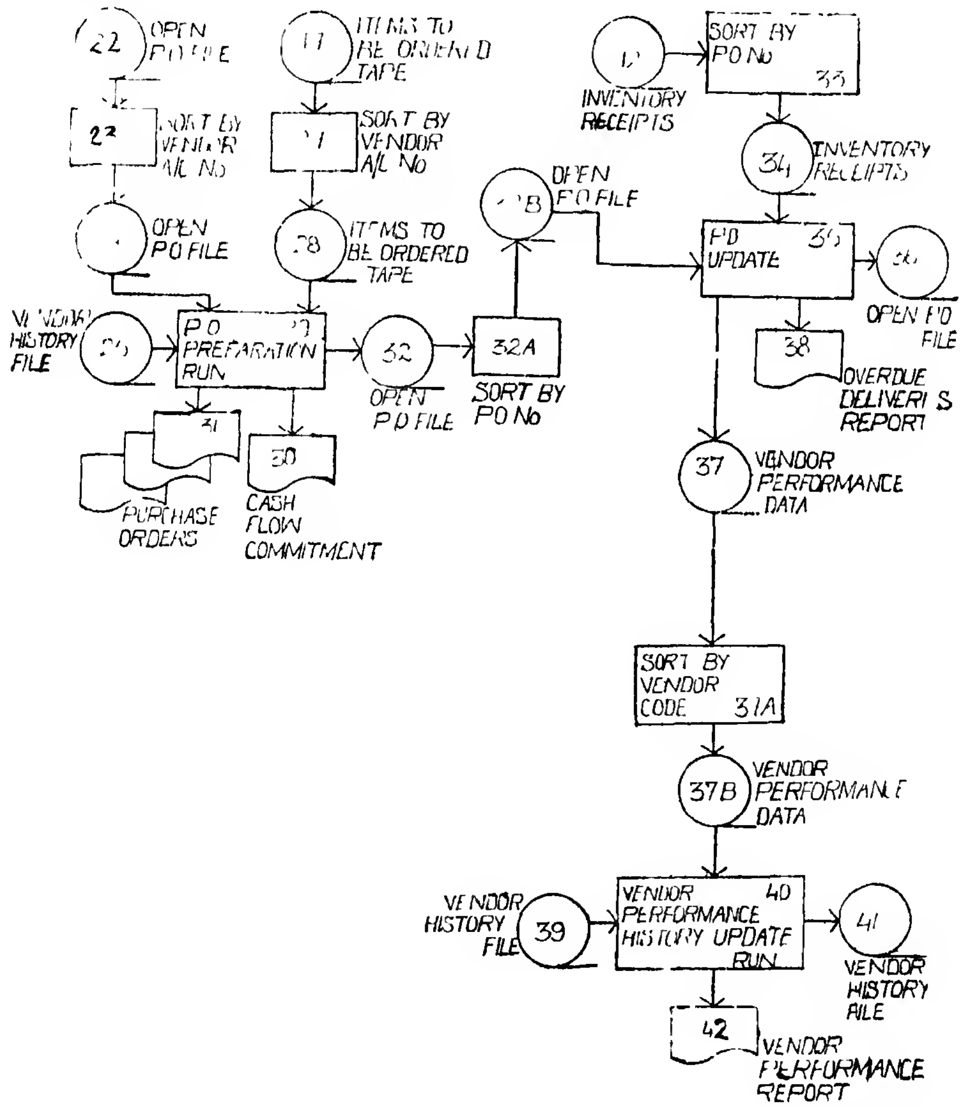


Fig. 102

29. The open purchase order file (24), the items to be ordered tape (28) and the vendor history fils (25) constitute the inputs to this run. The purchase orders (31) are prepared in this run, the particulars of the items to be ordered tape and the particulars of the vendor favoured with the order are got from the vendor history file. Also printed out is the cash flow commitment statement. The open purchase order file (24) is updated i e. new purchased orders are entered for the various vendor favoured with the orders.

33. In this run the inventory receipts tape (12) of fig. 6 is sorted by vendor A/c. No.

35. In this run, the inventory receipts tape (34) used to update the open purchase order file (32) i e. the receipts to date are accumulated and the fulfilled orders deleted. Also printed out is the overdue deliveries report (38) and the vendor performance data tape (37) is produced.

40. In this run, the vendor performance history file (31) (containing price index, delivery index, quality index, etc.) is updated and the vendor performance report (42) compiled.

Element of FORTRAN & COBOL

FORTRAN (FORmula TRANslation) was originally developed for solving problems of engineers/scientist/mathematicians but it is in fairly wide use in business situations, particularly for applications leaning heavily on methematical models and/or statistical analyses. FORTRAN has the following major 4 types of statements :

1. *Arithmetic Statements* for addition, subtraction, multiplication, division and exponentiation (raising powers). These are denoted by +, —, *, and ** respectively. These statements can operate on integer constants (e g. 17), real constants (with decimal point, e g 41.04) and both real or integral variables. Real variable (say, WAGE) would assume values in decimal and integer variable would assume only integral values. As per rules of the language integral variables must be symbolised starting with any of the letters I to N. For example, marks of students may be symbolised by MARKS but not SCORE since the latter start with S which is beyond I to N ; it is, however, being assumed that marks are awarded only in integral values. The general form of the arithmetic statement is

$$l = r$$

by which the value of the expression on R. H. S. is assigned to the variable on the L. H. S. Therefore, often the arithmetic statements are also called assignment statements in FOTRAN.

Examples : J=105/7 (*J assumes value of 21 Note that P=21 is wrong. Why ?*)

S=A * B

S=A+C

S=A * * 3 (R.H.S. is A^3)

X=Y−Z * A/B * * 2+C

[is executed like mathematical equation

$$X = Y - \frac{ZA}{B} + C \;]$$

X = (Y+Z)* (A/B) * * 2−P

[is executed like mathematical equation

$$X = (Y+Z)\,(A/B)^2 - P].$$

For such mathematical operations as taking square root or finding log or finding the cosine there is standard vocabulary : SQRT, ALOG, COS respectively.

2. *Input/Output Statements* are two in number : READ and WRITE. The former is used for reading a block from a peripheral into the CPU and the latter for transferring a block from the CPU onto an output media.

The READ Statement. Its general format is :

READ (*a*, *b*) list.

a stands for the number assigned to the input peripheral. *b* stands for the FORMAT statements number somewhere in the program. FORMAT statement would be explained later.

Example. READ (5, 11) P, Q, R which means,

"Read the values for the variables, P, Q and R via the Card reader (its assigned no. being 5) from a punched card. The format of these variables is to be had form statement number, 11.

The WRITE Statement has the similar format :

WRITE (*a*, *b*) list

Example. WRITE (8, 3) X, Y, Z which means,

"Print the current value of variables X, Y and Z on printer no. 8 using the format statement 3.

3. *Specification Statements* specify the format of the data to be read/written. They differ from one type of variable to another (real and integer), the number of data fields to be read/written, etc. We shall give a little program to clarify the use of the 3 major types of FORTRAN statements discussed thus far. We shall discuss only the FORMAT specification statement although there exist another DIMENSION statement that it uses for reading an array or matrix (with rows and columns) or cube (consisting of rows, columns and levels).

Program : Read two real variables' values from a punched card, add them and print the result.

READ (5, 20) R, S	Input Statement
20 FORMAT (2F 7.2)	Statement
Q=R+S	Arithmetic statement
WRITE (6, 20) Q	Output Statement
STOP	Control Statement
END	Control Statement

Let us explain each statement in turn.

READ (5, 20) R, S

"Read the values of two varibles R and S from peripheral 5 (which could be a card reader so that R and S read from a punched card) as per the format statement number 20. This number 20 is assigned arbitrarily to the FORMAT statement which may or may not follow or precede immediately by its READ Statement.

20 FORMAT (2 F 7 2)

Format statement must be numbered as 20, here, 2F means that 2 real variables are involved. The use of F is mandatory for real variables (i.e. non-integral variables) in the FORMAT statement, 7 2 means that the value of each of the two variables occupies 7 positions with 2 places after the decimal point.

Q=R+S

It is an arithmetic statement. The sum of values of R and S is assigned to Q.

WRITE (6, 20)Q

6 in the parenthesis stands for the printer which has been assigned this number. 20 refers to the FORMAT statement above i. e printing is to be done exactly in the format of reading in this case.

STOP

It is a control statement which stops the execution of the program (by the object program).

END

It is intended for the compiler only to finish translating from FORTRAN into the machine language.

4. *Control statements.* A couple of these have been used above. Another "jump" statements is discussed below.

The Go To statement

Example. Go to 45

It means go to statement 45 which may be above or below it in sequence by skipping intervening statements.

COBOL (Comman Business Oriented Language). It is a language for business applications since it uses plain English like statements so easily comprehensible in business circles and it is particularly developed to input, process and output inherently voluminous alphanumeric data in the business context. Its program are structured in the following four divisions.

I. IDENTIFICATION DIVISION
II. ENVIRONMENT DIVISION
III. DATA DIVISION
IV. PROCEDURE DIVISION

I. IDENTIFICATION DIVISION contains the program name and num programmer's name, application to which the program pertains and the date program was written However writing the program name alone is mandatory

II. ENVIRONMENT DIVISION usually contains the following two secti (*a*) CONFIGURATION SECTION (*b*) INPUT-OUTPUT SECTION.

(*a*) CONFIGURATION SECTION usually contains the name of the c puter on which the source program is to be complied and the computer that execute the program. However, it is usually the same computer that does compilat and then execution.

(*b*) INPUT-OUTPUT SECTION usually includes a FILE CONTROL p graph which assigns (programmer's supplied symbolic) names to both the input output files. This paragraph contains the following reserved (COBOL Vocabul words :

SELECT the particular file.

ASSIGN to the wanted input or output peripheral (number).

III. DATA DIVISION has usually two sections : (*a*) FILE SECTION (*b*) WORKING SECTION.

(*a*) FILE SECTION. For each file in the ENVIRONMENT DIVISI above the following details have to be defined by the programmer i.e. symbolic na have to be supplied by the programmer : (i) file name, (ii) record name and, layout of the records (iv) location of buffer storage for input and output.

Layout of the record contains for each data files its name location, size format. Towards this, X stands for alphanumeric characters and 9 stands for numeric characters. For example,

NAME X (25) may designate Employee's name data field that is 25 alphanur (alphabetic in the case, to be more precise), and
EMPNO 9 (5) may designate 5 numerics long employee's number.

(*b*) WORKING STORAGE SECTION. In it, the programmer supplies symbolic names for the working locations he puts to use, viz COUNTER, TOTA

IV. PROCEDURE DIVISION contains the step-by-step procedure that computer must follow to produce the designed output. Reserved words are i sec specific operations on symbolic names supplied by the programmer in the D DIVISION.

Example. Let us take up a very simple program that prints out a stuc name and address given below.

SUBIR BOSE
41 HIGH STREET
CALCUTTA-99

The program is given on the next page and is neatly self-explanatory, particula the light of discussion above.

```
IDENTIFICATION DIVISION.
PROGRAM-ID. PROG01.

*THIS PROGRAM PRINIS THE STUDENTS NAME AND
*ADDRESS ON THREE LINES.
* * * *

ENVIRONMENT DIVISION.
CONFIGURATION SECTION.
SOURCE-COMPUTER. IBM-370
OBJECT-COMPUTER. IBM-370
INPUT-OUTPUT SECTION.
FILE-CONTROL.
SELECT STUDENT ADDRESS ASSIGN TO UT-S-SYPRINT.

* * * *

DATA DIVISION
FILE SECTION.
FD STUDENT-ADDRESS
      LABLE RECORDS ARE OMITTED.
01 ALINE PICTURE X(40).

* * * *

PROCEDURE DIVISION.
      OPEN OUTPUT STUDENT-ADDRSS.
      MOVE 'SUBIR BOSE' TO ALINE.
      WRITE ALINE AFTER ADVANCING 2 LINES.
      MOVE '41 HIGH STREET' TO ALINE.
      WRITE ALINE AFTER ADVANCING 1 LINES.
      MOVE 'CALCUTTA-99' TO ALINE.
      WRITE ALINE AFTER ADVANCING 1 LINES.
      CLOSE STUDENT ADRESS.
      STOP RUN.
```

---

*in position 7 makes the following statement a mere descriptive material to be printed in the output of compilation run.

Type of COBOL Statements : These are four in number as discussed below briefly. The reserved words for commands are also given.

1. *Input Output.* OPEN, CLOSE, READ, WRITE.

An input/output file has to be OPENed before any processing can be performed on its records. In our sample program there is a single record file that is OPENed in statement no, 000270 (Please note that gaps in statement number are deliberate to permit insertion of more statement (s) upon afterthrough on the part of the programmer. The statement takes form of

$$\text{OPEN} \quad \begin{Bmatrix} \text{INPUT} \\ \text{OUTPUT} \\ \text{I-O} \end{Bmatrix} \quad \text{file name, file name......}$$

i. e. more than one file can be OPENED in a statement. Statement 000270 being

OPEN OUTPUT STUDENT-ADDRESS.

A file that is OPENed must be CLOSEd and that is examplified by statement no. 000340 and it takes the general form CLOSE file name ..

READ statement takes the general form of

READ file name

READ file name RECORD

e.g. if stock control file is named STOCKF these two statements would be as below

READ STOCKF

READ STOCKF RECORD

WRITE statement takes the general from

WRITE record name.

It is exemplified by statement nos. 000290,310,330, in the sample program.

The Statement WRITE ALINE AFTER ADVANCING 2 LINES will ensure double spacing before ALINE is written.

2. MOVE Statements move data from one storage area to another. Statement no, 000320 provides an example. It can also be used to clear a location, viz MOVE ZEROES TO COUNTER.

3. *Arithmetic Statements.*

COMPUTE Statements are used to perform arithmetic operations that are expressed in the form of mathematical formula. The operation symbols used are given here under :

+	plus
—	minus
*	multiply
/	divide
**	exponentiate

Example 1 COMPUTE X=(Y/Z)+P* * 3

2. COMPUTE TOPAY=NETPAY+ALLOWANCE

ADD, SUBTRACT, MULTIPLY, DIVIDE commands are explained by means of the following examples :—

COBOL form	Arithmetic euqivalent
ADD A, B TO C	C=A+B+C
ADD A, B GIVING C	C=A+B
SUBTRACT A FROM B	B=B—A
SUBTRACT A FROM B GIVING C	C=B—A
MULTIPLY A BY B	B=A×B
MULTIPLY A BY B GIVING C	C=A×B
DIVIDE A INTO B	B=B—A
DIVIDE A INTO B GIVING C	C=B÷A

A more practical example of an arthmetic statement follows :

MULTIPLY WGRATE BY HOURS GIVING TOWAGE (multiply wage rate by hourse giving total wage).

4. *Control Statements.*

IF Statement is conditional i. e. if the specified condition is met the program performs the requisite task otherwise it continues sequentially with the following instructions.

Example. IF MARKS IS LESS THAN O GO TO ERROR ROUTINE.

STOP Statement. If used literally as below

'STOP'

it causes a temporary halt 'STOP' printed on the console stationery, upon restart by the programmar the program will pick up from the next statement in sequence.

STOP RUN

As such the program will terminate permanently.

APPENDIX I

Sorting a Magnetic Tape file online

Unlike punched cards (which are sorted offline on a card sorter) the magnetic tape files are sorted online in connection with the CPU. Sorting, basically, involves enormous number of the comparisons within the CPU and this master is discussed in Part I. In this section, we shall assume that sorting in the CPU can be carried out. Because of a large number of comparisons involved sorting consumes a significant data processing time and it is therefore recommended by many experts that sorting is best avoided. But it cannot be obviated in magnetic tape installations where it may take as much as 30 to 50 per cent of the total data processing time on the computer. Transaction file would invariably be sorted in these installations by the key by which the master files to be up-dated by them are arranged. If there are several files to be up-dated the transaction file would have to be sorted over and again by the key of each of these. For example, the transactions on customer orders may first be sequenced by the customer account number as the key to up-date the account receivable master file, resorted to up date the inventory master file and finally resorted again to up-date the sales summary master file. One should always be alert to reduce sorting time which can be done if the transactions are pre-sorted manually to some extent. The clerical routines should therefore be so designed that they encourage pre-sorting of the transaction. Below we give a very simple example on the classical "two-way merge sort" technique.

There are two steps in sorting the magnetic tape :

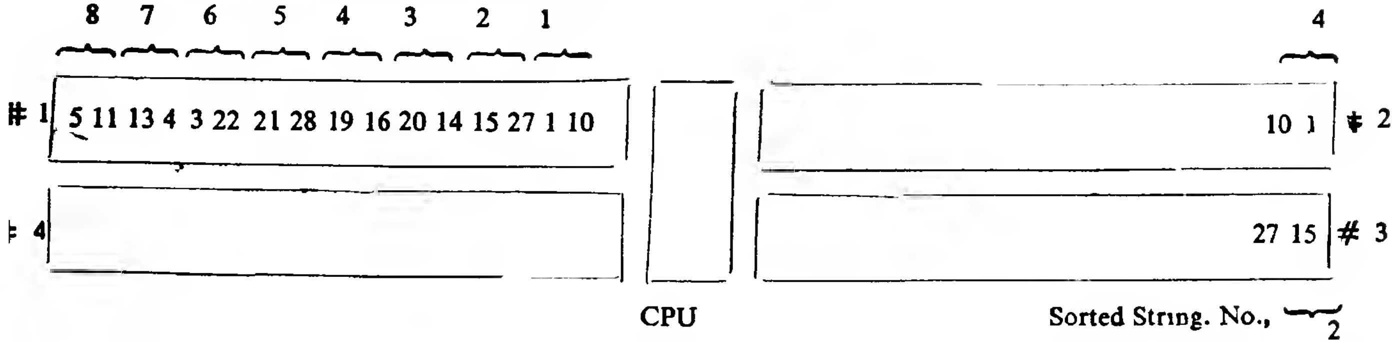
(i) Establishing the initial sorting strings.

(ii) Merging these strings into a single sequenced string

To explain these two steps let us consider a file holding 16 records with the following keys :

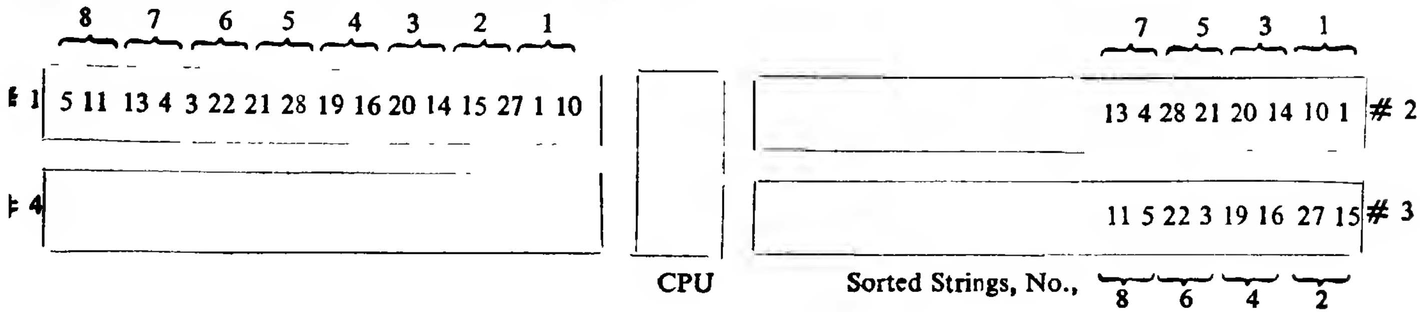
10, 1, 27, 15, 14, 20, 16, 19, 28, 21, 22, 3, 4, 13, 11, 5;

Step 1. **Establishing the initial sorting strings.**



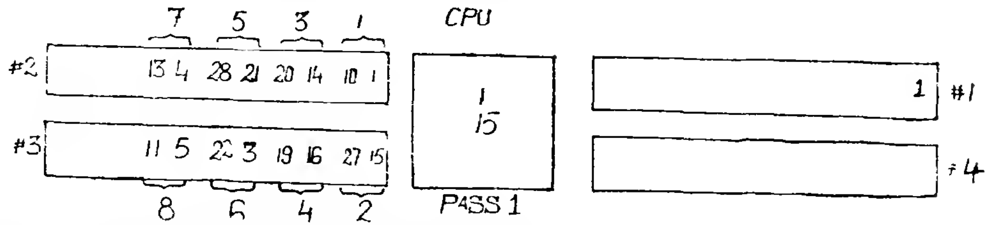
The file of 16 records is put on tape # 1. Three more tapes # 2, # 3, end # 4, would also be needed for ting this file online in connection with the CPU. We shall read a pair by pair of records in the CPU, compare ir keys and put them in the ascending order if not already so. Thus the first pair (known in the technical jargo a string of twos) 10, 1 is sequenced into 1, 10 in the CPU and is put on tape # 2. The second string of 27, 15 is uenced into 15, 27 and put on tape # 3.

This process is continued i e, pairs or strings are read into the CPU in turn, sequenced therein and are put on es # 2 and # 3 alternately. With this process (constituting our step (1) of establishing the initial sorting strings obtain 4 sorted strings each on tape # 2 and # 3 as below.

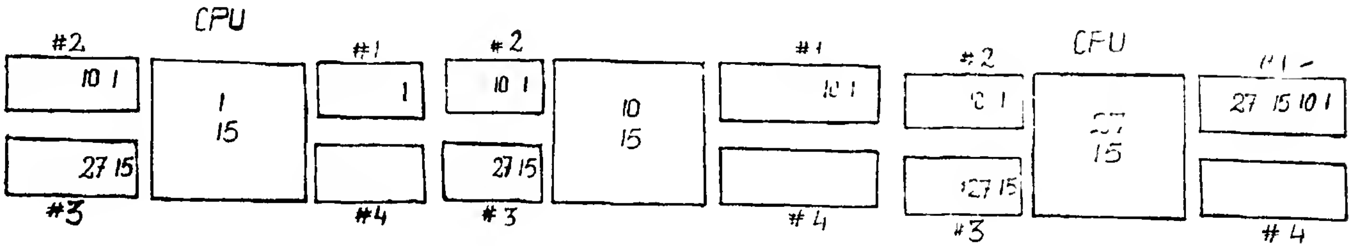


Step 2. **Merging of these strings into a single sequenced string.**

Tapes # 2 and 3 # are brought to the input side and # 1 and 4 on the output side.



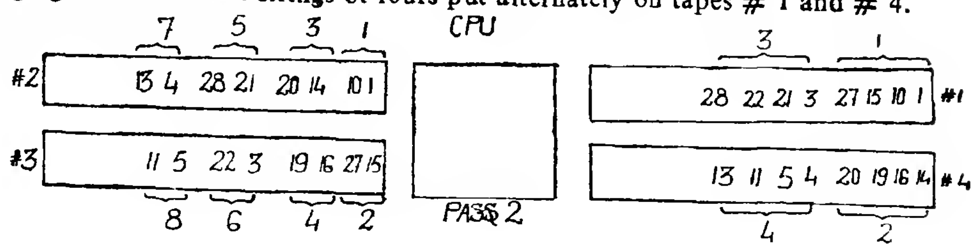
We now want to make string of fours. Towards this, (1, 2), (3, 4), (5, 6), (7, 8) would be merged in turn to get 4 strings of fours. For merging (1, 2) we read into the CPU their first numbers, 1 and 15, 1 being the smaller is written on tape # 1.



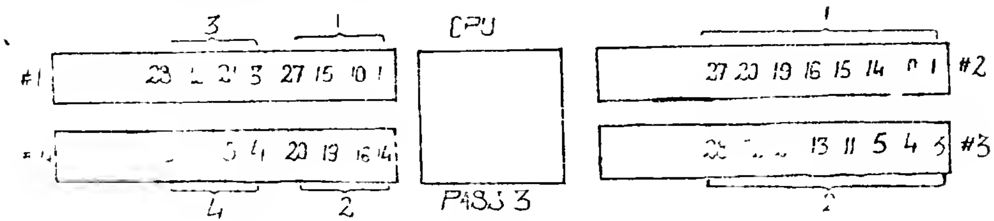
1 of tape # 2 having been put on tape # 1, we read in 10 of tape # 2 in the CPU. 10 being the smaller of 10 and 15 is put on the magnetic tape # 1.

The last (4th) number of the two strings 27 is brought into the CPU. 15 and 27 are put on tape # 1 in this order

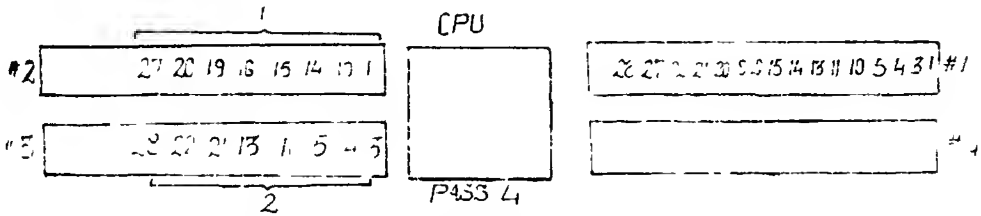
The follwing figure shows the 4 strings of fours put alternately on tapes # 1 and # 4.



Just as the strings of two's were merged into four's above the strings of 4's are merged into strings of eight's put alternately on tapes # 2 and # 3 as below :



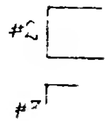
In the next (final) pass the strings of eight's (from # 1 and # 4) are merged into one sorted string of 16 as
ɔw :



We see that 4 passes were needed to sort the file of 16 records. The number of passes to sort N records is
al to $\log_2{}^N$. Thus the above sort of 16 records trkes $\log_2 16=4$ passes If magnetic tapes are available in suffi-
tly large number a "three-way merge" requiring only $\log_3{}^N$ passes may be performed. The principle of operation
sentially the same as two way merge sort i.e., three strings of sequenced records are created, merged and written
y the final string of sorted records is produced. In theory, it is possible to have a "4 way merge sort", and so on.
an be generalised that the more the work areas made available to the sort program the faster the program will

The "sort-merge" programs are supplied by the computer manufactures as a part of the utility program
ɛage.

Step 2. **Merging of t**
Tapes # 2 and 3 # a



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THE INSTITUTE OF CHARTERED ACCOUNTANTS OF INDIA

STUDY MATERIAL

F S P. (N) S.A. & D.P.—2
Combination B

FINAL COURSE (N)
F L O W C H A R T I N G
STUDY—II

Contents ·

Suggested Reading : Data Processing & MIS by *Anderson.* (M & E Pocket Book)

Flowchart, as used in data processing, is a graphic tool or model that prc a means of recording, analysing and communicating problem information. F charts depict flow of data, documents, etc. very clearly i.e., what are the steps to repeated (looping), and what are the alternatives (branching or switches) at a pa cular step are shown very clearly.

Flowcharts can be divided into 4 categories as below and as such they m be likened to the geographical maps with regard to the extent of detail :

1. Program flowcharts	discussed in this Study	(District map
2. Run flowcharts		(State map
3. System flowcharts		(National map)
4. System outline Charts		(Global map)

Part I. Program Flowcharts (Combination C students may leave Part I)

The program flowcharts are the most detailed and are concerned with the logical/arithmetic operations on data within the CPU and the flow of data between the CPU on the one hand and the input/output peripherals on the other.

Program flowcharting, to some extent, is a function of the language in which the program will be ultimately coded i e. it will vary to some extent from one language to another Therefore, it also depends upon the instruction repertoire or mix of the computer on hand Nevertheless the variations are minor The following hypothetical instruction mix (say, for computer X) will be utilised throughout as a basis for drawing the flowcharts

Logical/Arithmetic Operations

1. **Addition**

(i) Add the contents of two locations, say A and B, and put the results in either A or B, or any other location An example follows .

$C=A+B$ or $C=B+A$ means 'add the contents of two locations A and B and put the results in location C. However, this interpretation is assembly language oriented The compiler language orientation would be "C *becomes* the sum of A and B". In this interpretation by '=' is meant 'becomes' and not "equal to" Also in this interpretation C, A and B are treated as if they are variables. Most flowcharting in this study note is compiler language oriented. However, we shall use assembly language interpretations at places though rather sparingly.

$A=A+B$ or $A=B+A$ A becomes the sum of the previous value of A and the value of B.

$B=A+B$ or $B=B+A$ B becomes the sum of the value of A and the previous value of B.

(ii) Add a constant to the contents of a location or the value of a variable.

$C=A+13$ C becomes the value of A plus 13.

$A=A+13$ A becomes the previous value of A plus 13.

2. **Subtraction**

The student can interpret these on the lines of interpretations of the addition operations above.

B=A−B
C=B−A
A=A−B
A=B−A
B=A−14
A=A−14

3. **Multiplication**

The multiplication is best represented by asterisk in flowcharting so that it does not confuse with the widely used letter, X. The student can interpret the following operations himself.

C=A*B
C=B*A
A=A*B
A=B*A
A=A*7
B=A*7

4. **Division**

Two types of division can be carried out. Suppose, for example, that we divide 7 by 4. In one type, we get 1·75. In the other type we just get the quotient 1 and the remainder 3 is consigned to a location reserved for remainder by the computer manufacturer, say, REM. We shall use the same format for the two types as below

Type 1	*Type 2*	
Results in the location on L.H.S	Only the quotient in the location on the L H S. The remainder is consigned to standard location symbolised by REM.	
		Remainder in
C=A/B	C=A/B	REM
C=B/A	C=B/A	REM
A=A/B	A=A/B	REM
A=B/A	A=B/A	REM
A=A/131	A=A/131	REM
B=A/131	B=A/131	REM

5. **Transfer**

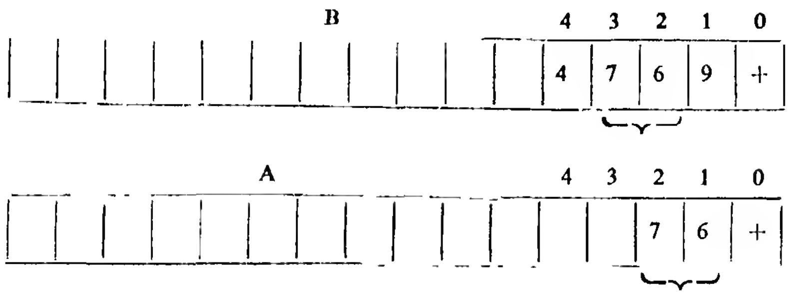
Transfer the contents of one location into another location. In other words, the variable on the L.H S. becomes (or assumes) the value of the variable on the R.H.S. Examples follows :

A=B If B were 13 and A were 7 or whatever, A would now become 13. The value of B remains 13 with this operation.

A=17 A, whatever its previous value, becomes 17.

6. Partial Transfer

It is possible with this type to transfer only some digits, (and not all for which operation type 5 above exists), from one location into another location. For example, if we want to transfer the middle two digits, 76 of location B below into A we can do so, anywhere in A



We would not give the format of this type We can state this in the flowchart in plain English "Transfer the contents of B (2nd and 3rd digit) into A in its 1st and 2nd cell".

All these operations 1 to 6 above, are depicted in the flowchart in a box For example, A=A−14 would be depicted as below

$$\boxed{A=A-14}$$

It may also be desired to designate a location or a variable by a suggestive symbol. Thus the step below means that we want to increment "COUNT" by 1,

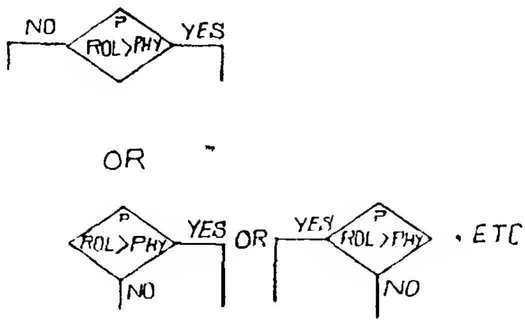
$\boxed{\text{COUNT}=\text{COUNT}+1}$ COUNT becomes the previous value of COUNT plus 1.

Desirably the length of symbols should not exceed six characters. Also they must start with an alphabet letter and never with a numeric or a special character. However, the following five characters could be of any kind. These symbols are devised by the person who draws the flowchart. Alphabets must be in capitals.

7. Comparison

In it, the values of two variables (i e. contents of two locations) are compared and one action is taken if the answer to the comparison is "yes" and another action if the answer is "no". A comparison is always shown in a diamond as below in which ROL (the symbol for reorder for level) is compared with PHY (the symbol for the

physical balance). If ROL is greater than PHY we would place the replenishment order, otherwise not.



The following types of comparisons are possible in most instruction repertoires.

	Constant on RHS	
A>B	A>13	(Instead of constants
A<B	A<13	alphabetic character(s)
A=B	A=13	or special symbol(s) can
A≠B	A≠13	be had on R.H.S.

8. **Print**

The following types of print operation formats are available.

(i) Print (Material) at position ...

e.g, Print "RAMU, 28" at 005

We want to print RAMU, 28 which constitutes the material. The continuous stationery usually can accomodate 160 characters. Thus, there are 160 print positions from 001 to 160 In the example above, we want to start printing at position 005

(ii) Print (Location or Variable) at position *e.g.*, Print A at 010, with which we want to print the value of the variable (or the contents of location) A starting at position 010

9. **Feed**

This means raising the continuous stationery by 1 or more lines for printing the next line. The format is as in the examples below

1 line C.S. feed

3 lines C.S. feed

We shall write other input/output instructions (viz. read a punched card) in plain English with some exceptions to be explained where the use is made.

Numerous examples on flowcharting follow The student should, however, familiarise himself with example on $\frac{27.7}{96}$ in Study I at this stage before proceeding with the following material

Example 1 Draw the program flowchart for finding the sum of first 100 odd numbers

Solution The flowchart is drawn as figure 1 and is explained step by step below The step numbers are shown in the flowchart in circles and as such are not a part of the flowchart but only a referencing device

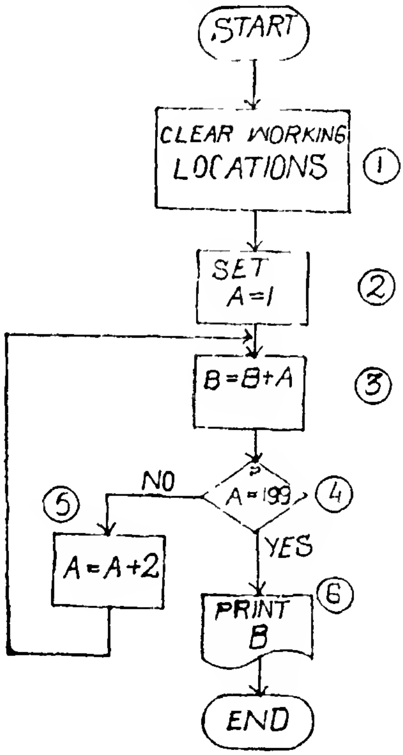


Fig. 1

Our purpose is to find the sum of the series 1, 3, 5, 7, 9 (100 terms). The student can verify that the 100th term would be 199. We propose to set A=1 and then go in incrementing it by 2 so that it holds the various terms of the series in turn. B is an accumulator in the sense that A is added to B whenever A is incremented. Thus B will hold

1
1+3=4
4+5=9
9+7=16, etc., in turn.

Step 1 All working locations are set at zero. This is necessary because if they are holding some data off the previous program that data is liable to corrupt the results of the flowchart.

Step 2 A is set at 1 so that subsequently by incrementing it successively by 2 we get the wanted odd terms 1, 3, 5, 7, etc

Step 3 A is poured into B *i e.* added to B B being 0 at the moment and A being 1, B becomes 0 + 1 = 1

Step 4 In step 5 we shall increment A by 2 So that although at the moment A is 1, it will be made 3 in step 5, and so on Since we have to stop at the 100th term which is 199 step 4 poses a question. "Has A become 199 ?", if not go back to step 3 by forming a loop Thus A is repeatedly incremented in step 5 and added to B in Step 3 In other words, B holds the cumulative sum upto the latest term held in A.

When A has become 199 that means the necessary computations have been carried out so that in step 6 the result is printed

Perhaps more suggestive symbols for A and B could be ODD and SUM respectively.

nExample 2. Draw the flowchart for finding the value of $|\underline{K}$ where K represents a integer whose value will be read into the computer each time the program is run

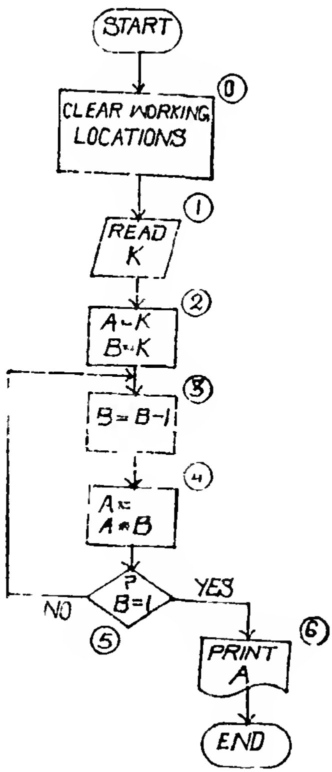


Fig. 2

The flowchart is drawn as figure 2 above. It may be recalled that we drew a flowchart for computing factorial of 7 but here we intend to generalise for any value designated by K Thus the number for which the factorial is needed is read into the CPU (via, say, a punched card) and this number is held in a location which is also designated as K Thus K may be given any integral value, *viz* 7, 17, 20 etc. This is done in step 1

Step 2 A and B are both equated to K. In the following steps we shall repeatedly decrement B by 1 and go on multiplying it with A successively so that A holds K(K—1), K(-1) (K-2) etc in turn and B becomes K, (K-1), (K-2), etc. in turn

Step 3 As already stated above B is brought down from K to K-1.

Step 4 A becomes the product of A and B *i e*, K(K-1).

Step 5 is a comparison step for looping. Obviously the factorial would have been computed when B, after having been successively decremented by 1, becomes 1 But since at the moment B has come down to K—1 and not 1 by looping we go back to step 2 by which B becomes (K-2) and A, in step 3, becomes K(K-1) (K-2); so on until A holds |K, which is printed in step 6

Example 3 Draw the flowchart for finding the value of K^N where K and N are read into the computer each time the program is run, N has to be an integer.

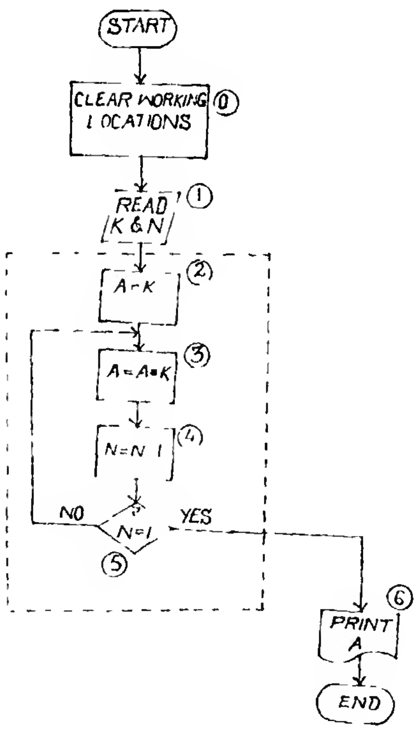


Fig. 3

Figure 3

Step 0. We zeroise all working locations.

Step 1. Values of K and N are read, say, via a punched card.

Step 2 A is equated to K. We shall subsequently go on multiplping A by K successively via a loop so that A is made A^2, A^3, etc in turn.

Step 3. A becomes AK *i e.* A^2 since K is equal to A.

Step 4. We have to carry out the multiplication of A by K $(N-1)$ times to get the value K^N. In this step, therefore, we decrease N by 1. Via the loop we shall continue to decrement it by 1 until it is brought down to 1.

Step 5. This is a comparison step where it is decided whether to continue with the loop or not When N comes down to 1 (in Step 4) A, which, becomes K^N, is printed in step 6.

Example 4 Draw the flowchart which will calculate the sum of the first N multiples of an integer K. (For example, if $K=3$ and $N=10$), then calculate the sum of $(1\times 3+2\times 3+ \quad +10\times 3)$ Make the flowchart completely general by reading an integer value for N and K each time this program is run.

Figure 4

Step 0. The working locations A, B and C are cleared *i.e.* zeroised to erase data sticking in from the previous program, if any.

Step 1. Parameters of the problem, K and N are read in via, say, a punched card.

Step 2. It is the intention to hold 1, 2, 3 etc., in turn, in A , therefore, A is incremented by 1.

Step 3. In B is held the first term of the given series which is A multiplied by K *i.e.* $1\times K$ to start with.

Step 4. It is the intention to add the terms of the given series one by one in C ; therefore, the first term, to start with, is accumulated in C.

Step 5. When A becomes equal to N we would print K, N, C as per step 6 ; but, at the moment we form a loop back to step 2 so that A is made $1+1=2$ to prepare the 2nd term in the following steps.

Example 5. There are three quantities : Q_1, Q_2 and Q_3 It is desired to obtain the highest of these in location H and lowest of these in location L.

Figure 5

Step 1. The three quantities, Q_1, Q_2 and Q_3 are read in via, say, a punched card.

Step 2 Any two quantities, say, Q_1 and Q_2 are compared. If Q_1 is greater than Q_2 we tentatively make $H=Q_1$ and $L=Q_2$ in Steps 3B and 4B ; otherwise, in Steps 3A and 4A we make $H=Q_2$ and $L=Q_1$

At Step 5 we are holding the higher of Q_1 and Q_2 in H and the lower of these in L. In Step 5 we see if Q_3 is greater than the higher of Q_1 and Q_2 in H. If it is obviously in Step 8 H is made Q_3 If Q_3 is not greater than H we compare Q_3 with L in Step 6

In Step 6, if $Q_3 < L$ we go to Step 7 and make $L=Q_3$, otherwise, the job has already been done prior to Step 5.

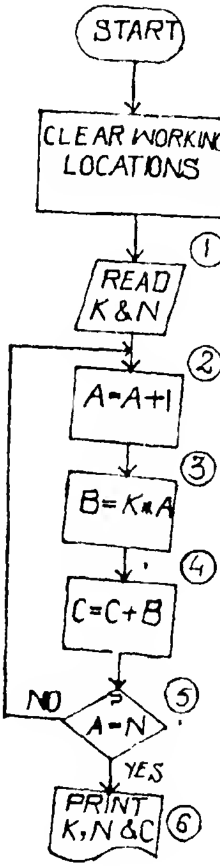


Fig. 4

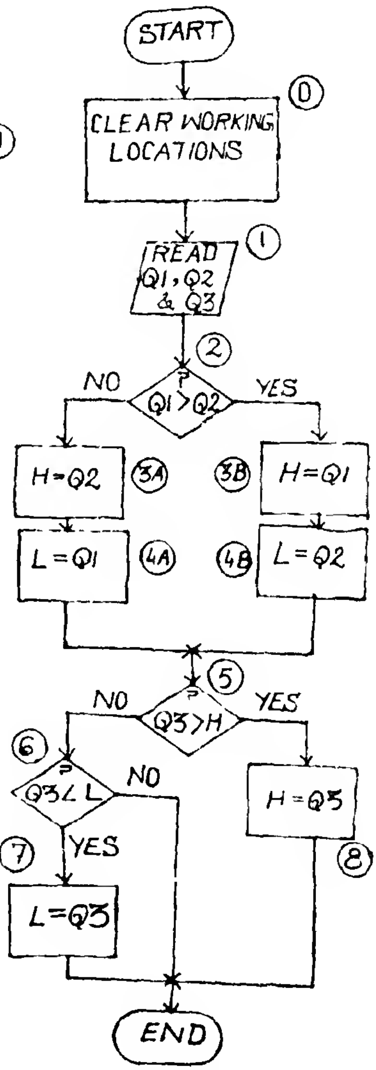


Fig. 5

Example 6. The square root of a number can be computed by an iterative procedure. The following computational steps are performed

1. Select a first guess for the desired square root A reasonable value for the first guess might be obtained by dividing the given number by 2

2. Divide the given number by the assumed square root.

3. If the quotient and divisor are sufficiently close, then the desired square root has been obtained to a sufficient degree of accuracy and the computation ceases.

4 If the quotient and the divisor do not agree, then a new guess must be obtained for the square root and the procedure repeated. The new guess is obtained by calculating the arithmetic average of the most recent divisor and quotient The computation then returns to step 2.

Say, N=the given number whose square root is desired.

D=the divisor.

Q=the quotient

R=the desired square root.

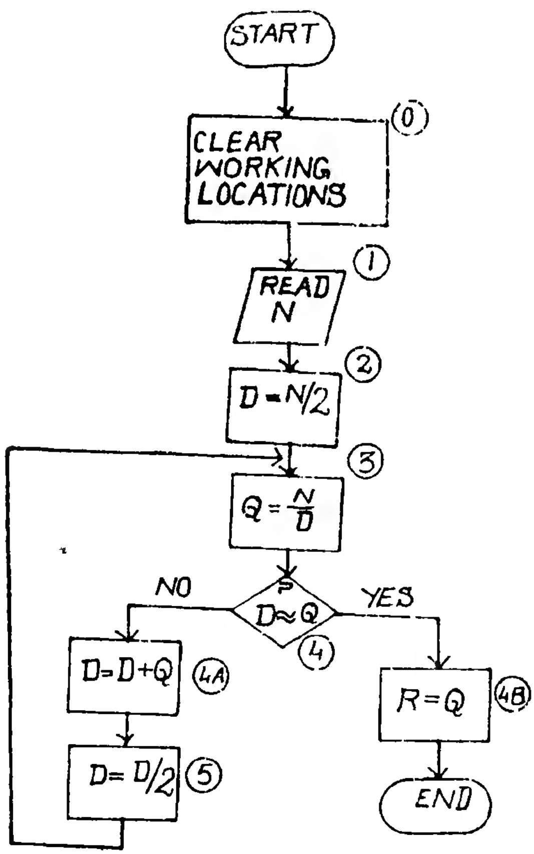


Fig. 6

[Let us now apply this method to a problem namely, computing the square root of 8 The computation will proceed in the following steps.

(a) $D_1=\frac{1}{2}(8)=4$ $\quad Q_1=8-4=2$

(b) $D_2=\frac{1}{2}(4+2)=3$ $\quad Q_2=8-3=2\ 6666$

(c) $D_3=\frac{1}{2}(3+2\ 6666)=2\ 83333$ $\quad Q_3=8-2\ 83333=2\ 82353$

(d) $D_4=\frac{1}{2}(2\ 83333+2\ 82353)=2\ 82843$ $\quad Q_4=8-2\ 82843=2{\cdot}82842$]

The (rather condensed) flowchart is given above in fig. 6.

Step 1. N, the number for which the square root is wanted.

Step 2 D is made half of N as the initial estimate of square root of N(=8 may be imagined)

Step 3. N(=8) is divided by the estimate D to get Q

Step 4 If D is approximately equal Q we have computed the square root which, however, is not the case as yet (since $D=\frac{N}{2}=4$ and $Q=\frac{N}{D}=\frac{8}{4}=2$ and $D\neq Q$) , therefore, go to steps 5 and 6 In these two steps we find the average of D and Q and put it in D This average $\left(=\frac{2+4}{2}=3\right)$ is taken in D as the new estimate of the square root and we loop back to step 3

*In fact, there does not exist any instruction in any computer by which we can compare if two quantities are roughly equal. What, therefore, would actually be done is to find the difference between D and Q and if it is $\leqslant$ prescribed difference, say, 0 001 we accept them as equal.

Example 7 Draw the flowchart for deriving the sum of the squares of first 20 odd numbers

The flowchart is shown in fig. 7

Step 0 All working locations are zeroised.

Step 1. In step 2 we employ the square subroutine (which is the set of steps enclosed in the dotted box in figure 3 for computing K^N.) ; therefore, we set N=2 for ever in this program and K=1.

Step 2. K^N i e. 1^2 is computed by the aforesaid subroutive (S.R.). A S.R. is always depicted in the hexagonal symbol in program flowcharting

Step 3. We accumulate the first term i.e. square of the first odd number, 1 in location C.

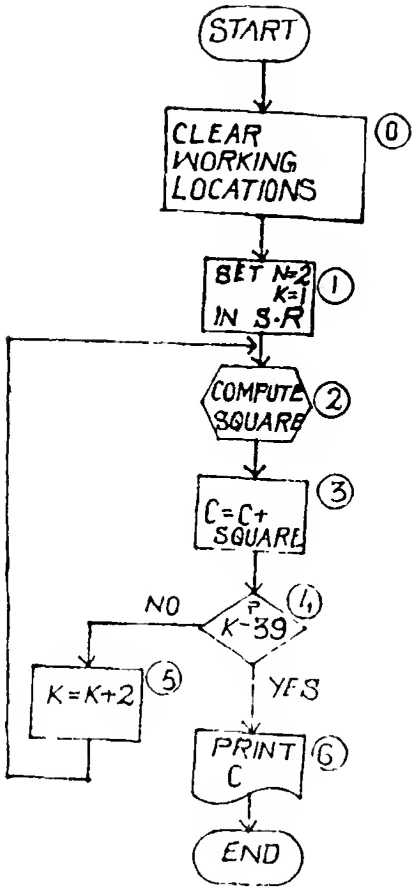


Fig 7

Step 4 The 20th odd number is 39 , therefore, in this step we see if A has become 39 from 1 (by step 5).

Step 5 K is incremented by 2 i e it becomes 1+2=3.

Example 8. The sine of X can be approximately calculated by summing up the first 100 terms of the infinite series

$$\text{Sin } X = X - \frac{X^3}{\lfloor 3} + \frac{X^5}{\lfloor 5} - \frac{X^7}{\lfloor 7} + \dots$$

Draw the program flow chart in which the value of X in radians will be read in and then the sine is computed

Figure 8

Step 0. Clear all working locations

Step 1. The value of X in radians is read in via, say, a punched card.

Step 2. In step 4, we shall use the power S.R. for computing K^N. In this step 2, we set A=1 and in step 3, we set K=X and N=A (i e.=1).

Step 4. K^N ($=X^A$) is computed by the S.R. (A having been at present set at 1 in step 2).

Step 5 In step 6 we shall compute the factorial of A (which is 1 at the moment) Since in fig. 2 we used the symbol K we set K=A in the S R of step 6, A being 1 at the moment

Step 6. $\underline{|A}$ i e. $\underline{|1}$ is computed. It constitutes the denominator of the first term, $\frac{X^1}{\underline{|1}}$,

Step 7. We divide X^A (i.e. X^1) by $\underline{|A}$ (i.e. $\underline{|1}$ and put it in location TERM.

Step 8 We count the number of terms in location COUNT which becomes 1 by incrementing it by 1

Step 9 We see if COUNT is odd or even. In the given series (on R.H S.) it is to be seen that odd terms are preceeded by plus sign and even terms by minus sign.

Step 10 A and 10 B. If count is odd we accumulate it in location SINX otherwise we subtract it from SINX Thus SINX holds the sum of the R H S. upto 1st, 2nd, 3rd , etc. term in turn

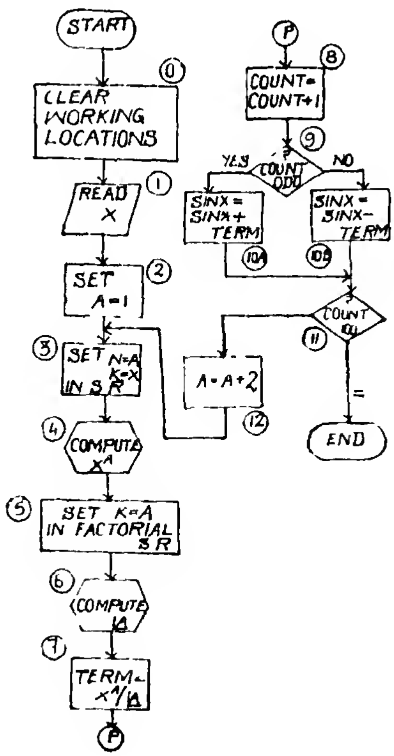


Fig. 8

Step 11. It is ascertained if the 100 terms have been processed. If so we end, otherwise, in step 12 we increment A by 2 (e g it becomes 3 from 1) and loop back to step 3 to process for the 2nd term, $-\frac{X^3}{\underline{|3}}$.

Example 9. Draw the program flowchart for computing the annual acquisition, inventory-carrying and total costs for lot sizes of 100, 200 2400 The various variables of interest are supposed to be there in the locations symbolized below.

REQ	Annual requirements of the item
ACQ	Procurement cost/order
COST	Cost per unit
RATE	Inventory-carrying rate, I

The flowchart is drawn in figure 9. The following symbols represent the working locations put to use by this flowchart.

LOTSIZ	Lot size
IVCOST	Annual inventory-carrying cost
AQCOST	Annual acquisition cost
TOCOST	Annual total cost

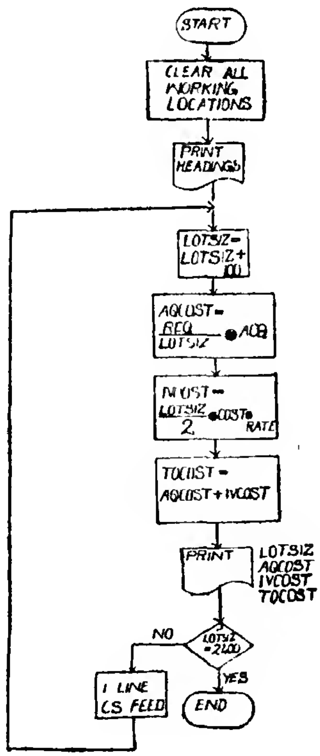


Fig. 9

Example 10. Draw the flowchart for finding the amount of an annuity of of Rs. A in N years. Rate of interest$=r\%$. $R=(1+r)$. This amount is given by the following series $A+AR+AR^2+ \quad AR^{N-1}$.

The flowchart is drawn in figure 10 The following symbols are employed.

TERM	To hold A, AR^2, etc. (i.e. the various terms) in turn.
SUM	In it is accumulated the sum of terms.
COUNT	Counter to count the number of terms accumulated.

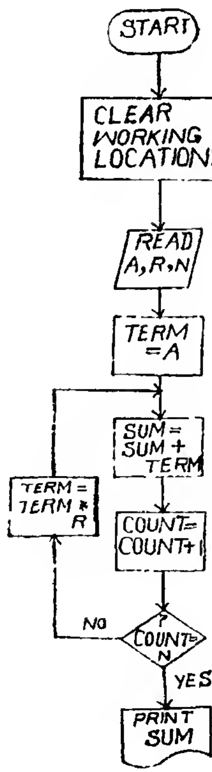


Fig. 10

Example 11. (On Computing Customs Duties) Assume that imported goods from foreign countries are classified into 4 categories for the purpose of levying customs duty. The duty rate for each category is as follows

Class No.	*Class of Goods*	*Customs duty (%) K, on Values of Goods V*
1	Foods, beverages	10
2	Clothing, footware	15
3	Heavy machinery	17½
4	Luxury items	40

Draw the flowchart for computing the appropriate customs duty

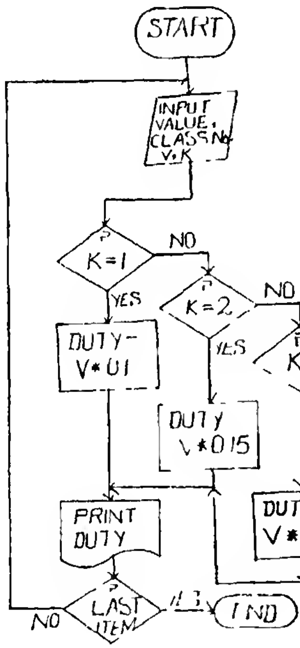


Fig 11

Example 12 The problem is to compute, for a series of transactions, the gross sales (G) ; the quantity discounts, (D), if any ; and the net sales (N). The raw data to be supplied in the program includes the quantity sold (Q) and unit price (P). The quantity discount schedule is as follows :

If quantity sold is	*The discount rate would be :*
less then 100 units	none
100 to less than 200	10%
200 and over	20%

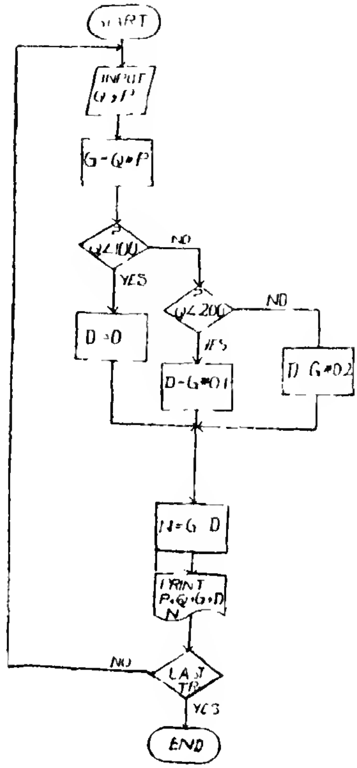


Fig 12

Example 13. Given the following set of data ·

Account No	*Age of Customer*	*Sex*	*Unpaid Balance*
13466	28	M	Rs 145 23
43156	20	F	49·50
33215	45	F	89 24
44178	19	M	115 23
56723	28	F	75·95
47892	33	F	25 78
24567	19	M	54 75
56783	55	M	24 78
43579	39	F	67 85
56782	30	M	150 97
79234	18	F	39 95
65423	29	F	69·95

Draw a flowchart to compute and print out the following :

Average Customer Age	*Unpaid Balance* Males	Females
Under 20	Rs. ×××.××	Rs. ×××.×
20 To Under 30	×××	×××
30 To Under 40	×××	×××
40 and Over	×××	×××

The program flowchart is given below M1 to M4 accumulate balances for the 4 age groups of male customers and likewise F1 to F4 for the female customers. Age is symbolised by A, balance by B and Sex is codified as M or F *The last record is a dummy*

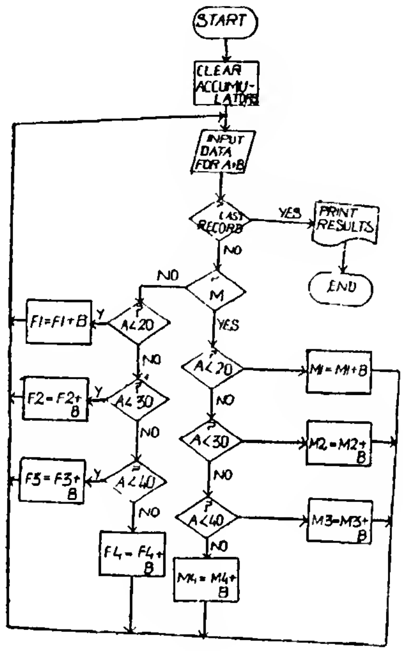


Fig 13

Example 14. Using the data of the previous example, draw a flowchart for computing and printing out the following statistics .—

Sex	*Average Unpaid Balance*
Male	Rs. ××× ××
Female	×××.××
Overall	××× ××

The program flowchart is shown below. The following Symbols are used.

MANO	Counter for males
FENO	Counter for females
MABAL	Sum of male balances
FEBAL	Sum of female balances

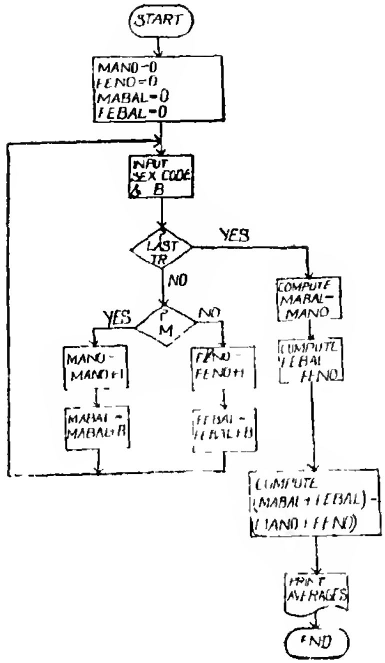


Fig 14

Example 15 Flowchart for Binary Search (Ref Magnetic Disc, Study I)

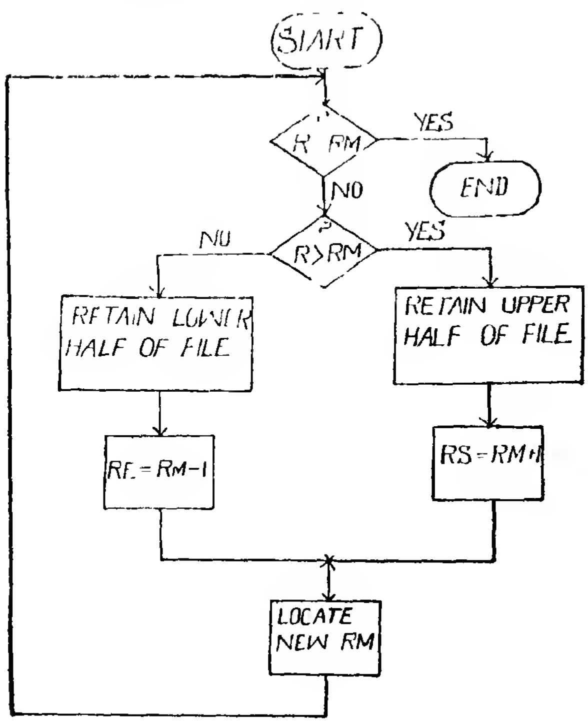


Fig 15

R = Desired Record
RE = Record at the end of file
RM = Record in middle of file
RS = Record at start of file

Modification/Initialisation Instructions

These instructions can change the value of a *variable location number* in an existing instruction during the program execution process The initialisation instruction can set or reset the value of this variable to any desired number The modification instruction can increment/decrement this variable during the loop execution by any constant (viz 1, 2, 13,).

Example 16 Marks of each student (in a class) in 12 papers are encoded on a punched card which are read (one card by one) into the CPU locations MARKS 001 to MARKS 012. You are required to draw the flowchart for computing and printing the average marks of each student

Solution MARKS 001, MARKS 002, MARKS 012, are holding the marks in 12 papers of a student We propose to accumulate them as ACCUM This could be accomplished in 12 instructions as below

ACCUM = ACCUM + MARKS 001
ACCUM = ACCUM + MARKS 002, etc to –,
ACCUM = ACCUM + MARKS 012

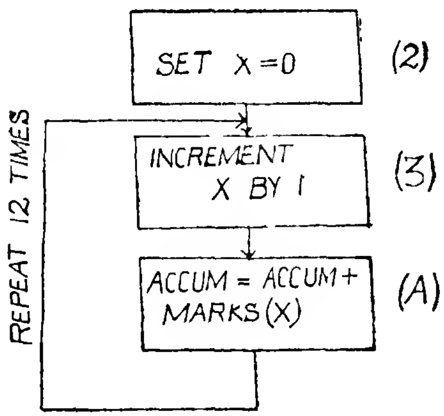
But we do not do this way. We shall adopt a cleverer approach which is made possible by the facility of what is known as the "modifying" instruction in the instruction repertoire.

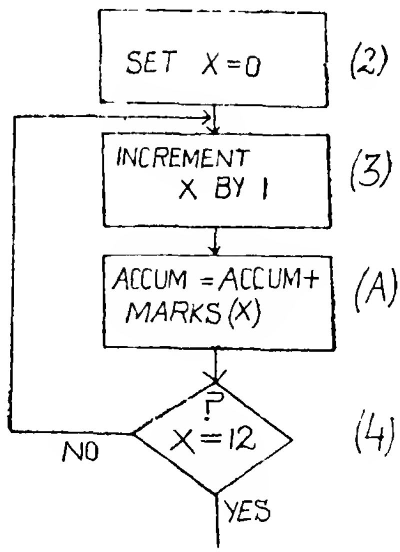
It is to be seen that the 12 instructions above can be generalised as

ACCUM = ACCUM + MARKS (X) (A)

We have to start with X = 1 and then go on incrementing it by one to generate the above 12 instructions. This we do as below.

Set X = 0 in (A) . (2)
Increment X by 1 in (A) (3)
ACCUM = ACCUM + MARKS (X) (A)





By step (2) we made X in step (A)=0 and by step (3) we incremented it by 1 so that MARKS (X) in A has been made MARKS (1) which is the same thing as MARKS 001. If we repeat steps (3) and (A) 12 times as per the left flowchart segment above, we, in effect, will have performed the aforesaid 12 instructions

But how do we repeat this loop 12 times ? This is accomplished by including the comparison step (4) as per R H S segment above In this step, we pose the question if X has become 12.

The completed flowchart is shown in Fig. 16

Step (3) in Fig B above corresponds to what is known as the modifying instruction since it modifies the instruction corresponding to step (A) The step (2) is a sort of initialisation step or instruction since it sets the value of the variable X at 0 for each student's 12 papers.

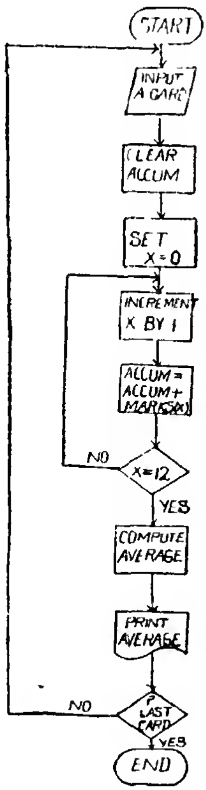


Fig 16

Modification of the 'Comparison' Step

Example 17. Prices for ten commodities in the current year are designated by J(X), X varying from 1 to 10. Likewise, their last year's prices are designated by K(Y), Y varying from 1 to 10 Draw the flowchart for finding the number, N of commodities of which prices have increased.

The flowchart is drawn in figure 17.

The crooked arrow shows the comparison step that is initialised and modified for looping. The following is the comprehensive list of comparisons of this type that are valid.

$$J(X) > K(Y)$$
$$J(X) = K(Y)$$
$$J(X) \neq K(Y)$$
$$J(X) < K(Y)$$

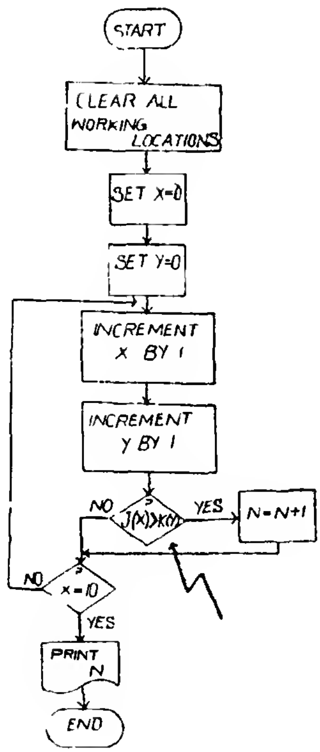


Fig. 17

Example 18. Prices of a commodity in ten major cities are designated by J(X), X varying from 1 to 10 The price pervailing in the capital is designated by C. Find the number of cities having the price less than that in the capital.

The flowchart is drawn in figure 18 with the crooked arrow showing the comparison step that is initialised and modified for looping The following is the comprehensive list of this type of comparisons possible in flowcharting.

$J(X) < C$	or	$C > J(X)$
$J(X) = C$		$C = J(X)$
$J(X) > C$		$C < J(X)$
$J(X) \neq C$		$C \neq J(X)$

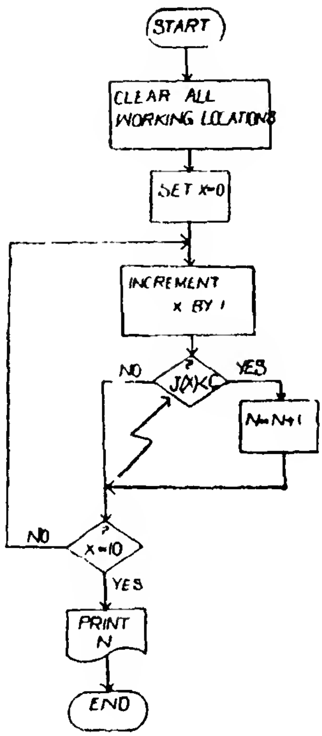


Fig. 18

Example 19. There are 1000 students in for an examination. The roll number of a student, his name and marks obtained by him in the 10 papers are encoded in a punched card Cards are input into the CPU one by one for finding for each student the number of papers, N in which he scored distinction by obtaining 75 out of 100 or more marks. Name is held by NAME, Roll number by ROLLNO and maks by MARKS (X), X=1, 2, 3 10.

The flowchart is shown in figure 19. The crooked arrow shows the comparison step of major interest. This comparison involves a constant 75 The following is the comprehensive list of this type of comparisons valid in flowcharting.

$J(X) < 75$ (On R.H.S, alphabetics or special symbols can also be had).
$J(X) = 75$
$J(X) \neq 75$
$J(X) > 75$

N.B. 75 > J (X), etc. is invalid.

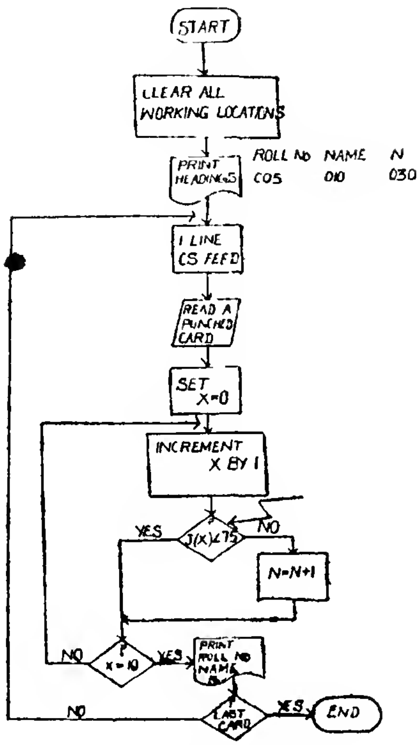


Fig. 19

Modification of the "Transfer Step".

Example 20. Assign a value of 37 to each of the array, J (X), X=1, 2...10.

The crooked arrow in the flowchart of fig. 20 shows the "transfer" step of interest wherein 37 is put in each of the 10 locations designated in general by J (X).

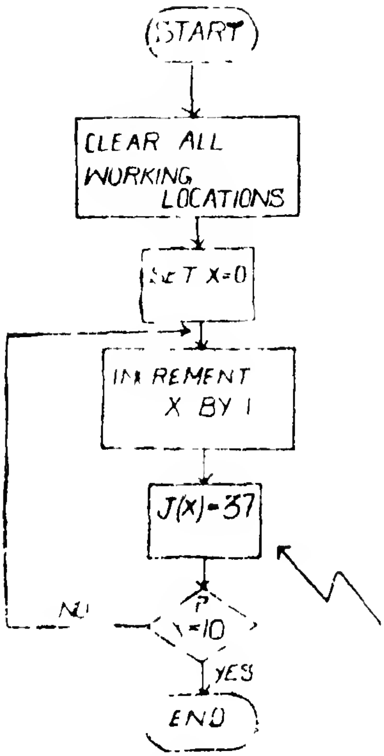


Fig 20

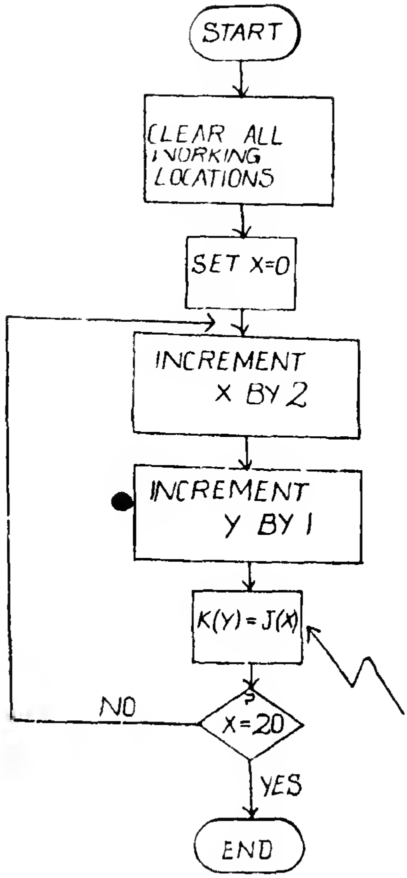


Fig. 21

Example 21. Transfer the contents of locations J (X), X=2, 4, 6, 8, 10 . .20 to K (Y), Y=1, 2, 3 10

The flowchart is shown in figure 21 The crooked arrow shows the step of major interest wherein K(Y)'s are successively equated to J (X)'s.

Example 22 Transfer the contents of location J (0) to each of the following 10 locations

The flowchart is shown in figure 22 for transferring the contents of J (0) to each of J (1), J (2), J (3) 5 (10) The crooked arrow shows the step of major interest.

Modification of Arithmetic Steps.

Example 23 It is desired to add contents of 10 locations J (X), X=4, 7, 10 ......31 and K (Y), Y=1, 2, 3 10 on a one to one basis and put the results in R (Z), Z=2, 4, 6 20

The flowchart is shown in figure 23 The step of major interest has a crooked arrow to it. The following is the comprehensive list of such types of steps.

R (Z) = J (X) + K (Y)	Such types are
R (Z) = J (X) — K (Y)	also valid :
R (Z) = J (X) * K (Y)	R (Z)= R (Z)+J (X)
R (Z) = J (X) / K (Y)	R (Z)= R (Z) / I (X), etc.

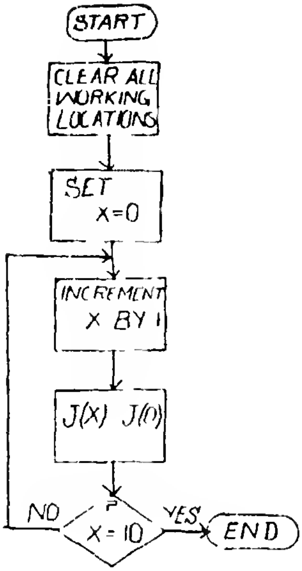


Fig. 22

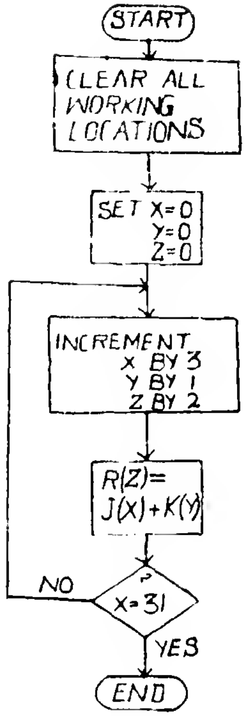


Fig. 23

More Examples on Modification of Arithmetic operations.

Example 24. Multiply R (Z), Z=2, 4, 6 20 and K (Y), Y=3, 6, 9 ...30 on a one to one basis and put the results in J (X), X=19, 18, 17 10

The flowchart is shown in figure 24. It is the intention here to bring out the fact that *decrementing* of L H.S, J (X) (in general even one or both designations on R.H.S) is also valid by 1 (in particular and in general any integer).

Example 25. Add Rs 45 (a constant) to the wages of 10 persons designated by J (X), X=1, 2 10.

The flowchart is drawn in figure 25 The crooked arrow shows the step for adding a constant to the contents of a location. Other permissible steps of this type are as below :

J (X) = J (X) −45

J (X) = J (X) * 45

J (X) = J (X) / 45

K (Y) = J (X) +45, etc.

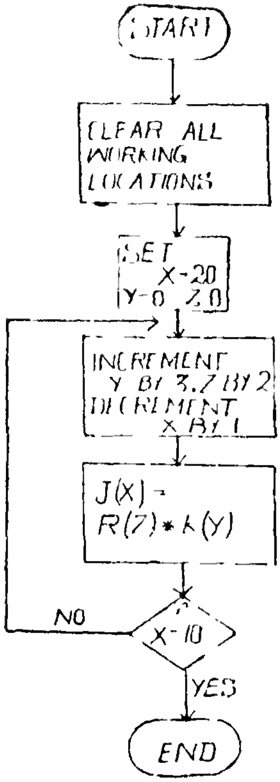


Fig 24

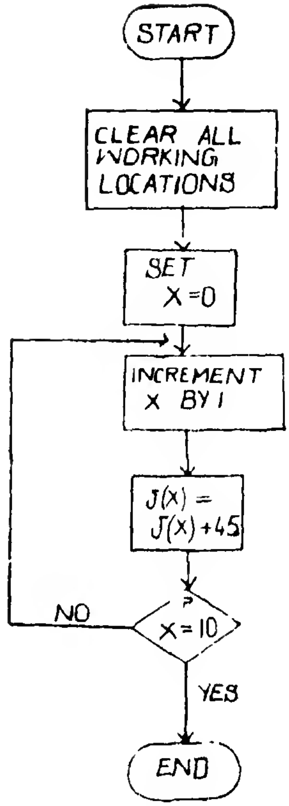


Fig. 25

Example 26. Print 6 'P's in the pattern below.

```
            012 (print position)
                P
               P
              P
             P
            P
           P
            007 (print position)
```

The flowchart is drawn in figure 26 The print instruction has the following two formats Printing is done in one line by an instruction

1. Print (Given material as 'P' here) starting at print position (001 to 160).

With this type of instruction the programmed material is printed. We want to print 'P' at starting print position 012 ; therefore, we give the instruction "Print 'P' at Y." Y is the print position on the continuous stationery which usually can accomodate 160 characters in one line. We want to print 'P' at the Y-th position. In the given pattern 'P' in the first line is to be printed at position 12, therefore, Y is set

at 13 and then decremented by 1 This is followed by "1 line CS feed" which means programming to raise the continuous stationery by one line so that it is set for printing the 2nd 'P' in the 2nd line By means of the loop Y is decremented from 012 one by one so that 'P's are printed at positions 012, 011, 010, 009, 008, 007 in successive lines

2. Print (Contents of a location) at starting print position (001 to 160)

The type is illustrated in the following example

Example 27. 64 locations J (X), X=1, 2 .64 hold 64 3-digit quantities It is required to draw the flowchart for printing these in an 8×8 matrix as below :

412 331 602 400 405 403 408 421
424 425 423 422 421 420 419 426

531 310 410 212 111 402 124 429

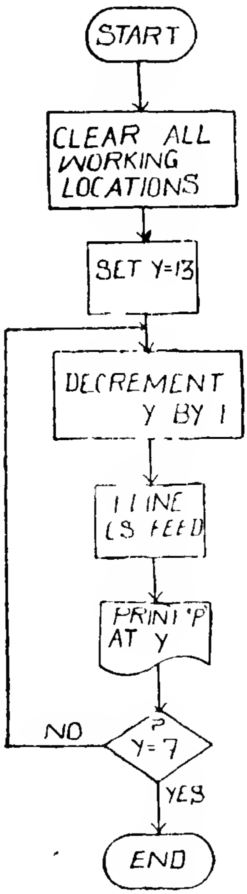


Fig 26

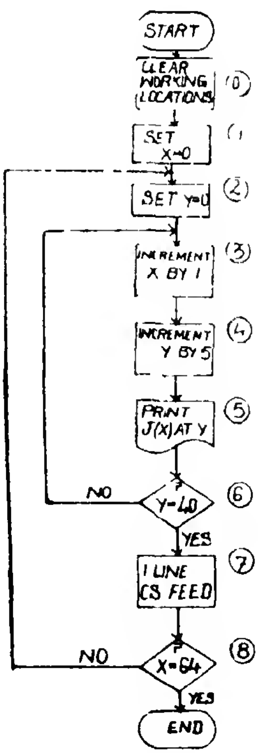


Fig 27

The flowchart is drawn in figure 27 The first row figures in the above matrix are the contents of J (1) to J (8) The second row figures in the above matrix are the contents of J (9) to J (16) Moving this way the figures of the last row are the contents of J (57) to J (64)

The first column is printed at start position Y=005, the 2nd column at Y=010, and so on, so that the 8th column has the starting print position Y=040. Thus in the flowchart when Y become 040 it is a signal that the printing of a line is over , therefore, the continuous stationery is raised by a line, Y reset at 005 for commencing the printing of the next line.

We are giving an increment of 5 to Y which is the minimum necessary It could be more but it should not be less because the three digits and the sign (for, say, debit / credit as + or —) would require 4 print positions and the fifth position would be left blank as a gap between two neighbouring qualities

Example 28 Q (X), holds 9 quantities Q1, Q2 Q9 Obtain the highest quantity in location H and the lowest quantity in location L

The flowchart is shown figure 28. In addition to the given symbols another symbol Q (which is ths same as Q (0)) is used to hold Q 3, Q 4 Q9 in turn

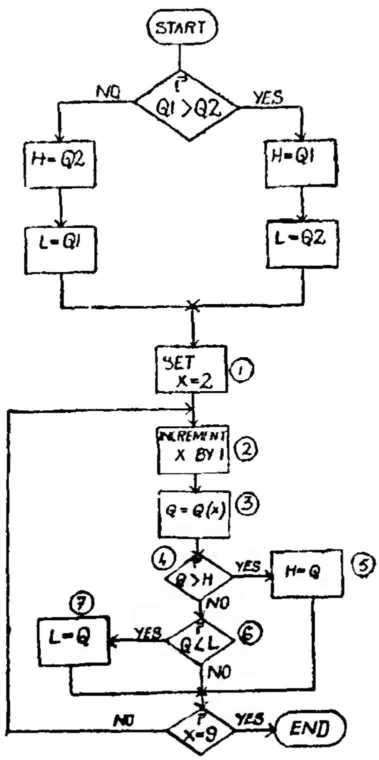


Fig. 28

We start in the manner of example 5 on page 9 Having put that way the higher of Q1 and Q2 in H and the lower in L tentatively, from step 1 onwards, we want to compare Q3, Q4 Q9 in turn with H and L In fact, steps 4 to 7 are similar to the later part of the flowchart of example 5 on page 9.

In steps 1 and 2 we prepare X=3 for step 3 which now reads "Q=Q(3)". What we have done is that we have put the contents of Q (3) in Q and in steps 4 to 7 we work on Q instead of Q (X). Why are we reluctant to work with Q(X) straight in steps 4 to 7 ? well we could do so and we would get the wanted results But by working with Q(X) directly in steps 4 to 7 these steps would read as below on the L H S rather than on the R H.S as in the flowchart

$$\left.\begin{array}{ll} \text{Step 4} & Q(X) > H \\ \text{Step 5} & H = Q(X) \\ \text{Step 6} & Q(X) < L \\ \text{Step 7} & L = Q(X) \end{array}\right\} \text{rather than} \left\{\begin{array}{l} Q > H \\ H = Q \\ Q < L \\ L = Q \end{array}\right. \text{is the flowchart}$$

So if we work with Q(X) straight in steps 4 to 7 we shall have to set X=3 in each of these steps But by having equated Q with Q(X) we can work with Q and we do not have any problem of setting X in steps 4 to 7.

Example 29 In locations J(X), X=1, 2 . 200 are held 200 quantities. Draw the flowchart for finding the ratio of the total number of quantities divisible by 10 to that of not divisible by 10

The flowchart is shown in figure 29 The following symbols are used in it

NONTEN	Total number of items not divisible by 10
TENNER	Total number of items disivible by 10
RATIO	Ratio NONTEN/TENNER.
J(X), X=1,2..	200
J	This is used to hold the last digit of a quantity.

Partial transfer (as we have done in this flowchart) of one or more consecutive digits from one location into another location is valid

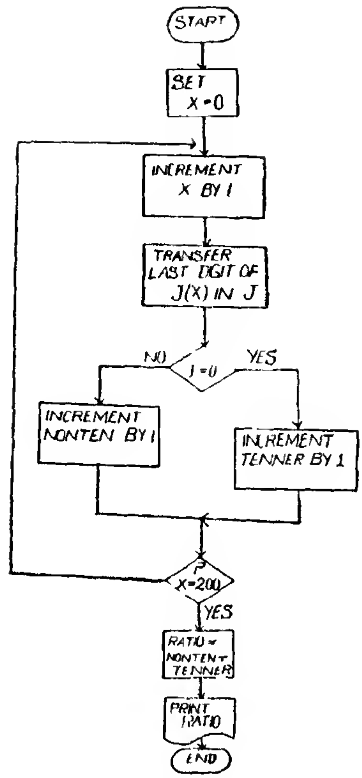


Fig 29

Example 30. It is required to compute the geometric mean of six past prices for each commodity is an inventory of 50 commodities, the 6 prices having been encoded in a punched card which are read into the CPU in locations designated by VALUE 001 to Value 006 Draw the flowchart. Use would be made of a S R to compute the 1/6 powers

Here again, modification instruction is put to use, which is always the case whenever an array or a list of variables are to be processed similarly.

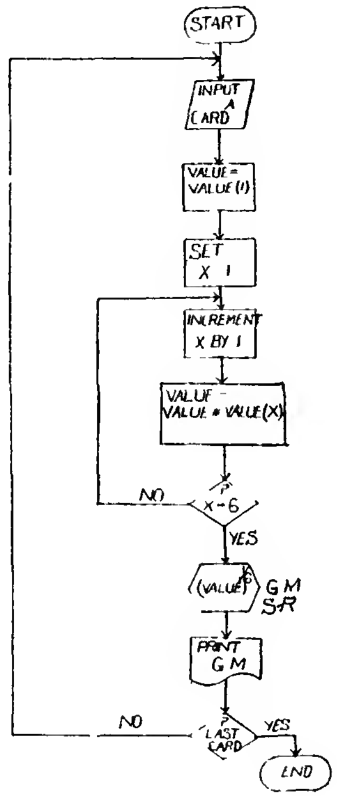


Fig. 30

Example 31. It is desired to sort 5 quantities in a list held in the CPU locations symbolised by LIST 001 to LIST 005 Draw the program flowchart.

Switching or Exchange Method of Sorting Within the CPU

The list X to be sorted is assumed to be as follows

X (1)
X (2)
X (3)
X (4)
X (5)

The logic of method depicted in the flowchart in fig. 31 may be summarised as follows :—

1. Check the first pair of values in the list X-that is, compare X (1) and X (2). If they are in the right order, X (1)$<$X (2), leave them alone and proceed to check the

next pair of values. If however X(1)>X(2) they are in the wrong order and need to be switched i.e. exchanged before proceeding to a check of the next pair, X (2) and X (3)

2. After one pass through comparing each neighbouring pair of the values of X, it is necessary to go through another pass to ensure that each pair of values is now in the right order (that is if no switching occurs during the pass) another pass is still required to ensure that a sorted list is being achieved

For illustration of how the sorting logic works, the list X is assumed to be as follows before sorting begins

X (1)=1

X (2)=5

X (3)=−2

X (4)=7

X (5)=4

The values of the elements of X during the first pass though the list are summarised below

	VALUE OF X DURING 1ST PASS				AFTER I LOOP
	I=1	I=2	I=3	I=4	
X(1)=	1	1	1	1	1
X(2)=	5	5	-2	-2	-2
X(3)=	-2	-2	5	5	5
X(4)=	7	7	7	7	4
X(5)=	4	4	4	4	7
? SWITCHING	NO	YES	NO	YES	
VALUE OF S	0	1	1	2	2

The value of I only goes from 1 to 4, since when I=4 the last pair of values of X, X (4) and X (5), will be compared.

As demonstrated above, after one pass though the list, the values in list X are not all in the right order since two switches occurred during the pass. Hence a second pass through the list is required. The value of the elements of X during the second pass are as follows :

	VALUE OF X DURING 2ND PASS				AFTER I LOOP
	I=1	I=2	I=3	I=4	
X(1) =	1	−2	−2	−2	−2
X(2) =	−2	1	1	1	1
X(3) =	5	5	5	4	4
X(4) =	4	4	4	5	5
X(5) =	7	7	7	7	7
SWITCHING ?	YES	NO	YES	NO	
VALUE OF S	1	1	2	2	2

In the third pass no switching would be there i.e. S=0, meaning that the list is sorted.

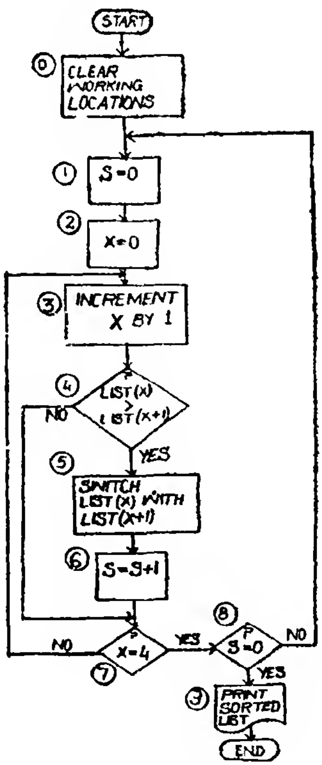


Fig. 31

The flowchart is drawn above and explained below

Step 1. The switch counter is set equal to zero

Step 2 This step initialises the loop to follow

In steps 4 and 5, we are using LIST (X) as the general symble for the five locations :

LIST (1), LIST (2), LIST (3), LIST (4) and LIST (5) holding the five numbers to be sorted in the ascending order. Naturally, therefore, if LIST (X) is made LIST (0) by setting X=0 in step 2 under explanation LIST (X+1) would mean LIST (1) Thus having set X=0 in steps 4 and 5, LIST (X) means LIST (0) and LIST (X+1) means LIST (1). Surely LIST (0) is not one of the five locations holding the five numbers. This is set right in step 3

Step 3 X is incremented by 1 in the following steps, 4 and 5. This makes LIST (1) of LIST (X) and LIST (2) of LIST (X+1)

Step 4. LIST (1) is compared with LIST (2)

Since we know LIST (1)=1 and LIST (2)=5 are in the right (ascending) order no switching is needed, therefore, steps 5 and 6 are bypassed

Step 7. Since X=1 and not 4 therefore, the program flowchart loops back to step 3.

Step 3 X is incremented by 1 so that step 4 reads "Is LIST (2) > LIST (3) Since we know LIST (2)=5 and LIST (3)=—2 and as such the answer to the question of step 4 is in affirmative We, therefore, proceed with step 5 and switch the contents of the two locations. In step 6, the switch counter is incremented by 1 to count that one switching has taken place.

In this manner, the loop is executed 4 times (X=4 in step 7) and then we take up step 8 which poses the question, "Is S, switch counter=0 ?" We know S≠0, therefore, the flowchart loops back to step 1 for the end pass.

Note 1 . If it were required to draw the flowchart for sorting those quantities in the *descending order* the above flowchart with step 4 modified as below would serve the propose

LIST (X) < LIST (X+1)

Note 2 . In the above flowchart, we have condensed the printing step 9. supposing the list were to be printed in the format below expand step 9 as an exercise.

—2 (Start print position 005)
1
4
5
7

Exercises Set 1

1 Draw flowcharts, one each for summing up the following serie

$1\,A + 2\,A^2 + 3\,A^3 + 4\,A^4 + \ldots$ N terms

$\frac{1}{2\,3} + \frac{2}{3\,4} + \frac{3}{4\,5} + \frac{4}{5\,6} + \ldots$ N terms

$\frac{1}{\lfloor 2} + \frac{2}{\lfloor 3} + \frac{3}{\lfloor 4} + \frac{4}{\lfloor 5} +$ N terms

$\frac{1\,3}{2\,4\,5} + \frac{2\,4}{3\,5\,6} + \frac{3\,5}{4\,6\,7} +$ N terms

$1\,A - 2\,A^2 + 3\,A^3 - 4\,A^4 +$ N terms

2 Draw a flowchart for finding the 16th root of a number.

3 Draw a flowchart for computing and printing the simple interest for 10, 11, 12, 13 and 14 years at the rate of 3% per annum on an investment of Rs. 10,000.

Miscellaneous Solved Examples

(After having gone through these the student may want to redraw these by closing the study paper)

Example 32. Physiologists claim that twice the height in inches of a person gives his normal weight in pounds roughly We have 50 hights as J (H) and corresponding weights as K (W) , both H and W=1, 2, 3 50 Draw the flowchart for computing the %age of over weights.

The flowchart is drawn in figure 32. The following symbols, in addition to the ones given, have been employed

RATIO	of height and weight of a person.
COUNT	Count of No. of persons.
PERCENT	%age of over weights.

Example 33. Salaries of 100 persons are designated by J (S), S=1, 2, 3.. 100. Draw flowchart for finding %age of the following salary ranges.

< Rs. 1500 (per month)

1500 to 3000

> 3000

The flowchart is drawn in figure 33.

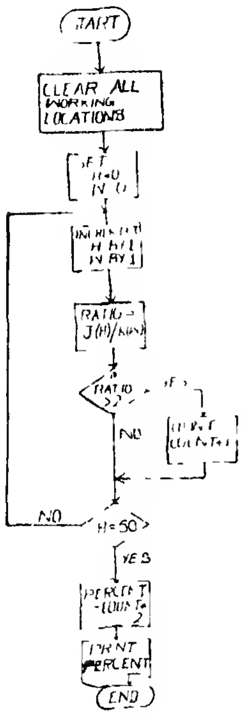


Fig. 32

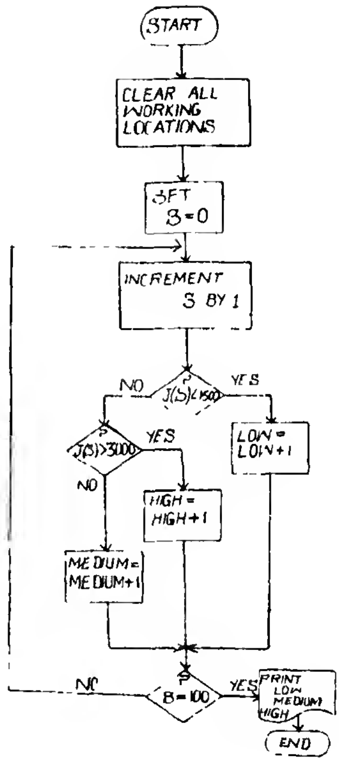


Fig. 33

(Example 34 to 36 should be done after having gone through OR Study VI)

Example 34. Referring to the first solved example in Replacement Theory in or Study VI draw the flowchart for finding the optimal replacement interval and the least cost. The following symbols may be used.

C	Initial Capital Expenditure.
S	Salvage Value.
CAPT	C-S (fixed for all periods)

Printout is to be had in the format below. Also, given are the necessary symbols.

Period	C–S	Running Cost	Cumulative Running Cost	Total Cost	Average Cost
Symbol N	Symbol	Symbol	Symbol	Symbol	Symbol
⋮	CAPT	R (X)	CUMR	TOCOST	AVCOST
⋮		X=1,2 .T			X=1, 2, T
.					
.					
Last period N=T					

The flowchart is drawn in figure 33. Stepwise explanation follows

Step 4. X of R (X) is set at zero. This initialises the loop from step 5 through step 13

In this loop, in step 8 R (X) are accumulated in CUMR one by one Total Cost is computed by adding C-S to CUMR in step 9, average cost is computed in step 10 by dividing total cost by N and the following are printed in step 11 in a line as above.

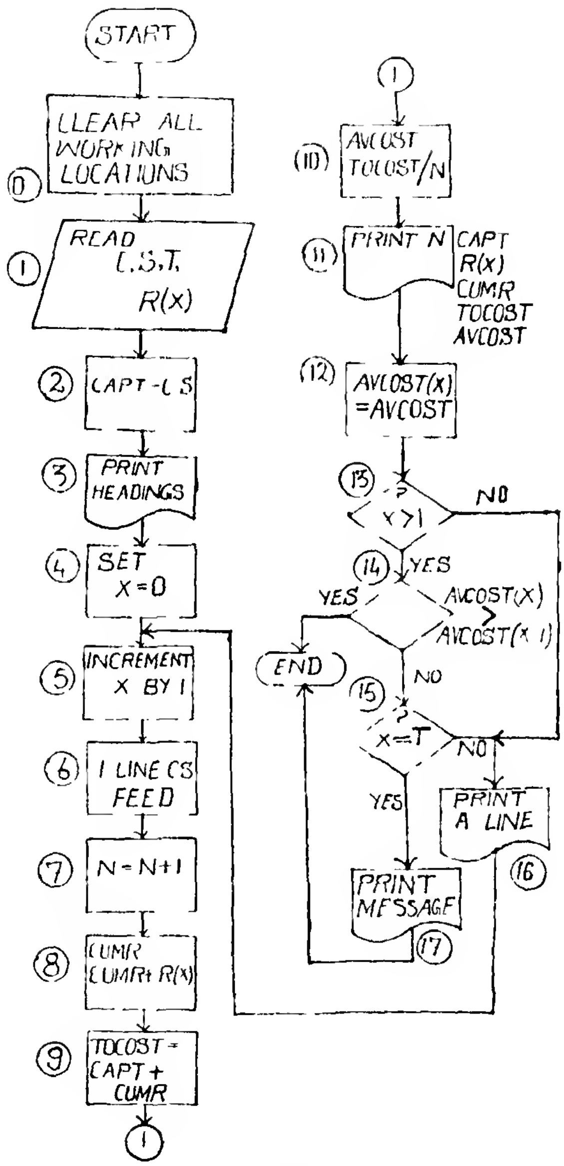


Fig. 34

Example 35. Draw the program flowchart for finding the optimal replacement interval for example 8 of the replacement theory, study VI of OR Rate of interest, r is given.

An excerpt of the solution of this example for Machine A is given below. Symbols used in the flowchart below are also given at the bottom of the excerpt. $R=1+r$

Year	Running Costs	V^{n-1}	Col. 2× Col. 3	Σ col 4	Total Cost	$\frac{V^n-1}{V-1}$ $=\Sigma V^{n-1}$	Col 2×Col 7 e g 2nd fig. of Col 2× 1st fig of Col 7
1	2	3	4	5	6	7	8
1	800	1 0000	800	800	5800	1 0000	800
2	*800	0 9091	727	1527	6527	1 9091	1528
3	800	0 8264	661	2188	7188	2 7355	2189
X=1 2 ..T	R (X)	F	F*R (X)		TOCOST (X)	CUMF	COL 8

COL is Accumulated in TOCOST

Step 1. Initial Cost, C, Salvage Value, S, the total years for which data is given, T, $R=1+r$ and R (X), the running costs are read via, say, a punched card.

Step 2. Let TOCOST=C−S

Step 3. Set F=1 and X=0. F would be made $\frac{1}{R}$, $\frac{1}{R^2}$ in turn in the loop to follow.

Step 4. X is incremented by 1 in steps 4 to 8 i.e , in the loop whenever it is used

Step 5. TOCOST (X) i e. each figure of Col. 4 is computed by discounting R (X) with the discount factor F F is 1 to start with In step 10 it would be made $\frac{1}{R}$, $\frac{1}{R^2}$, etc. in turn.

Step 6 Col. 6 figures are obtained in TOCOST which accumulates Col. 4.

Step 7. $\frac{V^n-1}{V-1}=\sum_{i=0}^{i=n-1} V^i$, therefore, it is obtained (i.e. figures of Col. 7) by accumulating F^{i} in CUMF.

Step 8 COL 8 (i.e. col. 8 figures) are derived by multiplying CUMF with R (X+1) i.e. running cost of the following row.

Step 9. COL 8 is compared with TOCOST.

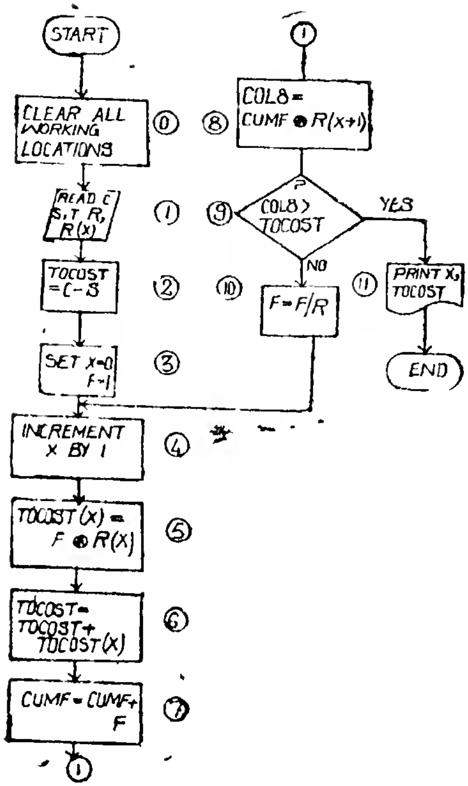


Fig. 35

Example 36 Draw flowchart for finding the optimal replacement interval for group replacement (for items that fail suddenly) with reference to example 9 on Replacement Theory in OR Study VI

The flowchart is drawn in figure 37. The following symbols have been used.

N	Number of bulbs in the installation.
GRCOST	Cost of replacing a bulb in group.
INCOST	Cost of replacing an individual bulb.
P(X)	Probability of failure (Not to be confused with the given "probability of failure to date" in example 9, OR Study VI) in X-th week, X=1, 2

The above are read in ·

T	Replacement Interval. T will assume values of 1, 2
N(Y)	No. of bulbs individually replaced in period Y=1, 2, 3, . .
TERM	N_0p_1 or N_0p_2, etc In general, it is N(Y) * P(X). It is to be noted that X+Y=T.

TOCOST Total cost. It holds the total costs for 1 week, 2 weeks, 3 weeks, etc in turn

AVCOST (T) Average cost for replacement interval T.

Step-wise Explanation

Step 0. Clear all working locations

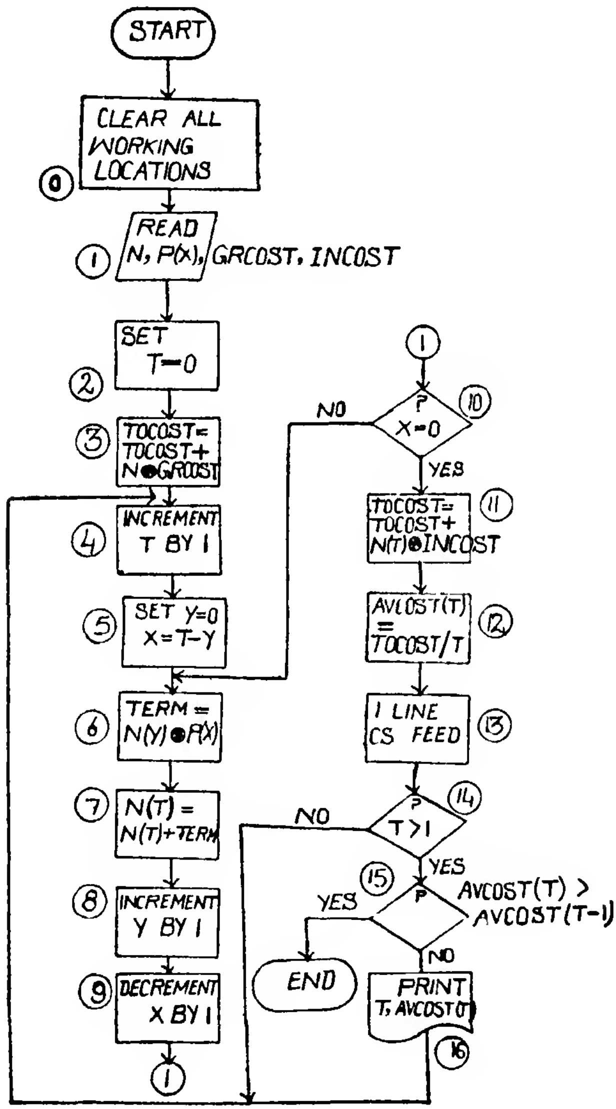


Fig. 36

Step 1 N, GRCOST and INCOST are read in via, say, a punched card. Also read in are all P(X) To tie P(X) to the solved example 9 of OR Study VI

$$P(1)=0\ 05$$
$$P(2)=0\ 08$$
$$P(3)=0\ 12, \text{ etc } \textit{i e} \text{ they are pre computed}$$

Step 2 T is set = 0 for the outer loop from steps 4 through step 14.

Step 3 Total cost is set at the periodic group replacement of N bulbs at $1000 \times 0\ 60$

Step 4 T is incremented by 1 i e to start with it is 1 weekly replacement policy. Thus step 4 through step 15 constitute the replacement policy loop.

Step 5 $\left.\begin{array}{l} Y=0 \\ X=T-Y \end{array}\right\}$ for the inner loop from steps 6 through Step 10 In this loop we shall compute

$$N_1=N_0p_1$$
$$N_2=N_0p_2+N_1p_1$$
$$N_3=N_0p_3+N_1p_2+N_2p_1, \text{ etc , in turn.}$$

Step 6 TERM = N(Y) * P(X)

Since to start with T = 1, Y = 0, X = T − Y = 1 ,

TERM = N(0) * P(1) i e N_0p_1

Step 7 We compute N(T) i e N(1) to start with as the sum of previous value of N(1) which is zero and TERM which is N_0p_1.

Step 8 & 9. Y becomes 1 and X becomes 0.

Step 10. For N_1 only one term , therefore X=0 makes it an exit from the loop

Step 11. The inner loop yields the values of N (1), N (2), etc in turn i e the number of individual bulbs to be replaced in each week chronologically To TOCOST is added the cost of individual weekly replacements.

Step 12 Average costs are computed and stored in AVCOST (T) i.e.
AVCOST (1), AVCOST (2), etc
AVCOST (0)=0, made in step 0.

Step 15. AVCOST (T) is compared with AVCOST (T−1). If it is more it means AVCOST (T—1) was optimal with (T−1) as the optimal replacement period ; otherwise, flowchart loops back to step 4 Since AVCOST (1) is more than AVCOST (0), the latter being zeroised in step 0 it is an exception to step 15.

Step 16. The interval T and the corresponding average cost are printed.

Example 37. A student has to take 5 subjects in an examination He is declared to have passed if he gets in the aggregate a minimum of 45% marks and a minimum of 40% marks in each subject. If the aggregate marks is 60% or more with 40% minimum marks in each subject, he is declared to have passed in first class If the student does not pass but gets 50% marks in any of the subjects he is exempted from appearing in that subject next time.

Write a flowchart to print the result, given the students names and mark in each subject as input.

The flowchart follows. The headings referred to in its step 1 are given below

Name	Marks in paper no					Total marks or "FAIL"	Division "FIRST" or left blank
	1	2	3	4	5		
Print Position 005	025	030	035	040	045	050	056

(N B. Marks followed by "E" means exemption in that paper)

The following symbols have been employed.

M(X),	M(1), M(2), M(3), M(4), M(5)	for marks in each of the 5 papers	MARKS	holds marks in each of the 5 papers one by one

TOTAL	Total marks (only if passed all papers)
FAIL	=0 means pass =1 means fail in one or more papers
NAME	Name of a student.

Explanation on the flowchart

Steps 6 7 8. 9 ascertain if a student has passed all papers or not If he has passed FAIL remains 0, otherwise upon failure in a paper FAIL is made 1

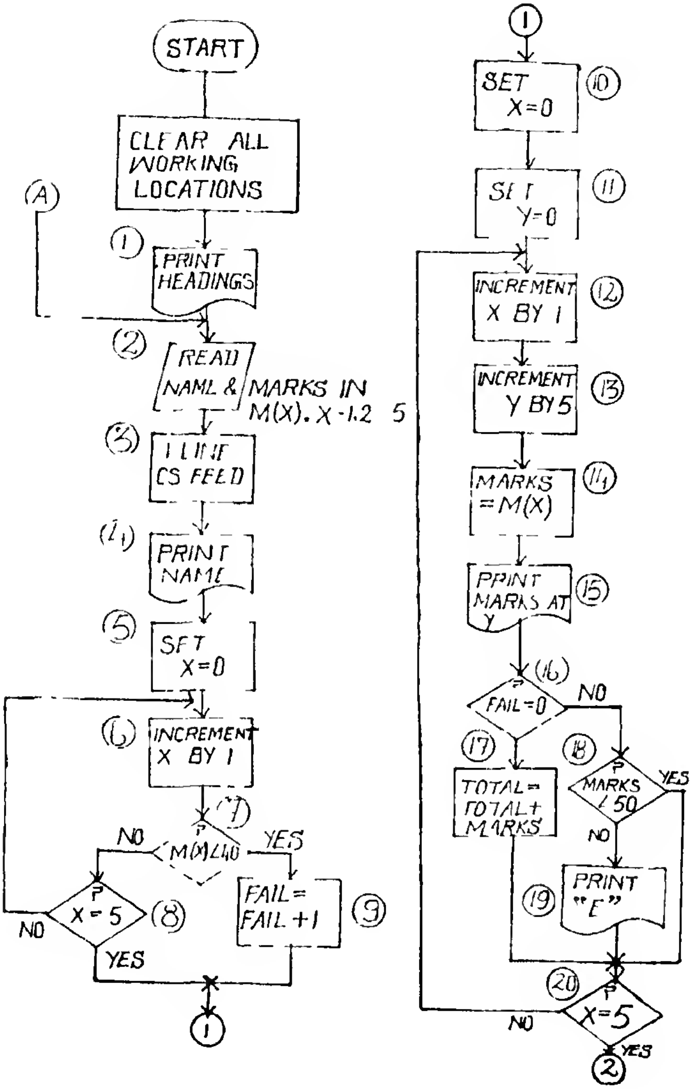
Steps 10 and 11 initialise the loop for printing marks in each of the papers for printing "E" where applicable and for finding the total marks of a pass candidate This loop being initialised runs from Step 12 through Step 20. X and Y are set at zero Also, TOTAL is zeroised

[In *Step* 14, marks in each paper, in turn, are copied in MARKS]

Steps 12 to 15 print the marks in each paper at print position Y=005, 010, 015, 020, 025

Steps 18 and 19 apply to "not pass" students for determining and printing E, exemption Step 17 accumulates the marks of a student in the 5 papers.

Steps 21 to 27 are self-explanatory.



(In step 10, please include TOTAL=0)

Fig 37 Contd.

Example 38. Write a program flowchart to process data in the format shown below ,

Employee No.	Job classification code	No. of years employed	Employee's current annual salary

Punched Card

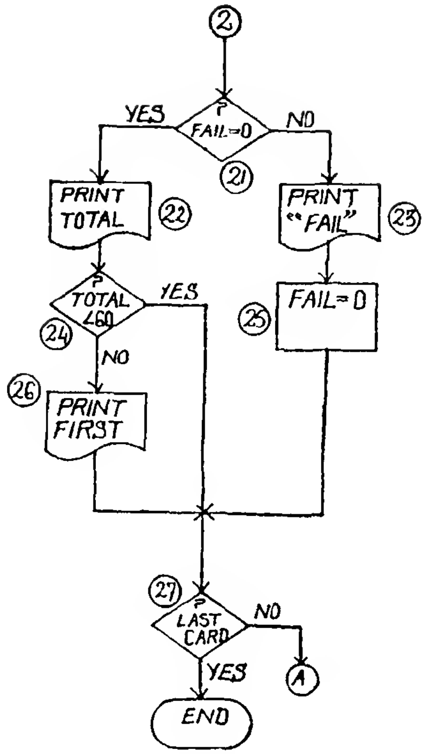


Fig. 37

Each employee is to receive a % age salary increase based on the job classification and the number of years employed The following table shows the % age increase for the different job classifications and years of employment

Years Employed	Job Class 1	2	3	4	5	
0 - 1	5%	5%	5%	5%	5%	J(1) to J(5)
2—5	10%	10%	15%	15%	15%	J(6) to J(10)
6—10	15%	15%	15%	15%	20%	J(11) to J(15)
11—20	20%	25%	25%	25%	25%	J(16) to J(20)
21—99	28%	28%	28%	28%	30%	J(21) to J(25)

The flowchart is to determine for each employee the rupee amount of the increase and print on one line the employee number, the old salary, the increase and the salary after the increase. At the end of the report print the total of all increases and the total of all new salaries.

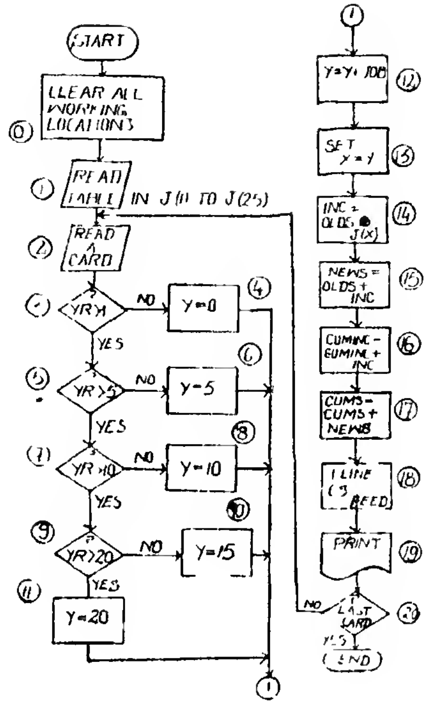


Fig 38

Solution The flowchart is drawn in the figure 38 above. The following symbols have been employed.

J(1) to J(25) hold the table of % age increase J(1) to J(5) are meant for the 1st row, J(6) to J(10) are meant for the 2nd row and so on.

YR	Years Employed
JOB	Job Class (1, 2, 3, 4, or 5)
Y	Working location to determine location holding the % age from the given table on the basis of the years employed and Job Class.
INC	Increase in Salary
OLDS	Old Salary
NEWS	New Salary
CUMINC	Cumulative Increase in Salary.
CUMS	Cumulative new Salary

Explanation. First, we shall explain how Y is assigned a value. Suppose there is an employee with 7 years of service and Job class 4. Looking along the yeras of service across the % age table this employee's % age is to be formed in the 3rd row i e. in one of the locations J(11) to J(15). Thus, if we first set Y=J(10) and then make it J(13) in view of his Job Class 3 we would have got J(13) as the address of the

location that contains the % age applicable to this employee Steps 3 to 11 set Y= 0, 5, 10, 15, 20 for the Ist, 2nd, 3rd, 4th and 5th rows respectively In Step 12 Y is incremented by the Job Class

In Step 13 we set X=Y for step 14 In step 14 we compute the increase in salary by multiplying the old salary with J(Y), the % age increase.

Example 39. Same as example 38 but intead of the years employed therein we have the following column.

Years Employed
0–5
6—10
11–15
16–20
21–25

Solution . The flowchart is drawn in the figure below. This time the years employed class intervals are equal , therefore, in step 3 of the flowchart Y is straighting computed as

$$Y = \left(\frac{\text{years employed}}{5} \right) - 1 + \text{Job Class}$$

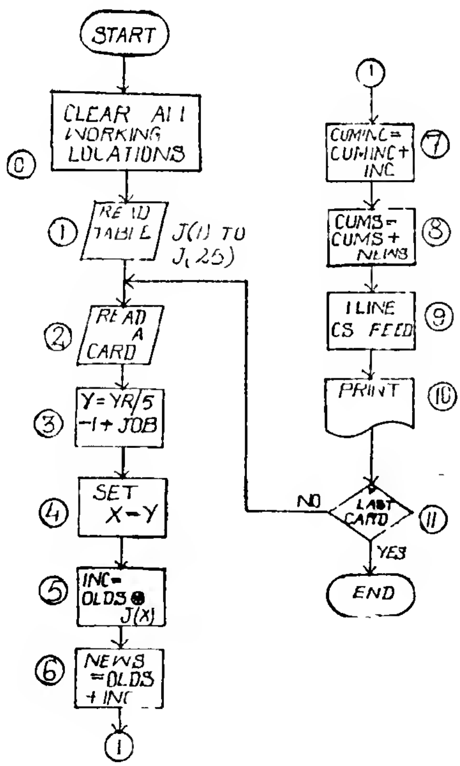


Fig. 39

Example 40. A Comprehensive Flowchart for Processing Sequential Files under Batch Processing Mode

(N.B. The flowchart of figure 13, Study I was deliberately kept simple by using dummy blank cards at the end of the master stock file and transaction file. However, this flowchart is comprehensive as well as usable in real-life situations).

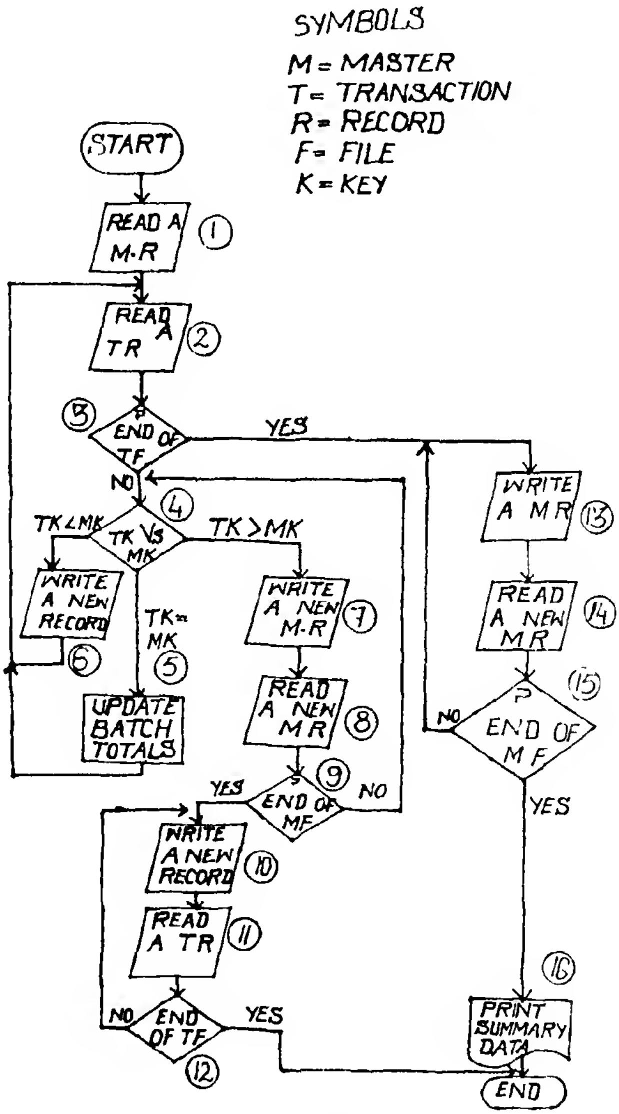


Fig. 40

To explain the various steps let us suppose that the following are the master file and transaction file record keys :

Master File	1 2 4 6 9 11 17 20 21 22 29 *
Transaction File	*1 5 6 7 9 11* *
	* means end of the file.

Also, please note that there are gaps in the keys of the master file Keys 5 and 7 in the transaction file represent new records created to be put into the master file appropriately in sequence

Please have a look at the definition of the various symbols used at the top of the flowchart.

Step 1 A master record is read Key=1

2 A transaction record is read Key=1

3. Is transaction Key * ?
No, it is 1 , therefore, step 4

4 Transaction Key Versus Master Key. They are equal , therefore, Step 5.

5. Master record is uprated. Also accumulated are batch totals from the transaction record. Back to Step 2

2 Read a transaction record Key=2

3 Is transaction Key * ?
No, it is 2 ; therefore, step 4.

4. Transaction Key (2) is greater than Master Key (1) , therefore, Step 7.

7. Updated master record 1 is written on the output (C/F) magnetic tape.

8. Next Master record is read Key=2

9 Is master Key * ? No, it is not , therefore, Step 4

4. Transaction Key (2)=Master Key (2) ; therefore, Step 5.

5 Update master record 2. Also accumulate batch totals for transaction 2. Back to Step 2.

2. Next transaction record is read Key=5

3. Is transaction key * ? No, it is not , therefore, to Step 4.

4. Transaction Key (5) > Master Key (2) ,
therefore, Step 7.

7. Updated master record 2 is written onto C/F tape

8. Next master record is read Key=6

9. Is master Key * ? No, it is 6 ;
therefore, back to Step 4.

4. Transaction Key (5) < Master Key (6) ,
therefore, Step 6.

6. New master record is created by
copying transaction record 5 on to CF
tape. Back to Step 2.

The student may proceed on this way until both the asterisks have been reached one by one. Likewise, he may "dry process" (just as we did above) the following. master and transaction files.

Master File	1, 2, 3, 4, 6 *
Transaction File	1, 4, 7, 8, 9, 10 *

Here new records 7, 8, 9, 10 have to be put at the end of the C/F tape.

Exercises, Set II. *(The Student must do these. Mere understanding our examples is not enough)*

1. Wages of 500 workers are held in J(W), W=1, 2, ... 500 Draw the program flowchart for compiling the frequeney distribution and printing it in the following format.

Class Interval	*Frequency*
< 300	
300 < 400	
400 < 500	
500 < 600	
600 < 700	
700 < 800	
800 < 900	
> 900	

2. Out of an array of numbers J (N), N=1, 2. . 100, 5 numbers are known to be zeros. You have to draw the program flowchart for squeezing the zeros out i.e. re-arrange the 95 non-zero numbers in locations J (N), N=1, 2 ..95.

Also, extend the flowchart for printing these 95 numbers in a 19×5 matrix assume that a number is at most of six digits

3. J(E), E=1, 2 . 42 contains 42 quantities of a 7×6 matrix. Draw the program flowchart for printing its transpose , assume that a number is at most of 5 digits.

4. Names of eleven cricket players are held in J(X), X=1, 3, 5 21 and their respective batting average in J(X), X=2, 4 22. You are to arrange the eleven players in the descending order of their batting average e g J(1) and J(2) would hold the name and average of the topmost player.

5. Draw the program flowchart for summing up 1, 11, 111, 1111 (10 terms).

6. J(X), X=1, 2 200 designate 200 numbers. Draw the program flowchart for computing the following.

    1. The % ages of negative, zero and positive numbers
    2. The sum of the -ve and + ve numbers separately.
    3. The sum of the absolute numbers.
    4. The sum of squares of all the numbers.

7. Draw the program flowchart for printing the following pattern on the continuous stationery.

```
M               M
 M             M
  M           M
   M         M
    M       M
     M     M
      M   M
       M M
        M
```

8. Referring to OR Study II, for the Newspaper Boy's problem draw the program flowchart for finding the expected payoff of each strategy and picking up the best strategy.

9. Assume that you opened a savings account with a local bank on 1-1-1980 The annual interest rate in 5 75% Interest is compounded at the end of each month Assuming that your initial deposit is ×rupees draw a flowchart to print out the balance of your account at the end of each month for two years.

10. Write a program flowchart to compute the mean and S.D of N numbers denoted by J(X), X=1, 2 .. N.

11 The median of a list of numbers is defined to be that item in the list that has as many items greater than it as less than it, if there is an odd number of items in the list the median is the average of the two middle items. Draw a program flowchart to read a list of numbers and print the median

[**Hint** : Sort the list first]

12. A + ve integer is called "perfect" if it equals the sum of its proper divisors For example, the numbers 6 and 28 are perfect, because,

$$6=1+2+3$$
$$28=1+2+4+7+14$$

Write a flowchart to decide whether a +ve integer is perfect.

13 There is a class of 150 students. Each student appears in 12 papers The marks obtained by him are transcribed on a punched card, thereby giving us a deck of 150 punched cards It is desired to draw the program flowchart for totalling and printing the marks of those students who pass all the 12 papers and printing "FAIL" for others Pass marks=40%.

14. In addition to the printing requirements of example 13 the following would also be printed :

Marks in each paper (whether or not he passed it individually)
Marks in a paper would be followed by E if a student scores more than 60% marks in it but has failed in at most one paper (E stands for exemption).

Marks in a paper would be followed by D if a student has scored 75% or more and also he has not failed in any paper (D stands for distinction).

Summary Data

No of pass (in all papers) students
No of failed students.
No. of exemptions in each paper.
No. of distinctions in each paper.

You are advised not to tamper with the flowchart of exercise 13 but to draw the flowchart for this example afresh.

[**Note** · *The student may send in at most 4 flowcharts of these 14 to us for appraisal*].

•Part II. RUN FLOWCHARTING

Several business applications (viz. Inventory Control) are discussed in this part by means of run flowcharts These, however, cannot be grafted on a given organisation i.e, they are just hypothetical in nature and are given here to explain flowcharting. All the steps of systems analysis and design discussed in delail in Study III have to be gone through for arriving at flowcharts for the various applications for a given organisation in practice.

•Combination C students need not concern themselves overmuch with flowcharting details ; but they ought to do carefully the inputs, processing and outputs of each application.

The transactions in these applications are punched on cards and processed periodically against the master files to produce the desired reports, summaries, projections etc. in a computer setup (also known as the computer run). The transactions may have to be sorted before the file up-dating run If these transactions are on punched cards these would be sorted by the key in which the master file has been arranged on a card sorter off line. If, however, the cards are on a magnetic tape they can be sorted online in several passes in what is known as a sorting run.

The punched cards have been used for the transactions files and the magnetic tape has been used for the master files throughout the study note merely for illustration Alternative media, however, can be employed Which media actualy to employ in a given situation is again a systems analysis and design matter and is adequately discussed in study III.

It is to be carefully noted that all the data fields appearing on the output reports, summaries, projections etc are after all either directy picked up from the transactions/master files or derived, after some arithmetic operations, from these. If therefore, the layouts of the outputs are well-defined and the input layouts designed to suit the former writing of the program for a particular up-dating run becomes self obviously simple and can be entrusted to the programmers. Incidently, we can also assert if the student is clear about layouts of outputs and of transactions/master files for an application the run flowcharts should flow out of his hands naturally. Besides, in the program have also to be incorporated some checks and controls to serve as, what we call, judgement For example, in a manual system if a clerk finds the price of a washer as Rs 50/- instead of 50 P, he would be suspicious, run around, get the correct price and amend the records accordingly. The computer does not possess any such judgement and it would accept Rs. 50/- as the correct price for a washer and even pass the invoices sent by the supplier. Therefore, there is a need to build in checks and controls in the program so that, such errors or omissions can be detected and brought to the notice of the computer users Invariably, in an up-dating run one of the outputs is an error list and summary information which contains such errors and the batch totals computed by the computer. These totals are then compared with the ones derived manually prior to processing as discussed in study I, and the errors detected via the checks and controls are scrutinised by the user departments and ultimately rectified and re-submitted for computer processing.

Another point which needs some explanation at this stage is the term, integration. Integration of sub-systems implies multiple use of the same transaction copies and files.

There are several gradations of integration. In a rudimentary way, it can be had even in manual systems. For example, a company has a simple integrated procedure for processing goods received notes. In this procedure, the receiving organisation puts a stamp on the back of the advice notes received along with the goods from the suppliers. This stamp carries all the usual particulars to be found on a goods receipt note This single copy of the advice note then travels from the receiving section through the inspection, stock control, purchasing to invoicing section etc. All these sections update their files with this single copy and make the appropriate notings on it This is an example of the integration of various departments i.e., all of them use the same copy of the transaction in contrast

to un-integrated systems where the goods receipt note is prepared in several copies and each department uses its own copy In all the applications discussed in this study note, this concept of multiple use of the same transactions is to be found The transactions may be put on punched cards or magnetic tape, etc, for computer processing but they are never multiplied in several copies Integration at a higher level also implies fusing of the various files into a data base and is discussed in more detail in study III

1. Sales

A computer-based sales order processing system is characterised by the following possibilities of integration —

1 Sales accounting and sales analysis are integrated with accounts receivable operation and records

2 Credit functions can be integrated as far as possible into the overall flow of automated processing

3 The sales application is closely related to inventory control processing

Integration of credit functions is based on the assumption that credit review contains elements that are capable of being handled by a computerised system This assumption is valid if credit review involves the application of a set of criteria in a systematic way without any resort to subjective considerations Nevertheless, even if the credit review cannot be completely formalised and automated the bulk of routine computations can still be performed by the computer The rejections would contain the special cases that would be reviewed subjectively later.

The flowchart in figures 1 to 3 depict a typical sales order processing system

1 1 Input Preparation

The sales department prepares the sales order in duplicate upon receipt of the customer's purchase order or telephonic information from a salesman or the customer. after satisfying themeselves that the customer's account is not delinquent. One of the outputs of a computer run to be discussed subsequently is the list of delinquent accounts which the sales department consults to establish the credit-worthiness of the customer. One of the copies of the sales order is sent to the customer as an acknowledgement of the receipt of his purchase order The other copy is sent to the shipping department as an authorisation to ship the goods to the customer. The format of the sales order should be so designed that it facilitates error-free and quick entry of data from the custmer's purchase order as also transcription of data from it on to punched cards subsequently. The sales department enters the quantity shipped and quantity back ordered against each line on the order, i e, for each stock item on order, the price having already been entered by the sales department. The shipping department assembles the sales orderes in daily batches and compiles the following batch totals for it on an adding machine, perhaps · record count, financial total of the values of items shipped and hash totals of quantities shipped, quantities back-ordered, customer's accounts numbers and stock items numbers The batch of the sales orders together with the control slip bearing these batch totals is then forwarded to the data processing department for transcription on to punched cards.

Likewise, the mailroom assembles the daily bath of remittance advices received from the customers and compiles the following batch totals for it record count, financial total of the amounts remitted and the hash total of the customer account numbers. The batch together with the control slip bearing these totals is forwarded to the data preparation section.

We are dealing with only two types of transactions for this application for simplification of illustration though in practice there would also be the following inputs :

(i) Addition of new records to the file

(ii) Credits for sales returns and allowance

(iii) Account right-offs

(iv) Changes of addresses and other routine adjustment and corrections

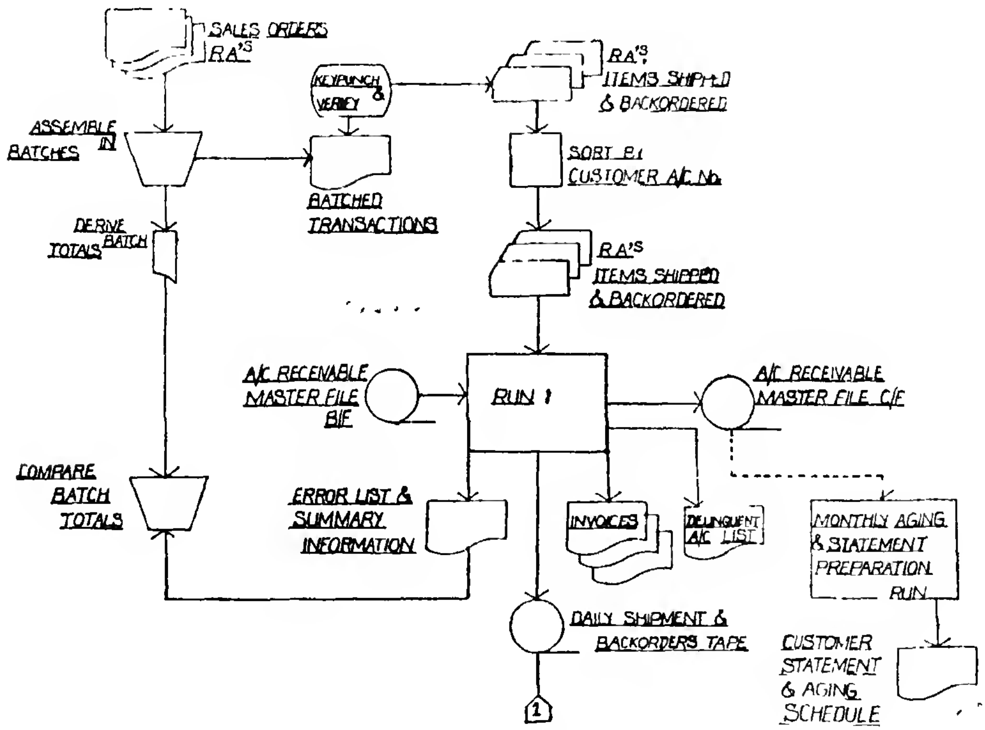


Fig. 1

The data preparation section of the data processing department prepares a punched card for each item back-ordered, each item shipped and each remittance advice on the key punch. The deck of punched cards is then given to the verifier operator who verifies the cards by re-entering the transaction data via the verifier keyboard. For economies in verification time it may, however, be desired that just the critical data fields on the punched cards are verified.

The deck of punched and verified cards is then sorted off-line on a card sorter with the customer account number as the key

The sorted deck of punched cards consitutes the transaction file which is used to update the accounts receivable master file by the accounts receivables update program in a computer run In this run, the customer accounts are updated for the sales Typically, the accounts receivable master file contains the following fields

(i) Customer account number (control field).

(ii) Customer name and address.

(iii) Credit rating.

(iv) Credit limit

(v) Balance due as of last monthly statement

The following particulars of each transaction since then

(i) Transaction type code.

(ii) Document number.

(iii) Date

(iv) Amount

(v) Current balance

Incorporated in the program are also the various control checks discussed in detail in study III which check the picture, limits etc. of the various data fields. The outputs of this run are as follows :—

(i) The updated A/c receivables master file

(ii) Delinquent accounts list which contains the particulars of those customer who have crossed either the credit limit or the credit rating assigned to them

(iii) Invoices in as many copies as desired. It has to be noted that invoices would be prepared for only the items shipped. A specimen of the invoice is given below .—

XYZ Manufacturing Company

15, High Street, Sometown Tx   Tel   . Invoice No ...

INVOICE

Customer Order No	Date	Salesman Code	Cust Acct No
. . ..		.	

Sold to	Ship to
ABC Mfg Co	Same
13, Nehru Road	
Allahabad	

Shipper	Date Shipped	Invoice Date	Terms of Sale
. . ..	. ...	.. ... .	.. .. . .

Item Code	Description	Qty Ordered	Qty. Back-Ordered	Qty. Shipped	Unit Price	Total Price

(iv) *Daily shipment and back-orders tape* The punched cards data is put on to magnetic tape for a subsequent speedier resorting by stock item number as well as speedier processing against inventory file the discussion on which would follow.

(v) *Error list and summary information* This contains the rejects i e, those transactions which could not pass the control checks. These are ultimately investigated by the user department, rectified and re-submitted to the data processing department for reprocessing. The summary information

consists of all the batch totals derived by the computer. These batch totals are compared with the ones derived manually prior to processing and entered into control slips travelling with the batches of the transactions.

The accounts receivable file is also processed every month to produce the customer statements and aging schedules a specimen of which is shown below.

Statement
XYZ Monufacturing Company
15, High Street
Sometown

To

ABC Limited Date Account No.
13, Nehru Road
Allahabad

Date	Invoice Number	Charges	Credits	
				Previous Balance .... Current Account .... Total Amount

Past Due Amounts
Over 30 days .
Over 60 days .
Over 90 days.... .

1.3 Inventory processing

The daily shipment and back-orders tape obtained as an output of a run in the previous flowchart is used to update the finished goods master file according to the concept of integration discussed at the outset of this part. Since, however, the finished goods master file is sequenced by the product stock number as the key, the daily shipment and back-orders tape is also sorted in this order. In the daily finished goods update run, the sorted daily shipment and back-orders tape constitutes the transaction file and by updating finished goods master file produces the following outputs :

(i) *Finished goods master file* is updated for the various stock balances in it. Back-orders sub-records may also be added.

(ii) *Back-orders and replenishment orders list.* The items for which the stock balance has fallen below the reorder level would find place in the replenishment orders list. 'Backorder' means a customer's order for an item against which goods cannot be supplied now but would be supplied as and when they are received from the works.

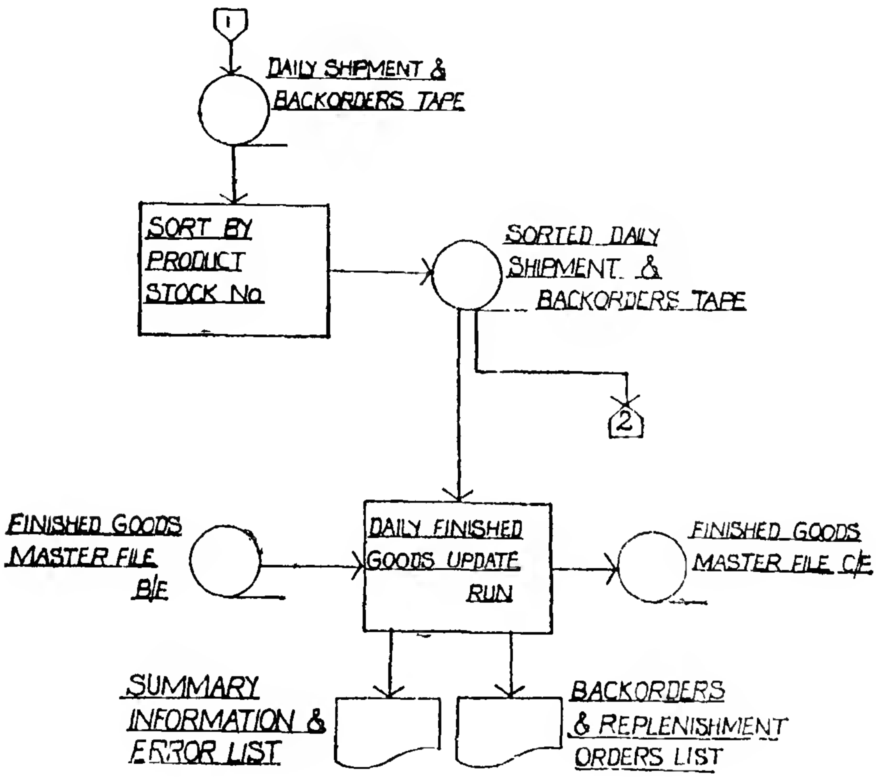


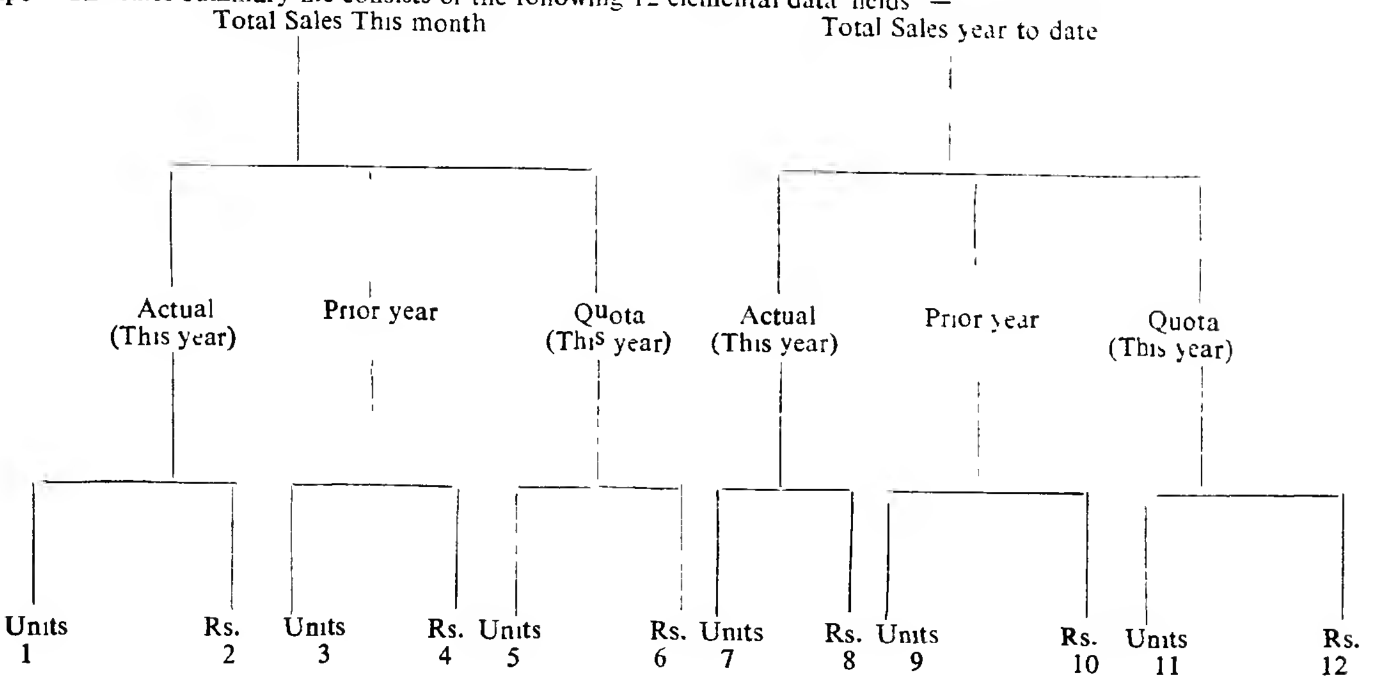
Fig. 2.

(iii) *Summary information and error list.* The summary information consists of the various batch totals derived by the computer These will ultimately be compared with their counterparts derived manually prior to computer processing. The error list would contain erroneous transactions which could not pass the various checks built in the computer program.

The sorted daily shipment and back orders tape, in consonance with the concept of integration, is used in the following flowchart for sales analysis runs.

1.4 Sales analysis runs

The sales analysis are made by updating the sales summary Master File by the daily shipment and back orders tape. The sales summary file consists of the following 12 elemental data fields —



This set of 12 data fields appears against each combination of a product, a customer and a salesman in the sale summary master file maintained on magnetic tape For example, these 12 data fields would be there for Ramu (a salesman) who is selling refrigerators to P (a customer) Likewise, these 12 data fields would be there again for Ramu who is selling refrigerators again to Q (another customer), and so on In other words, for each combination of a salesman, a product and a customer this set of 12 data fields would be there

The layout of this file sequenced by the product stock number as the key is pictured below

Refrigerators	*PX	PY	PZ	QX	QY	QZ	RX	RY	RZ	Air Coolers	PX	PY	PZ	QX
	1-12	1-12	1-12	1-12	1-12	1-12	1-12	1-12	1 12		1-12	1-12	1-12	1-12

(Contd)

(*) PX 1-12 stands for 1 to 12 data fields of the chart above for refrigerators being sold to the customer P by the salesman X

The Layout of the same file sequenced by the customer number is pictured below,

PX Ref	*PY Ref	PZ Ref	PX Air	PY Air	PZ Air	PX Exh	PY Exh	PZ Exh	QX Ref
1-12	1-12	1-12	1-12	1-12	1-12	1-12	1-12	1-12	1-12

(Contd)

PY Ref 1-12 stands for the 1-12 data fields for refrigerators being sold to the customer P by salesman Y

The layout of the same file sequenced by the salesman code is pictured below

PX Ref	QX Ref	RX Ref	*PX Air	QX Air	RX Air	PX Exh	QX Exh	RX Exh	PX Ref
1-12	1-12	1-12	1-12	1-12	1-12	1-12	1-12	1-12	1-12

(Contd)

(*) PX Air 1-12 Stands for the 1-12 data fields for the Air Coolers being sold to the customer P by the salesman Y

(Note : In an actual situation several of these sub-records would be missing e g , if the customer P does not buy air coolers the air coolers sub-records against this customer would not be there.)

It is assumed, however, that to start with the sales summary master file is sequenced by the product stock number and, therefore, it is straightway updated in run 1 by the daily shipment and back-orders tape As an output, the sales analysis report by product stock number is obtained

The sales summary master file is then sorted by the salesman number as the key and in run 2 a printout of the sales analysis by salesman is obtained.

The sales summary master file is finally sorted by the customer number as the key and in run 3 a printout of the sales analysis by customer is obtained.

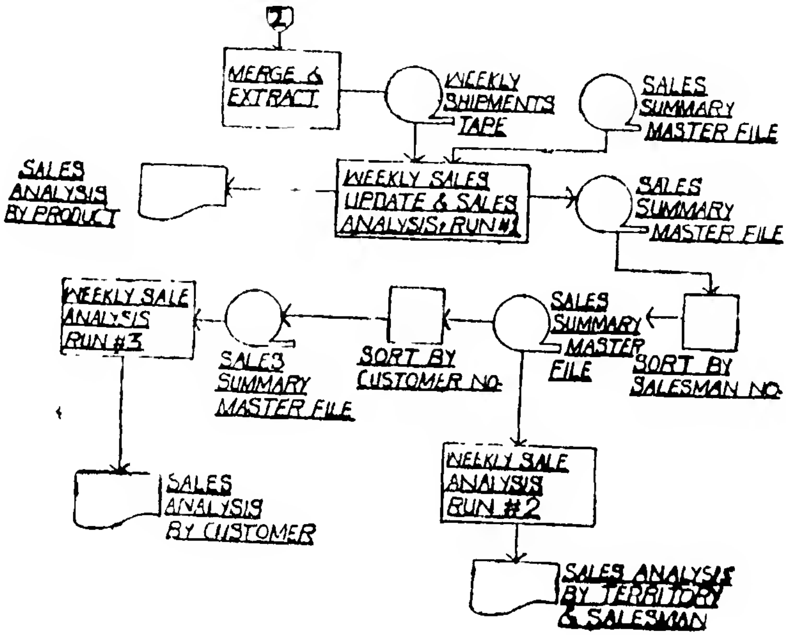


Fig. 3

SALES ANALYSIS BY SALESMEN

Period Ending · August '79

Salesman	Period	Sales		% Change	Quota	%Variance from quota
		Actual	Prior year			
		Rs.	Rs.			
Pannikar	This month	7,500	8,000	−6.25%	8.000	−6 25%
	Year to date	60,000	75,000	−20%	80 000	−25%
Natrajan						
⋮						

2. Payroll

The basic purpose of the payroll system is to produce payslips and pay-cheques for the employees every month. Towards this purpose, the employees payroll

file is run against their attendance cards, both being sequenced by the employee number The attendance sheet of each employee may include the following data fields :

Transaction Code	1 Characters
Employee No.	6 Ch
Deptt. No.	2 Ch.
Job No.	6 Ch.
No of days worked	2 Ch.
Special Deductions (Loan, etc.)	6 Ch.

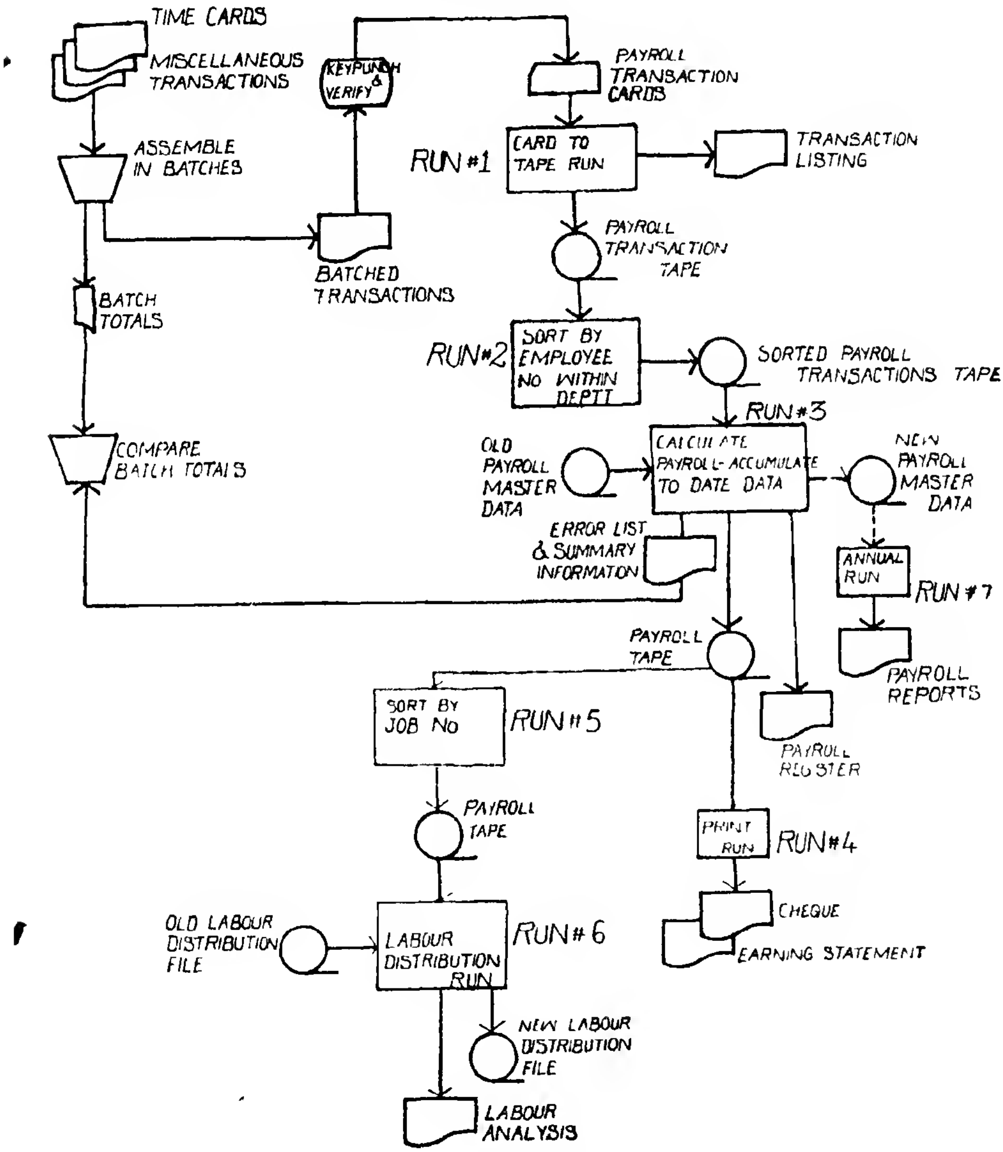


Fig. 4

Since each attendance record carries $6+2+6+2+6=22$ characters it would be desired to put three records in one punched card.

Besides the attendance records the transactions would also include additions, delections and amendments to the payroll file originating from the payroll section These are also laid out on the punched cards

In run # 1, the payroll transaction punched cards are put on magnetic tape for subsequent speedier processing Also printed out is the transaction listing for control and audit purposes

In run #2, the payroll transaction file is sorted by the employee number within departm nt number

In run # 3, the sorted transaction file is used to update the payroll master file on magnetic tape The latter has the following typical fields

Employee Number
Employee Name
Designation
Department Number
Job No
Marital Status
Basic Pay
Type of cost code
Amount of authorised deduction (several fields)
Year-to-date earnings total (several fields)

The type of cost code in the account number indicates the nature of the account to which the employees' gross pay is charged—direct labour, indirect labour, etc

Gross and net pay are calculated in this run year-to-date and period-to-date data are accumulated and written on the new payroll master file The data necessary for printing the payslips and the paycheques is put on a magnetic tape constituting another output of this run Also, printed are the Error List and Summary Information and the payroll register

In run # 4, the payroll tape is used to produce payslips and paycheques.

In run #5, the payroll tape is sorted by job number, which in run # 6 is used to update the labour distribution file with a specimen format given below .

Labour Distribution File on Magnetic Tape

Job No.	Trade 01	Value	Trade 02	Value		Trade 40	Trade
6 Ch.	2 Ch	6 Ch	2 Ch.	6 Ch.	.	2 Ch.	6 Ch.

(It can be seen that a maximum of 40 trades could be there for a job).

The cumulative values of various trades *as at the end of the last month* are updated to *as at the end of this month* by means of the payroll tape

The labour Analysis is printed for each job as another output of this run.

In run # 7 (which is undertaken annually), the payroll master file is used to produce annual payroll reports.

APPENDIX I

(*N B. . Combination C. Students may leave appendices*)

Sorting a Magnetic Tape file online

Unlike punched cards (which are sorted offline on a card sorter) the magnetic tape files are sorted online in connection with the CPU. Sorting, basically, involves enormous number of comparisons within the CPU and this matter is discussed in Part I. In this section, we shall assume that sorting in the CPU can be carried out Because of a large number of comparisons involved sorting consumes a significant data processing time and it is therefore recommended by many experts that sorting is best avoided But it cannot be obviated in magnetic tape installations where it may take as much as 30 to 50 per cent of the total data processing time on the computer. Transaction files would invariably be sorted in these installations by the key by which the master files to be up-dated by them are arranged If there are several files to be up-dated the transaction files would have to be sorted over and again by the key of each of these For example, the transactions on customer orders may first be sequenced by the customer account number as the key to up-date the account receivable master file, resorted to up-date the inventory master file and finally resorted again to up-date the sales summary master file. One should always be alert to reduce sorting time which can be done if the transactions are pre-sorted manually to some extent. The clerical routines should therefore be so designed that they encourage pre-sorting of the transactions Below we give a very simple example on the classical "two-way merge sort" technique.

There are two steps in sorting the magnetic tape .

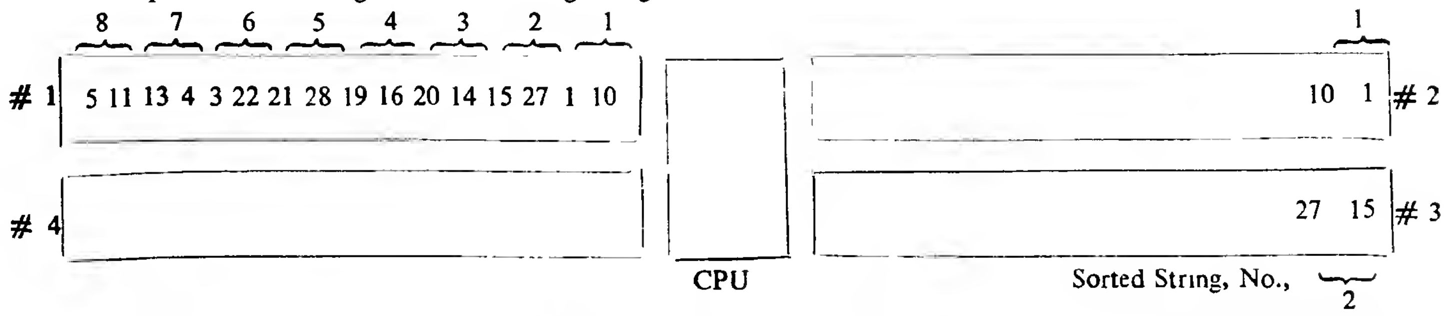
(i) Establishing the initial sorting strings.

(ii) Merging these strings into a single sequenced string.

To explain these two steps let us consider a file holding 16 records with the following keys :

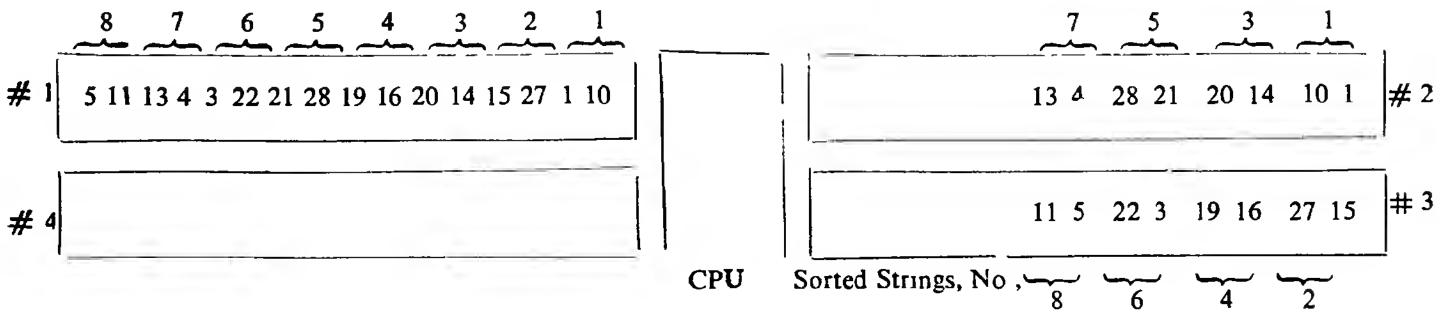
10, 1, 27, 15, 14, 20, 16, 19, 28, 21, 22, 3, 4, 13, 11, 5 ,

Step 1. Establishing the initial sorting strings



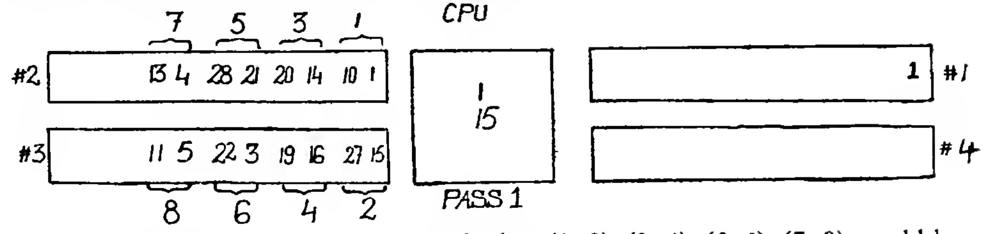
The file of 16 records is put on tape #1. Three more tapes # 2, # 3, and # 4, would also be needed for sorting this file online in connection with the CPU. We shall read a pair by pair of records in the CPU, compare their keys and put them in the ascending order if not already so Thus the first pair (known in the technical jargon as a string of twos) 10, 1 is sequenced into 1, 10 in the CPU and is put on tape # 2. The second string of 27, 15 is sequenced into 15 27 and put on tape # 3.

This process is continued i e pairs or strings are read into the CPU in turn, sequenced therein and are put on tapes # 2 and # 3 alternately. With this process (constituting our step (1) of establishing the initial sorting strings) we obtain 4 sorted strings each on tape # 2 and # 3 as below.

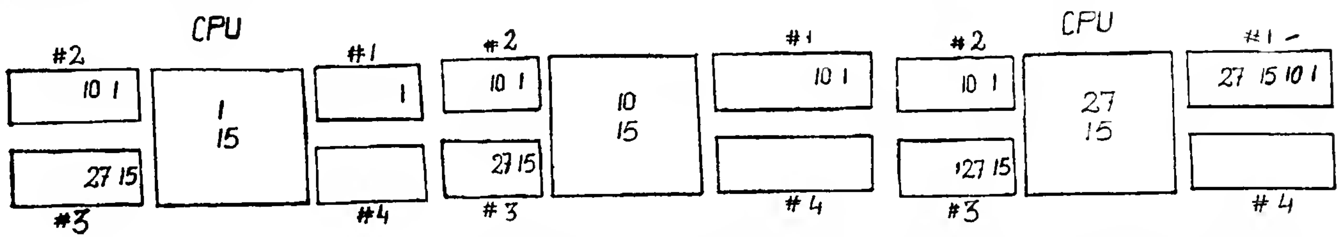


Step 2. Merging of these strings into a single sequenced string

Tapes # 2 and # 3 are brought to the input side and # 1 and # 4 on the output side.



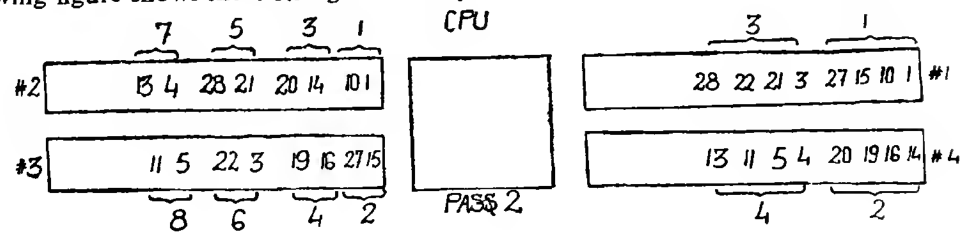
We now want to make strings of fours. Towards this, (1, 2), (3, 4), (5, 6), (7, 8) would be merged in turn to get 4 strings of fours. For merging (1, 2) we read into the CPU their first numbers, 1 and 15, 1 being the smaller is written on tape # 1.



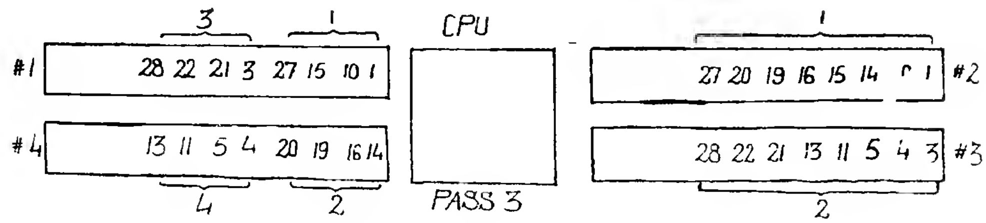
1 of tape # 2 having been put on tape # 1, we read in 10 of tape #2 in the CPU 10 being the smaller of 10 and 15 is put on the magnetic tape # 1

The last (4th) number of the two strings 27 is brought into the CPU. 15 and 27 are put on tape # 1 in this order

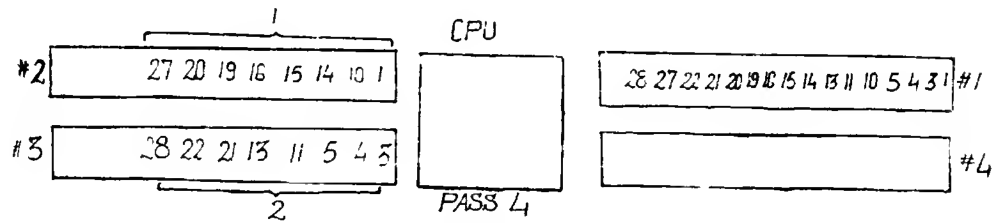
The following figure shows the 4 strings of fours put alternately on tapes # 1 and # 4,



Just as the strings of two's were merged into four's above the strings of 4's are merged into strings of eight's and put alternately on tapes # 2 and # 3 as below



In the next (final) pass the strings of eight's (from # 1 and # 4) are merged into one sorted string of 16 as below



We see that 4 passes were needed to sort the file of 16 records. The number of passes to sort N records is equal to $\log_2 N$ Thus the above sort of 16 records takes $\log_2 16 = 4$ passes If magnetic tapes are available in sufficiently large number a "three-way merge" requiring only $\log_3 N$ passes may be performed. The principle of operation is essentially the same as two way merge sort i e , three strings of sequenced records are created, merged and written. Only the final string of sorted records is produced. In theory, it is possible to have a "4 way merge sort", and so on. It can be generalised that the more the work areas made available to the sort program the faster the program will run.

The "sort-merge" programs are supplied by the computer manufacturers as a part of the utility program package.

APPENDIX II

Dry Run and Debugging the Program

In Study I, we stated that any program of some scope even written with great care is likely to contain some mistakes known as bugs in the technical jargon. There, therefore, is a need to remove these mistakes or debug the program Debugging should start with the review of the flowchart through review of the program code and finally testing the program with fictitious data in one or more computer setups

Review of the flowchart is also carried out by means of fictitious data and as such is known as the *dry run* since no computer setup is involved. We shall take up the flowchart of example 5 to elucidate the means to carry out the dry run This flowchart is concerned with picking up the highest and the lowest of three quantities . Q 1, Q 2 and Q 3 putting them in locations designated by H and L respectively

The flowchart of fig 5 is reproduced below in fig A except for the deliberate mistakes indicated by the crooked arrows in this figure (A) "Yes" and "No" have been interchanged, i e. bugs have been deliberately introduced Now let us see how these bugs are detected by means of the dry run.

We shall try three sets of values for Q 1, Q 2 and Q 3 as below

	Q 1	Q 2	Q3	
Set 1	6	2	14	Q 3 > Q 1 > Q 2
Set 2	3	7	15	Q 3 > Q 2 > Q 1
Set 3	2	4	3	Q 2 > Q 3 > Q 1

In Fig A the data of the Ist set have been 'flown' in the flowchart and it flows across the dotted lines in the flowchart Ultimately, we end up with 14 in H and 2 in L This is correct since we can see for ourselves that in the first set 14 is the highest and 2 is the lowest.

In Fig B on p 73 the data of the 2nd set is flown and it flows across the dotted lines in this flowchart Again, we end up with the correct result, 15 as the highest and 3 as the lowest in H and L respectively

In Fig C on p 74 the data of the 3rd set is flown Here we end up with 4 in H and 3 in L which is wrong since we can see that 2 is the lowest in the 3rd set whereas we are getting 3 as the lowest This arouses our suspicion and we would carefully scrutinise the lower portion of the flowchart until we detect the bugs.

Following this up we shall rectify the flowchart and the program code.

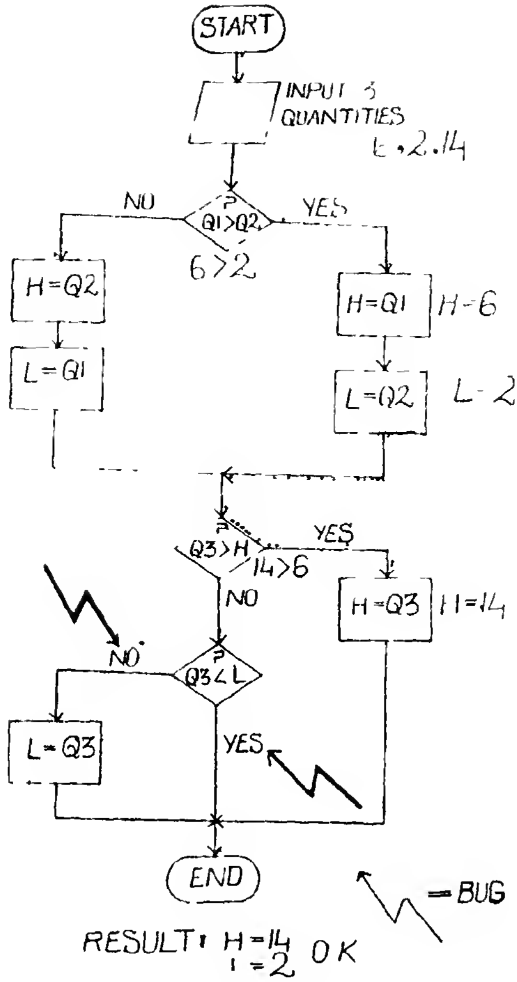


Fig. A

[By means of such dry runs the student may want to verify the flowcharts for the exercises he draws].

In larger flowcharts with many more branches we shall not be content with a mere dry run. We shall actually set up the computer with the given program loaded is its memory, input the test data, and compare the results output by the computer with the ones computed in longhand. The task of debugging is formidable indeed.

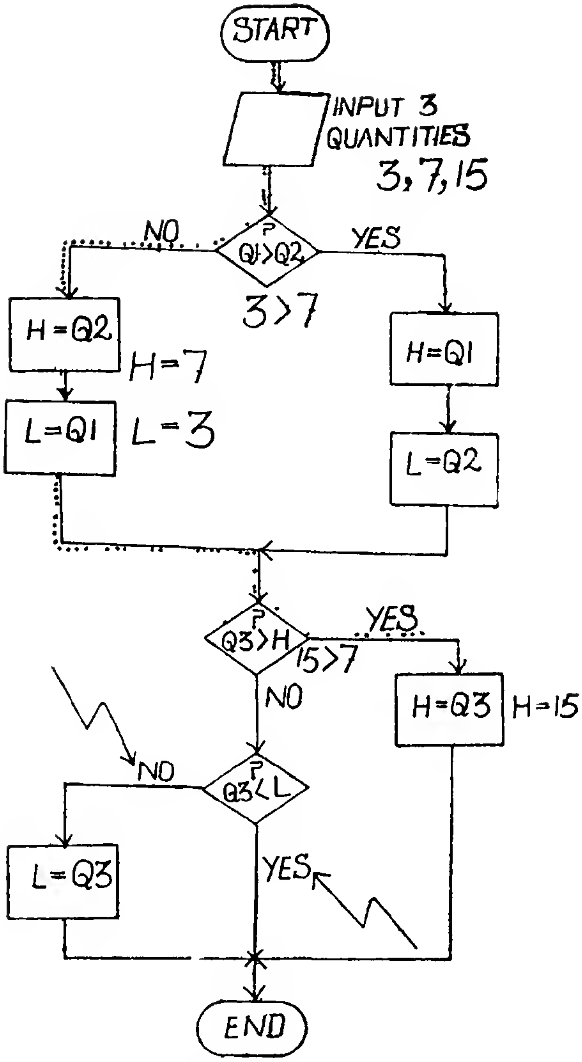


Fig. B

In complex programs there may be tens of thousands of different paths through the program. It simply is not practical (and may be not even possible) to trace through all the different paths during testing Boehm determined for the rather simple looking program flowchart of figure D on p 75 that the number of the different paths is as astoundingly high as 10^{20}. He further observed, if we could some-

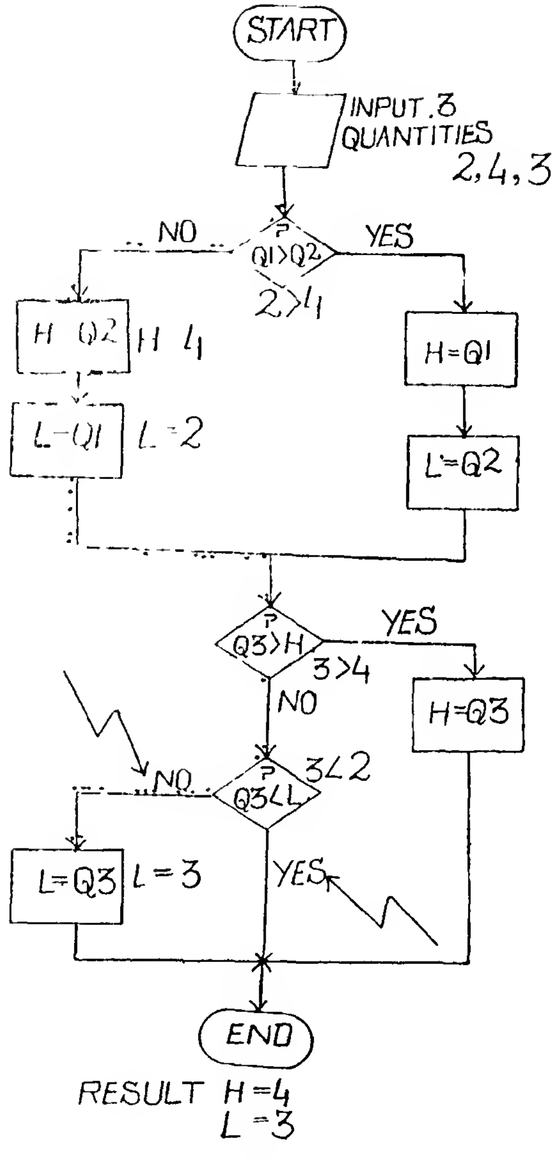


Fig. C

how check out one path per nanosecond, and if we had started our testing in th year 1. A D , we would only be about be about half done at the present time !

It is to be noted, however, that removal of the syntax errors diagnose by the compiler is not the part of the debugging procedure. The programmer compile the test data deck which should contain (1) typical data which will test the generall

used program paths ; (2) unusual but valid data which will test the pragram paths used to handle exceptions, and (3) incorrect, incomplete or inappropriate data, which will test the error handling capabilities of the program

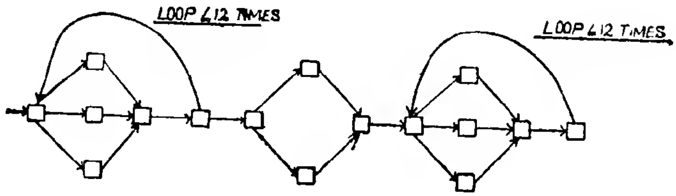


Fig. D

The programmer, after the dry run, loads the computer with the program to be tested, inputs the test data and obtains the output results which he compares with the results derived by him in long hand prior to processing If the program does not pass the test, i e , the results do not tally, the programmer may do the following :

1. Trace through the program, a step at a time at the computer console , but this facility is, usually, available with smaller and mini computers only

2. Call for a trace program run The trace program prints the results of executlon of each instruction in detail. It is thus comparable to console checking. However, less machine time is required.

3. Call for a storage dump when the program hangs up (i e computer hums) i e obtain a printout of the contents of the primary storage and registers at the time of the hangup. The programmer can then study this listing for possible clues to the cause of the programming errors.

However, more bugs may come to notice upon parallel running, which is done upon program implementation with live data.

APPENDIX III

Decision Tables and Flowcharting

Decision table is a table which may accompany a flowchart, defining the possible contingencies that may be considered within the program and the appropriate course of action for each contigency.

Decision tables are necessitated by the fact that branches of the flowchart multiply at each diamond (comparison symbol) and may easily run into scores and even hundreds. If therefore, the programmer attempts to draw the flowchart directly he is liable to miss some of the branches.

Decision table is divided into four parts (see the table on p 77) : *Condition Stub* (which comprehensively lists the comparisons or questions to be posed in the program). *Action stub* (which comprehensively lists the actions to be taken along the various program branches), *condition entries* (which lists, in its various columns, ths possible permutations of answers to the questions in the condition stub) and *Action entries* (which lists, in its columns corresponding to the condition entries, the actions contingent upon the set of answers to questions of that column).

Example. Below is given the decision table for wage calculations in an organisation. Gross Pay (G.P.) is derived from the Guaranteed Minimum (G.M) as follows :

$$\begin{aligned} GP &= 1.05\ GM \text{ when quantity produced, } && Q \geqslant 100 \\ &= 1.15\ GM \text{ when } && Q \geqslant 120 \\ &= 1.25\ GM \text{ when } && Q \geqslant 130. \end{aligned}$$

Also awarded is the quality bonus if a certain level of quality has been attained by the worker. However, in case $Q \geqslant 130$ and the worker also attains the aforesaid quality level his gross pay GP = 1·25 GM+Quality bonus is subject to an overall maximum check which is performed by means of subroutine, SR 2 with the details of which we would not concern ourselves for the limited purpose ahead. In the program also incorporated is SR 3 which validates the wage no, in a transaction i e , it checks if the wage no, is correct Here again we would not be concerned with the details of this subroutine which are beyond the scope of this discussion. The exit from the program is made to a subroutine, SR 4 the details of which we ignore This description has been captured in the first two parts of the decision table below

Part I contains all the possible questions, 5 in number.

Part II lists all the possible actions.

Part III lists the 9 feasible sets of answers. For example, the first set has 'yes' to all the 5 questions and the last set has 'No' to the first question and it by passes the other questions which is noted by dots in its column.

Part IV indicates, by means of crosses (×) the actions to be taken for each set of condition entries. For example, under the set of answers, 1 (all yes) there are 4 actions to be taken as noted by 4 crosses in the action entry column below it.

The system analyst/programmer will first compile this decision table and therefrom draw the flowchart because he can set out the table without any likelihood of ignoring an answer set. In this section, our endeavour is to explain how the flowchart is drawn from a given table. We shall take this table as an example.

	Conditions	Rules 1	2	3	4	5	6	7	8	9	
Part I Questions	Valid wage no. ?	Y	Y	Y	Y	Y	Y	Y	Y	N	Part III Sets of Answers
	Qty. produced ⩾ 100 ?	Y	Y	Y	Y	Y	Y	Y	Y	.	
	Qty. produced ⩾ 120 ?	Y	Y	Y	Y	Y	Y	Y	.	.	
	Qty. produced ⩾ 130 ?	Y	Y	N	N	.	.	.	.	.	
	Quality bonus ?	Y	N	Y	N	Y	N	Y	N	.	
Part II Actions	Gross pay = GM	.	.	.	.	.	.	×	×	.	Part IV Sets of Actions
	GP = 1·05 GM	.	.	.	.	×	×	.	.	.	
	GP = 1·15 GM	.	.	×	×	.	.	.	.	.	
	GP = 1·25 GM	×	×	.	.	.	.	.	.	.	
	Add quality bonus	×	.	×	.	×	.	×	.	.	
	do max. check SR. 2	×	.	.	.	.	.	.	.	.	
	do invalid wage no. 3	.	.	.	.	.	.	.	.	×	
	go to this table	.	.	.	.	.	.	.	.	×	
	do deductions calculations SR 4	×	×	×	×	×	×	×	×	.	

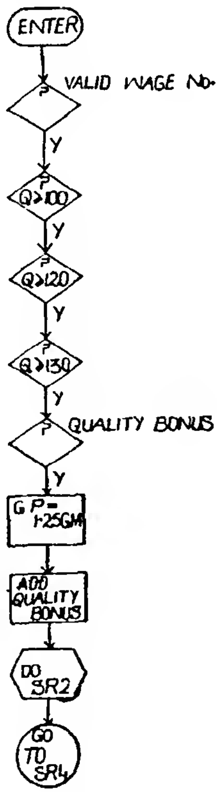


Fig E

In fig E we draw the segment of the flowchart for the answers and actions of column I in the table

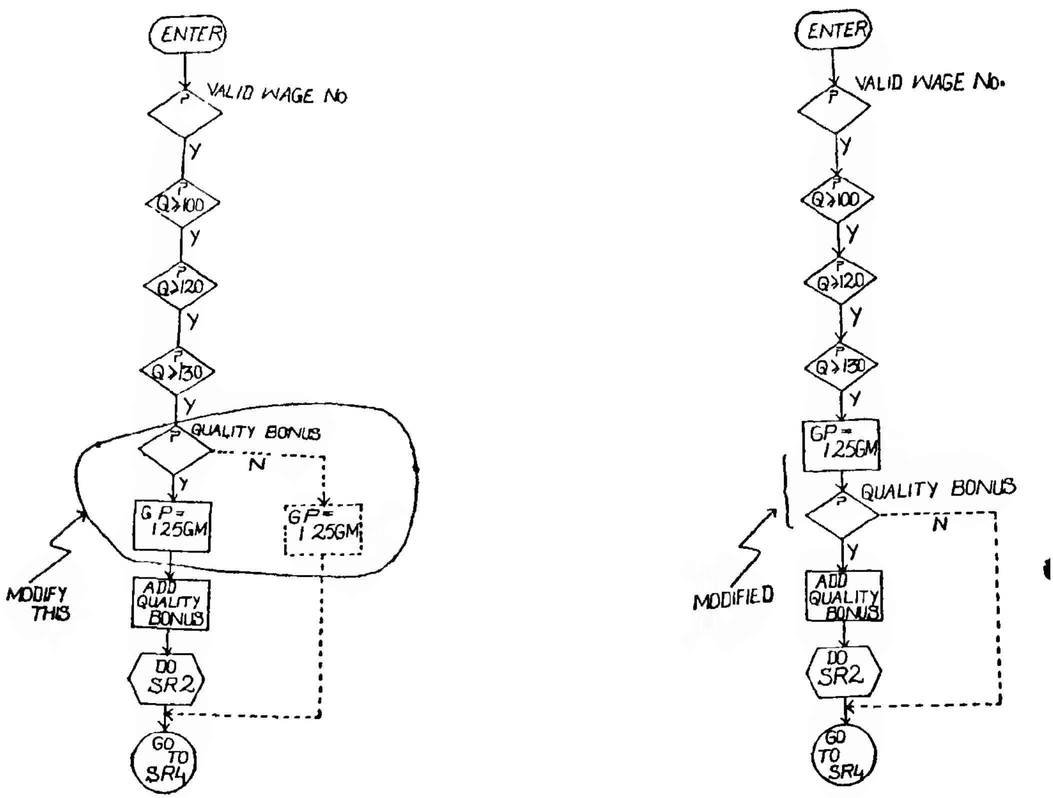
In fig F we endeavour to superimpose the segment of column 2 (shown in dotted lines in fig F) on fig E. But we see for the question "quality bonus"? that for both 'yes' and 'no' to it we find

$$GP = 1.25\ G\ M.$$

Obviously then we should first compute

$$GP = 1.25\ G\ M$$

and then pose this question i e fig F needs modification which has been done in fig G.



Figs. F and G

In fig H, we have superimposed the segment for col 3 (in crosses) onto that of fig. G and we notice that the question "quality bonus ?" has to be posed once again.

In this manner, we continue column wise superimposition of segments until we end up with the final flowchart as in fig I below.

This shows that ths flowchart is drawn by trial and error from the given table and quite a few erasures and rework would be involved Also when the final flowchart has been drawn it can be verified against the given decision table

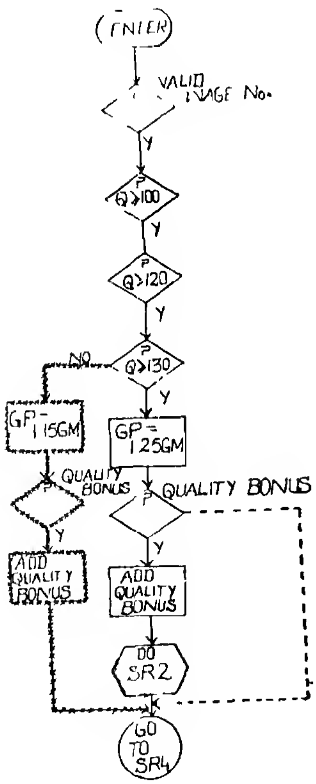


Fig. H

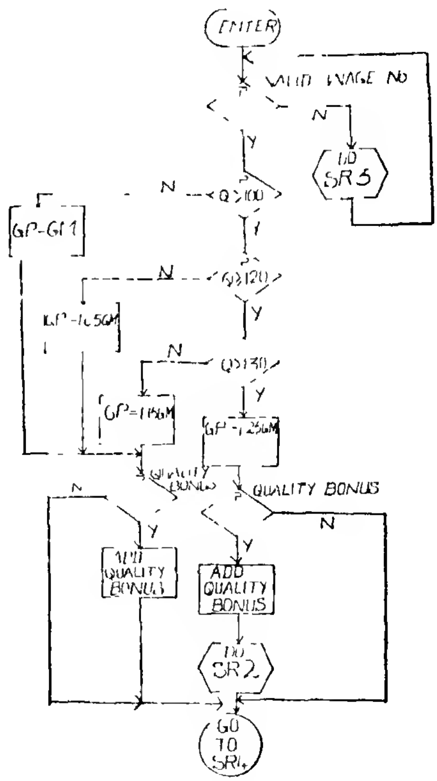


Fig I

Exercise 1. Draw the flowchart for the following decision table.

A/c Receivable Penalty Procedure	1	2	3	4	5	6	7
Number days balance overdue	<30	<30	$\geqslant 30$	$\geqslant 30$	$\geqslant 60$	$\geqslant 60$	
Number days balance overdue			<60	<60			
Unpaid balance $\leqslant$ Rs. 200 ?			Y	N	Y	N	
Last record ?	N	Y	N	N	N	N	Y
Calculate 2% penalty			×				
Calculate 3% penalty				×		×	
Add penalty to customer A/c			×	×	×	×	
Write customer statement			×	×	×	×	
Print warning on statement	×	×			×	×	
Go to next record	×		×	×	×		×
stop		×				×	

Exercise 2. Compile the decission table for the flowchart of figure 40, page 50. Also modify the flowchart if the gaps in the keys of the master file are not to be filled i e new records can be put only towards the end and not in the gaps

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F. III. B-9

THE INSTITUTE OF CHARTERED ACCOUNTANTS OF INDIA

STUDY MATERIAL

F S P (N) SA&DP—3
COMBINATION—B

FINAL COURSE (N)

Systems Analysis & Data Processing

STUDY—III

Systems Analysis and Design

Contents :

INTRODUCTION—PHASES OF SYSTEMS DEVELOPMENT—PRELIMINARY STUDY—FEASIBILITY STUDIES

Master Development Plan :

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Systems Analysis & Design :

TECHNIQUES—DECISION ANALYSIS APPROACH—DATA ANALYSIS APPROACH—OUTPUT DESIGN—MASTER FILES—INPUT DESIGN—FORM DESIGN—CONTROLS AND AUDITING—CODIFICATION DESIGN—DOCUMENTATION—GUIDELINES FOR DESIGN—PROGRAM PRODUCTION—PROCEDURES DEVELOPMENT

Systems Implementation :

EDUCATION—FILE CONVERSION—TRANSITION
SYSTEMS MAINTENANCE

Systems Analyst :

Prescribed Reading :

SYSTEM ANALYSIS AND DATA PROCESSING
BY SUBRAMANYAM AND MENON
WHEELER PUBLISHING

Introduction

Computer is a *machine*, but a *unique* one at that. Other machines extend human muscles (*viz.* the car performing the function of human legs) and do some physical work. Computer is, however, a way apart in this regard It, in contrast, amplifies human intellect since it can perform arithmetic at a phenomenal speed in particular and symbol manipulation (*viz.*, extracting the Works Order data from one or more files and embodying it in a printout of the set of Works order copies) As such it controls the working of men and machines Since it is a machine it has to be acquired on the basis of cost/benefit analysis as is the case with any other machine As we shall see subsequently costs and benefits in its case are spread over a span of quite a few years By applying the DCF technique(s), then, ROI is computed and presented to the management so that they can compare it with alternative investment ventures and depending upon its ranking they may or may not approve computerisation. Incidentally, therefore, clerical tasks (elaborated below) and planning and control tasks of the management are not to be automated (*i.e.*, computerised) merely for the sake of automation or on such justification that a competitor has brought in a computer. The ultimate end here is to produce legitimate information for the management at the strategic planning, management control and operational control levels from the transaction processing sub-system illustrated in Study 1 by progressive summarisation. If this can be accomplished manually more economically computerisation would be dismissed without any distress except in so far figure growth in the business and consequently increase in the data processing volume favours computerisation.

Two Parallel Systems in a Manufacturing Organisation

There is no movement of materials in the manufacturing system and other inputs (energy, labour, capital etc) that is not accompanied by paperwork, sometimes quite voluminous. Acquiring materials requires floating enquiries and distributing purchase order. Receiving materials requires preparation of Goods Receipts notes, their distribution and subsequent action by inspection, updating by stock control of stock records, noting by the buying department for quantity received/rejected, etc Issuance of materials from stores would also be accomplished by Issue Notes, Material Requisitions, Work Order copies, job tickets, etc ; movement of a batch of components is accomplished by means of job tickets , and so on. Thus there are two systems in operation in parallel : Manufacturing System and MIS (with the data processing as a sub-system). The latter, in a way, supervises the former.

However, the intention here is to bring out the fact that paperwork becomes very much voluminous and in larger organisations, the transaction frequency being high, it may reach a threshold where manual system finds it difficult and error-prone to cope with. Also the nature and volume of paperwork is ultimately determined by strategic planning *via* management control and operational control. If thrust of the strategic planning is on penetrating new market, marketing reports would have to be redesigned and possibly increased in number Likewise, if this thrust is on boosting the morale of employees and recruiting first grade workers and managers, forms design for the personnel function would assume a great importance. Anyhow, whereas paperwork becomes more voluminous and transactions become more frequent planning (including forecasting and control) tend to assume more and more complexity.

Now, then, the need for the computer is felt because computer can be applied, in general wherever human mind is at work, but in particular in business situations whenever data processing assumes volume and complexity of a high order.

But the manual system that is proposed to be computerised had been designed (or more likely grown haphazardly) to suit the human capabilities and limitations. Computer reinforces human intellect in planning and control and has ability to now process data at a speed heretofore unimagniable This (together with the following few points) requires that the system is overhauled or redesigned to exploit optimally the human and computer capabilities and complement limitations of each with the strengths of the other In other words, man and machine abilities have to be harmonised. A analogy in this regard would clarify the point Just as the car requires a metalled road with smooth turns and is liable to accidents on a *kacha* (unmetalled) road where heretofore bullockcarts were driven. Similarly computer requires a system (MIS) designed with its capabilities and limitations specifically in view ; otherwise, it may lead to disastrous results if used for systems based on human capabilities and limitations The need for systems overhaul and convincing the users to accept new system is further emphasized by the following abilities and inabilities of the computer *vis-a-vis* human being.

Computer is integrative in nature. It can, for example, perform sales order vetting, credit appraisal, stock updating, placement of orders (on vendors or works), machine loading and scheduling at one stroke in a matter of moments. As a consequence, it cuts across departmental jurisdictions that are created to make up for human limitation to perform so much in one stroke. Human beings divide such work into bits, specialise in these bits create departments for such specialisations, then require co-ordination and expediting, employ ledgers or files that may carry much of the same data (*viz* customer A/c No. appearing in customer master file, A/c receivables file, stock ledger, etc.) which is both time-consuming and liable to be discrepant. The integrative ability of the computer, therefore, has a few implications set out below.

(a) It weakens the departmental jurisdictions. Therefore, it can perturb the managers used to working in water tight compartments. Because of this and other reasons as well the manager may dread computerisation. In many organisation, the bureaucratic manager has cultivated patterns of hiding, manipulating and covering up formation and he fears being exposed upon computerisation. For example, the production manager would not have time to avoid shortfalls and invent/discover reasons for shortfalls for defending himself since his boss would have got the information about this by the realtime access from terminals before even him, which in manual system, he or his staff would prepare (and occasionally cook *e.g*, Work managers do sometimes add up to production figures the goods finished but not yet inspected and okeyed). The computer is surrounded with mysteriousness because of the "hidden" data in the magnetic media. Also, the manager must have heard stories (a couple of which would be narrated later) of failure of many a computer effort. Quite a few manager are rather orthodox about their own style of managing, and if any technique comes along which is incompatible with that style it will largely be ignored or resisted. The manager also does not approve the "uppishness" of the "youngish" computer staff. Much of the decision-making will now be done by the computer that seems to "reduce" the importance of the manager and the pride he takes in his job. He is liable to feel that he is merely a cog for supervising pre-

paring the data for the computer. This, however, is misfounded notion. If the student at this stage goes through "Programmed and non-programmed decision-making" in SADP Study VIII or MICS Study V he would notice that much of programmed decision-making ought to be entrusted to the computer to enable the operating manager to concentrate his expertise, judgement, intuition and time on ill-structured, one-shot non-programmable decisions

Likewise, the non-supervisory employees may resist change because they will, as they think, (*i*) lose their jobs or be downgraded in relation to the computer professionals (*ii*) transferred away (*iii*) be unable to acclimatise themselves with the computer environment, and (*iv*) lose the status and prestige. The management can take the following actions to avert resistance on the part of the managers and other employees at large :

(*i*) Keep employees abreast of effects of the change in their jobs. *viz.*, the impending redundancies and measures to mitigate these by transfers, absorption in the *d.p.* deptt , not filling new posts, etc.

(*ii*) Encourage the employees to participate in the systems effort. This gives them a sense of making a success of what they themselves largely developed.

(*iii*) Make manager's ability to handle change an important criteria of rewards or punishment.

(*iv*) The various scheduled deadlines for computerisation should be flexible rather than rigid The employees should be given enough time to absorb one change before another is implemented.

(*v*) Educate the employees by means of seminars, bulletins, lectures, etc.

(*vi*) Co-ordination with vendors and customers so that formats of the inputs documents from them and output documents to them are agreed upon. This is with the view to win the sympathy of the customers and vendors, who are an important party here, to accept proposed computerisation.

(*b*) The integrative ability of the computer offers the possibility of a data base or a data bank that stores the data in the most compact and comprehensive form. This obviates discrepancies inherent with the manual systems. This also offers the possibility of aggregation of data, experimentation with data in various ways As an example of aggregation of data numerous sales analysis can be conducted by products, customers, territories, salesmen, kinds of package, brands, terms of payment, profit, revenue, etc , and combinations thereof. It is incidentally not the suggestion that so many analyses should be performed by each and every organisation A small organisation without much substantial sales activity may require just a few of these. In Ackoff's opinion, for example, the present day manager suffers more from overload of information rather than its paucity. However, a large organisation dealing in consumer goods with widespread market can avail of this ability of multi-aggregation. As an example of experimentation, the manager can pose such "what

if" question as: what happens if the advertising budget is slashed by 5%? The computer provides the consequences or answer over which the manager may ponder and, perhaps, realise 5% is far too much and then pose another question on slashing the budget by 2%. The computer according to the appropriate stored program would yield the consequences of this as a printout or on the CRT. Thus the manager (human being) and the computer (machine) interact in doing planning, the human being exercising his judgement and the computer doing the "dirty" calculation work. This way man and machine iterate down to the most satisfactory solution This is how human machine capabilities are harmonised or finely blended. [As another example, provided the student has gone through Hertz's simulation model in OR Study VI, the manager may want to know the NPV internal rate distributions for some specified increase/decrease in the selling price, etc]

Also, the data base pools together all the data, otherwise maintained department-wise, comprehensively and most efficiently for retrieval The complete directory of data circulated by the data base administrator is a wealth for the planner, forecaster, modeller, and manager who can discharge their functions much more efficiently by means of experimentation with total data since, for example, the production function is not restricted merely to its own data but can retrieve pertinent data of other functions They can probe for data, items, compare data, analyse, test for various conditions, evaluate, execute statistical calculations and provide answers to many "what if" questions.

(c) The phenomenal speed of the computer enables the manager to use mathematical models for forecasting and planning instead of hunches, guesses and rules of thumb he employed in manual systems However, again, it is not a suggestion that mathematical models are always superior. This should be evident from OR Study I where many difficulties associated with the E.O.Q models are given as an example. Nevertheless, the computer does offer the possibility of their use and where feasible, which is usually the case in larger organisations, they can be used to great advantage The student may also refer to SADP Study VII or MISC Study V for the uses of computer.

(d) Computer has its limitations, one of these is that it lacks human judgement as cited in previous Study in the case of a washer which was inadvertently priced at Rs. 50 instead of the correct price of 50 N P This can be circumvented by incorporating what are known as software checks discussed in detail in a subsequent section.

(e) Computer works strictly and pedantically according to what was programmed into it. The MIS that is evolved for it, therefore, has to be written down with scientific precision and to the minutest detail that is embodied in what is known as the Systems Manual or the Specification Manual.

Systems design then forms the basis of this study note. Each organisation has its own peculiarities and it is usually not advisable to graft the existing design of one organisation onto another In fact, the systems analysts and designers should be cautious in overplaying their experience. It is rather a exhaustive task, no doubt, to carry out the systems design for an organisation. It is expensive, time-consuming and requires a great deal of expertise. The computer circles have been watching

with dismay exorbitantly high costs of systems development in contrast to the steadily diminishing costs of the hardware This trend, it appears would continue All that one can do is to account for the systems design expenses in the cost-benefit analysis. Without proper study and design of the system, the computer can prove a real beast. A great deal of investment, and morale, may go down the drain.

It is also to be noted that systems analysis, though expensive, is a blessing in disguise. Companies spend millions on research and development of products but the managements may perhaps increase their effectiveness in a far more telling manner by a sustained study of themselves. The faculties of planning, forecasting, control, modelling as well as systems and procedures can surely be improved spectacularly if only managements subjected themselves to scrutiny by systems analysis. With much less expenditure than on R & D of products the return on investment in terms of profitability and competitive strength may well prove phenomenal. Systems analysis is thus applicable to management's own bailwick It is also an open secret that many of the benefits of computerisation do not owe their origin to the computer itself but the systems analysis studies preceding its installation.

The need for systems development may arise from, (*i*) expansion of the business, (*ii*) the management being forced to undertake systems development in response to such market developments as a competitor announcing dramatic cuts in delivery times. Besides, such questions as, "Is our equipment outmoded"? Or "Does our product design need simplification"?, Management may want to know if it is possible to shorten delivery times by improvements in the information systems, (*iii*) the system maintenance activity may call for design of some of the information systems and finally, (*iv*) the following symptoms in a manufacturing organisation (Ref. Information System Design by D.I Scheraga):

1. A high ratio of indirect to direct manufacturing costs.
2. Poor delivery performance relative to promise made.
3. Frequent stockouts at all levels of inventory.
4. No *statistical techniques used in inventory control* and *sales*.
5. Frequent changes to short-range sales forecasts.
6. Excessive raw and work in process inventories.
7. Long manufacturing cycle relative to the sum of setup and processing times.
8. Much obvious paperwork.
9. Long response time to inquiries on order and inventory status, or poor quality of status information.
10. Frequent schedule changes.
11. Low degree of component standardisation between products.
12. Unsatisfactory labour and machine utilisation.

13. Excessive confusion and stock obsolescence resulting from engineering changes.
14. Poor or non-existent labour standards
15. Excessive overtime.
16. Low inventory turnover.

MIS Development is a fairly comprehensive activity and is carried out in the following 6 phases [We shall use MIS development, systems development, systems overhaul, Systems Analysis, Systems Design and Systems Analysis Design all synonymously though the individual phase IV below is also known as Systems Analysis and Design in itself].

I. Area Selection
II. Overall Analysis, Gross Design and Feasibility study
III Master Development Plan
IV. Systems Analysis & Design
V. Systems Implementation
VI. Systems Maintenance or Audit.

Systems development should be viewed as *a continuous iterative process that* recycles through each stage until a system is discarded. Further, almost all the stages overlap each other.

Systems Development, Stage I—AREA SELECTION.

The main factor in determining the success of a computer installation is the right choice of applications along with the skill of their development and implementation. Most computer installations are unprofitable because they are doing what is easiest to implement, as opposed to what is most profitable, doing what is most obvious and follow the line of least resistant. Too often applications are chosen and programmed without adequate study and without understanding the management principles. This invariable leads to disaster. Thus selection of right kind of application is of utmost importance for the success of any installation.

The following applications are generally prospective for computerisation.

(*a*) Applications to large-cost areas

Accounting and administrative areas are generally low cost areas and savings in computerising such bread and butter applications as payroll or inventory accounting may not substantially decrease the costs. Thus a 10% saving in administrative cost may be insignificant as compared to even a 0 1% saving in the cost of materials. But it is unfortunately a historical accident that computer in the past *was* applied to accounting applications One of the reason for this being that the accountant *knew* what he wanted from the new system. His applications worked well in pre-computer days and it was a simple matter to computerise these since hardly any effort is needed in redesigning them. They can virtually be put on the computer the way they worked in manual systems. And this is almost a principle in computerisation that applications that work well in manual systems will fetch little benefits. In

contrast, things were just otherwise with production and inventory control. Production and inventory control is required to monitor stocks of thousands of items and thousands of operations and control hundreds of men machines and tools amidst a highly volatile internal environment of machine breakdowns, tool breakage, scrappage, losses, absenteeism, late deliveries by vendors, changes in due delivery dates by customers, engineering changes prompted from within and so on. It has been a chronic problem with manual systems and it is a fact that it was virtually impossible with manual capabilities of data processing and re-processing. It was ill-managed even in moderate size organisations. The near-astronomical complexity of the problem left the production controller wondering what to expect from the computer which can assist the management in really managing men, machines and inventory. In manufacturing situations usually, according to an authority, the payoffs are lucrative. Material Requirements Planning that is far superior to traditional production and inventory control techniques works across the complex product-structures and requires massive data processing every week, and even continuously indeed, making computer a necessary adjunct.

Transportation is another large expense for certain kinds of companies. If the same product is produced in a number of factories, an optimisation programme can determine the quantity of each product to be produced in each factory and transported to individual warehouses. The location of the warehouses can also be examined to minimise the total transportation cost from factory to warehouse and transported to individual warehouses. The location of the warehouses can also be from warehouse to stockists or customers. Such a system can easily yield savings upto 10% of the total transportation cost.

(*b*) Applications to "Main Line" Functions

Above we made a general statement that production and inventory control is usually a lucrative area for computerisation, but this would not always hold good. Consider, for example, a company that has few equipment of its own and is mainly engaged in assembling of procured components. In this case, then buying is *the* main line function highly prospective for computerisation Vendors can be periodically rated for quality delivery time and prices of their supplies This would lead to the right selection of the vendor for a given order. In a consumer industry with hundreds of saleables, marketing is the mainline function and in a speciality manufacturing company R & D is the main-line function This concept is similar to the principle of applying the computer to the largest cost area, except that sometimes the most important area may not be the most expensive

'A paint manufacturing company had an unprofitable computer effort for quite some time before they tried the main-line approach A computer system was developed to prepare an optimum production plan based on stockists' projected requirements, raw material availability and machine capacity. The system took a couple of years to program and implement, but produced a 10% increase in profitability, paying for computer costs many times over'.

Similarly the marketing division of the Indian Oil Corporation has a savings potential of a few crores of rupees per annum from a computer systems applied to

calculate the optimal distribution pattern of oil products from its refineries and to schedule its tank-trucks.

(c) Things not otherwise done

One of the interesting factors about computers is the fact that some of the most profitable applications come from analyses and systems that are not possible without them.

Thus, the obvious applications—the automation of clerical tasks are not the most profitable It takes considerable study and expertise to determine the really profitable applications where the computer works to assist the thougth processes of managers. These are briefly discussed below :

(*i*) *Manually uneconomic systems*: Computer application to "new" systems can be classified into systems that are manually uneconomic and those that are manually impossible. An example of the former would be detailed costing systems. Many companies still do not have costing systems The cost of preparing the data and keeping it up to date is an important cause of this is lack of management awareness. Even where manual costing system exists, costs are ascertained in a few standard ways, not nearly as detailed as would be required for good management decision-making. Often, situations arise where special cost figures are required and since the costing system is not geared to these, they have to be laboriously compiled resulting in extra cost, delays and, in extreme cases, non-availability.

A computerised costing system can retain the break-up of costs in many varied ways even if these are required only rarely. More important, it can keep all corporate data in a set of generalised files which can be queried at short notice to answer any question relating to costs. This "data bank" concept of storing all relevant information in a selectively retrieveable form to answer occasional queries is made possible by the powerful and inexpensive data storage and retrical capability of the disc packs A storehouse full of old office files is also a data bank, but one that is very difficult and expensive to work with

Other examples of systems curtailed due to labour costs are "Accounts Receivable" and "Material, Ordering". In most companies the follow-up on outstanding accounts inadequate, reminders are late and too few. A thorough system of screening outstanding accounts and sending reminders of increasing severity to defaulters is made possible by inexpensive computer processing Such system can have a dramatic impact on outstandings.

Similarly, it is difficult to keep track of the stock of each individual item in the inventories of a company in manual systems. Usually, ordering is too infrequent. Merely by making these functions cheaper and providing a system that carries them out routinely, inexorably, and with no chance of forgetting the computer improves the situation and leads to substantial savings. Decreases in inventory holdings of upto 25% are not uncommon after the introduction of a computer system.

Note that in such cases the computer system is not replacing a manual system but considerably increasing its capability and its power. No staff surpluses are

created ; indeed sometimes even a few new hands may be required ; but the management benefits from the system are much larger than the expenditure on it.

(ii) *Manually impossible systems* As an example of analysis impossible to perform without a computer, consider the simplified case of a toiletries manufacturing company with four factories around the country. With a few exceptions, each factory is equipped to make almost all of the company's ten products. Variable costs of production differ somewhat at each factory owing to variation in the local prices of raw materials. The factories supply goods to 20 warehouses covering the entire country.

The problem to be solved every month is the quantity of each product to be produced at each factory and the quantity of each product to be shipped to each warehouse so that the cost of transportation and the total variable cost of production is minimised This is an extension of the transportation problem (discussed more fully in OR Study III and MICS Study IV) which only minimises the cost of transportation. No doubt a solution to this problems can be arrived at manually and many companies have and do run without the benefits of computer system. The management may even examine a few alternatives to try to decrease the cost, but it is very difficult to arrive at the optimal solution that minimises the total cost. Indeed a search for the optimum solution would involve the evaluation of a very large number of alternatives, a task that would defeat any office staff, however, large. On the computer such a system would take some time to implement ; but the monthly optimization would take only a few minutes Since the generation and evaluation of the alternatives and the search procedures are all standardized sets of simple arithmetic operations, the computer zips through them in an infinitesimal fraction of the time and cost it would take an army of clerks. In addition, the company has a model of the process on which it can test alternative corporate strategies and evaluate their implications on profitability on an integrated, comprehensive basis. A number of such systems, incorporating a large number of alternatives, are in use by the international oil companies as planning models.

Construction companies usually have to level a plot of land before they can start building. The cost of levelling can vary significantly with the strategy adopted for the redistribution of earth. Again, a large number of alternatives are possible. While any contractor can work out a plan for levelling the land the computer can examine all alternatives and work out of the cheapest method and save a great deal of money.

Another problem that falls into this category of system is truck or tanker routing system. A company having a fixed fleet of carriers is faced with the problem of getting the maximum utilisation from or, alternatively, meeting all commitments at the minimum cost. A computer system supplied with the routes, requirements, costs and other constraints, can considerably improve schedules and save a large fraction of the operating expenses. Again, manual working is possible but at the cost of operating much below optimal efficiency.

What kinds of applications are profitable : This is a common question, but its answer is complicated by the varieties of possible applications and the varied needs of organisations. What seems to make the difference apparently is whether or not an application improves the quality of the management systems it impinges on,

The importent benefits of the computer come from its ability to improve decision-making Either it can make the decisions according to clearly specified impersonal rules, in which case its consistency itself is an improvement over human decision-makers. or it can supply the data required, pre-processed to whatever degree possible, to form the basis for human decision-making It is this improvement of the management process, this ability to control operations and perform analysis not possible before that produces the true and substantial payoffs from the computer But better data for quicker and mere scientific decision-making is not enough The outputs produced by the computer have also to be integrated into the corporate management process. The managers have to understand the outputs and use them and have a clear ideas not only on how the reports are produced, but also what their assumptions are and what else the computer could produce.

Of increasing importance are integrated systems that take many different kinds of information and integrate them to produce comprehensive reports for management, each piece of data being correlated in all outputs where it is can be useful.

Viewed in this way, the mechanization of even some routine manual systems can be profitable. A financial accounting system can pay its way by producing profits and loss figures soon after the close of every month The promptly produced data can be used by management to control and improve instead of merely serving as a late post-mortem. Similarly, a payroll system can provide figures for efficiency and productivity calculations.

Consider the matter another way. A control system that monitors outstanding. payments, say, in a hire purchase company, is a rewarding application. When such a system is implemented, it is found hat the data for the control system can, with a few addition, be used to produce the accounting statements as well. In this case, the accounting system can be justified by the control system. In a number of cases, an important application will give rise to offshoots ; other applications that may be implemented with a minimum of additional data They may not be profitable in themselves but the incremental cost being small, it can be easily covered by the additional benefits.

Another situation in which the mechanization of manual system may be justified is when they serve to build up a data base on which more sophisticated optimization systems can be built. All operations research models require cost data and unless this has been carefully collected, sorted and compiled, optimiziation will not be possible. Thus, a good costing system is a must for almost all corporate computer efforts. Similary, other simple administrative systems may be justified not for what they are but for what they can support on their shoulders.

Scientific and Engineering applications : Scientific and engineering applications are usually easier to implement than management applications as they do not necessitate changes in the human systems around them. For organisations involved in research, prodtict development of any kind of engineering, computers can provide very valuable assistance to the technical staff and make important contribution to cutting product cost and improving performance.

In competitive markets, it is not adequate to produce a good product. The product design must be optimized to be the best possible under the external constraints of cost and function. Optimization in engineering, as in management, is difficult and time-consuming. Many calculations are required , thousands of alternatives must be examined. This is best done by a computer. In chemical industries, optimum reactor design have saved millions.

Other programs can assist the design engineer by merely doing his routine computations for him An architect, for example, or a structural engineer can examine many more alternatives if the routine calculations are done rapidly and conveniently on the computer. For the electronic circuit designer, a number of programs exist to simulate the operation of different circuit configurations under varied conditions This relieves him of the burden of actually building and testing many prototype models.

These are only a few simple examples of what the computer can do for the engineer and the scientist Many other applications of greater sophistication exist such as numerically controlled machine tools, that can be programmed to perform complex operations automatically and automatic warehouses that store and retrieve labelled packages without human help. These are often "main-line" applications and their potential should be examined in detail in the light of the individual stances of each company.

Systems Development, Stage II

OVERALL ANALYSIS, GROSS DESIGN & FEASIBILITY STUDY

Earlier, we discussed area selection in general *i e.*, the aforesaid considerations would apply to all organisations proposed to be computerised. The purpose of the overall analysts is (i) to establish rapport between the computer professionals (known as systems analysts) on the one hand and the managers, supervisors and employees on the other, (ii) to enable the systems analysts (or analysts more briefly) to earmark weak spots and (iii) to familiarise the analysts with the organisation to enable them to evolve gross design subsequently.

There are 3 methods of carrying out the overall systems analysis study. In the *survey* method, the existing system is studied in depth by means of interviews, questionnaries and observations which leads to compilation of a large number of flow-charts in minute detail and massive documentation This is all with the intention of spotting weakness in the existing procedures and methods and earmarking the areas for improvement. But the executives for whom the flowcharts and documentation are meant may find it both time consuming and distasteful to plod through details. More seriously this study is likely to *colour* to *bias* both the executives and systems analysts to think in terms of the existing ways which are suited to human abilities and naturally ignore the computer capabilities and limitations For example, the computer, because of its phenomenal speed, can utilise sophisticated OR models for forecasting, planning and control whereas manual systems depend upon mere guesses, estimates and hunches in this regard. To avoid this bias, therefore, it is argued that no time need be wasted in studying the existing system.

The second method viz *Questionnaires Method* consists of asking the executives what information they require for forecasting, planning and control from the proposed system. The drawback of this method is that, not unusually, executives do not really know what they expect from the new system and, if even they knew what they *want* it may not be what they actually *need*. For example, the Works Manager may insist upon "scientifically" computed reorder levels and safety stocks whereas computer can plan and control inventories far more efficiently by the recent technique of Material Requirements Planning.

The 3rd method, viz *Observation Method* which is the best, consists of determining the information needs of the business. The systems analyst gets the organisational objectives from the top management and derives the Management Information System objectives therefrom. For example, if an organisation is planning to divert much of its output abroad then sales and shipping procedures would have to be redesigned. This method enables the executives and systems analysts to think in terms of not only the present information needs but also the future needs of the company. But dismissing the existing system altogether from mind is not advisable. First, a *limited investigation* of the existing system can bring out the strengths and weaknesses of the organisation (including the managers and other personnel at large). Second, minor details and procedures, which otherwise are likely to be overlooked, can be ensured, in the proposed system.

Gross Design : An analogy on engineering design may be quite useful here. A new racing car is to be built. Two teams of engineers are assigned this work. One team comes up with a 3-wheeled car design with the engine in the rear and with the specified width, breadth and height etc. The other team proposes a 4-wheeled car with the engine in the front and with specified dimensions again. Both the designs are created to meet such objectives as speed, range, maneouverability etc. These, then, are the alternative designs. The systems analysts also conceives such overall or gross design alternatives to start with. Consider a company with a manufacturing unit and a head office with several branch warehouses scattered all over the region. The systems analyst may propose such gross designs as follows.

1. All the customer orders will be received centrally, credit appraisal too will be conducted centrally by the computer, perpetual inventory records will be maintained centrally and aggregate forecasts for all the warehouses would also be undertaken centrally and the customer orders will be passed on to the nearest warehouse.

2. Each warehouse prepares its own forecast and transmits it to the head office, and maintains its own inventory records.

These, then, are the MIS gross design alternatives. Several more alternatives could be conceived. These have to be evaluated against such criteria as costs, flexibility, practicability, etc. Occassionally, none of the alternatives may be found satisfactory against these criteria in which case more alternatives would have to be conceived.

Now the detailed engineering design, say for the 4-wheeled car above, would consist of specification in the form of drawings and specification reports for system

as a whole and for all components in the system. The mathematical analyses and test reports, too, may be appended. The detail should be fine enough to enable shop floor production of the car. Likewise, for the MIS design the input/output/file layouts, information requirements tables, flowcharts, decision tables and other such documentation (discussed in Section IV) constitute the MIS design specifications which are necessary for program production to proceed smoothly.

Feasibility study is then undertaken. Feasibility of the proposed system development projects may be viewed from three angles: technical, operational and economic *Technical Feasibility* is concerned with whether the systems proposed for development can be computerised. The hardware developments are so fantastic that there is hardly any business job that cannot be computerised. In fact the hardware being marketed is far too sophisticated for business applications. This has changed the emphasis in technical feasibility from *whether* to *how* to computerise. This calls for a look on the software and brainware (computer experts) rather than the hardware The organisation then needs to know if there *are* people within the organisation or (in consultancy firms) who can computerise the proposed projects. *Operational feasibility* focusses on the willingness and ability of the management, employees, customers and suppliers etc., of an organisation to operate use, and support a proposed system.

Economic feasibility is basically cost benefit analysis of the proposed system development and is usually decisive in undertaking or not undertaking the development. It is relatively easier to measure costs than benefits which are usually intangible and, therefore, hard to quantity. The costs may be decomposed into the major components which can then be further split as below :

Start up Costs

— salaries of the system analyst and programmers during system development.

— Cost of conversion and preparation of data files and systems manual and other supportive documents.

— cost of the new or additional computer hardware.

— cost of training employees.

Operating costs

— hardware/software renal or depreciation charges.

— salaries of the operating staff.

— salaries of the maintenance staff.

— cost of input data preparation.

— cost of D.P. supplies.

— installation maintenance costs.

— Overhead charges of the business firm.

There are also intangible costs, *viz*, those of disruption of the business during systems development.

Benefits would include :

(*i*) Direct savings—

cost of equipment replaced

reduction in clerical staff

reduction in accommodation costs

reduced overtime charges.

reduced communication charges.

(*ii*) Better management information

(*iii*) Better planning

(*iv*) Improved computational and data experimentation ability.

(*v*) Improved competitive position *i.e.*, better and faster response to competitor's action.

(*iv*) Improved employee's morale.

Intangible though it is, nevertheless, it is important to put a rupee and paise tag to each benefit for purposes of profit and loss statement which can be done with diligence on the part of the operating managers. The operating manager has a homework to do here. The analyst can estimate for him the proportion of time the computer would save for the chief Buyer, for example, by taking over his routine tasks and programmable decision-making It is now for the chief buyers to deliberate and come out with how best to utilise this time He can, for example, undertake more extensive sourcing, spend more time on negotiations, participate more heavily in the value analysis effort and so on ; therefore it is he who has to quantify the benefits there from in monetary terms This has the dual advantage that subsequently during systems audit his performance can be appraised *i e*, to what extent the payoffs he anticipated were actually realised upon computerisation. However, it must be mentioned in passing that not all benefits are so hard to

quantify. A computerised payroll system, for example, would likely displace some clerks and it is easy enough to measure benefits in the case.

System Development, Stage III—MASTER DEVELOPMENT PLAN

Master Development Plan basically is a schedule of various applications to be computerised *i e* start and finish dates of systems analysis, design, implementation and maintenance activities for each application. Because of independencies amongst applications this planning (and the associated physical and cost control) is best done by the network (CPM/PERT) techniques. However, the application scheduled has to be supported by manpower, hardware and financial schedules All the schedules, however, would ultimately be influenced by the organisational objectives *via* MIS objectives It is usual not to take a big leap forward in computerisation *i e*, it is not desirable to switch over from a manual system to an advanced system at once Usually, start is made with batch processing, then later the files are knit into a data base to permit random inquiries, this would be followed by building models and the model base for operational control, management control and strategic planning in this order This matter is dealt with more fully in a subsequent section on "Bottom up approach". Thus the aforesaid schedules may run quite a few years into the future and their periodic revision is naturally essential in consonance with the actual progress and the fact that the world does not stand still while MIS implementation is going on. We shall soon take up this topic in more detail but prior to that it must be brought to the notice of the student that often in the past computerisation had been done without any such pre-planning and the consequences were naturally disappointing or occasionally even disastrous.

Another allied point is that the managements often tended to keep themselves aloof from the computerisation project thinking implicitly that it is susceptible to a purely technical solution in the way of installing air-conditioning equipment for which engineers/technicians of the service organisation come and do the entire job on their own without at all bothering the management except with the bill But unfortunately this is not so with computerisation. It encompasses and pervades in the entire MIS and the management's involvement, in view of their knowledge about the future goals, growth and direction of the company as also the technical expertise of the operating managers who alone can best spell out their information needs, is of utmost importance. There is a case in the literature of a European team of top executives of a company manufacturing motors visited the U S to consult the IBM experts for the proposed computerisation of their MIS. The experts naturally advised the team to involve themselves fully in the systems development effort. But the chief of the team thus countered the useful advice out of ignorance, "We are in the business of making motors, not programming and installing computers" This man mistakenly thinks that the computer can be installed by systems analysts and programmers without bothering his operating managers who are far too busy in their operating problems The statement is pernicious since it looks so logical on the face. A layman will readily tend to agree with him why he and his deputies should divert their attention from their routine to computer technicalities. But as already emphasized it is far from truth.

There is yet another case in the computerisation folklore. A company intended to computerise its commercial applications. The Vice-President of Engineering had his own giant slide-rule type computer suited to his engineering computations. Having heard stories of many a failure of computerisation of commercial application he thought it fit to keep himself aloof lest his own department's work is fouled up as he thought. Thus commercial computerisation proceeded without his participation and it did collapse ultimately. The Vice-President congratulated himself for his "far-sightedness". But the real reason for the failure was his non-participation. He did not allow anyone to modify the Bills of material file, which is the most basic to almost all functions as we shall see several pages later.

The importance of management involvement in planning and control of information systems development was brought out convincingly in a survey of computerised organisation undertaken by the U S. consultancy film, Mickinsey and Company in 1968. The sample companies were divided into two categories: successful and unsuccessful with regard to computerisation Planning and auditing of the E D P activities were pinned down as the major factors for success or failure of the system. The result of this survey are summarised below.

	More Successful	*Less Successful*
Companies that plan E.D.P. activities and audit results against plans.	9	3
Companies that plan E.D.P. but do not audit results.	7	3
Companies that neither plan E.D.P. nor audit results.	2	12

Thus the right course for the top management in this regard is to constitute a steering committee that draws its members from the departments being computerised and systems analyst (from within or without) to supervise systems development. This would synthesise or blend the business expertise and insights of the former with the technical expertise of the latter. Ideally, the operating manager himself should join the committee or at least he should be represented by his *ablest lieutenant*. He cannot immerse himself in the daily routine of petty operating problems when his future (information system) is going to be decided and redesigned. Participation of the operating managers also motivate them to make success of the system which they themselves created as otherwise they would resist it on the pretext of NIH (Not Invented Here). And in no case, the upper hand in MIS overhaul should be given to the technical personnel, the systems analysts. If so they are liable to give a system which they participated in for some other organisation i e, invent a square wheel since each organisation has its own peculiarities and is unique. They are also liable to pursue technical elegance for the sake of elegance. They may design a highly

sophisticated (and therefore expensive) data base which, in fact, is not needed. As often-repeated the ultimate end is legitimate information. If it can be had without an advanced data base no real need for it then.

Need for the Master Development Plan

Since the late sixties the idea of the total system has been envisioned by several computer experts It implies that not only the obvious and suitable jobs would be automated but even the unrelated individual procedures, *i e.*, the entire business would be treated as one automated unit. Both the information generators (field sales officers, shop floor personnel, quality control etc) and the information recipients (suppliers, customers, management control, personnel, etc.) would have on line linkage with the central processor. This has also the ultimate possibility of integration starting from the supplier through the organisation to the customers, *i.e* , computers of the three talking to each other directly. This would mean standardisation of firms' documents and message formats. Except for the hard copy exception reports the system would approach the ideal of paperlessness. The computer system would then serve as follows :

(*i*) *Universal Data Base* that would store all product specifications, operating records and financial records.

(*ii*) *Universal Data Receiver* that would accept data from both within the organisation and without through various input and transmittal media.

(*iii*) *Universal Data Processor* that would vet the received data and aggregate it with the stored data.

The idea of the total system is surely not ideal. Several sophisticated installations tend to approach this ideal. But the case with the majority of installations is otherwise. Often individual applications have been allowed to grow under pressure from the concerned executives without much regard for the payoffs they yield or their precedence relationships. The haphazard growth made it as difficult, expensive, time-consuming and disruptive to seek to integrate them as to work *ab initio* This has impaired communication between different departments in the same organisation and there has been a great deal of duplication of effort and data storage.

Below we list the various advantages of working to a Master Plan.

1. It forms a sound basis for the selection of the computer configuration.

2. By scheduling applications by priorities in terms of payoffs they yield, ROI can be maximised. Also, having sorted out dependency relationships amongst applications it leads to good co-ordinations of progress.

3. The plan can be suitably amended for any charges in goals and objectives of the organisation, or any technological breakthroughs which are all too common in the striding computer technology.

4. Progress can be measured and compared against the plan so that corrective action is initiated for any significant slippages. This also leads to stringent cost control against realistic budgets.

5. It reduces confusion and the number of isolated, non-compatible subsystems, which, as stressed earlier, may require expensive rework latter on.

Which Way to Proceed (Application Priorities) ?

The MIS structure for a typical manufacturing organisation is depicted in the figure below.

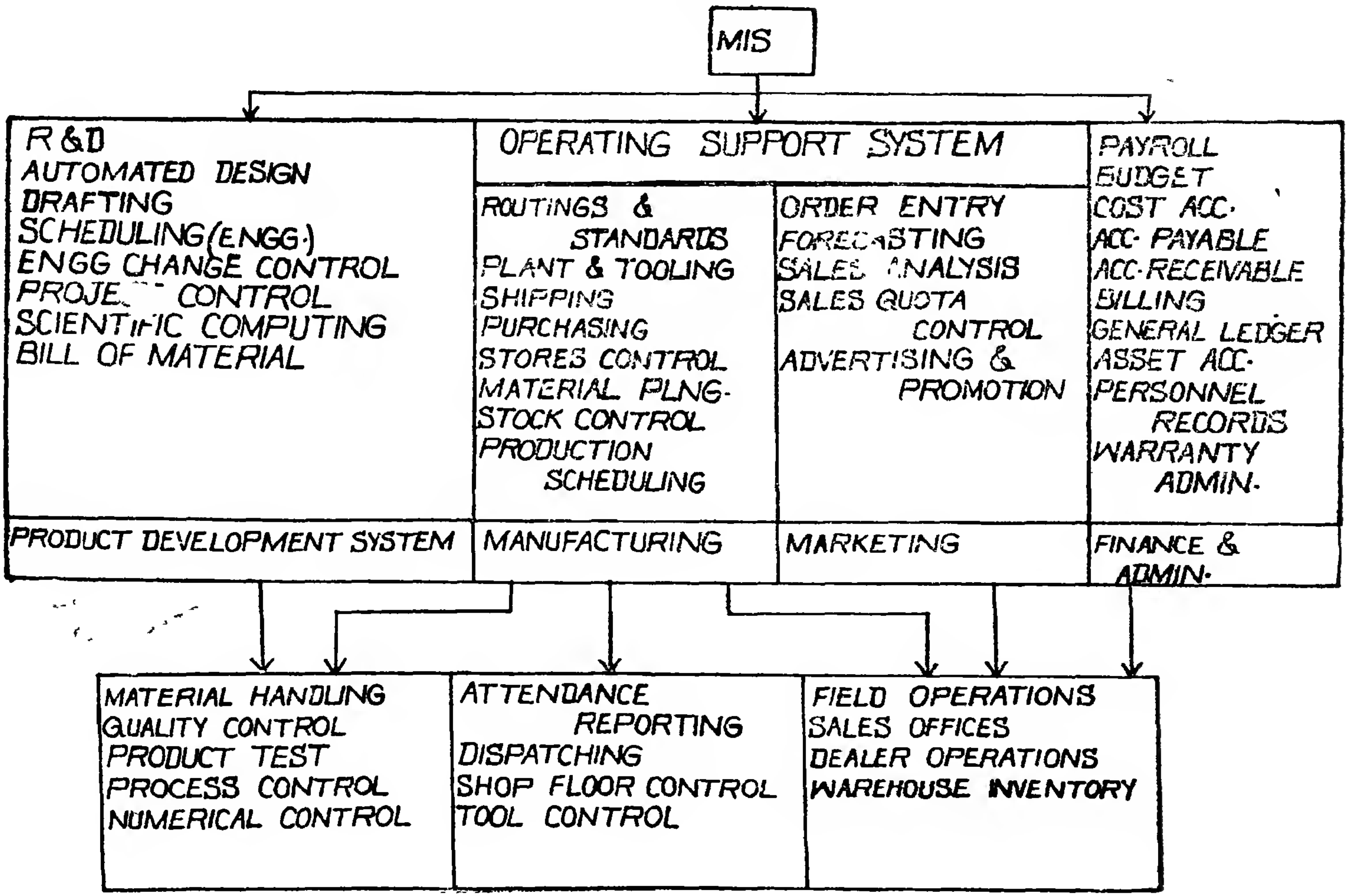
At the bottom of this stucture are the data or information generators This is the operational or execution level of the factory and the field In the middle, management control level (offices) are the functional information sub-systems The MIS at the top level represents a superstructure straddling all planning and control systems and having direct information pipelines to the execution level Where to start for computerisation and which way to proceed then ? Seemingly, one should proceed in the descending order of application payoffs This is quite justified but here all precedence relationship For manufacturing organisations, the following sequence as an instance is recommended.

1. Forecasting
2. Materials Planning
3. Inventory Control
4. Manufacturing Operations scheduling
5. Shop Order release
6. Dispatching
7. Collection of production (feed back) data.

Thus the master development plan should not only take into account the expected payoffs but also the aforesaid precedence relationships It is to be carefully noted that accounting applications do not figure in the above sequence although they have usually been the first to be computerised. This has been so because it is easy to computerise them. The accounting practices are well structured and meticulous. It does not require much of effort in systems analysis and design to computerise these And the fact is that they offer very little payoffs In industry, the following three areas when successfully automated, consistency yield the greatest benefits :

1. Planning and control of finished goods inventory
2. Planning and control of the use of men, machines and materials.

MIS Structure for a Manufacturing Organisation



4. Planning and control of the material procurement functions

The accounting functions should therefore be treated as supportive only and computerised as by products

There are several approaches for preparing the master development plan. We shall discuss the bottom-up approach here.

Bottom-up Approach : It consists of the following five steps :—

1. Individual functional applications (discussed in Study II) are planned separately and they consist of transaction processing, updating of files and sample reports.

2. The files of the various functional applications are integrated by means of indexing and chaining into a data base. On-line inquiries can now be handled by means of the random access disc storage. The information requirments of the operational level are thus completely met at this stage.

Such decision-making and planning models for the various functions as follows are then added to operate on the data base at the management control level, production planning by linear programming, etc.

4. Integration of models into one model base having a wide variety of analysis, decision and planning models The model and data base may be enlarged to include data for the expanded use. The ROI model, for example, straddles over different functions and it would not be possible to compute ROI in one stroke under the scheme of step 3. (See Study Paper No. I, p. 8 in Advanced Accounting).

5. Strategic planning data and planning models are added to the information system data which should be added to the two bases in this final stage.

Components of Master Development Plan

The master development plan consists of the following four sections :

(a) MIS Objectives (General and Specific)

(b) Inventory of Current Capabilities

(c) Forecasts of developments affecting the Plan

(d) The Specific Schedules (discussed earlier).

(a) *MIS Objectives* : Rapport with the top of management by systems analysis is to be established by means of interviews mainly. The questonnaires and observation methods are applicable at lower levels only. The purpose of the rapport is to involve the top management in systems development. Besides the objectives established by the top management are gotten, based on which the systems

development objectives are laid down. In other words, the objectives of systems development should be supporting to the overall organisational objectives. The systems development objectives can be classified as general and specific. The former are common to most organisations and may be set out as below :

(*i*) Integrate the six levels of MIS *i e.*, the interactions amongst the forecaster, planner, the supervisor, the system analyst, the programmer and the data base administrator should be harmonised (Ref. MICS study V or SADP VII).

(*ii*) Keep MIS adaptable. This means that MIS should be flexible enough to,

1. accommodate changes in frequency, format and contents of output statements and displays.
2. decrease or increase in storage capacity,
3. changes in models and programs,
4. changes in the type of data and relationships between various data,
5. changes in method of data input and updating, etc.

(*iii*) Keep MIS cost effective. This calls for careful analysis of each output with a view to eliminating those which do not justify costs.

(*iv*) Control by exception.

(*v*) Automation of routine decision-making *viz.*, computing E.O Q.'s, setting reorder levels, statistical quality control etc. The decision-making rules can be programmed and generally do not require human judgement or intervention. Such decision-making should be automated

(*vi*) Aid in data analysis and experimentation for non-routine decision-making.

(*vii*) Minimum of paper output.

(*viii*) Speed in receiving, processing and dissemination of information, etc.

Specific objectives : however, relate to the given system. Where possible specific objectives should be stated in quantitative terms because this ensures a rigorous comparison of alternative designs. Further, during routine operation of the system upon implementation performance can be realistically appraised. Some examples on specific objectives follow :

1. All financial statements should be furnished within 48 hours of closure of the period.

2. So automation the operation of the superstore inventory control system that order sizes and ROL's are optimally computed.

3. Pay 100% of invoices before due date.

4. Identify cost and quantity variances within one day to permit initiation of corrective action.

5. Establish a system that can simulate different work schemes for an acceptable workload on the shop.

(b) *Current Capabilities* : for information handling need to be carefully appraised. This is analogous to strengths and weaknesses analysis in corporate planning An appraisal of the various equipment, software, applications and personnel expenses and facility utilisation should be undertaken. These should then be critically appraised.

(c) *Forecasts of Developments*: Computer data processing is a striding field Within less than a quarter of century we have witnessed three generations, of computer and the fourth is on its way. It is, therefore, highly desirable to be well versed with the computer market and technological developments. The hardware is known to have had a great deal of influence on the MIS centralisation and decentralisation issue. The first generation computers were high in costs and large in size. With the result the information system were sought to be centralised to derive benefits of hardware economies. The second generation compurers were substantially cheaper and the trend was towards MIS decentralisation The third generation however offered communication capabilities and the use of remote terminals.

The trend was reserved to centralisation again. This was further reinforced by data base and minimal programming effort accruing from centralisation.

(d) *The Specific Plan* is a schedule of the following. It may cover several years ahead.

1. A hardware and purchased software schedule (may firm up only after Stage IV).

2. Application development schedule.

3. Schedule of software maintenance and conversion effort.

4. Personal resource plan.

5. Financial resources plan.

Maintenançe of the Master Plan is to be done by means of periodic reviews. The short range plan is updated and the long-range plan adjusted in the light of latest developments. For example, miniaturisation of computers would have impact on data preparation and direct entry of input. Dependency relationship discussed

above may necessitate planning and controlling by means of CPM and PERT. The reporting of progress can be had both activity-wise and milestone-wise, *i e.*, which milestones have been achieved. It is to be noted that the original plans may be modified great deal during reviews This is because the world does not stand still whilst MIS implementation is going on. Cost Control of MIS development project is important.

Systems Development Stage IV—SYSTEM ANALYSIS AND DESIGN

The earlier three stages were concerned with all the application collectively. The following stages, however, pertain to each application individually as elaborated under a section below on "Application Life Development Section". This would be so when they are being developed for batch processing mode. Subsequently they, too, may be taken up collectively when data base and model base, etc are built.

Also as stated earlier this stage is intended for detailed application-wise analysis and design ; gross design and overall analysis having been covered in Stage II.

Application Life Development Cycle : Each application typically follows the following steps of systems development mentioned on page 6 *i e.*,

Systems Analysis

Systems Design

Systems Implementation

Systems Maintenance.

Where the applications are to be integrated they can be considered together for these phases.

SYSTEMS ANALYSIS AND DESIGN TASKS

Systems Analysis : There are two approaches to conduct systems analysis.

Decision Analysis Approach

1. Indentify objectives and/or current decisions or processes *viz.*, in inventory control the decisions may be *how much* and *when* to order and processes may be stock balance updating.

2. Identify or formulate a decision-making process. In the above example, the decision-making model may be E.O.Q. model.

3. Identify the data required for the decision model or the process model, The E.O Q. model is given below :

$$\text{E.O.Q.} = \sqrt{\frac{2C_aA}{C\,I}}$$

Where C_a = Aquisition Cost/Order

A = Annual Demand

C = Cost of the item

I = Inventory carrying charge.

4. Specify the accuracy and availability of the limits for the data needed.

The advantage of this approach is that the analyst and the manager are not coloured or biased in favour of the existing system and procedures. They can express in precise terms what exactly are the information requirements for the decision or the process model.

Data Analysis Approach :

(a) Collect all data, reports, files etc. currently in use

(b) By interviews and experience identify additional data that is not being currently collected.

(c) By interviews and analysis, seek to eliminate data for which no need can be perceived.

Techniques Systems Analysis & Design

These apply to both the existing (whether manual or computerised) and the proposed system.

1. *Interviews*

2. *Questionnaries*

3. *Observation and Work Study Flowcharts*

(Already discussed in Stage II).

4. *Documentation Flowcharts* depicts the flow of documents across various departments. As such they indicate the sources of originating data and the nature and disposition of outputs. An example on such a chart for handling the customer order is given in figure 2. Vide Page 26.

5. *Systems Outline Chart* merely lists the inputs, files, processes and outputs without regard to any sequence whatever. An example of this chart on Sales Order Processing is given on p. 27.

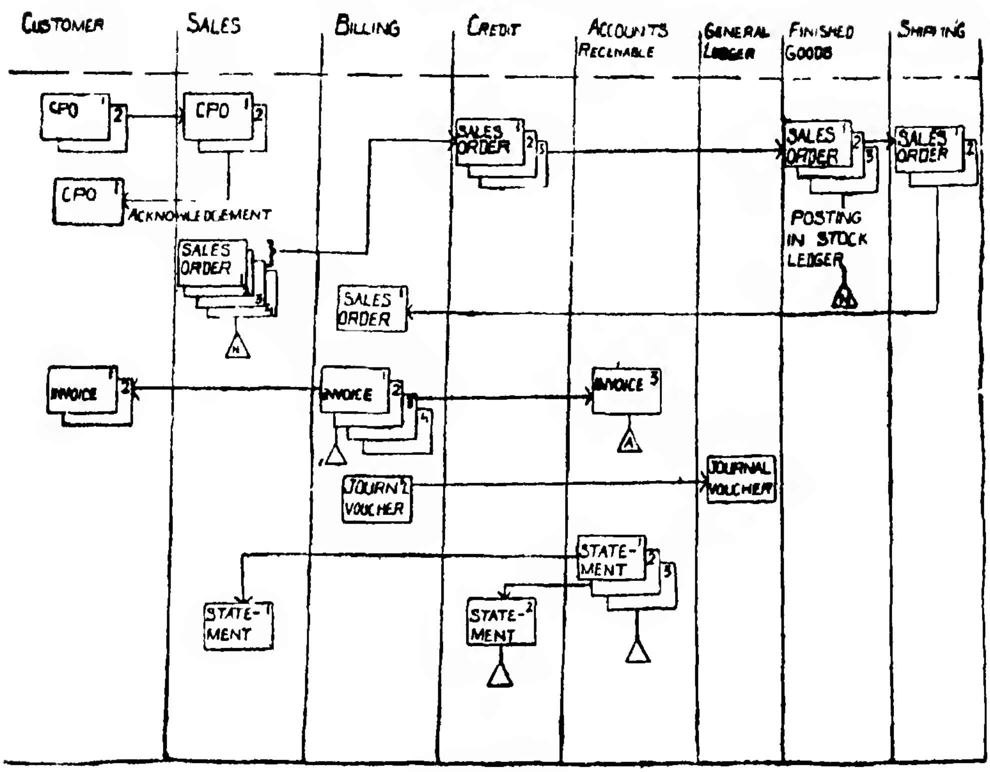


Fig. 2

6. *Run Flow Chart* (Syn. Block Diagrams) as discussed in Part II of Study II.

7. *Program Flowcharts* as discussed in Part I of Study II. These, however, are usually drawn by the programmers in the following stage. Occasionally, however, the systems analysts may draw these for some complex problems.

8. *Decision Tables* as discussed in Study II.

9. *Input Layouts* as discussed in the context of punched card in Study I for example. Such layouts would have to be prepared for each and every transaction on all kinds of transaction file media.

10. *Master File Layouts.*

11. *Grid Charts* link the data items to reports, documents and files and inputs. An example of a grid chart is given vide page 28. The grid chart helps in spotting redundant data items and facilitates consolidation and elimination of forms, files and reports. A couple of more realistic examples would be found in SADP Study VII, or MICS Study V.

Title *Sale Order Processing (SOP)*	*System* *S*	*Document* *3 1*	*Sheet* *1'*
Inputs Customer Order Details		Processes Order entry (clerical) Order acknowledgement (computer) Despatch (clerical) Despatch update (computer)	
Files Product Catalogue Customer Index Cards Doubtful Cost List Delivery Cost List Factory Stock List Customer N/A Card Product Card file Outstanding Order File Product Order Book Order Ledger		Outputs Error reports Balance Order Set Advice Notes Set Invoice Details Tape	
Notes, Cross reference			
Issue : Date :			

12. *Information Requirements Tables* describe all data processing activities present in an information system and the contents and the input/output and storage media used. An example on it is vide page no. 28-29.

13. O.R. Models.

14. *Other Tools and Techniques* : Organisation Charts, work distribution charts, financial statements and reports may also be compiled or collected by the systems analyst.

Grid Chart

Reports / Documents and Files — Data Items	Customer File	Stock File	Despatch Note	Etc.
Customer Name	√		√	
Customer Address	√		√	
Quantity Orders	√	√	√	
Discount Code			√	

Information Requirements Form

Sub-system Name	*Activity Name*	*Output*	*Data File*		*Input*
			File name	*Data-element*	
Order Processing	Edit, credit check, and book	Open order file, unacce-pted orders	Customer master	Cust. No. Cust. name Cust address Credit code	Customer Order
	Check stock availability, price and print acknow-ledgement.	Customer acknowled-gements	Part No. and stock master	P/N Available Bal. Description Qty. Received List Price Discount code	Open Order
			Customer Master	Cust. no. Cust. name (Bill to) Cust. Address Cust. name (ship to) Cust. Address Date last trans.	

Continue—

Print shipment documents and invoices	Packing lists Picking lists invoices	Part number and invoice master	P/N Description Location Code Pieces/Package Package type List Price Discount Code
		Customer master	Cust. no. Cust. name (Bill to) Cust. Address Tax Code Insurance limit Shipping instrn.

The analyst may employ the various techniques of systems analysis and design for information analysis. He may, for example, prepare systems outline chart, systems flowcharts, computer run charts, procedure flowchart, decision tables, grid charts etc. for the existing system which, later he can study in his office. He analyses the existing system under the data analysis approach and proceeds for system design. Under the decision analysis approach he can straightway proceed with the system design, *i.e.*, without analysing the existing system. The outputs of the system design are :

System outline chart	
System flowchart	
Computer run chart	
Program flowcharts	
Procedure flowcharts	Audit Requirements
Layouts of the outputs	Software Checks
Layouts of the files	Forms (Sample Copies)
Layouts of the inputs	Codification Schemes

SYSTEMS DESIGN

1. Output Design

The major concern of the user in the systems design effort is properly designed output that is intelligible and decision-impelling If, therefore, the output design is poor the entire systems developments is likely to be jeopardised. Once the output reports formats and contents have been fixed the systems analyst can work backwards

and devise the input (transactions) and master file layouts as also the computations to be performed to derive the figures in the output reports. Going further back he would evolve suitable procedures and design the forms to capture the input data at points of organisation. At this point we must emphasise that any application spawns both the clerical staff and their superior for data creation and computer for data processing This, in fact, should have been quite clear from several applications discussed in part II of Study II. Printed output can be bifurcated as below.

(i) *External Results* i.e the outputs that are to be distributed amongst the user departments, e g. invoices, ageing schedules, list of replenishment orders, sales summaries, payslips, etc.

(ii) *Internal Results* comprise transaction listings for control and audit (purposes) and error lists and summary information (*i.e.* control totals).

Format of the information reports for the users (*viz* variance analyses or sales reports) should be so devised that it assists in decision-making, identifying, solving problems, planning, initiating corrective action and searching Therefore, all the rules of compiling statistical tables should be followed More often the executive would simply ignore the report that is not readily comprehensible. For *ad hoc* reports required on demand (usu. by the top management) it is not a bad idea to supplement the computer printouts with summarisation by means of diagrams, curves, histograms, graphs, etc. However, where graph plotters are available as output peripherals the computer can do this bit of job too. In order that reports are readily comprehensible, codes and abbreviations are best avoided. Reports should preferably be supplied on an exception basis to save the manager from an overload of information. Occasionally, reports may be required in entirety, *viz.* for the auditors even in which case significant divergences or slippages or variances or deviations can be highlighted by means of printing asterisks As mentioned several pages earlier data (in case of marketing) can be classified in myriads of ways but only those analyses which are relevant to decision-making should be provided for in the software. Likewise, for such complex measures as ROI or productivity not only the summary figures but the entire hierarchy of their data components should be provided so that the manager can spot weaknesses (or strengths) at once.

For design of the output too collaboration between the user (executives) and systems analyst (s) is of great importance. The latter knows the computer capabilities in data and symbol manipulation and printing/display and the former knows what he needs, if not explicity by proper probing by the analyst. It is highly desirable to go about this problem by what has come to be known as the black box approach. It is a highly creative approach and is best carried out in the spirit of brain storming sessions (Ref : OR Study V) amongst the executives and analysts. The idea is to dismiss the existing outputs from mind and think afresh what is (reports) needed to support the managerial activities of directing, organising, planning and control. The participants are not constrained to be pragmatic On the contrary they are encouraged to be dreamy. Economy is forgotten for a while though it will subsequently figure in as it should Quantity of ideas rather than quality is the objective. With this approach such seemingly bizarre output media as audio response units would not be overlooked which is very likely going to happen in conventional approach. And the audio-response may well turn out to be an excellent mode. Following this the analyst 'scrutinises each idea for its economics and even feasibility.

The reports collected during the systems analysis phase can now be subjected to a close scrutiny for their validity. It is usually the case in most organisations that 10 to 30% of the reports do not serve any purpose and these are merely a carry over from some past events. Thus elimination of such reports is called for Some of these can be discontinued as an experimentation and if no one demands they should be abandoned It is also desirable to merge two or more reports where possible.

It is also to be ascertained if the cost of a report is justified by the benefits. The cost factor is not difficult to establish Using costing techniques total cost of each statement can be derived but the benefits are hard to quantify In fact, assessment of benefits is very subjective and their conversion to objective units of measurement is nearly impossible However, to resolve this problem we can classify all the managerial statements into a few categories with regard to importance, say, (a) absolutely essential, (b) necessary, (c) normal, (d) extra The last category statements ought to be eliminated and the (c) category statements should be viewed with skepticism. If the cost of such a statement is excessive it should be eliminated.

Frequency of reports is another aspect to be determined by the systems analyst Too frequent report hide the broad trend or pattern and undue importance is given to change fluctuations in progress. On the other end of the scale too infrequent reporting is liable to miss important fluctuations owing their origin to some assignable cause (s). Considerable deliberation is, therefore, needed to strike a balance though daily, weekly, fortnightly, monthly, quarterly, etc. are usually (sort of) rounded off periods.

Timeliness is yet another aspect of reporting Obviously an annual report can be presented within a few days whereas a weekly report would have to be presented within hours, if not less. Anyhow, the systems analyst should ascertain the maximum possible interval between the production and presentation of the report. Too short an interval would unnecessarily burden the staff.

After these aspects have been covered the analyst comes down to detail regarding the contents. For the existing reports he can go in by plus/minus i e. add new contents that are considered necessary by him (in conjunction with the operating manager) and delete the unnecessary data items. For altogether new reports the contents in entirety would have to be conceived. Each data item would be defined as numeric or alphabetic or alphanumeric The picture would be defined completely and this would be more clear if the student refers back to COBOL in the previous study II. It is to be noted that some numeric data items can assume negative values and provisions would have to be made for printing the sign. Likewise monetary figures may be printed in one of several ways some of which are given below.

Rs. 4708.21	or	Rs 4708.21	or	Rs. 4708.21
Rs. 0506.26		Rs. 506.25		Rs 0506 25
Rs. 0000.00				Rs. ****.**
Rs. 0014.00		Rs. 14.00		Rs. 0014.00

In the 2nd case non-significant zeros are omitted. In the 3rd case for no amount stars are printed so that a defrauder is frustrated whereas in the 2nd case he could type in some figure. Thus for each data field appearing in an output the following specifications should be clearly spelt out.

(i) Whether fixed length or variable length.

(ii) Whether it is to include significant zeros, non-significant zeros, spaces or special characters (*viz.*, oblique in data).

(iii) Whether it is left-justified or right justified.

(iv) Whether it is purely numeric or alphabetic, or alphanumeric.

(v) Its maximum size, including sign, decimal point, comma (s), sign etc.

(vi) Its maximum value (viz. Rs 40,000 for limit checks to be discussed subsequently).

For each output liable to run into more than one page of continuous stationery it is to be specified if in the sheets in continuation headings are to be printed or not and how many blank lines are to be left after headings.

Based on the aforesaid considerations, for each report the following output analysis form and printer layout sheets are compiled.

Typical System Output Analysis Form

(*Incomplete*)

System Output

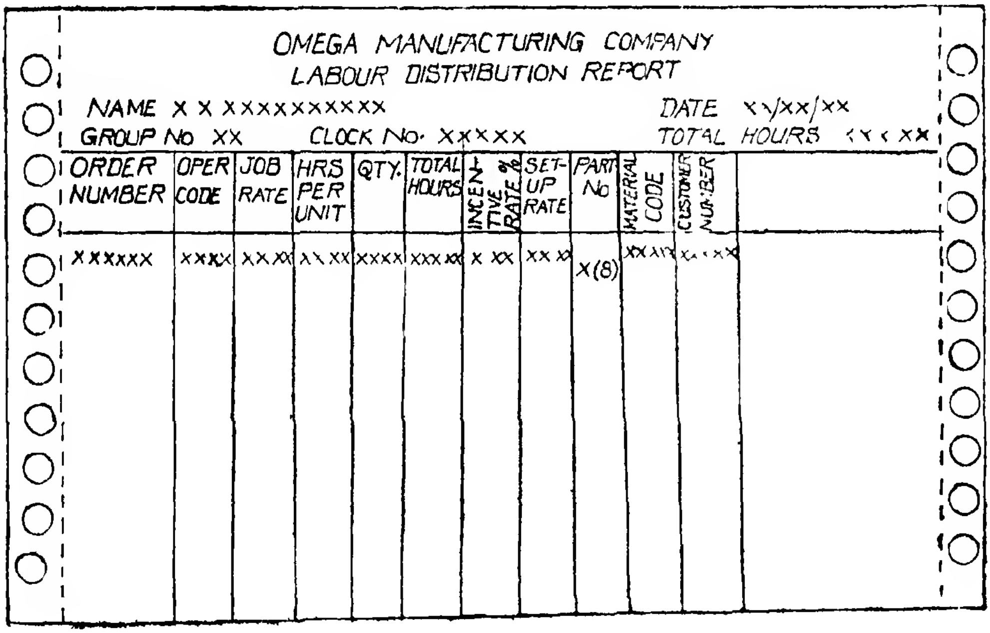
Page 1 of 1
Date : July 25, 1982
Prepared by : A Pannikar

Type of Output : Monthly Sales Report by Products

Data Items	*Type of Characters*	*No. of Characters*	*Comments*
Product Code	Numeric	5	
Product Name			
Monthly Sales (Units)			
Sales Price	Numeric	4	Edit with decimal point
Monthly Sales by product (Rs.)			
Total Monthly Sales for all products (Rs.)			

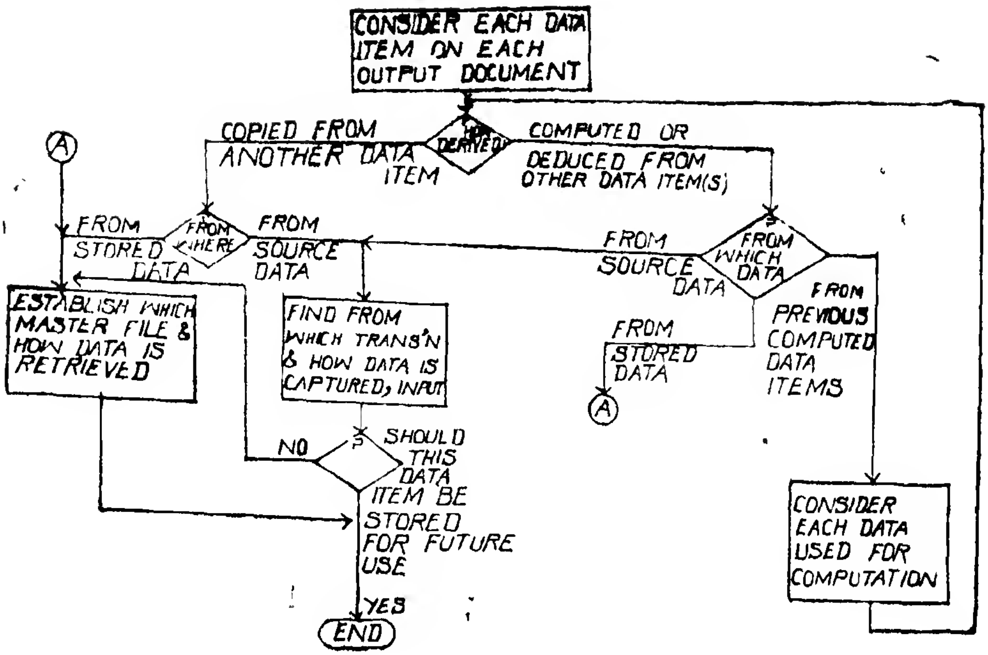
Comments : The computer program is to generate the report and column headings. Also the current date and page number are to be printed at top of each page.

The printer layout is available in graphic form Horizontally, each grid measures 1/10th of an inch and vertically it measures 1/6th of an inch Columns are headed from 1 to the maximum print position available. Such charts facilitate portrayal of the output highly useful to the programmers subsequently and as documentation Layout should include a statement whether the output is to be had on continuous statio ery or pre-printed forms It is to be remembered that numeric fields that can attain —ve value require an extra print position. A specimen print layout of Labour Distribution Report follows.



Output reports, having been thus fixed, are now put to use to derive the data fields to be incorporated in the inputs and master files by the logic adumbrated in the flowchart *vide*, page No.34 Also to be established at this stage are the file media for the master files and inputs.

2. **Designing Files :** There exist different kinds of files (master file, transaction file, reference file) as also the various media viz , punched cards, paper tape, magnetic tape, magnetic disc, magnetic ledger cards and occasionally magnetic drum. File organisation refers to the relationships between the keys of the adjacent records. If these are in serial order, the organisation is said to be sequential Punched cards, paper tape and magnetic tape are inherently sequential media If there exists no relationships between the keys of adjacent records the file organisation



is said to be random. Magnetic disc is the most common such media though magnetic drum (that is much more expensive) and magnetic ledger cards (that are much slower) are also in use. All these random (or direct) access media can also be used as sequential media whereas punched cards, paper tape and magnetic tape can only be used as sequential media and never as direct access media. The criteria to organise the files in sequential or random order are stated below :

(i) the cost and ability of the media ;
(ii) complexity of systems design and programming involved and the difficulty of operation ; and

(iii) the effect on overall running time of the computer.

The choice of a media within a particular file organisation would depend upon the following :

(a) The transfer speed, punched cards, for example, being very slow in being read into the CPU can waste the fast internal speed of the computer.

(b) In some applications e.g , fixed order cycle stock control systems the file organisation should obviously be sequential since all the records would be reviewed. So is the case with payroll file in general but occasionally a manager may want such information as the list of the persons who are below 35 and at least B.A. in qualifications Such an inquiry can only be expensively had on the magnetic tape etc. But the magnetic disc can

manage both periodic processing and such random inquiry handling. Therefore, due regard to the nature of the application has to be paid whilst deciding upon a particular medium.

(c) Volume of information that the medium can hold conveniently. Take punched cards as an instance. If, in an application, the records do not exceed 80 columns in length, this medium could be further considered, but if the records are longer, say, 90 columns, two cards per record would have to be maintained. This would not only make it a wasteful proposition but also programming would be more complex.

Further, the arrangement of the data fields within the chosen medium is also worth consideration at this stage Fixed length records are wasteful in utilising the storage of the secondary medium but keep programming simple Variable length records, on the other hand, utilise the storage space most economically but make programming rather complex.

Files are continually amended, *i e.*, new records are added and some existing records may be purged Punched cards are the most convenient in this regard since such additions or deletions can be simply made manually. In magnetic tape and sequential access on magnetic disc, insertions are made by copying existing records until the key of the insertion record is reached. Deletions are made by copying records from old file to new file by omitting unwanted records from the new file or by inserting a deletion mark on the unwanted record—obviously time consuming and therefore expensive. In magnetic disc with direct access file organisation, new records may be simply added at the end by amending the location index. And unwanted records can be simply over-written by new records.

Having made the choice of the medium and organisation the systems analyst ought to specify each file as below :

Identification : file number :

file name.

Purpose : a short description of the reasons why this file is needed

Characteristics : e.g.

—master file ;

—transaction file ;

—inactive file.

Contents : a listing of :

—the kinds of records appearing in the file, with connecting record identification ;

—average and maximum number of each record for each file item ;

—sequence of the records

Volume : the average and maximum number of records which can occur in a file, the fixed, average and maximum sizes for each type of record, and the average and maximum number of characters (=number of records × record length).

File organisation : the type of access, e.g ,

—sequential ;

—index sequential ;

—direct addressing ;

—relative addressing and the sequence of sequential files in terms of major/intermediate minor fields.

Planned storage media : punched cards ;

: magnetic tape ;

: disc ;

3. **Input Design** : A placard carrying the message "GIGO" is hung in many computer installations It stands for "Garbage In Garbage Out". This should bring out the importance of the subject of input design Unless raw data is captured and input in a suitable form no sensible processing can be performed. Unlike process control where input emanates from thermometers and pressure gauges, in business systems it emanates from human beings and conflicts are inevitable Great care should, therefore, be exercised in balancing system requirements against personnel capabilities. Inputs may be had in three ways

Direct Entry for subsequent processing : The transactions are entered directly into the computer via terminals or the transaction is edited immediately but processing is postponed until a sizeable batch has accumulated Great many types of errors can be detected during editing and referred to the input device for on the spot correction. The disadvantage of direct entry for subsequent processing lies in the fact that files are not always up to date.

Direct Entry for immediate processing : The transaction is both edited and processed immediately upon entry. The input devices would respond with either the message of completion of procession or notification of errors discovered during editing. The files are always up-to-date , but this mode of entry and processing is costlier than batch processing. Also, audit trail (Ref : Study VI) is rendered sketchy and safeguards are necessary to save the files against accidental or deliberate destruction during online updating.

Periodic processing of batched transactions : This mode of processing is also known as batch processing and is contrasted with immediate online processing. The transactions are accumulated into batches. The frequency of processing for the

accumulated batch may be hourly, daily, weekly, monthly etc depending on the application and cost benefits involved. Obviously batch processing is less expensive than on line processing Greater the batch size, lesser the setup costs and per transaction cost of processing. Editing can be performed on the entire batch and this allows better control over processing accuracy and better protection against file destruction. But the disadvantage of batch processing is that master files are out of date It is to be noted that magnetic tape is inherently suited to batch or sequential processing In contrast, magnetic disc may be used for sequential processing and non-sequential processing.

Extent of Integration : A point of great importance is the relevance of an input to more than one sub-system For example, customer orders may have effect on accounts receivable file, inventory control file, production scheduling etc, etc. The extent of such integration, *i e*, multiple use of data is to be determined by the analyst. The more integrated the system, the more difficult and expensive it is to develop, but the computer is used more efficiently. Instead of developing and designing an integrated system straightway it is also possible to design independent sub- systems to start with —subsequently upon implementation these can be combined in stages.

Types of Data : Three main classes of input data to a computer system can be recognised :

(*i*) External data—hours worked, quantity received, part numbers, quantity sold, etc.

(*ii*) Internal uses data for inquiries

(*iii*) Internal system data such as console typewriter instructions.

The external data can be further sub divided into current and file data upon which the former operates during transaction processing Files are also amended for insertions, deletions, and amendments. It is necessary to process the file against batches of amendment data before file updating is undertaken for current data, *i.e.*, transactions. It is desirable, however, to put the file amendments at the head of the transaction files

Data Capture and Design : Most employees come into contact with the computer for inputting data Therefore the input methods should be as simple as possible. If the employees regard this provision of input as a tedious and unrewarding activity, they will be resistant to data processing activities and the quality of information provided will suffer. Thus care in data capture has to be reconciled with the need to present data to the computer in a form and sequence that is acceptable. Depending upon the method of data capture some or all of the following stages will be met :

(*i*) Original recording

(*ii*) Transmission of data from the place of original recording to the data processing centre.

(*iii*) Transcription of data in a suitable input medium.

(*iv*) Verification of transcription .

(*v*) Sorting of the records by some key.

(*vi*) Control validates that all the original records have been transcribed.

(*vii*) Input.

Ideally, the data should be transcribed at the place of origin But this tends to be expensive. Also, shortage of equipment and qualified, experienced staff would call for centralisation of the data preparation, (transcription and onwards) activities.

The methods of recording data are numerous. These are listed below :

(i) Card punches.

(*ii*) Paper tape punches.

(*iii*) Magnetic tape encoders. Data is encoded via keyboards. Early encoders had input block limitations of 80 to 160 characters This is because they were intended to replace the punch cards. Now-a-days, however, the equipment is capable of producing blocks upto 150 characters

(*iv*) Indirect keyboard machines—which produce paper tape as a by product of normal keyboard operation to prepare hard copy. Examples are found in typewriters, accounting machines and cash registers.

(*v*) Mark-sensing punches—in which pencil marks made in pre-determined positions in a punched card are converted automatically into punched holes on the same card, thus eliminating manual punching of data.

(*vi*) Magnetic ink character readers.

(*vii*) Optical character readers.

(*viii*) Mark readers—akin to mark sensing readers, the only difference being that mark reading punching is performed on normal sized paper forms instead of punched cards.

(*ix*) Document reader—which accept and convert information from documents produced by high speed printers, typewriters, adding machines credit printers and cash registers.

(*x*) On-line terminals.

The factors to be considered for adopting any of these methods are :

(*a*) *Type of Processing*—It may be batch processing (groups of records) or direct processing (individual and distinct records) : serial (sorted) or random (unsorted) It may be recalled that if records are transcribed on a paper tape they cannot be sorted. Hence processing would be random.

(*b*) *Speed* has to be considered for input Punched cards and paper tape have low capture and preparation speed but very fast input speed. On the other hand, OCR documents need little preparation but are slow in inputting.

(*c*) *Accuracy*

(*d*) *Verification* can be performed (*i*) by sight checking form hard copy, (*ii*) arithmetic methods (check digit : see page, 47), (*iii*) automatically by program, (*iv*) batch totals verification. Each input media has to be considered for these possibilities of verification.

(*e*) *Operator requirement*

(*f*) *Cost.* These can be spilt into two components ; (*i*) capital cost of equipment and (*ii*) cost per character of input. The latter is composed of stationery, floor space, staff and overhead.

Of all the input media discussed, the two principal contenders are the punched cards and paper tape. Some of the advantages of the two media are given below :

Advantages of punched cards :

(*i*) The use of standard 80 column is univeral. In contrast there is a conversion problem for a wide range of formats in 5, 6, 7 and 8 channel tape.

(*ii*) Records can be inserted or deleted merely by inserting new cards or removing the existing cards.

(*iii*) For program input, use of cards is easier.

(*iv*) Cards can be loaded in the card reader hopper whilst the computer is operating. In changing reels of paper tape, time may be lost.

(*v*) Cards can be sorted off-line and can also be processed off-line by auxiliary equipment.

(*vi*) Card files are tidier than unreeled tape.

(*vii*) Cards can be interpreted and read by eye.

(*viii*) Error correction is easier—just pull out the incorrect card and insert the corrected one in its place.

(*ix*) Cards are more robust than paper tape.

Advantages of paper tape :

(*i*) It is easier and cheaper to produce paper tape which also produces a hard copy as a by product *viz*, on typewriters or accounting machines.

(*ii*) It is less bulky.

(*iii*) Records cannot be lost or get out of sequence.

(*iv*) Variable length records are possible to accommodate

(*v*) Parity check can be provided.

(*vi*) It is telex compatible.

(*vii*) It is a cheaper media.

(*viii*) Equipment is cheaper.

(*ix*) It can be read in reverse.

(*x*) Errors during punching can be designated and ignored during processing Cards, in contrast, have to be re-punched.

(*xi*) Paper tape reel is easier to feed to the computer than its card equivalent.

4. Form Design

The analyst may have to design forms for revised procedures he produces. He should also be thorough with the considerations for design of the punching documents.

Office forms are instruments of office procedures ; they have no life of their own. They exist in order that information arising out of a procedure may be conveyed in a predetermined and orderly fashion. Thus the first question to settle is whether the procedures which give rise to the form are valid "have they been reduced to the minimum consistent with an efficient job ?". The question 'why have a form at all' ? must be converted into the question 'why should this information be provided or conveyed, at this point in the process, in this manner'? The design of a form introduces the skill of conveying information by documentary means : it is part of the 'how' of the procedure, but it comes last. At the earlier questions : What is being done (and how much), why it is provided, and when and where it should be provided must first be answered.

Detailed Specification of Requirements : What is required should always be drawn up. This should include information concerning :

1. Who creates the form and in what circumstances, for example, a clerk on a desk, an inspector as he moves around, a workman on a bench, etc – and in what form and sequence is the information normally obtained—for example by copying from other information, extracting or, editing, or making verbal statement ?

2. Who uses or processes the form afterwards ?—what is done ?—for example check, edit, analyse, calculate, select, summarise , the sequence in which these actions are normally undertaken.

3. How will the form be filled up writing and/or typing, imprinting, addressing, by means of an accounting machine, tabulator, line printer, etc. ? What limitation—for example, size of layout—does the particular method impose ?

4. The precise contents of the main items of information to go into the form namely :

   (a) Heading, if any, and whether it must be a document—for example, for an invoice—or merely as identification as for example, on an internal advice

   (b) Space, if any, for name and address or recipient , how to be inserted and if in a box—for example, whether to suit a window envelope.

   (c) Whether any of the entries can be preprinted ?

   (d) Columnar arrangements for money entries , number of digits likely to appear in price and value column , provisions fortotals and sub-totals.

   (e) Maximum length of lines of data ; number of lines in the form of the rough distribution of frequencies.

   (f) The intention and working of an introductory, explanatory or instructional material—where, on the form, this should appear and what weight it should be given ?

5. Any special features of form—for example, for security or control purpose.

6 The number of copies required , the purpose of each ; the minimum amount of data in the original which must appear on the copies, material which must appear on copies—for affecting the choice of the copying method, legibility, permanance, handling, de collation etc.

7. Limits on the size and shape, for example, for handling, sorting, filing, insertion in binders, etc. Forms sent through the post must comply with certain regulations.

8. Usage per year How should forms be provided—singly, in pads, in the form of continuous stationery for computer printing ?

Before moving further, the designer should satisfy himself that a new form is really necessary—for example, that it is not possible for an existing form to be added to or adopted to serve the purpose. Generally, the aim should be to reduce the number of forms in use and, whenever, possible, to combine forms. Care

should, however, be taken not to complicate a composite form so much that it becomes difficult to use it.

The forms design can make a very worthwhile contribution to the success of office procedures by the skillful agreement of that material so that it has the appearance of a well contrived and attractive form. There is also scope for considerable ingenuity in devising methods of documentation that best suit the system This can result in considerable saving in the cost of producing and the much larger cost of filling the form

Drafting the Form · With the specifications of requirements clearly in mind, the broad structure of the form should be decided upon Various major aspects of design must be considered, for example :

1. The order in which the groups of introduction and information should appear ; the amount of space each should occupy in order to serve their purpose adequately.

2. The need, if any, to give special weight to particular parts of the form, for example for prestige reasons or to draw special attention to directions or requests.

3. The method of creating, handling or processing the form—the design should aim at facilitating the use of the form within the procedure to which it relates

A rough sketch of the main features of the form, suppressing detail, is required in order to be able to visualise it as a useful instrument of the procedure and make sure that it holds the promise of being an attractive form. At this early stage it may be worth showing the rough draft to the user or customer to make sure that he is likely to be satisfied with it in its broad outline.

In drafting the form in detail many considerations must be kept in view ; for example :

1. Provide the form with a means of identification, usually a title which makes it distinguishable.

2. Arrange the material in a logical order so that it is easy to fill and to understand and see that the working is sufficiently clear so that doubts and queries will be avoided.

3. If questions are asked, put them in simple terms, arranged in the expected sequence for easy comprehension.

4. Put notes and instructions where they will be read before entries are made ; preliminary general notes at the head of the form, and specific ones near the point at which the relevant entries are to be made. Avoid putting notes overleaf, since it makes reference to entry difficult. If there are complex instructions about the completion of the form, put them in a separate note rather than clutter up the form.

5. Arrange for all the entries on the form to be made from left to right, the natural way of writing Use a columnar arrangement. With horizontal descriptions, for example for price or value

6. Use boxes or panels for single entries : make an estimate of anticipated answers to questions and see that sufficient space is allotted. Put subheadings in the top left hand corner.

7. Written entries require lines at 6 mm space ; for both writing and typing 8 mm. No lines are needed for typewritten or accounting machine entries ; these should be allotted space at multiples of 4 mm

8. Introduce emphasis by shading columns, heavy lines, etc. If the form is to be used for a specific clerical operation, for example copying or checking, see that the detail is arranged and spaced to provide maximum help to the operation—for example, in relation to a document from which card punching will be undertaken.

9. Decide how many copies of the form are to be created and produced —for example by the use of coated paper, carbon banding of patching, hectograph masters, dyeline or offset masters—and whether copies are to be provided in sets, singly or in continuous.

10. Use smooth paper if entries are to be made in ink and rougher paper for pencil entries. Add colour flashes for easier identification of copies.

11. Consider whether the form needs to include aids to reading (arrows or marks), copying (lines or codes) tabulating (starting and tabulating) line, folding (folding guides).

12. Check that security aspects have been covered—pre-numbering forms, double side carbon, etc.

When the form will be used in an office machine, take account of requirements of the machine and of the limitations it imposes, for example :

1. *Typewriters.* Reduce the number of lines by arranging data in horizontal form whenever possible.

(a) Minimize carriage movement and reduce the number of starting and finishing points involving the use of tabulation stops.

(b) Whenever practicable, arrange the vertical spacing on the form in such away that the tabulation stops can be continued down the form. Avoid entries close to the top or bottom of the form. In a complex form print guidelines to facilitate the making of entries.

(c) Space entries according to the size of type face. Arrange vertical spacing at multiples of 4 mm.

2. *Accounting machines.* Requirements, in general, are the same as for typewriters: on same machines there are fixed points for entries relating to money registers, entry of data, and other standard data

3. *Addressing machines.* Overall size of documents · distance of edge of form from plate , number of lines of characters in the plate and the spacing within the plate so designed that all needed data can be entered in all the required places.

Design of documents used for punching data

Data may be punched directly from original documents—that is, source documents—or form specially prepared data sheets into which selected information has previously been entered The design of any source document must be largely influenced by the work requirement at its point of use Data sheets can, however, be designed to promote the greatest efficiency in the punching operations.

The question whether data should be punched from source documents or from data sheets is a matter deserving close consideration The aim is not merely to reduce the number of operator key depressions but to minimise the time and cost required to convert the data into machine readable form. The decision is usually based on the characteristics and degree of complexity of the source document

Size and shape of the source document. Is this likely to slow down punching, particularly if, in manipulating the document, the operator needs to use both hands ?

Distribution of data on the source document Is data distributed in such a way that it is difficult for the operator to select the required data ? Does the selection involve considerable eye movement and is therefore fatiguing ?

In some cases, the information to be punched is so small a part of the content of the document that the preparation of data sheet is obviously the best plant. For large scale operations, it may be worthwhile to specially preprint data sheets and arrange for their completion at the point of origin of the data—for example, at an order receipt and edit point (it should not be forgotten that the creation of an inter mediate document doubles risk of miscopying and it is desirable that the information on the data sheet should be carefully checked) The disadvantages of using a source document, in terms of the extra costs of slow punching speeds, should be calculated and compared to (a) the clerical costs of transfer of data sheet, and (b) the cost of punching from those sheets. The forms designer will often be faced with the problem of balancing the question of the efficieney of the originating document at its place of use against the efficiency of the punching operation. Normally, the former will carry greater weight.

Whether a source document of a data sheet is used for the punching operation, certain points affecting design are worth considering namely :

1. Full advantage should be taken of the possibility of using any by products from machines used earlier in the system—for example, accounting machines capable of producing paper tape as a by-product Whenever possible, use turn around documents produced by the computer at an earlier stage, to secure basic data.

2. Pre-print or make entries on documents by means which enter the verified information, or codes, are available in punchable form for example, from address plates, edge punched cards, etc

3. Avoid the entry of data which can be calculated, derived or retrieved by the computer.

4. Do not punch anything that can be established as a norm for the whole area of work or for sections of it, restrict punching to variables or exceptions. When using punched cards make use of the facility for pre-punching common or standing data

5. Ensure that entries in the punching media are made in the same sequence as in the source document—that is, from left to right and from top to bottom of a page (It is not necessary to follow the sequence of processing since the computer can sort the data into any order before processing)

6. Establish punching conventions in consultation with the punching room, for example :

   (a) Double vertical lines signify end of block.

   (b) Single vertical lines signify end of field.

   (c) Dotted line or break marks are used to assist spacing.

   (d) Indicators are provided when zeros must be punched

   (e) Rules are laid down concerning punching of decimal points.

   (f) Block or field numbers are entered on documents.

7. Consider devices for distinguishing data not to be punched—for example, cross-hatching of areas of the document, using pale coloured printing to make written entries stand out.

5. **Controls and Auditing** : We have referred to several distinct capabilities of the computer at the outset The system that is designed must exploit these. But computer is after all a machine It has its limitations too. One of these limitations that figures importantly in system design is that computer possesses no judgement which, though, can be built into its programs—of course, at some cost. Consider, for example, an error in goods receipt note—Rs 50 as the price of a washer in place of the correct price of 50 paise per piece In manual system, the stock control clerk is likely to view this with suspicion. He may run around and get the correct price Computer possesses on such judgment It would accept the wrong price, enter it in the stock records as such and pass incorrect invoices. Because of this, computer systems are also known as GIGO systems meaning "Garbage In Garbage Out" systems Such errors can arise during manual recording and transcription and there must be means to detect these to maintain the integrity of the files and produce correct outputs. This is usually accomplished by building in checks in the computer programs that can signal for such errors in the form of printed

errors reports which can be inspected by the staff, rectified on the input medium and resubmitted to the computer staff for input. Some of the important software checks are discussed below .

A. FIELD CHECKS

(1) *Limit Checks* may be applied to both the input data and the output data. The field is checked by the program to ensure that its value lies within certain predefined limits (in the program) This applies to both input and output fields considered to be important. Several examples follow :

(i) Hours worked by an employee in a week cannot exceed, say 60. The program would verify this for each employee's clock card and where this limit is exceeded an error message would either be printed or displayed This, then, is a safeguard against both inadvertent errors and fraud.

(ii) Limit on weekly earnings of an employee.

(iii) The highest vendor number is 3429. If the program encounters the vendor no. 4329 error message would be printed.

(iv) All customers with code 2, cannot purchase in excess of Rs. 10,000.

(v) No pay when there is sick leave.

(vi) Physical balance can never go below zero. Likewise, wages, forecasts cannot go negative.

(vii) Lowest and highest prices of stockitems to ascertain that the price on each transaction is not beyond these.

(viii) Date is usually put in the format DD MM YY. Checks can be built to ensure that DD$\leqslant$31, MM$\leqslant$12 and YY=84.

(ix) Transactions may be checked that they are not backdated say more than a week.

(x) If the quantity invoiced by a supplier exceed twice the average size the fact should be brought out as an error message.

(xi) The range of code number, for example, could be fragmented too. The permissible code number, could be 1 to 10, 15 to 20 etc , so that codes such as 8, 10, 15, 17 would be accepted whereas 11, 13, 14, etc. would be rejected.

(xii) Combinations of various fields may also be subjected to limit checks. Suppose the limts of purchase price are Rs.1 to 65 and that of quantity 1 to 100. A purchase order for 80 items at Rs 55 each could be detected by applying a combination check with a maximum of say, Rs. 4,000 to the value of the purchase order. The combination limit check is equivalent to a limit check on intermediate or final results and

particularly useful for detecting output fields that have become far too large for printing space allocated to these.

(2) *Picture Checks.* These check against entry into processing of incorrect characters.

Example All department numbers may be made up of numerics. An incorrect deptt. no. 4D3 would thus be filtered.

(3) *Valid Code Checks.* Checks are made against predetermined transaction codes, tables or other data to ensure that input data are valid The predetermined codes or tables may either be embedded in the programs or stored in (usu direct access) files.

Example (i) In an inventory updating application the following may be comprehensive list of transaction codes :

I = Issue

R = Receipts

C = Allocations against customer orders

A = Amendments

D = Deletion

N = Addition of new records

S = Stocktaking adjustments.

Any other code (misc. punching for example) would be brought out in the error message.

Example (ii) Every time a payment is made to a vendor, the number of the vendor must match a number in the table of valid vendors.

Example (iii) A code 19 is assigned to the preventive maintenance If one or several time card codes from preventive maintenance did not agree with this code an error would be displayed.

Example (iv) In the codes table for the chart of accounts the current assets may be designated form 100 to 199, where cash is 100. If cash receipts are being processed all cash credits or debits must contain the code 100.

(4) *Check Digit.* Check digit is an extra digit that is computed from a code itself and placed along-side it for subsequent checking Whenever the code is transcribed from one document to another this check is to be effected. Consider

the number 3721 for example. The weightages as below are assigned to its various positions Other steps to derive the check digit are also given.

Step 1 : 3 7 2 1

5 4 3 2 (Weightages)

Step 2 : $1\times2+2\times3+7\times4+3\times5=51$

Step 3 : $51-11=4$ with a remainder of 7.

Step 4 Check digit= $11-7=4$.

The check digit may be placed at the end of the number, giving 37214· (If the check digit turns out to be 10 letter A may be taken as the equivalent).

When the number is checked for validity the following steps would be gone over.

Step 1 : $1\times4+1\times2+2\times3+7\times4+5\times3=55$

Step 2 : $55\div11=5$ with a remainder 0.

If the number had been mispunched as 73214 the remainder of 4 (and not 0) would be obtained, thereby causing an error messager. The check digit described above is known as the 11—modulo check digit for the reasons which should be apparent now It is to be noted that on some errors the check digit may fail us. There are numerous other check digit schemes but the 11—modulo check digit scheme has been empirically established as the best to detect the following types of errors commonly encountered in data processing situations.

(i) *Transcription*, where the wrong number is written completely, e g., 2 for 8.

(ii) *Transposition*, where the correct numbers are written but their positions are reversed, e.g , 3419 for 4319.

(iii) *Double transposition*, where is an interchange of numbers between columns, e g , 21963 for 26913.

(iv) *Random*, which is a combination of two or more of the above, or any other error not sepcifically listed here.

In the table on page 49 are given the survey test results for the 11—modulo *check digit scheme* (Ref : *Basic training in System Analysis* by Daniels and Yeastes).

Performance of 11—modulo check digit scheme

Number of entries—1,00,000 Total No. of errors = 1,000

Type of error	No. of errors	% efficiency	% Errors detected	Errors undetected
Transcription	860	100	860	0
Transposition	80	100	80	0
Double transposition	10	100	10	0
Random	50	91	45	5
			995	5

Efficiency—5 errors undetected in 1,00,000 entries, i e, 99 995% coverage of errors

(5) *Arithmetic Checks.* Arithmetic is performed in different ways to validate the result of other computations or the value of selected data fields.

Example (i) The discounted amount for Rs. 4,000 at 5% discount may be computed twice by the following different ways :

$$4000-4000\times\frac{5}{100}=3800 \text{ or}$$

$$4000\left(\frac{1000-5}{100}\right)=3800$$

Example (ii) In payroll processing, it is usual to accumulate gross pay, deductions and net pay. At each employee's record whenever those totals are accumulated the Gross Pay must equal Net Pay + Deductions.

Example (iii) If x is to be divided by y not only this is done but also for verification the quotient is multiplied by y to verify the result.

Example (iv) If fairly homogeneous items such as steel, rolled stock or a lubricant are shipped to a customer, the billable amount can be checked for accuracy. A standard price in place of the actual fluctuating one say Rs. 100 per unit may be multiplied with weight/volume to derive an approximate billable amount. If this amount is say, not within 4% of the billed amount, then an error message is given for investigations.

Example (v) For arrays of numbers, both horizontal and vertical totals may be summed up for verification.

(6) *Cross Checks* may be employed to verify fields appearing in different files to see that the results tally For example, the quantity received, quantity invoiced by the supplier and quantity ordered may be compared. Incidentally, it may also be necessary to effect a check on the units of quantity. Fraud is possible if the unit is gm on the transaction whereas the standard unit in the master file is kg.

Transaction Checks

(i) *Sequence Checks* are exercised to detect any missing transaction off serially numbered vouchers (subsequently transcribed for computer processing) or erroneous sorting

(ii) *Format completeness checks* are used to check the presence and position of all the fields in a transaction. This check is particularly useful for variable data field records.

(iii) *Redundant data checks* are used in sequential processing Matching keys of the transaction record and its master record may not be deemed enough. One may, in a sales application for example, want to compare, say first five characters of the customer's name

(iv) *Combination checks.* Credit against shipments is invalid and ought to be rejected.

As another example, consider the following set of codes :

Type of Transaction	Type of Customer
A (Payment on Credit)	1 (Individuals
B	2
C (Cash payment)	3
	4

In theory, there are 12 combinations, but, in practice, such a combination as A1 could be invalid.

(v) *Passwords* are issued to the various users in on-line systems for processing their inquiries. For example, the financial manager may have to enter via the terminal his code no., his name and say, the height of his son which was issued to him as his password The program will compare this against the table of passwords so that any fraudulent attempt by any other employee to retrieve sensitive financial data is frustrated. It is desirable to periodically change the passwords.

(vi) Once a user has been identified in an online system, it remains to be seen what he is authorised to access, read, write etc.

Towards this an authorisation table like the one below has to be embedded in the data base.

		Authorisation for file A			
User	*Read only*	*Write only*	*Read/Wrire*	*Delete*	*Add*
1.	X			X	X
2.		X			
3.	X		X		

Depending upon the sensitivity of data the validity of the user may be further verified or authenticated by periodic requests for further information or reverification from the user or disconnecting the terminal and dialling back if the right terminal responds that will detect any masquerading terminal.

6. Coding and classification of accounts in computerised systems

It is necessary to identify such objects as human beings stock keeping items etc. uniquely and the descriptions are unsuited in this regard. For example, "bearing" does not tells us which specific bearing *i e*, its type and size. That there exist different types of objects should suggest the necessity of classifying these.

Descriptions are particularly unsuited for computerised applications. They are usually far too long and would require much higher computer time for processing than do the codes. This falls on the system analyst's responsibility to devise the appropriate coding schemes. Although there exist coding schemes in manual systems also it is usually necessary to modify these to suit the computer capabilities. Also, human beings can manage with bad and disorganised coding schemes but not the computer. The following are the characteristics to be desired of a coding scheme :—

Individuality. The code must identify each object in a set uniquely and with absolute precision To use one code number for several objects in a set would obviously cause a great deal of confusion. Furthermore, the code should be universally used over the entire organisation.

Space. As far as possible a code number must be much more brief than its description.

Convenience. The formats of the code numbers should facilitate their use by people. This implies that the code number should be short and simple and consist of digits and/or upper case alphabets. It is better to avoid the use of such special symbols as hyphens, oblique, dot, etc.

Expandability. As far as possible future growth in the number of objects in a set should be provided for Therefore, whilst introducing the scheme longer number of digits/number than necessary at present may be adopted as the code length

Related items must use fundamentally similar numbers As an example, the pattern number, the casting number and the finished part number of a component at various stages of processing should have code numbers which mostly resemble but for a digit/letter alteration to suggest whether it is a pattern, casting or the finished part. This is quite convenient when, for example, cross indexing which casting has to be ordered on the foundary for a particular part.

Suggestiveness The logic of the coding scheme should be readily understandable. Also, the letters or numbers should be suggestive of the item characteristics *i e*, whether it is made from a casting or rolled stock, whether it pertains specifically to such and such model or it is used commonly by more than one end-

product. But this should not be carried too far in lengthening the code since it would defeat the purpose of brevity.

Premanence Changing circumstances should not invalidable the scheme or invalidation in the future should be sought to be kept to minimal.

Coding Schemes

Tailor-made classification schemes are available. For example, a classification scheme that is applicable to a wide cross-section of manufacturing firms for stock-keeping items has been evolved by Brisch and his associates. The following is a typical class breakdown for an engineering organisation :

0 —Organisation and operation.

1—Primary materials.

2—Bought out commodities.

3—Components (single piece parts) to user's own design.

4—Sub assemblies and assemblies to user's own design,

5—Tools and portable equipment.

6—Plant and machinery.

7—Building service and utilities.

8—Scrap and waste.

9—Reserved.

All the items are first allocated under these headings and further sub-categorisation may be had. This classification would depend upon an individual firm. The next stage is to assign a code by allocating digits to symbolise the classified feature Each digit is supposed to designate and/or suggest some feature of the item. The codes are usually kept to maximum length of 8 digits and are divided into two parts. The first part is called the surname or the family name and the second part is known as the christian name.

Block Codes. A block of codes is assigned to a family of materials *e g.* 100 — 200 for turners, 201 — 250 for fitters, 251 to 300 for drivers etc. The individual objects may then be assigned numbers from these blocks arbitrarily or with further sub- blocking. The block coding scheme has the virtue of brevity and simplicity but suggestiveness of the features of a part is lost.

Mnemonic Codes. These are suitable where the codes have to be remembered by people *e g* , BL may stand for bolts. Mnemonic coding is particularly suited to tool stores since tools can be divided into few categories and each category may be assigned a mnemonic code *e.g.*

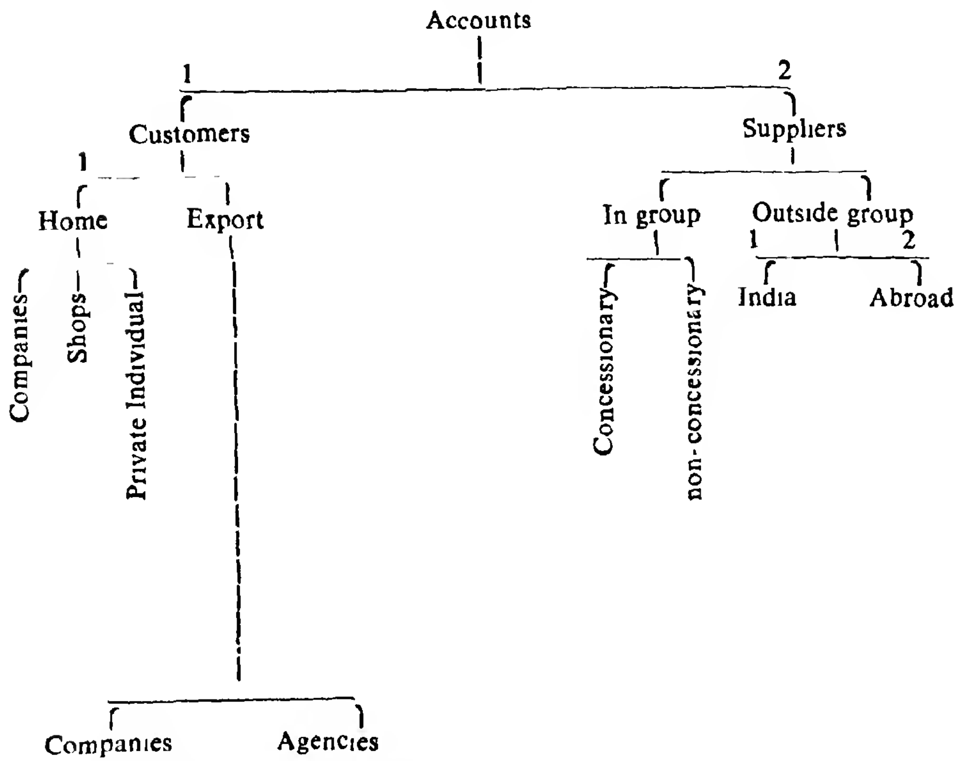
DR for drills

SW for saw blades.

GR for grinding wheels etc.

The prefix (*e.g.*, DR) may be followed by size *e.g* , 0.2 meaning 2 centimeters drill.

Heirarchical classification A couple of examples are given below :



The faceted version of the classification is shown below :

1st digit

1 — Customer

2 — Supplier

2nd digit

0 — irrelevant

1 — home or India

2 — export or abroad

3rd digit

0 — irrelevant

1 — company

2 — shop

3 — agency

4 — private individual

4th digit

0 — irrelevant

1 — inside the group

2 — outside the group

5th digit

0 — irrelevant

1 — concessionary

2 — non-concessionary

This classification method can be developed for use in almost any business and for all kinds of office records.

Example on Coding

A building company operates job costing for direct labour and materials Fach job account contains the following headings under which it is required to classify costs and each heading falls under one of the three general descriptions, substructures, superstructures and internal operations :

Internal drainage	(sub-structures)
Pre-casting	(super-structures)
Roofing	(super-structures)
Plumbing	(internal operations)
Elevation brickwork	(super-structures)
Site clearance	(sub-structures)
Electrical	(internal operations)
Foundations	(sub-structures)
Joinery	(internal operations)
Glazing	(internal operations)
Plastering	(internal operations)

You are required to :

(*a*) Draw up an appropriate numeric code list for direct labour costs relating to the above headings (max 4 digit code) as related to job number 459. The company wishes to control idle time.

(*b*) Name the type of coding system you have used, together with names of the two other types of coding system.

(*c*) Design a form of labour cost summary, suitable to act as a record of original entry into the costing records. The company normally operates a maximum of five jobs at any one time.

Solution :

Numeric Code List for Direct Labour Costs

Cost	*Description*	
200	Direct Labour	—Total
210	Substructures	—Total
211	Substructures	—Site Clearance
212	Substructures	—Foundations
213	Substructures	—Internal Drainage
220	Superstructures	—Total
221	Superstructures	—Elevation Brickwork
222	Superstructures	—Pre-cast Units
223	Superstructures	—Roofing
230	Internal Operations	—Total
231	Internal Operations	—Joinery
232	Internal Operations	—Glazing
233	Internal Operations	—Plumbering
234	Internal Operations	—Electiical
235	Internal Operations	—Plastering
240	Idle Time	—Total
241	Idle Time	—Bad weather
242	Idle Time	—Waiting for Material

Job number 459 would appear as a prefix.

It is assumed that, for control purposes, it is more useful to identify id time with the job and analyse its incidence due to cause. *Alternatively, id time could be assigned that digit '3' (instead of '2' and follow analysis by operation, e.g., 321 signifies idle time relating to elevation brickwork. A further digit t signify cause could be used*

This type of code is known as Progressive Code. The first digit signifies its man class, *Direct Labour. The second digit signifies the sub-group and the third* digit, in this case taken in conjunction with the second digit, signifies subgroup.

LABOUR COST SUMMARY

Code		Job Number 459	471	473	482	490	Total
	211						
	212						
	213						
Sub-tatal	240						
	221						
	222						
	223						
	241						
	242						
Sub-total	240						
Sub-total	200						

Other types code are ;

Serial numbers ;

Block codes ;

Progressive decimal codes ;

7. **Design and Documentation.** Once the specification and standards for inputs, output and files are laid down all that remains is to link them by computational requirements and processing This will create the new system design that, however, remains to be comprehensively documented by the techniques of systems analysis and design. i.e., system outlines, system flowcharts, run charts, program flowcharts, decision tables, grid charts, file storage plans etc. Several guidelines for system design are explained in a subsequent section which can be ignored only to the peril of entire system development effort It is to be carefully noted, however, that the initial design the analyst comes up with is generally only tentative He may further scrutinise the draft himself for soundness or may discuss it with the operating managers and MIS development committee It is not at all unlikely that it is rejected and he has to develop a series of alternative designs Generally, the criteria against which these designs alternatives are evaluated are: cost, efficiency and accuracy, practicality and flexibility. However, importance of each criteria or objective may differ from one organisation to another The analyst has, therefore, to settle the weightages for the given organisation at hand very carefully.

9. **Breakdown of system Design** The system can be broken down into routines and runs. A routine can be regarded as a piece of data processing work that achieves result that is usable outside the system A routine may be a part of an application or may cover the application in entirety It itself is made up of one or more computer runs A run is a piece of work on the computer that is carried out as a whole and in a continuous fashion. Though it is possible to interrupt a run but generally it is not done as it is highly inefficient. In one routine, the output of one run, in the form of magnetic tape, disc or occasionally in core store, constitutes the input to another run The output may also be printed documents that are inspected and amended manually before being repunched or re-read to form the input to another run.

To further elucidate the discussion thus far consider an inventory accounting application. It can be broken down into three routines as below :

Raw material update (Fig. 3)

Finished Goods Update—Receipts (Fig. 4)

Finished Goods Update—Issues (Fig-5)

Each routine consists of two or more runs.

The first routine provides for use outside the system, stock movement and valuation reports for raw materials. The 2nd routine provides completed production order summary. The 3rd routine provides stock movement and valuation reports for the finished goods.

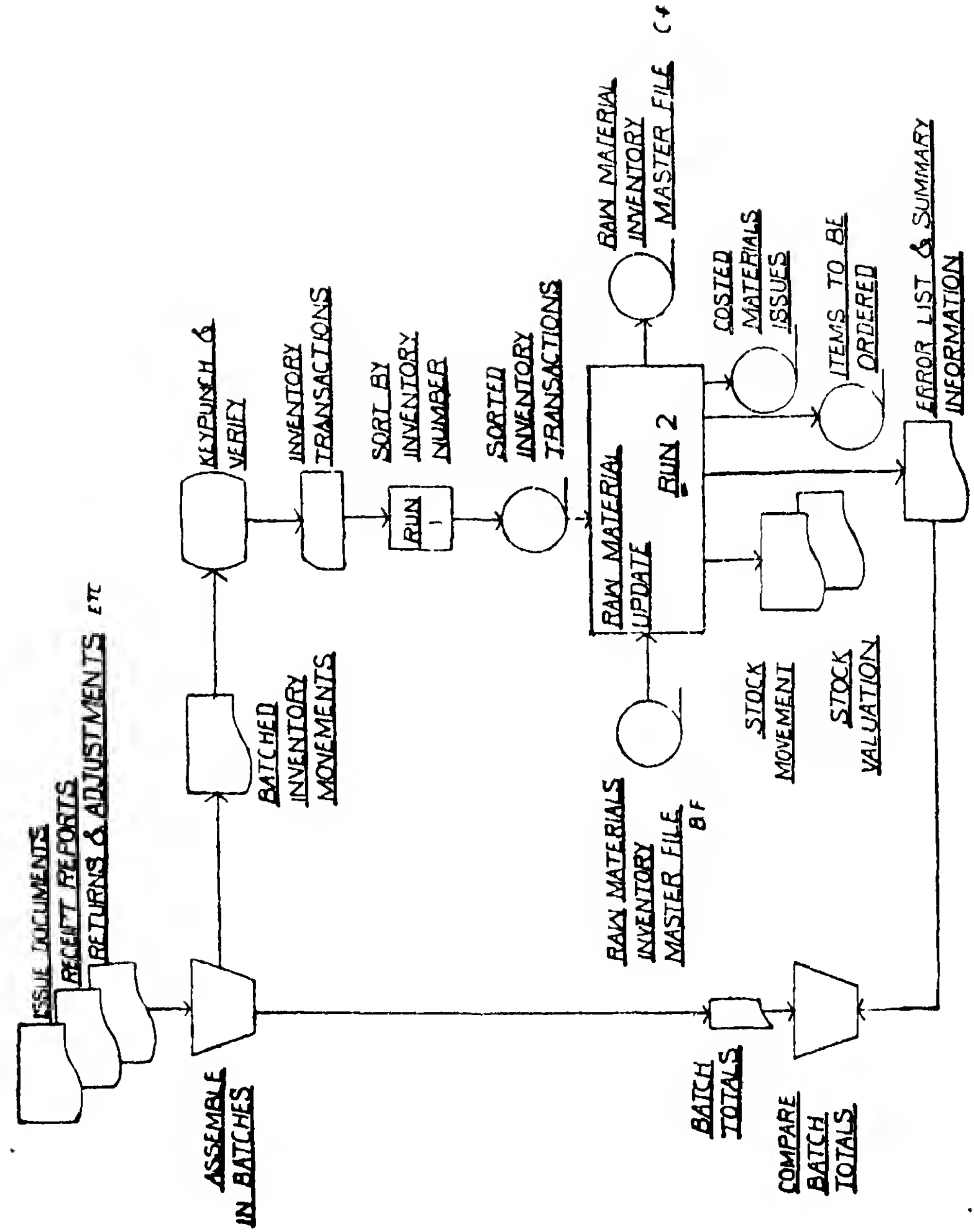


Fig. 3 Routine—1

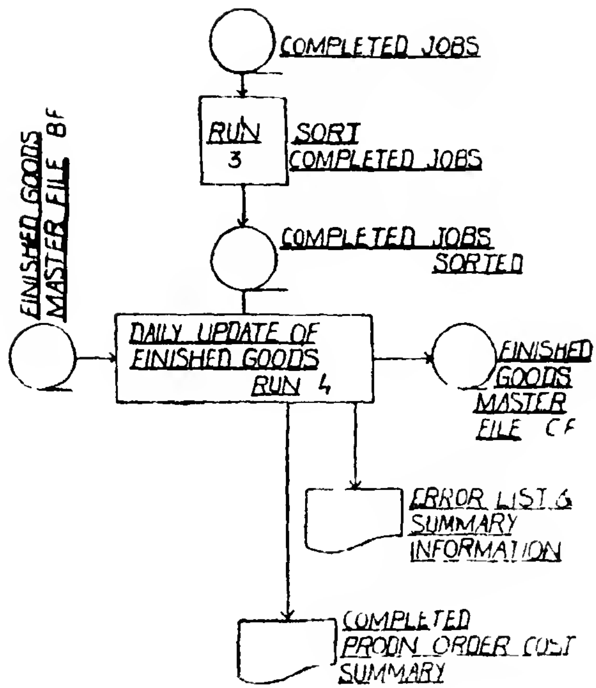


Fig 4 Routine – 2

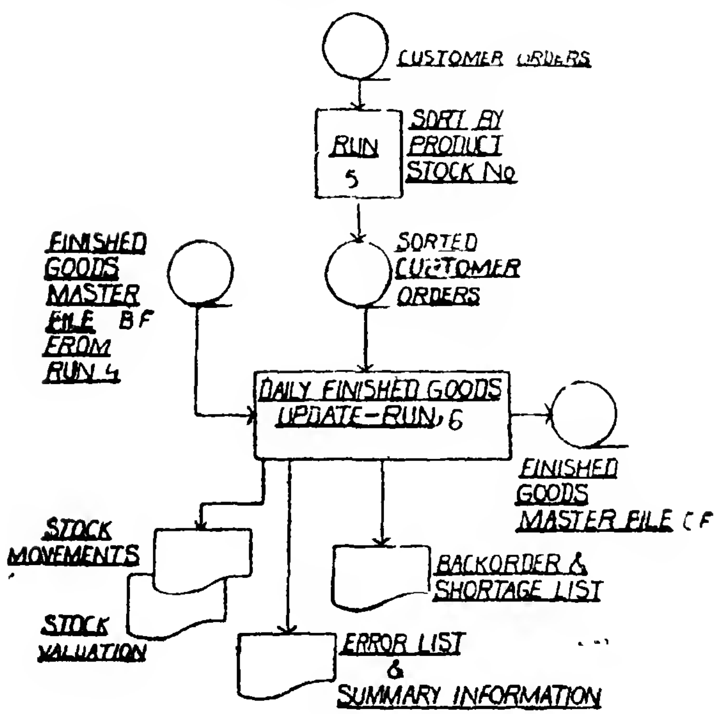


Fig 5 Routine—3

Processing per computer run should be as large as possible, subject to the following :

1. The computer's ability to hold the main program. If, for example, it is a run for stock updating all the following transactions should sought to be covered in one run : customer order, receipts, issues, returns, stocktaking adjustments file amendments It may be noted that it is generally possible to hold the lesser-used segments of the program in backing storage, usually, on magnetic drums.

2. Availability of the necessary peripherals, particularly magnetic tape reels

3 Printer capacity If the reports are too many and exceed the printer width the run may have to be split. *i e*, some of the (report) data being put onto a magnetic tape for subsequent printing in another run.

4 The ability of the run to cope with expansion. The customer orders, for example, may be handled by exponential smoothing forecasting with trend correction sub routine for demand forecasting. It may latter be envisaged to incorporate a similar sub-routine for seasonals instead that occupies more storage. If the persent program is almost fully occupying the primary storage, this expansion would not be possible.

5 The run's ability to detect and deal with error messages.

6. The run's ability to meet control and audit requirements.

If one or more of the above conditions are not met, the run would have to be split.

10. Systems Manual The basic output of the system design is a description of the tasks to be performed, complete with layouts and flowcharts. This is called the *job specification manual or system manual.* It contains.

(*i*) General description of the existing system.

(*ii*) Flow of the existing system.

(*iii*) Outputs of the existing system The documents produced by the existing system are listed and briefly described, including distribution of copies.

(*iv*) General description of the new system—Its purposes and functions and the major differences from the existing system are stated together with a brief statement of the reasons of advantages of the change

(v) Flow of the new system—This shows the flow of the system from and to the computer operation and the flow within the computer department.

(*vi*) **Output Layouts.**

(*vii*) Output Distribution—The distribution of the new output documents is indicated and the number of copies, routing and purpose in each department shown The output distribution is summarized to show that each department will receive as a part of the proposed system

(*viii*) Input Layouts—The inputs to the new system are described and complete layouts of the input documents and input cards or tapes provided.

(*ix*) Input Responsibility—The source of each input document is indicated as also the user department responsible for each item on the input documents.

(*x*) Macro-Logic—The overall logic of the internal flows will be briefly described by the systems analyst, wherever useful

(*xi*) Files to be maintained—The specifications will contain a listing of the tape, card or other permanent record files to be maintained, and the items of information to be included in each file There must be complete layouts for intermediate or work file , these may be prepared later by the programmer.

(*xii*) List of programs—A list of the programs to be written shall be a part of the systems specification.

(*xiii*) Timing Estimates—A summary of approximate computer timing is provided by the systems analyst.

(*xiv*) Controls – This shall include type of control, and the method in which it will be operated.

(*xv*) Audit Trail—A separate section of the systems specifications shows the audit trail for all financial information. It indicates the methods with which errors and defalcations will be prevented or eliminated.

(*xvi*) Glossary of the terms used.

11. **Guideline for Systems Designs.** Systems Design is analogous to preparing the architectural plan of a building and subsequent program production to construct the building. If architectural plan is bad, no amount of effort in construction can mitigate it So is the case with systems design and program production. Therefore, the best talent should be employed for systems design The deficiency in systems design cannot be made up during programming The systems design should be outlined in the aforesaid documentation in meticulous detail so that the programmers can translate in into programs without turning to the systems analyst for further explanation. Some of the important guidelines for a successful systems design are explained below.

(A) *Product structure* is prescribed by the engineering department and it depicts the geneology of various products. It is a basic document upon which several functions, indicated below are performed.

1. Forecasting the demand of components of variable or optional product features.

2. Net component requirements calculations.
3. Inventory oontrol.
4. Purchase of raw materials.
5. Cost accounting
6 Spare parts requirements calculations.
7. Assembly instructions
8. Quality control.
9. Catalogue and field service
10. Warranty administration, etc.

If, therefore, the product structure is not put properly in the computer file and is not updated promptly for any changes by the engineering department, which may be highly frequent, all these functions will go astray. The systems analyst must take utmost care for designing the product structure files and the allied updating procedures.

(B) *Develop clear program specifications.* Computer can solve only well structured problems, *i e* , problems whose solution procedures are sufficiently well known beforehand Therefore, what the system is intended to do and how it is to do it should be explained in meticulous detail and unambiguously. Furtheremore, all the contingencies that can arise in the execution of a program should be covered in the flowcharts or decision tables Exceptions may have to be handled by turning to special programs or by signalling for human intervention.

If this is not so, the programmers would be looking to the systems analyst who may not be easily available or even may have left. The points brought by the programmers may call for reconsideration of the systems design—an expensive and disruptive situation indeed ! This would imply that the systems design was either not well thought out or was not well documented by means of clear specifications. Some of the documentation for stating the program specifications has already been discussed under techniques of systems analysis and design. There are quite a few recommended methods and formats for documenting the design. But this is not an important point The systems analyst may even write the program specifications in plain English. The quality of program specifications is in inverse proportion to the number of times that a programmer will have to come back to the systems designer for a sytems design decision, explanation, or clarification.

(C) *Integration.* This matter was highlighted at the outset of this Study Paper. Computer can perform several functions in one stroke by virtue of its superior information handling power Integrating means knitting together more than one application encompassed by the system. Integration does not necessarily mean that all the applications are integrated together. A few applications may be so integrated that they are open ended to allow subsequent knitting together of more applications Such a system too is an integrated system. The following are the basics of integrated data processing :

(i) *Data base.* No data field is duplicated in data storage. Duplication is uneconomic as also error-prone.

(ii) *Source data* is entered once and once only and thereafter is comprehensively processed for all applications. The customer order, once input, may be processed for credit appraisal, followed by stock updating, followed by placement of replenishment orders, etc. It would be inefficient to input the same data over and again. Likewise, the data from a production job ticket should enter the system once only and processed for job costing, payroll, work-in-process and so on. Obviously, such comprehensive processing requires various files suitably linked by means of indexing and chaining into a data base It is desirable to input the source data via magnetic tape that can be read very much faster than the source data media.

(iii) *Flexibility.* The system should be open-ended to permit fusion of further routines and applications. Also, the data base should be restructurable to allow such expansion.

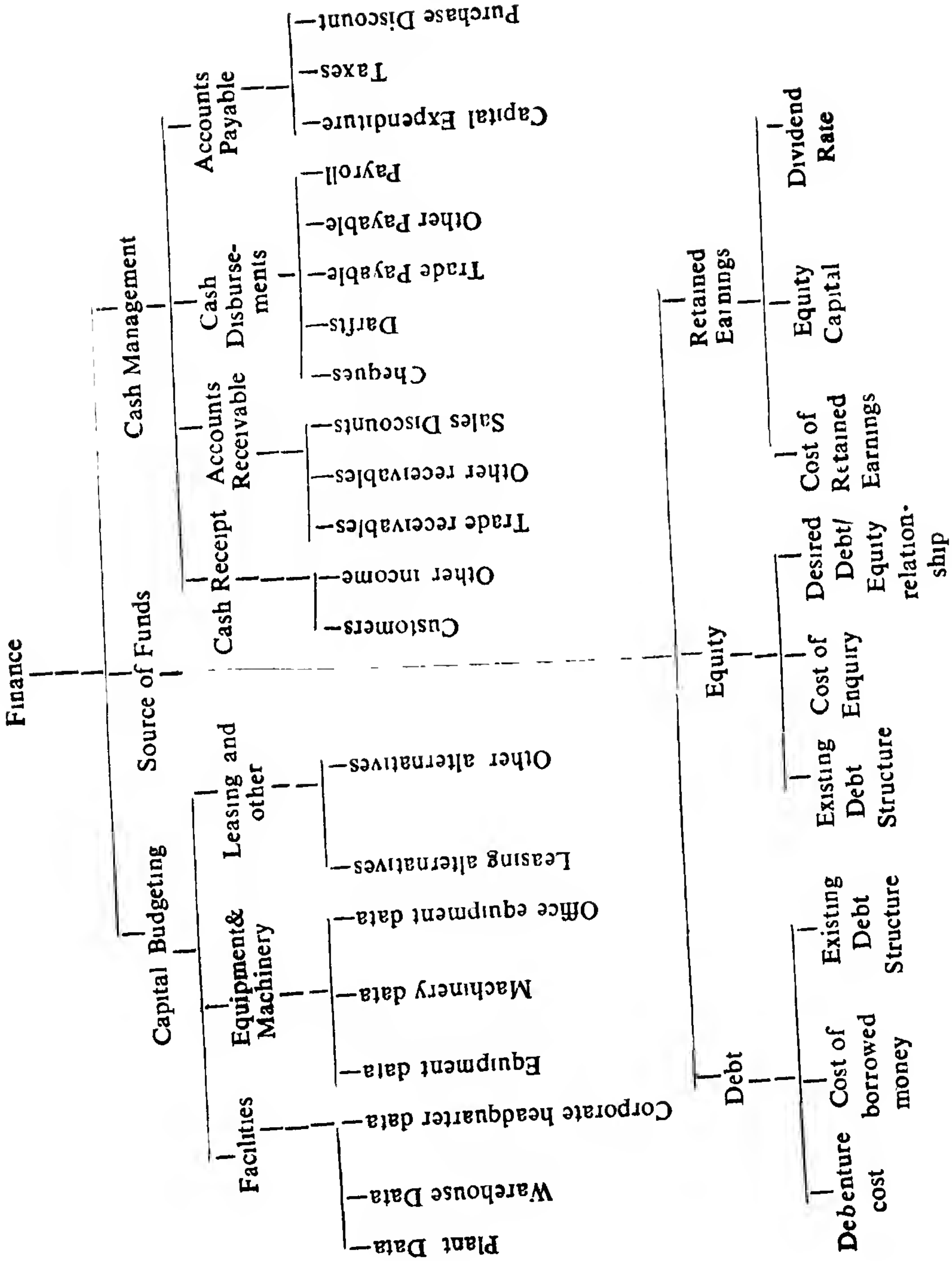
Integration, however, adds to complexity Cost and delay are the problems associated with complexity Also, there always lurks a fear of major system failure. The more integrated the system the more time consuming it is to implement it Also, it is extremely difficult to modify it to suit the changing environment One answer to these problems lies in properly documenting its design and logic The other answer lies in modular design

(D) *Modular system* is a concept borrowed from modular production In modular production, several end-products of different models and sizes are assembled from a few standard sub-assemblies The modular system concept is a method of breaking down a system and its programs into their lowest level component parts, *i e.*, modules so that the modules can be logically grouped for implementation and ease of making changes For example, the finance function can be sub-divided into three intermediate modules which can be further sub-divided into minor and basic modules as depicted on. Page 64

There are three major advantages of this approach.

(*a*) It enables to develop a system, and applications within it in a planned and orderly fashion.

(*b*) The individual program modules, segments or sub-routines can be altered and substituted without the entire system collapsing like house of cards Also, this facilitates allocation of programming work amongst several programmers Programme perparation is made easier in many way , flowcharting is simplified, documentation is made easier , and debugging is facilitated because errors requiring correction of one module should not affect others, this being so because each modules performs a separate function Generally, therefore, these modules should have no interaction with each other, interacting primarily, if not exclusively, with the program's control module. Thus, a program using modules will essentially consist of a small control section that does


Finance
Source of Funds
Cash Management
Accounts Payable
Purchase Discount
Taxes
Capital Expenditure
Cash Disburse-ments
Payroll
Other Payable
Trade Payable
Darfts
Cheques
Accounts Receivable
Sales Discounts
Other receivables
Trade receivables
Cash Receipt
Other income
Customers
Capital Budgeting
Leasing and other
Other alternatives
Leasing alternatives
Equipment& Machinery
Office equipment data
Machinery data
Equipment data
Facilities
Corporate headquarter data
Warehouse Data
Plant Data
Retained Earnings
Dividend Rate
Equity Capital
Cost of Retained Earnings
Equity
Desired Debt/Equity relation-ship
Cost of Enquiry
Existing Debt Structure
Debt
Existing Debt Structure
Cost of borrowed money
Debenture cost

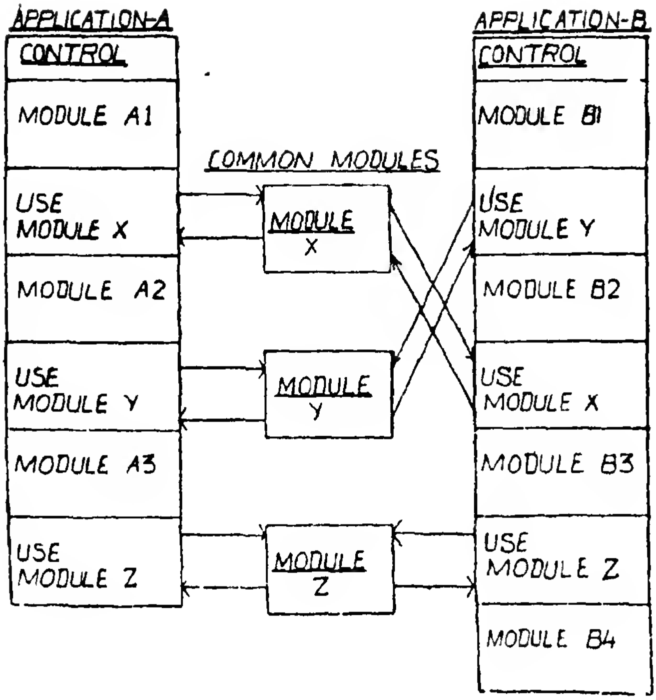


Fig. 6

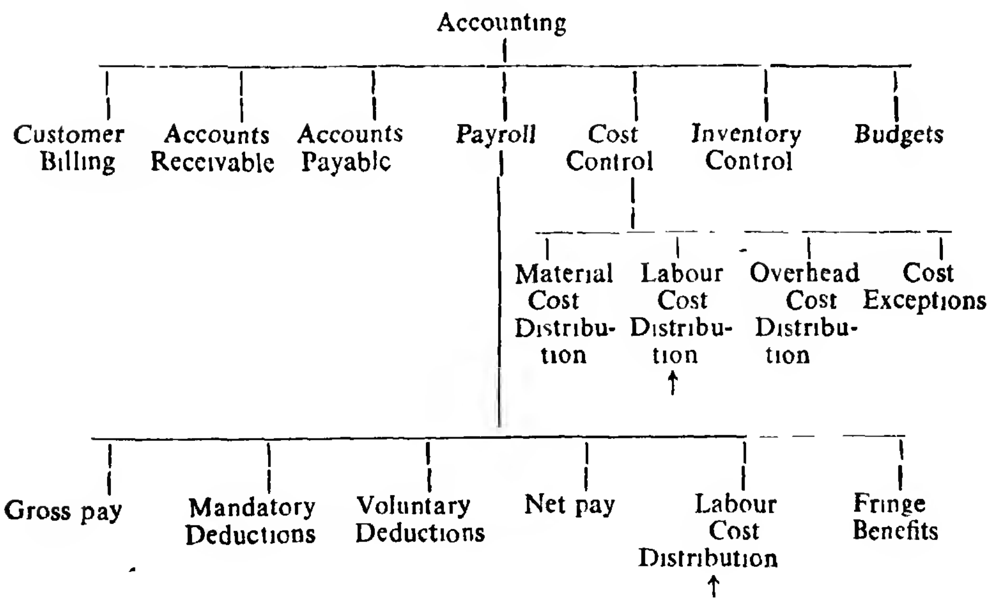


Fig 7

(Complete sub-division is not shown)

initialisation for the required application and calls the program modules in the proper order. This is depicted in the figure 6 on page 65.

(c) Some of the modules may turn up at more than one place, the programming effort is reduced since the programs already prepared can be used again. To illustrate this point a modular break down of an accounting function is depicted in Fig 7 on page 65. It can be seen that the "Labour Cost Distribution" module (indicating by arrows) appears twice.

(D) The concept of modularity offers a breakthrough in grappling with the exorbitantly rising costs of software development The modules that have been tested over several diverse organisations can readily be assembled for another organisation without much of initial systems analysis studies. The development is thus analogous to that of pre-fabricated concrete slabs.

(E) *Level of Automation* A fully automated system is the one that calls for no human intervention whatever. But automation is not to be pursued for its own sake There are distinct human and computer capabilities that should be harmonised or synergised. This matter has received due attention in MICS Study V (and SADP Study VII) The higher the echelon the less is the possibility of computerising decision-making, planning and control. The motto is "do not automate what the people can do better."

(F) *Avoid need for informal system.* Expediting in production control is an example of the informal system at work. Several progress chasers may be engaged on expediting the manufacture of components against their assembly lists They may rush up the manufacture of their components without realising how it affects the plans of other components Soon after expediting is undertaken, the original schedule would be outdate. The very connotations of expediting suggests that it is undertaken to make so for the system's in efficiency in planning. Need for such an informal system should therefore be avoided in systems design. This would require that priorities of various orders are scientifically established. And if some components are delayed for some reason, all the related components should then be deferred so that capacity is created for other products. Likewise, if a customer wants an item earlier then originally, the implications of this change in priority should be worked down to the entire schedule, *i e.*, priorities of all other scheduled items contending for the same man/machine capacity should be suitably adjusted. These and related matters are dealt with in the MICS Study V and SADP Study VII.

(G) *Exploit computational capacity.* Computer possesses an astoundingly high computational speed. Advantage ought to be taken of this by employing mathematical models where feasible. Operations research offers a wide ranging models that can be usefully applied in production control, marketing, design, finance and accounting and other functional sub-systems. Linear programming can be used for establishing optimal product mix, production planning, production scheduling, transportation costs from factories to warehouses, assigning salesman to different territories on an optimal basis, etc. Simulation technique can be used for optimal allocation of advertising budget, by assessing viability of introducing new products amidst probabilistic cash flows, for asking "what if" questions, etc. Queuing theory

can be used to establish the optimal maintenance gang size, number of servers at the tool crib, etc Replacement theory can be used to determine the optimal replacement intervals for items that fail suddenly, *e.g* , electric bulbs and items that deteriorate with time such as trucks. Sequencing theory can be used to derive optimal production scheduling rules and physical distribution schedules etc.

Computer can also make comparisons at a great speed. It may, therefore, be desired to build in diagnostic checks in the program.

(H) *Avoid tailor made applications.* Each manager has his own style for decision-making and may therefore differ in frequency and format of reports Some manager would plod through each and every bit of information and others may want just a brief summarisation and a list of exceptional items Although applications can be tailor made to suit each and every manager's predilections there is a danger that the system may soon be outdated or rendered obsolescent upon and shuffling in the organisation. In fact, the system should be kept independent of even the organisation structure since it itself is liable to change.

(I) *Computer's effect on the environment* is to be borne in mind when designing the new system Computer, because of its great data processing speed, can easily react to change in engineering design, changes in schedules, etc. This is likely to encourage the users to change the design and schedules more often since in the earlier system they were restrained to do in view of the inability of that system to respond to frequent changes. Thus the new design should provide for more frequent operation on changes.

Systems Development Stage V. SYSTEMS IMPLEMENTATION

Systems implementation can be as expensive as systems analysis and design phase as is to be realised from the diagram in Fig 8, page 68 The various tasks entailed in systems implementation are listed hereunder :

(1) design specification reviewed ;

(2) space arrangement completed ;

(3) determination of equipment additions ;

(4) training programmes established ,

(5) plans for work flow and floor layout completed ;

(6) design of forms for data collection ;

(7) development of files ;

(8) software completed ;

(9) formal training of supervisors completed ;

(10) completion of formal training of operating personnel ;

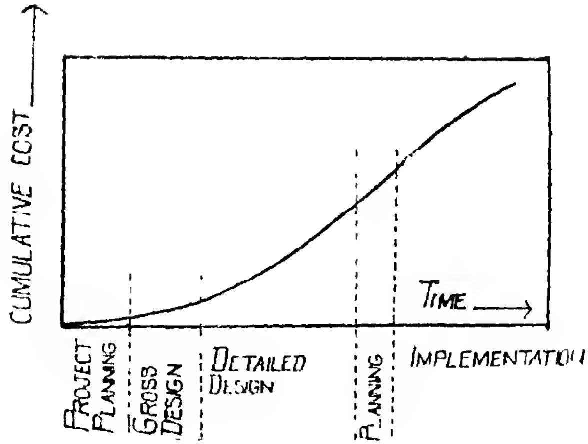


Fig. 8

(11) file documentation completed ;

(12) system simulation checks completed ,

(13) system into segments cut ,

(14) test of total new system completed ,

(15) operational testing and evaluation completed ; and

(16) new system documentation completed.

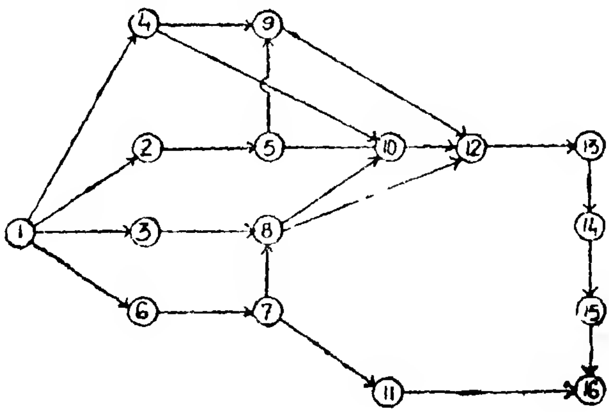


Fig. 9

The dependency relationships between these tasks are shown by means of the milestone chart—the numbers denote events listed above.

(This system implementation sub-network is a part of the entire system development network).

Planning Systems Implementation

As hinted above systems implementation is a fairly complex and expensive process involving numerous inter-dependent activities It, therefore, requires careful planning and control that can be best carried out by CPM/PERT The physical control, i e., control on the physical progress of various tasks can be carried out in the Gantt Chart version (an example given in Appendix A) of the network Besides, cost control has also to be exercised by apportioning the activity costs period-wise and accumulating them into an S-curve. [The MICS students may want to refer to MICS Study IV for details of the budgetary and cost control aspects of projects planned by networks The S-curve is an excellent device to combine the physical and cost dimensions of a project network Not only the physical slippages or achievements can be fruitfully depicted but also the cost variances for which periodic variance reports may be compiled and circulated to the concerned]

Some of the leading tasks of the implementation phase are discussed below

Acquire space and plan space layouts It would be foolish to be stingy on layout expenses and human environment when so much is being expended on system analysis design and implementation. A bad layout can not only drastically reduce the productivity of the D.P. department but even that of the entire organisation as a whole. The MIS project manager should prepare a rough layout, make cost estimates and get the budget approved from the management. [The interested student may also want to refer to MICS Study II for further details of space layout] Layout planning must be done well in advance order to permit acquisition of long leadtime items like air-conditioning equipment and training of the operating personnel. The following factors should be taken into account for space planning :—

(i) Space occupied by the equipment.

(ii) Space occupied by people.

(iii) Movements of equipments and people.

Education and training. It was emphasised at the outset of this study paper that the computer can alter the jurisdictions of the existing departments some of which may even disappear. The changeover can indeed be a traumatic experience since it will disturb the existing daily routine Those affected by the change have therefore to be educated for the new setup. The education phase may start after procedure development is over and may continue sometimes after the new system is operational. Education is to be imparted to the department carrying out the change over (*viz*, production and inventory control departments directly affected, accounts department which itself is not yet included for computerisation) and senior/middle management who will be indirectly affected. Ways of achieving this are:

(i) Articles in staff magazines.

(ii) Organising visits to the computer room.

(iii) Discussion meetings inside or outside the company's premises but at the company expense, aimed at broadening people's minds to accept the change dispelling the mystique of the computer and allaying any appre-

hension There has also to be a comprehensive training programme intended to teach the employees how to do the new job in new ways. Such training program will require:

Handbooks and manuals,

Lectures and demonstrations,

Examples of new documents, etc.

Development of Files This activity normally develops into the following sequence of events .

(i) Record data in the existing files on specially designed input documents.

(ii) Transcribe the data on a suitable medium *viz* punched cards and verify.

(iii) Use a tailor-made program to read transcribed data and then output the required files in the format needed by the user program.

A major problem at this stage crops up. It is that of converting a live file *e g* , stock file. It must be converted at a specific date and thereafter an amendment correction procedure adopted This calls for still more clerical work Besides, the converted files cannot usually be more accurate than the original files and, perhaps, less accurate. The analyst must therefore continuously strive to satisfy himself on these counts Are the data in the source file correct ? Is staff available to check source records ? What can they be checked against ? Is there any chance of getting control totals ?

Software development would comprise two parallel activities ; program production and procedures development The former is adequately discussed in Study I Not only the routine computer operation upon implementation has to be supported by procedures but even some of the implementation phases. Supporting procedures for routine operation would include, for example, issuance of production documentation ; work orders, operations lists, requisitions, bills of materials, reporting of progress on production, etc. Supporting procedures from implementation phases would, for example, include those for testing the system discussed below. Also important are the conversion and auxiliary procedures.

Conversion procedures are intended to provide smooth transition of change-over from the existing system to the new system. These are in the nature of one shot *i e* , once the new system is operational they are discarded.

The auxiliary procedures consist of back up and recovery procedures. Back up procedures deal with how the system is to be operated perhaps manually in the case of a major hardware failure. If, for example, there is a failure of the real time system, the organisation may have to resort to batch processing instead of on-line processing. Recovery procedures deal with the transition from failure of recovery.

Testing the System. Tests would consist of system component tests, sub-system tests and total system tests. The components may comprise equipment, software, procedures, reporting formats. With progress on installation of the com-

ponents the sub-system may be tested. These tests require verification of multiple inputs and complex programming logic. Towards elucidation of these consider computerisation of the following organisation :

Manufacturing organisation X has a standard costing system which is used for pricing and manufacturing accounting. The system is not satisfactory Reports are delayed, standard costs are not accurate and analysis of variations with actual has become difficult. Products manufactured have multi-level sub-assemblies which are either purchased or manufactured. Changes in design are frequent

The multiple inputs here, for example, would be engineering changes and progress on operations on the shop floor for the affected products Likewise, re-scheduling of co-components of a product upon delay in completion of a few of its components and related products because of the capacity thus created would constitute quite complex a programming logic that requires extensive testing Also it may be desired to verify if human beings are interfacing satisfactorily with the various computer equipment. For example, it may be seen if the operators are happy with inputting data for completion operation via CRT's or such turnaround documents as partially per-punched cards. Incidentally, this also provides a check on the quality of training being imparted. It goes without saying that such component and sub-system testing may call for some redesign of the sub-system Finally, the total system has to be reviewed by means of simulation experiments etc, (Ref : Appendix to Financial Management Stidy V) and this may continue over several months from once the new system is operational. Programs may be debugged with real life data, files cleansed and forms redesigned.

Transition to the new system may be had in one of the following four modes :

(*i*) Instal a new system in a new organisation which has just been designed

(*ii*) Cut-off the old system and instal the new one in its palce on sudden basis. This method is particularly attractive for smaller organisations where implementation can be done within a couple of days or so However for larger organisations too advantage can be taken of the plant shut downs and implementation carried out during that period.

(*iii*) Cut over by segments. Under this approach the system is implemented by sub-systems and therefore system implementation can run in parallel to system design, *i.e.*, one application is being implemented other sub-systems are concurrently being designed. This method has the disadvantage that alternative systems cannot be evaluated It is possible to adopt this approach for such systems as inventory control, for example. Then it is called pilot conversion Some of the inventory items are put on the new system and the result assessed. Subsequently the entire items are covered by the new system.

(*iv*) Operate in parallel. Such a sub-system as payroll has similar outputs in the old as well as the new system ; therefore the two systems can be run parallel and the results compared. But implementation has to be considered more broadly. There are numerous other activities of implemen-

tation which have to be taken into consideration. We assumed that systems specifications and details of facilities and equipment are required as the output of the system design phase The details of the facilable and equipment, is a specialised topic and is discussed in following studies under choice of equipment and vendor

Systems development Stage VI SYSTEM AUDIT

Audit of the EDP or information system function is desirable at periodic intervals the audit should evaluate the performance of the function for all activities The audit team may comprise the representative of the management committee of information systems group and EDP audit group. An audit is necessary because of the following .

(*i*) The costs involved in systems development, installation and implementation are very large

(*ii*) Systems development requires co-ordination of several departments and activities which may have a great and complex deal of interindependence.

(*iii*) Inadequacy in controls and planning may lead to serious errors.

Audit can be conducted for the following aspects ;

(*i*) Adequacy of management.

(*a*) Information for planning, forecasting and control

(*b*) Personnel problems.

(*ii*) Performance *vs* the plan which should cover results achieved against the target performance

(*iii*) Evaluation of existing applications.

(*iv*) Adequacy of controls for protection against errors and loss :

(*a*) Back up provisions.

(*b*) Applications controls.

(*c*) Error handling procedures.

(*d*) Risk insurance.

(*e*) Auditing.

The audit and review should show the potential usefulness of the information system and any misdirected developments.

For auditing the cost performance, regular, weekly or monthly-reports have to be prepared giving the plan and actual figures.

Evaluation of existing applications can be carried out by periodically reviewing how effectively the user is using the various reports. It can also be reviewed whether the system is catering to the changes that may occur now and in the

environment system. The system may be evaluated for integrity at various levels as below :

System integrity : This requires ascertaining how well the various sub-systems are integrated into the total system Is the system flexible ? Is the system expandable *i.e* , open enough to permit fusion of further applications ?

Operating integrity · Are operating personnel sufficiently skilled ? Is there enough planning and provision to take care of any vacancies liable to occur in the event of the key personnel leaving the organisation ?

Internal integrity : How well the system does what it purports to do *i e.,* is it producing the valid system outputs ? Is the system secure against theft, sabotage, errors etc.

Procedural integrity : Are the system and the procedures well documented in the form of manuals ? Are the procedures being followed in practice ?

Regarding controls and protection, periodic checks should be made at the computer centre of availability and storage of back up files Application controls can be checked by examining how various audit lists, control sheets etc , are maintained. Without informing operating staff, errors in the operating data may be deliberately introduced and later it is seen whether the error handling procedures are followed or not.

Though the computer department may be manned mainly by computer professionals, it is better that people in other disciplines are trained in computers and personnel are rotated between the system department and the user departments.

Summary : The need for system development may arise from pressing information problems or tempting opportunities But the need has to be substantiated by a realistic cost/benefit analysis in terms of, say, ROI This may require a preliminary analysis of the existing system and conception of the new system in outline.

Once, on the basis of this feasibility study, the green signal is given by the management a master development plan should be compiled The prospective applications may be scheduled with priorities generally in terms of the associated pay-offs but with due regard to any precedence relationships. Because of this and the fact that the various stages of systems development contain inter-dependent activities, CPM/PERT are recommended for planning and control of systems development. Besides, there must be budgetary control over the entire development cycle.

Once the master development plan is ready, systems analysis and design can commence for various applications. Systems design is mainly concerned with input, output, file and forms, controls and codification design Systems specifications must be prepared in most meticulous manner in the form of a systems or specifications manual which should be fully documented with input and output layouts, file storage plans, flowcharts and decision tables etc., so that program production proceeds smoothly.

Once systems design is over implementation of the new system can be undertaken This would require education of those affected File conversion is an important activities of this phase. Operation of the new system may be planned in parallel to the existing system or on a pilot basis, etc.

Self-Examination Questions

1. Distinguish between economic, technological and operational feasibility.

2. "The tools and techniques of systems analysis and design are used in every stage of system development as analytical tools, design tools and documentation methods " Describe several tools and techniques which substantiate this statement.

3 Explain how forms control is related to systems analysis and design.

4 ' Information systems development should focus on the analysis and design of the input, processing, storage, output and control components of the proposed information system." Discuss the rationale for such a statement.

5. Discuss several input processing, output, storage and control considerations of one of the business computer applications discussed in Study II.

6. Explain how the effectiveness of information system is influenced by the decision maker Explain the advantages and disadvantages of individualised information systems.

7. List the fields you would expect to find on a main file used in a payroll-personnel application. Quote the number of characters in each field, and define whether they are of fixed or variable word length. Arrange the fields in the most logical order for processing, and draw a file layout for one complete record on magnetic tape.

8. What three categories of cash flow estimates should be made during computer feasibility study ? List several individual elements within each category. Once these estimates have been made, what basic framework should be used in evaluating the feasibility of alternatives ?

The Systems Analyst

The profession of system analysis has come into existence. During early days of introduction of computers, such completely definable application as payroll were chosen for computerisation. The concerned organisations managed to do so by employing a few brilliant programmers. Since the applications were already well structured under manual systems the programmers neither experienced any difficulty in transferring these to the computer nor felt any need for communications with their managements. Production and inventory control applications for example were, however, quite different These applications are notorious for not having worked well under manual systems. Programmers, well-versed though they are in the com-

puter technicalities, could not understand how they operated, not to speak of how they should work. Managements were unaware of the computer capabilities, if not dreadful of its mysteriousness, and could not communicate to the programmers what to expect. There was thus a blind spot subsequently filled by systems analyst who were specialists in both information systems and computers They were aware of integrative power of the computer and could take a systems view of the problem as opposed to the programmers who had too narrow a view of the problem and the management who were usually interested in compartmentalisation.

Scope . System analysts are not concerned with such physical systems as power stations They deal exclusively with information systems whether it be a commercial, manufacturing or consultancy firms, computer manufactures and suppliers or government and educational institutions. Nevertheless, the broad principles guiding his approach, whatever the type of organisation are the same They are also variously known as systems designers, systems analysts and designers, systems investigators, system implementers, information systems experts, etc,, etc Though they are usually associated with the introduction of a computer system or replacement of a computerised system with a more advanced one, now a-days, they may also be engaged on merely overhauling a manual system without bringing in the computer or unit record equipment.

Duties : The list of the system analyst's various activities can be compiled by the student in the light of the previous discussion on systems development. Any how, his range of activities is outlined below :

1. Investigation of the existing system by means of interviews, questionnaires and observation This is done with a view to spot inefficiencies, bottlenecks and problems, to elicit systems objectives as well as support from the management and to establish constraints and criteria for evaluation of the alternatives of the system design.
2. Conducting the technical, operational and economic feasibility study and participating in the development of the master plan.
3. Analysing the existing system, determination of information requirements for applications embodied in the master development plan.
4. Designing the new system for inputs, outputs and files and documenting it with flowcharts, decision tables, file layouts, etc.
5. Assisting with implementation and maintenance of the new system.

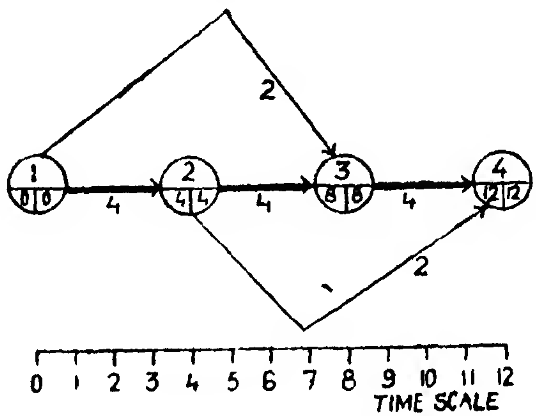
The range however would vary depending upon the size of the organisation. In smaller and even some larger, organisations, the duties of a systems analyst and a programmer may be combined and entrusted to the systems analyst. This has the advantage of annihilating the communication problem between the two and the consequent delays However, it is usually difficult to find a person who is both extrovert enough to be able to work to others extensively in their "languages" as well as be able to work individually on programming. Rarely one finds an individual possessing the capabilities.

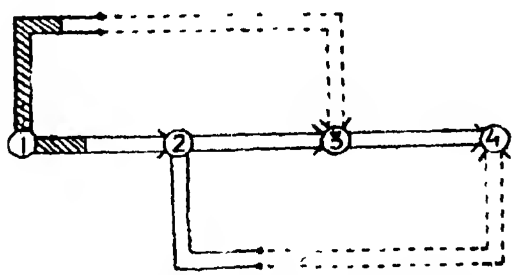
Backgronnd : It is expected that a graduate in business of computer sciences should become an able systems analyst. Besides the educational background, is desirable that he has had practical experience in diverse organisations and departments. Ideally, much of the experience should have been gained in

accounting, costing, production planning and control. Persons with long experience in one department alone tend to be biased in outlook. Likewise, it is not useful that programmers become successful systems analyst.

Knowledge · The system analyst should possess some knowledge of programming and the leading programming languages. More important, he should be thoroughly conversent with the range of computer equipment currently being marketed and the associated software He should also be familiar with various office procedures and O & M. He should also possess knowledge of preparing questionnaires, undertaking sample surveys and analysing the results.

APPENDIX A





GANTT CHART VERSION OF THE NETWORK

The network at the top has been laid down in a square fashion to scale to facilitate indication of progress by hatching the activity bars periodically. The systems implementation sub-network and even the entire systems development network may be so laid down to facilitate progressing.

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FINAL COURSE (N)

SYSTEMS ANALYSIS AND DATA PROCESSING

STUDY—IV

Contents :

Suggested Reading :

Computer Data Processing by Gordon B. Davis (Asian Edition)

1. INTRCDUCTION

Study Paper III was concerned with the systems analysis process. Systems analysis was singled out for special attention in Study Papar III because of its critical importance to the success of the data processisg effort. This study paper discusses the organization of a data processing department and shows how various aspects of its operation should be monitored and controlled.

2. ORGANIZATION OF THE DEPARTMENT

The single most common and most expensive error in computer management is to leave policy-making entirely in the hands of computer experts devoid of exposure to and expertise in ma gement and without organization status and authority essential to make extensive changes in procedures and policy. The general management of the computer department must be entrusted to experienced managers who appreciate the special characteristics of the company as well as the potentialities of the computer. To be sure, systems and programming specialists have an important part to play but they do not have the perspective to decide what will be useful to the company. As far as possible, their role should be restricted to advising on what is technically feasible and how best it should be processed on the computer.

Historically, the computerisation often starts in a particular department, the computer having been acquired in view of clerical overloads or the special problems and characteristics of that department, and this fact colours the choice of applications and the utilization of the computer for all time. In two out of three cases the computer starts in the accounts department because it is here that company expansion is quickest to make manual system creak and gr an With such beginnings applications tend to be accounts oriented and these, being mainly concerned with replacing men by machines, may not be very profitable. Also, the computer becomes the exclusive preserve of the accountants and the other departments may stay away from using it. They may even be actively discouraged. Similarly, computers that start in the projects, laboratory or the research and development wing tend to stay focussed on engineering applications and may miss the most profitable opportunities. In fact, the most useful systems often cross departmental lines and an interdiscipilinary approach to the application of the computer is essential

From such experience has emerged the concept of locating the computer in an independent department, which may also cover other management services, headed by a "Top Computer Executive" (TCE) who is responsible for the totality of the working of the computer department. This ensures independence of the computer centre and establishes its image as a corporate aid for use by any department which needs it. It is the responsibilty of the TCE to screen, if not initiate, all proposals for computer systems, to supervise their analysis and implementation and, in retrospect,to assess their effectiveness. He should also be responsible for periodic audits of the computer efforts

and for preparing and updating the implementation plan. Typically, the TCE reports to top management if he is not a part of it.

Usually, the TCE is also responsible for activities other than computer applications. These, typically, include operations research activities, organization and methods studies and improvement in clerical systems and procedures. Some TCEs may also be involved, in corporate planning and financial budgeting and audit. The department is usually called Management Services or Information Services and covers a variety of staff functions.

Perhaps the most important function of the TCE is to work with managers in other parts of the corporation and plan applications with them. He may have to "sell" computer methods to them, bring them down to earth on present day computer capabilites or just toss some ideas around It is these managers, after all, whose departmental functioning is going to be affected be the computer systems and unless they are involved in planning the progeny is apt to be ill-born or malformed

A study conducted by McKinsey and Company suggests that line management involvement is a crucial factor in the success of computer effort

"Advanced computer application concepts with potential impact on the central activities of a corporation, must have sponsors high in the management pyramid to plead their case. It takes enthusiastism in management leadership to gain the commitment of operating men—and it takes teamwork between operating men and computer professionals to turn imaginative concepts into practical reality. Indeed, knowlegeable and well motivated operating managers are likely to be a better source of ideas for for profitable changes in operations than are computer professionals.

In almost every industry, at least one company can now be found that is pioneering in profitable new uses of computors In such companies, the key to success has been a strong constructive interest from corporate operating executives who have put their heads on staffs to work on computer development project. It may soon be universal practice to transfer operating staff to computer development projects, either by making them members of a project team or by attaching them for a year or two to the corporate computer staff."

The study also showed that in the more successful computer users the line management was involved in 70% of the organizations with identifying computer opportunities, in 43% with specifying payoffs, in 80% with staffing and/or managing projects, and in 40% with accounting for results. Among the less successful users there was considerable less involvement, with only 22% companies using line management to identify opportunities, 15% to specify payoffs, 29% for staffing and managing projects and only 15% in accounting for results.

1.2, Structure of the Computer Division: An efficient computer department requires that a certain number of distinct functions be performed by trained personnel who can

be assigned responsibility for them. The aggregation of these functions into convenient executive designations is less important, and the structuring can be done in any number of ways, depending on the size of the department, the personalities involved and the relevant corporate policies. It should also be remembered that the computer division will usually be a part of larger management services department and the computer manager will typically report to the TCE who will also have managers for other services, reporting to him. This is probabely the most efficient way of organizing these functions. In a small department, the TCE may have computer personnel as well as operations research and other personnel reporting directly to him.

The role of the TCE is to provide centralized planning and control. If a company has a number of geographically distant computer installations he must provide a unified corporate policy and co-ordinate all development. In what is probably the best organization, applications are approved centrally with the major part of the systems analysis and programming also being carried out centrally under the guidance of the TCE. The computer staff merely run the programs. The staff may maintain the programs and carry out minor amendments from time to time, but the basic authority for systems maintainance must be centralized. Staff may be loaned from the central pool to various installations as and when required for implementing applications. While central control may lead to some delay in approving and implementing systems, it is essential to prevent expensive duplication of effort among the various installations

The question arises to whome the TCE should report in the company organization. As already emphasised the earlier practice was to make the TCE report to the controller or a financial executive. This was, perhaps, also the legacy of the *unit record equipment whose incharge was placed under an accounting manager. But computer is not just an extension of the unit required equipment. Its applications transcend the mere accounting transaction position for stock control, payroll, A/c receivables, etc. In fact, the accounting applications. in a well-planned computer system, would constitute only a secondary fraction, i.e., usually a by-product of computerising the main - line functions. Therefore, it is increasingly the trend now-a-days to delineate data processing from the accounting function and make the TCE to report to the chief or the deputy chief of the organization. And this trend, it seems, woulld continue inexorably.

The table of fig. 1 lists the various functions for which the TCE is directly or ultimately responsible. In this table, concerning ourselves for a while with level 3 and below, three major groupings of functions as below exist :

1. $\left\{\begin{array}{l}\text{Systems Analysis}\\\text{Programming}\end{array}\right.$

*Unit record equipment is discussed in the Appendix to study v.

Internal Organisation of the Data Processing Department

Management Level 1 (TCE, Manager Management Services)	Objective-setting Cost-benefit Audit Overall Control			
Management Level 2 (Computer Manager)	Application Selection, Development and Evaluation, Budgetary Control, Resource Planning, Physical Planning, Personnel Selection and Training			
	Systems Analysis	Programming	Operation	Others
Management Level 3 (System Manager, Chief Programmer etc)	Systems Selection Resource Allocation User Interaction Supervision	Resource Allocation Program Planning Supervision	Operating Control Management Reporting	Documentation Standards Stocking of computer accessories
Technical Staff Systems Analysis and Programmers)	Feasibility Reports Systems Design Systems Specification Systems Maintenance Supervision of input/output controls	Program Organisation Programming Standards Management Supervision of input, output control Supervision of systems running	Machine Scheduling	
Operating Staff (Machine Operators, Punch Operators, etc.)		Coding and Testing	Work Planning Machine Operation Input Control Library Maintenance	

Fig 1

2. Operations.
3. Others.

In evolving the data processing organisation for these functions two points as below have to be borne in mind.

1. Efficient communication amongst these functionaries as well amongst them on the one hand and the user departments on the other.

2. Productivity of the computer system.

It might appear surprising later on that productivity of the computer data processing department, systems and programming in particular, is not too much affected by how this department is internally organised but how the user department are organised !

We shall first confine our attention to the functions of systems analysis and programming (SAP) and d.p. operation comprising key-punching, and other auxiliary equipment and computer operations. First of all it is highly desirable that the heads of these two functions (SAP and Operations) report to a common superior because they have a high degree of mutual impact, e.g., the documentation evolved by SAP would be used by Computer operators programs would be put on punched card decks etc. by the keypunching operators and as such there would be a great deal of communication between the two. Also, it is necessary for them to be headed by one person because the user would not care whether his problems are to be attributed to SAP or Operations.

Now let us take the internal organisation of the SAP staff. Three possibilities exist here.

Organisation by Applications as depicted below :

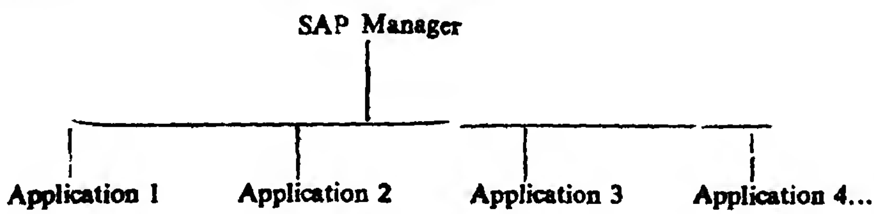


Fig. 2

This organisation has the following disadvantages and, therefore, it is not usually employed in practice.

Applications, as triggered by the systems audit phase of systems development, come and go ; therefore, there is a problem of perpetual reorganisation. Also the systems analysis and programmers cannot gain indepth knowledge of any business function, like marketing, etc.

Organisation by Internal Function (Systems and Programming function).

Under this scheme, the systems and programming function is sliced up into such sub-functions as systems analysis, systems design, systems implementation, programming, program maintenance and assigned to different systems analysts and programmers on a permanent basis, as depicted below.

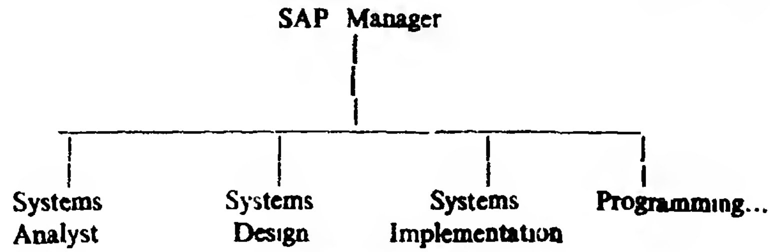


Fig. 3

This arrangement is, too, fraught with a few practical disadvantages, as below :

(i) The personnel still cannot gain any in-depth knowledge of business functions.

(ii) There arise severe communication problems between systems analysts and systems designers ; and systems designers and programmers etc. This, then, leads to a large amount of documentation as well as confusion and wastage of time. The user would not know to whom to contact for his problems

Organisation by Business Function as depicted below.

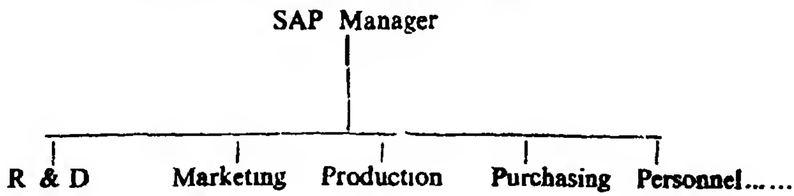


Fig. 4

Here a team of systems analysts and programmers is assigned to each business function. This, then, brings the communication problem down to a minimum and the user, too, is sure about to whom to contact for his problems since, in a way, each user department has its own team of the systems and programming talent. In other words, effective liasion with departments is established naturally. One problem that can be foreseen in this type of arrangement, however, is that of big projects which transcend business functional boundaries In that case such a project can be assigned to any one of the team, preferably the one to which it predominantly belongs. This would lead to some communication problems because of the project's use of other teams too.

This, in practice, is circumvented by employing more liasion men to liase with these teams

Overall Organisation of the D.P Department

On the last arrangement of systems analysts and programmers can now be superimposed other functions for operations, etc., and the first two levels in the table of p 5 The following organisation chart thus derived is found most suitable in practice.

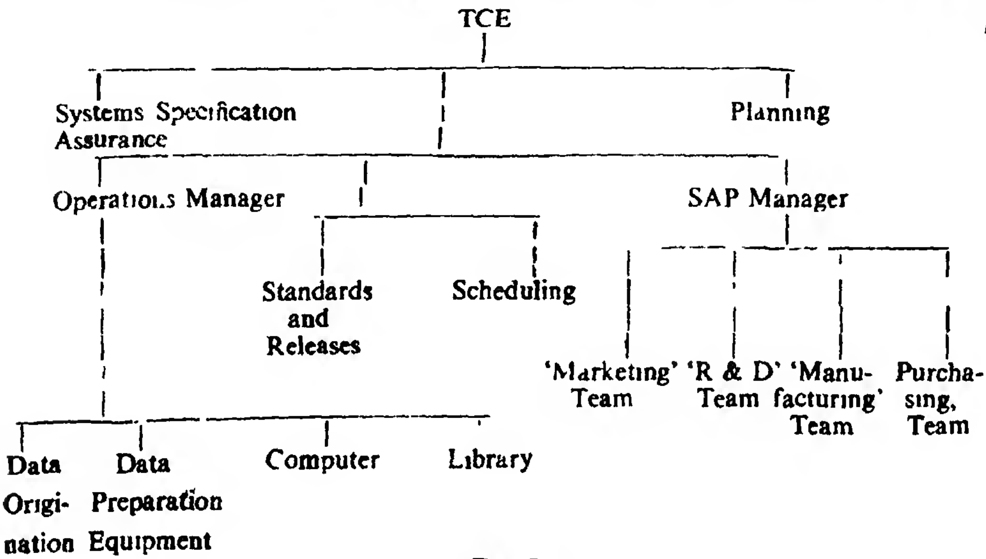


Fig. 5

This organisational arrangement is easily understood and best appreciated if the data processing department is considered as a workshop producing information from data. As such the operation manager is in the "line" capacity and the SAP manager in the "staff" capacity.

The latter, the "staffing" SAP staff performs functions (systems and programming) which are analogous to engineering, product development etc , in a factory.

It is to be noted that "scheduling" function has been kept independent of operations, i e., it has not been assigned to the operations manager, who would tend to do scheduling in a biased manner and is liable to ignore or play with the priorities of the jobs to be processed. The priorities are best established by an independent scheduling section. "Standards and Releases" are interposed between systems and programming on the one hand and operations on the other. This will ensure that the documents prepared by the former, systems and programming for operations is reviewed for quality, integrity and conformance to standards by this section of standards and releases.

The function of systems specifications assurance represents a form of high level quality control.

The function of planning would include level 2's functions listed in the table above

It is to be noted that operations manager department's organisation is somewhat complicated by the fact that each of his sections may work for more than one shift per day ; therefore, his sections may have to be headed by shift supervisors for each working shift. The operators are usually too few to have a supervisor but some among them may be designated senior operators and given responsibility for the shift.

We mentioned earlier that the productivity of the computer department does not depend so much upon the internal organisation of the computer department but that of the user departments. Strange, though, it is a fact. The reason lies in the fact that the user departments tend to bombard the SAP staff with illbaked job proposals for computerisation partly with the motive of getting too much of their work computerised and partly because of their lack of knowledge of the computer with the result that no inspection is performed on these proposals for their quality and resonableness which has to be done by SAP Thus SAP staff spends most of their time in evaluating these proposals for computerisation which results into backlog of projects and requests for additional service. The users have a homework to do here, which they cannot do if they lack in computer knowledge. Thus it is to be strongly recommended thab each department has one of its senior member exposed to computer who can appraise the proposals and forward only those to the d.p. department which are reasonable

The ideal organisations for small and medium installations are shown in figures 6 and 7 respectively However, in multi division corporations it could become very large. The outlines of the Computer Services Organisation of an American Oil Company in Fig 8 shows the basic structure of a large organisation.

This company has seven distinct computer installations, called Data Service Centers, to emphasize their role of providing a processing facility for systems applicable to the various departments. The Data Service Centres are under the control of managers equivalent to the Operations Manager in Figure 2. Each of these managers is supported by staff for computer operations, input/output and data preparation and is responsible only for the running of the various systems on the computers, minor pieces of systems modification and the general adminstration of the installation. They report to a Manager Operations, Data Service Centres, who reports, in turn. to the General Manager, Information and Computer Systems. An additional management layer is introduced between the Computer Manager and the Operations Manager to reduce the number reporting to the Computer Manager.

All systems development is centralized and headed by a Manager, Information Systems reporting to the General Manager. Four managers, heading Technical Systems, Business Systems—Head Office, Business Systems - Field, and Equipment Systems report to him. Each of these managers has other managers reporting to him, these being in-charge of an even more limited class of systems such as the Systems

Manager—Engineering, Systems Manager—Chemical, etc. The Business Systems Managers have rather extensive organisations below them. A Supervisor, General

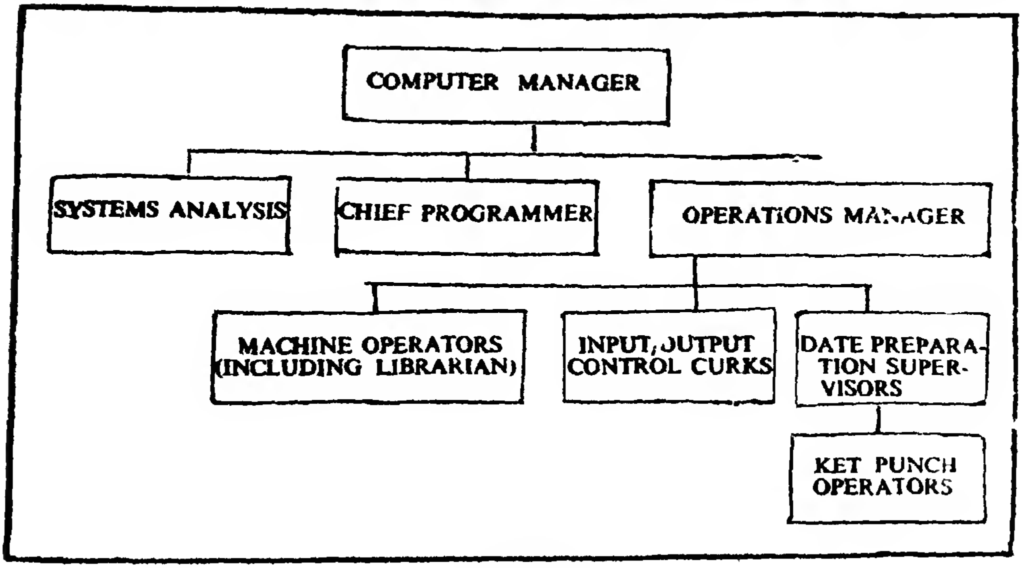


Figure 6

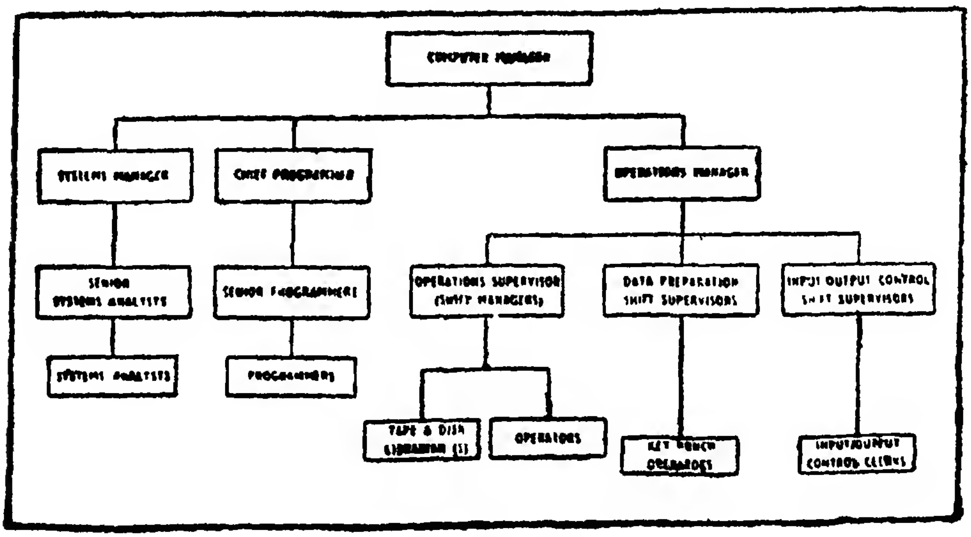


Figure 7

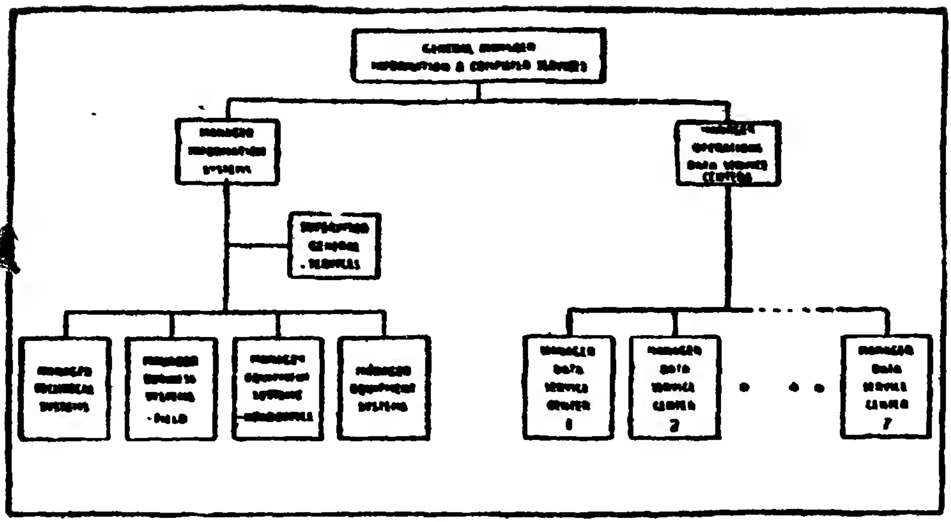


Figure 8

Service, also reports to the Manager, Information System, and is responsible for centralized records management, maintaining an information centre, and general administration and control.

In addition to these systems organization, there are several committees or inter departmental organizations for developing information systems in the specific corporate areas of personnel, transportation and supplise, marketing, finance, purchasing, and manufacturing, and one to develop systems for a subsidiary company. Each committee is headed by the appropriate manager and composed of operating executives and personnel from Information Systems. These teams are jointly responsible for developing and implementing new systems on a centralized basis. Later, the systems are handed over to the regional Data Service Centres for running.

The company has had an extremely successful computer effort. A large part of the success is due to the key concepts that are implemented ; an independent information services department and the inter-departmental systems organization that developed systems with line management co-operation and top management support.

2.2 Dual Reporting. If the student thinks for some time about the organization structures that are recommended in this Study Paper for the EDP department he, will realize that a dual organization structure is proposed. On one level the personnel are responsible to the EDP department and report to superiors in this department. On another level, systems analysis and programmers are constantly assigned to one or more projects and report to the project manager who is usually a manager for one of the other corporate departments.

While the dual reporting structure does create some problems, a central data processing department is the only way in which a pool of professional talent can be maintained, with proper channels for advancement. On the other hand, the project organization is necessary to allow the systems analysis and programming staff to learn about the systems from the departmental staff. It also involves the departmental manageres in the creation of the new system. In doing so, it ensures their involvement and their commitment and control over its design. This allows them to change the procedures and the functioning of the department to make the best of the technology. It is these changes that lead to truly rewarding systems Merely mechanizing existing manual processes is rarely very profitable.

2.3 Using Consultant and Package Programs : Whan a new system is to be computerized, the manager should consider alternatives to getting it done by his own staff. The two alternatives that are gaining increasing acceptance are the use of consultants and the use of package programs

Consultants can be used to do some of the systems analysis and programming work Usually they are called to alleviate peak loads in programming and to assist in the design and implementation of the first few systems They are, however, more expensive than company staff but may provide a higher standard of technical competence and introduce techniques and refinements not readily known to in-house personnel Also, consultants make better change agents, as the opinion of an outside 'expert" always carries more weight.

Systems anylysis demands close interaction with company managers and acquaintance with company procedures. Thus, consultants require a period of acclimatisation and fact finding before they begin to be useful. The extra competence they provide must be able to compensate for this additional cost.

A company shackled in an antiquated wage structure may find it difficult to get good people at the salaries offered. In such a case consultants can be hired on a retainer basis for the top computer management positions Consultants also tend to increase the probability of meeting deadlines. Their experience tends to reduce the research time required to produce a system and their planning is usually better. Unlike company personnel, they have to agree to fixed deadlines and can be held accountable for them.

The introduction of a computer in a company may follow the pattern of a large initial effort, while the basic systems are assembled, tapering down to the implementation of subsidiary systems and systems maintenance after a few years. A computer department planned for the starting peak may be considerably overstaffed after a few years. In a country with high unemployment and low job mobility such as India, it may be difficult to reduee the staff after the initial period. On the other hand, a smaller staff would delay the implementation of systems. A better alternative to contract effort is perhaps the best way to use consultants for other reasons also. The

benefit of their experience can be very useful in preparing the overall implementation plan for the installation.

A large number of "package" programs are now available for the more routine and most management functions. In addition to the computer manufacturers, many independent companies have produced packages, and some of them have proved efficient, reliable and easy to use. The most successful packages have been produced for limited applications such as payroll, order entry, and billing instead of more general applications such as inventory control and production planning where practices and requirements differ significantly from company to company.

An excellent way of implementing the more standard systems is, therefore, to look for software packages that may do the job, and if a suitable package can be had it can start working in a few days instead of the months required to prepare the programs in the company.

Before a package is required, however, it is important to ascertain whether the data input and output correspond to the user's particular requirement and whether the processing algorithms are meaningful and useful in his environment. For example, all sales forecasting packages use details of past and current sale as input for the future, but it is important to ensure that the particular techniques employed are relevant to the user's business The ability to conveniently add or modify facilities considerably enhance the applicability of the package.

Self-Examination Questions, No. 1

The student should test his understanding of the preceding material by attempting to answer the following questions Answers may be submitted for correction.

A Why should the manager of the Data Processing Division report directly to top management ? What are the disadvantages if he does not ?

B A survey carried out in Britain a few years ago showed that 66% of the computers started in the accounts department. Was this desirable ?
What are advantages and disadvantages ? Discuss from the viewpoint of a manager and an accountant.

C Who should be deciding on which applications get implemented on the computer ?

D. Who should be responsible for deciding on which programs are run on the computer each day ?

3 MANAGEMENT STANDARDS

Systems analysis and programming have been considered to be "artistic", "intuitive" and "creative" and, therefore, difficult to standardize, and control . It is essential, however, for management to be able to control the systems and programming

effort and to set budgets. Standards prescribe how certain types of jobs are to be carried out to make it easier for personnel to communicate with and evaluate each other.

Two distinct kinds of standards have been developed for EDP activities—methods standards and performance standards. The former specify time and cost schedules for the work. While the former may be incomplete, i.e., cover only part of the work, performance standards must be complete and comprehensive. It is difficult therefore to develop performance standards for loosely specified jobs such as systems analysis. They are a little easier, however, to develop for programming but data conversion (keypunching) and computer operations are quite amenable to performance standards.

To educate personnel regarding standards and assist in their enforcement every EDP department should have a Standards Manual or a Manual of Standards or Operating Policy, having at least five major sections

1. Systems Analysis Standards,
2. Programming Standards,
3. Operating Standards,
4. Documentation Standards,
5. Performance Standards.

Self Examination Questions No 2

A. Why are standards important ? What is their purpose ?

B. What kind of standards could be set up for an accounting department ?

3.1. **Systems Analysis Standards** : The functions to be standardised in systems analysis include :

(a) Definition of all terms and abbreviations used in the department.

(b) Layout of cards designs, report formats and file organisation.

(c) Review of input and output documents.

(d) Control Coding : The assignment of meaningful numbers to programs, systems, reports, files and the like.

(e) Flowcharting,

(f) The program specification manual : The final output of the systems analysis process.

Technical terms and abbreviations should have standard meanings. One should prepare a glossary of general data processing terms, and a glossary of industrial terms. Further, the systems analysis staff should specify identification procedures such as header and trailer labels for tape files, and identification codes for cards. The Systems department must also specify usage procedures such as :—

•All programs will be stored on a master program tape which will be updated every week and maintained in duplicate.

*No more than one file of data shall be stored on a reel of tape

*The standard input/output system supplied by the manufacturer will be used in all programs.

Systems standards include the layout of cards, tapes and reports. These are concerned with the preparation of layouts, standard ways of making certain indications such as multiple cards for single records, the organisation information on each document and mandatory information that must be included.

3.1.1. **Control Coding** : One of the first items to be standardised is the numbering scheme to be used for the programs, systems, forms, tapes and other files A simple system may be the following :

1. All application systems are assigned a letter code (A, B, C, etc)

2. Programs for each system are identified by the letter code followed by a number followed by a letter (D, W, M, Y) that indicates how often it is to be run Thus, A03W mean that it is the third program of System A and is run weekly.

3. Reports are numbered consecutively within each system and the number followed by the number of the program that creates it. Thus, the second report of a system A created by A03W may be coded as 002A03W.

4 Tapes are identified by a number This will be preceded by the concerned program, e g., file 002 created by A03W will be assigened A03W00 '

3 1 2. **Flowcharting conventions** : To ensure uniform, readable and unambiguous flowcharts, the systems department must prepare standards on the kind of flowchart, format, method of preparation and the information to be included.

3.2 **Programming Standards** : The programmer translates the specifications in the job specification manual into machine instruction. His responsibilities include ,

1. Creation of a program flowchart.
2 Writing the program
3. Checking the program manual (desk checking .
4. Preparation of test data.
5. Testing of programs by running with test data.
6. Documentation of the program and its operation.
7. Installation of the program as part of the application system.

For each of these tasks rigid discipline is necessary to ensure a uniform product that can be read and understood by the programmers It is also necessary for scheduling purposes and for evaluation of the programmers.

The standards should specify the kinds of flow diagrams to be drawn, their format and level of detail as well as the methods of diagramming. Since program flowcharting is capable of tighter specification than systems flowcharting, a numbbr of specification schemes have been developed for it. One of these may be selected as standard for the installation.

Programs are notoriously difficult to read and understand. To alleviate this situation coding standards are specified to include coding format, coding method and program organization.

Coding format is conditioned by the computer system and the facilities it provides, but can be improved by setting standards on the languages to be used, the manner in which programs are organised and the program code is written on the coding sheets. Another very useful convention is the inclusion of identification information and comments alongwith the coding information.

These should be simple rules so that two programmers referring to the same point in the program will use the same label. Standard conventions for naming labels and constants may be used for the purpose.

A standard for program organisation is of utmost importance. It improves communication as well as leads to more efficient programs. In general, each program must be organised along a single main line of logic Complex processing in this line should be related to subroutines. Similarly, exceptions and special conditions should be treated by branches from the main logic and in subroutines. All functions that are repetitive and occur more than a few times in programs should be standardised. In the same way, the passing of information from one program to another should be standardised.

The programmer is also responsible for creating a set of operating instructions for the operations staff. With this and messages from the program he is responsible for minimising operator errors The programmer should specify for each program the instructions to be followed to set up the program, run it and take down the tape reels and the special paper that may have been loaded for it. Similarly, standards have to be set for messages that indicate possible halt conditions due to input or program error and due to machine error Corrective action to be taken in each case should also be specified

3 2 1. **Audit and Control Standards** : It is difficult to establish a general set of standards for auditing. However, some specific rules that could be applied to most installation are the following ·

1. All operator actions which can have any effect on the information bein processed must be recorded on a type writter or magnetic tape log.
2. There must always be two operators, or one operator and one supervisor i the machine room when critical data is being processed to avoid deliberat misapplication.
3. All information entered into the system which alters any type of mone balance must be recorded separately on an audit control tape or ledg records.

4. All journals, ledgers, or other critical listings must have a sequential numeric page number Page 001 will be first page, and the last data page shall force a separate page to be created carrying the message "LAST PAGE ."

5 All data files have a minimum of two controls, verified at predetermined control "break". One of these controls shall be sum total of all money being processed, the other shall be a hash total of the field which controls the sequence of the file

6 In all financial progrms the audit trail shall be clearly defined as a separate part of the documenration

7 Machine check points shall be taken at all control points

8. All magnetic tape files shall be retained for a minimum of three processing cycles for purposes of protection and back up.

3.2 2 Testing : Testing of a program occurs in a number of phases. First, the program is checked manually for completeness and adherence to standard. Then, using some sample data the programmer goes through its processing manual This is called desk checking After this has been done and any errors corrected, he gets a set of cards punched for the program and tries to assemble it i.e , see if the computer and its associated software accepts it. A number of errors may be found at this step, generally in the syntax of individual instructions of the program. After this he prepares some sample data and runs the program with that data to test for errors in logic. Finally, the complete system is tested by itself and parallel running with the system to be replaced.

It is important to ensure that desk checking is carried out and the program is reviewed by another person at this stage, preferably the programmer's supervisor otherwise, one may end up wasting enormous amount of human and machine time, later

Test data to be prepared falls into two categories :

(1) To test each block of the program

(2) To test if system requirements are met

The programmer should be responsible for creating (1) and the system analyst for creating (2). Once the data have been created, these should be punched on standard cards with appropriate identification. A set of test results should also be prepared and a formal method of testing the program and comparing it with the test result and making appropriate analysis should be followed.

After all of the prepared test data is processed accurately by the program, it is ready for production testing Production testing should be performed the same way in which the program will ultimately operate. There should be no temporary corrections in the program. All of the input information must be "live" data.

The systems analysts who designed the system is responsible for checking output of the production test of each program. If the program is accepted by the analyst as producing satisfactory data, it is ready for a systems test. The systems test should be run similarly to the production test on approximately the same volume of data The final outputs of the systems test should he evaluated by the systems analyst and the user The systems test shouid include a run-through of the entire system, with fake end-of-quarter, conditions simulated, if necessary

Upon agreement by the systems analyst, the programming department and the user, a parallel run shall be initiated For programs with a run frequency of between one day and one month, the parallel run shall operate for a minimum of one weekly cycle and a maximum two months, before the old system is discontinued If the parallel run is successful the new system should be temporarily discountinued, if the parallel run produces accurate and suitable output, the old system may be discontinued provided that control personnel continue to evalute all output data Any run which has not been paralleled after the old system has been discontinued must be run under complete control of the uses department for the first two times.

All programmers should be given a minimum of two weeks on-the-job operator training, by acting as operators within the installation for that period of time. This will familiarise them with operational requirements, and enable them to translate their understanding of operator problems into meaningful, coordinated programs and programming manuals

All program testing should be done according to the "closed-shop" principle having the operator run the test under explicit instructions from the programmer. The programmer must prepare complete instructions.

An operating priority system should be established providing for at least three priority levels : priority production runs, normal testing and normal producton. "Crash" programs should be given priority in testing and guaranteed a maximum turnaround of two hours.

All programmers should be allowed to observe their test runs if their schedule permits. All programmers should be required to observe the first production test of their program

The programmer is responsible for two sets of documentation :

1. An operating manual
2. A Programmer's manual.

The operating manual instructs the operators in the details of operating the program ; what is to be done and how to handle exceptional and error conditions. The programmer's manual describes each program in great detail It includes flowcharts, descriptions of the logic of the program and the program listings.

3 2.3. **Management of Program Changes:** The most serious problem faced by an operating installation is the accurate maintenance of all programs used in the operation. Operation programs undergo changes for a number of reasons :

1. Corrections of latent errors uncovered by subsequent running,
2. Changes caused by time such as tax rate,
3. Changes in parameters,
4. Changes in output format, or procedure caused by a change in management requiremeuts,
5. Fuuctional expansion,

It is necessary to set up and enforce a realistic control over all the required changes. An analogous situation exists in the maintenance of a blue print file in a manufacturing company, where a separate department (Engineering Change Administration) guarantees that only the latest revisions of the prints are used. One cannot build a macnine or instrument with out-of-date prints or run a data processing system with an out of-date program or to attempt a change to a program whose documentation is not current

It is recommended that a change be initiated by the user when an output of procedure change is desired, or it may originate in the programming group when an error is found or a correction otherwise deemed desirable. The following procedure should be followed in making the change :

1. Establish the data or time when the change is to become effective.
2. Determine the cost for the change and the leadtime and advise the user.
3. Determine the effect of the change on the programs, and ascertain the best approach to making the change.
4. Fill out first section of a change request form This contains the reason for the change, its purpose, and its effect, and the method to be used to implement it.
5. The assigned maintenance programmer shall then make the change, based on the outline method
6. Create test data to properly test the effect of the change. Incorporate this in the test data already established for the program, and completely test the program using all the test data.
7. Update the existing documentation to show the effect of the change.
8. On the effective data of the change, replace the obsolete information with the current or new material.
9. As a part of the program library system, the change should be accepted by the librarian, after insuring that all material is properly updated. It should be recorded on the program history card, and the change request should be filled by programmer in revision number order.

Self Examination Questions No 3

A. What are the different types of standards required for systems analysis and programming ? What is the utility of each type of standards ?

B. How would you evaluate systems analysis and programmers using the standards described in preceding section ?

C Can you think of standards that may apply to the work of the systems manager and the chief programmer (programming manager) ?

3.3 **Operating Standards** : The computer is an expensive piece of equipment and to some extent, its utilisation depends on the operating staff

Operating standards should include rules regarding conduct in the operating room.

1. There should be no smoking in the machine room Besides fire, tape reading and writing problems may arise.
2. The machine room should be kept orderly at all times, day or night—there should be no loose papers ; cards or other materials left on the floor at any time
3. Machine room operators should be neatly dressed on all shifts
4. For safety and control there must always be at least two machine operators in the room when power is "on" in any unit.
5. Cards and program deck used for operation should be properly replaced immediately after use. The program deck must be returned to the program library ; all other cards are to be kept in trays, and returned to the data coordinator.

3.3.1. **The Machine Log** : If equipment and operating performance is to be accurately evaluated, careful records must be made of the exact utilization of the machine. These records must be maintained by the operators and recording methods should be carefully spelled out The operating staff should record the utilisation of each computer system on an individual log. They should record the time at which each job is started, the facilities used (for example, four tape units, two disc drives) and the outcome (successful machine error, operator error, etc). They should also record the time at which job ends, the time required to set up the next job.

3.3.2. **Control Function** : The operating department has five basic control functions :

1. *Scheduling* : assignment of job priorities ; and the establishment of a daily equipment schedule.
2. *Dispatching* : insuring that the jobs are performed in the assigned sequence.
3. *Data coordination* : obtaining all the required input and output information, and making it available to the operating group.
4. *Data control* : validation of output against predetermined totals.
5. *Report distribution control* : insuring that all reports are deleaved, decollated and bursted as required, are distributed to the users.

A schedule for the operation of a computer is usually established weekly, for each day's processing The schedule must account for all activities that can be anticipated ; recurring, production, special production on the basis of requests, testing, assembly, preventive maintenance, training and demonstration.

The first in the establishment of a schedule is review and processing of daily requests for production and testing time These should be submitted on special forms

After the total number of jobs to be run has been established, the scheduling group assigns priorities on the basis of the requested time, the urgency of the request, the user, and other factors that may be relevant The general priority system establishes a "rush" (first priority), a normal processing operation (second priority), a test or assembly having a pre-determined turnaround time, and an "immediate" priority for jobs that must be run the instant they are received. The last is generally not in the schedule and the schedule should allow buffer time for these. If no buffer is provided, it will have to be absorbed in normal processing at the end of the processing period

The scheduling system can get very complex if a number of computers have to be scheduled with more than one of them able to run a partieular job The complexity is further aggravated if each machine is capable of running more than one program at a time In such circumstances, a special scheduling group becomes necessary, often assisted by a set of computer programmers that create the basic schedule

The dispatching, or expediting function is often performed by the supervisor of the operating group. It is his responsibility to see that the schedule is observed and delays properly accounted for. A routine slip. or ticket, made out so that the operator will have the necessary instructions as to where to get input data and where to send output reports The routing ticket must accompany all input and output data.

The data condition function is often combined with dispatching. The data coordinator's objective is to see that all materials required for a job are gathered in one place so that the operator can take over without delay. It is very expensive to keep the machine waiting for information, while the operator is out getting tapes or forms The data coordinator, therefore, has an extremely valuable function. He must obtain the following information for every job, on the basis of the documentation provided :

Program deck of tape
Input cards if any
Parameter cards required, if any
Input tapes
Blank cards for possible output, if any
"Free" or "Scartch" tapes for output
Necessary blank or preprinted forms

Any required carriage control tapes .

Job documentation

Any other information required.

This may be done according to the documentation kept in the liabrary, or it may be done on the basis of a job dispatching sheet, maintained by the dispatcher or the data coordinator.

Computer output should be validated before being sent to the user. This validation is not performed item by item ; it is done through the use of control total or total. This function is an integral part of the responsibility of the operating group. The output be verified to detect machine failures and omissions of operation, data or other vitals functions Controls are maintained on money fields by carrying group totals On non-money fields hash totals or check sums are carried and verified to a predetermined value

One of the more important functions of the operating department is the handling of printer output This includes margin removal, decollating, bursting and binding, but more importaet, it also includes accurate distribution of copies of all personnel indicated. To perform this task satisfactorily within the time scheduled a separate distribution sheet is often used in the report control section for each job or procedure

This function is becoming more and more important as printers become faster and more versatile. A 1,000 lines per minute printer produces 1,000 pages per hour (printed 50 lines to the page) This will prodnce the staggering total of 4,20,000 pages of information for a two shift operation in one month ! The tr nd towards exception reporting will reduce the volume of information required but will correspondingly increase the importance of accurate distribution and control over confidential or secret information.

3.3.3. **General Machine Operation** : The manual of standards should also contain a section describing the basic rules of normal console operation and the responsibilities of the operator at each step. For each job that is to be run, the operator should follow the operating instructions. If a programmed halt occurs the operator should look up the documentation and take the approriate action. In addition to these basic functions, the operator should so schedule his work as to minimise set up times, and other non-productive time on the computer.

1 During normal operation the operator must watch the processing to detect malfunctions or usual machine actions. He must replenish the supply of input and output cards without stopping the machine, if possible and remove and replace all cards the machine has read or punched.

2. The operator under no condition may alter memory of an operating program or run any program other than one authorised for operation on the current schedule. He should not alter any program, program deck or program tape without the explicit approval of both the operating and the programming manager.

3. In case of a machine failure, data error, program error or operator error, the operator should follow the instructions outlined under emergency procedures. He must not return without authorisation or use any self constructed utility program in an attemot to correct the file

3.3.4 **Exceptions and Emergency Procedures** A specific section of the Standards Manual should be devoted to the writing of exception and emergency procedures *Exceptions* procedures refer to the occurrence of an unexpected machine caused condition—a program error, a machine failure, an operator error, or a data error, *Emergency* procedures refer to the steps to be followed in case of a real emergency—fire, attack, and the like.

EXCEPTION PROCEDURES

1. If a programmed halt occurs, the operator should note the halt number and the status of files, cards, tapes and the like. The operator should then look up the halt number, to determine the cause and action to be taken In the event the halt is "endless"—without corrective possibility—the operator should notify the supervisor of the occurrence immediately, and proceed to the next program.

2. If a machine error occurs, and the machine stops, the operator should record the occurrence on the log and notify his supervisor and the maintenance engineer. The program may contain a "restart" procedure , if so, he must follow the restart instructions If no restart procedure is available the operator should start the run over from the beginning If the halt occurs again, he should turn the machine over to the maintenance engineer

3. Whenever an exception condition occurs the operator must note all existing conditions on a "console" page. The same form must be used for all exception console conditions, including the occurrence of a program error, a data validity error, or an operator console error (This form is often designed as an image of the console enabling the operator to record the information rapidly and accurately)

4. In the event of a program error the operator should notify his supervisor and the programer responsible for the program.

5. In the event of a data error, the operator should notify his supervisor.

6. In the event of an operator error the operator should notify his supervisor, if the error has destroyed pertinent information, the supervisor should also notify the programmer responsible, to assist in recapturing the required information.

Program Library Organisation The main functions of the program librarian are to retain program records, issue program decks or tapes to operators and to maintain program revisions Ih ʌ

3.3.6. **Tape Library Organisation** : The tape librarian must keep records on the history of each tape and information about the files and their retention cycles. He must protect the files and issue them only to authorised personnel at the proper time. The librarian shall maintain the records in accordance with the following rules :

1 *All tapes must carry an identification label.*

2. *The librarian* should prepare a tape record card for each tape as it is added to the tape library, to indicate the information curtrently available on the tape It may also contain a record of errors encountered in reading the tape

3. The librarian shall assign tape numbers to the tapes in accordance with their use The library should be maintained in tape number order.

4 As the tape retention cycle expires, the tape becomes available for use.

5. If tape developes errors in the first 100 feet (this is the most likely place to experience excessive wear, because of magnetic labelling procedures which double the wear at the front) the librarian should strip a lengh of tape when the tape again becomes available for use The strippiug should be recorded on the reel

6 Tapes used by specific programmers or maintenance engineer are to be kept in a separate part of the tape libray

7 Tapes should not be released or taken to any area except the computer centre. All tapes must be transported in standard carriers and handled with extreme care The area selected for tape storage should be carefully tested for magnetic influence from surrounding equi, nt such as burglar alarms, elevators, and the like. The area should be fireproof and waterproof

8 Smoking should be prohibited in the tape library. Dust should be kept to a minimnm.

3.4 **Documention Standards** : Proper documentation is essential in a good data processing establishment The various documents are discussed below :

3.4.1. **Job Specification Manual** : The contents of the job Specification Manual have been described in Study III. Mandetory changes will have to be made, but it may be possible to delay them until the system is operational Desirable changes should be carefully evaluated.

3.4.2. **Program Specifications.** The program specifications should be prepared by the programming departments based on the general description in the job specification manual to provide a description of the system at its most detailed level, and spell out the structure and logic of each program. Program specifications are directions to the programmer. As such, they must be complete and unambiguous. The specifications for each program should be self-contained.

Estimates of programming time required and computer time required for testing will be based on the program specifications.

Programming specifications should provide the following minimum information :

General program structure.

Input formats and input volumes including special conditions.

Output formats, including special conditions.

File layout for all magnetic tape and disc files

Addressing and seeking magnetic tape and disc files.

Input and file processing routines to update files and produce outputs

Program module fragmentation.

Standards for opening and closing files, starting and ending the run

Halts to be provided under special conditions.

Communication between modules and with other programs

Interaction with the operating system

Detailed test data and programme for system cheek out

The setting down of program specifications serves as a freeze point for the program structure and design. After this, all changes must go through a formal procedure of submission and evaluation before they can be implemented

3.4 3. **Program Documentation :** Computer systems are usually developed in a hurry which often proves harmful

Program documentation is essential for continutity and systems maintenance though it is difficult to devise standards for it as a great deal depends on the use it is intended for and on the complexity of the system It should contain the following :

List of personnel on the job (internal copies only).

Contents and summary.

Job specifications.

Programmer's description of jobs and explanatory notes on programs.

Operating instructions.

List of flowchart symbols (customer's copies only.)

System flowchart

Outline program flowchart.

Program listing.

Test data listings.

Expected results

The job specifications are reproduced as received. The programmer's descriptions are somewhat more detailed, taking the place of program specifications and giving details of how each program has been put together The operating instructions contain direction to the computer operator on how each program is to be [illegible] special stationary and files are to be used and what to do in case of unusual messages and halts.

Programmers and systems analysts prepare flowcharts, as initial descriptions of the logic of each program, before writting the detailed instructions for the operations

represented. The levels of flowcharts are required in a standard system of documentation ; macro flowchart showing broadly the inter relations between the programs, a more detailed flowchart for each program showing its charactetistics in two or three pages ; and a very detailed flowchart with a box (an operation or an activity) for about evèry ten instructions In addition to this is the list of program instruetions, the most detailed level of all preferrably annotated with a large number of comments to aid understanding

It is customary to test programs with a sample input data called test data. The results expected from this data are calculated by hand and compared with the results produced by the program To be effective, the data should be written to cover every conceivable "legal" condition that may arise while the program is running. even though some of these may be very artificial

3 4 4. User' Manual : Usually user's documentation explets, if anathing, too much instead of too little. One specifiation it does not explicitly mention, however is client's manual detailing how the data should be supplied to the cumputer installation and how control totals, master file alterations and other special conditions should be coded This information may, however, be included under some other heads such as job specifications

It is useful, however to have a separate client's or user's manual giving the procedures, time schedules and responsibilities for the preparation of data to be sent to computer installaction This provides the clerical and supervisory staff complete instrucrions on how to run the system without any detail on how it works.

Self Examination Qusstions No 4

A. What is the purpose of standards for machine operation ? What would happen if these were not iollowed ?

C. Why is good documentation so important for successful computer system ?

3 5 Performence Standards

3 5.1 Equipment Standards and Standards for Operating Personnel : The time that a computer is switched on can be divided into time during which it is doing productive work and time during which it is idle for one or more reasons. The productive time generally includes :

Program running time

Testing time

Return time

Assembly or compile time

Demonstrations

Training

Non-productive time falls into these categories ;

Set-up time

Assembly set-up time

Testing set-up time
Scheduled maintenance
Unscheduled maintenance
Power failure
Idle time
Tape testing time

To ensure an efficient computer use there must be methods created for estimating each of these different times for each working day

In a business application, the productive or machine operating time varies almost directly with known and measureable parameters. Thus, in a printer or tape-limited system the productive time is in direct relationship to the printing or the amount of tape processing required by the program For any given application these factors may be calculated in advance, so that the onl do-to-day variable is volume, or number of records

With manufacturer supplied programs, a general timing formula is usually available The variables which effect the calculation are the file volume, the record length, and blocking factors Calculation of the standard time therefore entails simple airthmetic once the parameters are known.

If the approach is general, the objective is to establish standard percentage of compiling time in such a way that analysis of total machine usages indicates where total assembly or compile time has exceeded the norm Thus, if the monthly machine utilization analysis showed the following ;

Productive Time	154.3 hrs	40 2%
Compiling Time	15 4 hrs.	4.0%

the analyst, or the manager, would be able to recognise this as "acceptable" or not depending on the standard The standard, therefore, can be a percentage of total time, or percentage of productive time, or total number of compiling hours that can be considered acceptable under normal operating conditions

The specific approach relies on estimating the number of compilitations required for each pragram based on its size aud complexity It then estimates the number of programs that will be in various stages of preparation during a typical week and uses this to set a standard for compiling time

The standard time required to set up a machine system that does not operate under monitor control is a direct function of the number of equipment unit to be set up. There are two aporoaehes to developing standards for set up time The general approach standardises set up time as a percentage of productive time This is determined by study of the setistical time for each set-up operation such as mounting a tape, setting up a card reader, loading a printer, etc. The set-up operations and the running time are then determined for the average program and used to set the standard. The specific method uses the same device set-up times but computes them for

eech program and then, knowing the programs to be run, determines the set-up for each day

Many installations use a monitor program to assist program loading and scheduling When a monitor is used, a program tape is prepared. following by a tape containing all of the input for the series of programs to be run. Thus only set up necessary is the mounting the two input tape. printer and console set-up required for each run

In this case the set-up time is proportional to the number of monitor "runs" made By evaluating historical records, it is possible to standardise both the number of monitor runs and the number of programs each monitor run will use. A standard set-up time can then be derived.

The set-up times for a compiler run has very little in common with the standard for the program to be assembled. It is in direct relationship to the number of input and output units used by the *compiler*, and it will be the same for each and every compilation As a result it is possible to establish an exact set-up time for each compiler opeaation.

A simpler approach when using a multi-program compiler or assembler is to set aside a given time each day when all programs to be compiled will be run. This means, in effect, that there will be only one compiler set-up per day.

The standard used for the set-up time required for testing will approximate the set-up time for the same program after it becomes operational. The same unit must be set-up except that in testing the program it is usually in card form rather than one of tape and additional time must be considered for the loading of the program cards.

To determine a realistic standard for "down" time or unscheduled maintenance, the frequency and average length of occurrence should be used These figures can be obtained from the manufacturer of the equipment.

Return time is time lost because of an error. If an error occurs in the middle of a production program, the time lost is the time already expended. There are four distinct causes of lost time :

Data error
Program error
Operator error
Machine error

It is important to retain the distinction between these ; the last three reflect upon the quality of personnel performance ; the program error upon the programmer, the machine error upon the maintenance personnel, and the operator error upon the operator. It is extremely difficult to set a specific standard for retrun time.

The computer may be used to prepare analytical reports from the logs that records the time usage for each computer system. The major objective of analysis are :

To compare the actual performance in each category with the established standard.

To determine the effectiveness of personnel.

To account for and charge to the appropriate departments for services and to determine total rental due.

To determine trends, and recognize their impact on future data processing requirements.

To indicate management action where performance is not satisfactory, to optimise effectiveness of management policies, and to provide measures of the effect of such management action

3.5 2 Personnel Standards The performance standards for equipment discussed in the preceding sub-section can be used to derive performance standards for personnel directly associated with the computer For example, operator performance can be determined by how well the standards for productive time are met

Before the input goes for punching, it is usually scrutinised as input/output by Edit Department to ensure that it is complete and that controls have been provided for. Hash totals may be taken and document figures may be sample checked by this department Similarly, the editing of computer output or completeness and accuracy is as important as checking the input, the Edit Department should make certain that all controls were properly obeyed and that nothing has been added or omitted They are the final check output from the computer centre

In some computer centres the Edit Department is given the complete responsibility for the running of various jobs They collect the source documents, arrange for their punching, requisition for computer time, run the job, check the output and send it to the required department or customers It is their responsibility to ensure that all outputs are produced on time. This can be used as a performance standard for the edit department.

Performance standards for programming can be derived from the ability of programmers to meet established deadlines as well as the listing and compilation time used by their programs and their subsequent performance with the user.

The program parameters listed earlier—size, complexity and input/output complexity are also used to measure programming tasks. Similar parameters may be used for systems analysis but this is much more difficult and performance standards for systems analysis are rarely used.

The key factor is experience. All estimates are based on records of past performances. The most popular method is sub-dividing the programs into modules similar to jobs done before. Each module is then costed directly from past figures. The index of debugged instructions per hour is used as a performance standard rather than for costing.

Usually, two or three senior programmers are asked to estimate each job independently and the final estimates decided in a meeting to explain the differences. Progress is monitored by monthly reports contrasting actual and planned progress, highlighting items behind schedules and prescribing remedies for catching up Some companies use GANTT charts and even small PERT charts for this purpose.

The problem of estimating programming time is difficult enough even when based on good programming specifications derived from comprehensive systems reports. Without them or with poor specimens it becomes a nightmare—causing the chaos and the large time and cost overruns that have characterized a number of software projects

The quality of the program specifications also affects productivity Other factors, such as the availability of computer time for testing and programmer motivation also affect it in ways that are difficult to take into account in the estimates. Rigid, though realistic schedules must, however, be set and enforced Flexible schedules never do anybody any good

Conclusion

This study paper has approached the management of the data processing department through the establishment of standards for various aspects of the work to be performed This emphasises the importance of standaros in the management of the data processing effort but does not touch upon a few of the data processing manager's functions

The training of personnel is, for example, a very important function of his but can be taken care of, mainly by making use of classes offered by the computer manufacturer. Similarly he must ensure that personnal gain in experence in depth and breadth of knoweledge without being bogged down for ever in a routine job like payroll programming and without sacrificing the quality of the systems produced.

The safe custody of files has been discussed in the sections on programming and operating standards These procedures are adequate for most organisations but in some cases where computer files are the only records for the business far more stringent methods may be justified. An American insurance company, for example, makes duplicate copies of every file updated each day and stores them in a safe deposit vault at a secret location.

Similarly, we have discussed controls that constanly check for errors and fraudulence in the running of routine systems Again, these devices such as keeping totals of all records added and updated are normal businesses but may have to be radically extended in the case of application where errors or frand is more likely or more expensive

Self Examination Qnestions No 5

A Why is it difficult to create performance standards for systems analysis ? For managers ?

B. Would performance standards for operations and programming be different for different programming languages ? Justify your answer.

C. What steps would you take as the manager of an EDP instalation if you found you were getting insufficient production time ? What if there was excessive maintenance times ?

F. III B-11

THE INSTITUTE OF CHARTERED ACCOUNTANTS OF INDIA

STUDY MATERIAL F S P. S A & D P—5
Combination 'B'

FINAL COURSE
SYSTEMS ANALYSIS AND DATA PROCESSING
STUDY V

Contents :

Suggested Reading · Selecting the Computer System
By Chorafas (Gee & Co)

1. INTRODUCTION

It is widely believed that computers are very expensive and that a company must be large and prosperous to be able to afford the luxury of computer processing. It is also believed that smaller companies can make a start towards data processing by using punched card equipment or "unit record" equipment (Ref: Appendix) as it is sometimes called

Technological improvements have invalidated both these beliefs. A few hours of computer time can be hired at low cost for selected application and properly conceived and implemented, the effort can be very profitable. Developments in computer technology have led to better facilities at lower cost while unit record equipment has remained unchanged for many years. This has eroded the unit record market and today it is difficult to justify the use of these machines, save for special applications

A manager usually decides to investigate automatic data processing when his clerical department begins to show signs of overload through delays, errors and customer complaints Sometimes he may feel that certain aspects of the organization, like the sales effort or inventories are inadequately controlled or that some functions such as production planning, selection of an investment portfolio, or distribution planning could be improved through better analysis Or, more rarely the manager of an i' ˆant company realizes that he will ultimately have to rely on data processing and decides to build up his method and procedures with this in mind This is the best way to start, since it avoids the traumatic change in work patterns and thinking that accompanies the replacement of manual system by computer system It also avoids the problem of surplus labour that has held up data processing proposals more than any other factor except, perhaps, management's unfamiliarity with computers

Having decided to consider data processing, the manager faces the following alternatives :

1 A manual system (if data processing is not found to be justified)
2 Installation of punched card equipment.
3 Contracting the implementation of the system to a consulting organisation
4 Hiring computer time from a service bureau.
5 Acquiring a computer system
6 Sharing a computer installation with one or more organisations.

For the last three alternatives, staff will be required and a department built up to apply and run the data processing equipment. Consultants may or may not be retained

Manual systems can boast of intelligence behind every action. This gives them tremendous flexibility and allows them to cope with widely different conditions, correct many unforeseen types of errors and cater to a fantastic diversity

of requirements They do, however, have their limitations and these increase as the volume of processing increases Moreover, they are expensive and unless flexibility is required, there are cheaper ways of carrying out the routine processes that occupy most of the effort of manual systems

Mechanisation, however, leads to rigidity and specialization This is true of whatever human function is mechanized Man's two legs enable him to walk on hard roads, over sand, through quarries, construction sites, slush and jungle A particular mechanical transport may be faster and more efficient under any one of these conditions but will be very inefficient under others.

Thus, all mechanization requires is that the scope be closely defined and such detail carefully specified Effectiveness and advantages are related to particular requirements A detailed examination of the system and procedures to be mechanized is therefore necessary This is very expensive and to it is to be added the cost of programming and the cost of putting the existing information on the computer Some staff retaining and other "change cost" may also be involved All these are initial costs that have to be borne before the system is operational. The total initial, or one time, cost may be quite large in comparison to the recurring cost of producing the required outputs and reports This initial investment is required for EDP systems before they yield results and often, even before the manager really knows what to expect This makes the decision to mechanize a very difficult one Going in for EDP often requires leap of faith on the part of management.

1.1. Installing an in-house computer Vs. hiring computer time

Service bureaus provide data processing services to clients on the equipment which they own, usually on batch processing basis. The clients are charged on time or item basis at standard rates to transport their source input documents to the installation and collect the outputs The bureaus, usually, offer generalised programs but a special program at an extra fee can also be written The following points may prompt an organisation to go in for such services

Installing an in-house computer or hiring computer time is basically a capital budgeting decision. The ROI's for the two alternatives have to be computed and compared. However, such an analysis may not be explicitly conducted by smaller organisations who may obviously find an in house computer an expensive proposition indeed Similarly, larger organisations with a fair deal of complexity in their operations may dismiss computer time straightway In between there are medium type organisations and large organisations with rather straightforward operations who would have to carefully weigh the two alternatives on cost benefit grounds Whilst making this comparison, planned growth or termination of the organisation has to be kept in view. Besides, the following points may favour hiring computer time

(i) An organisation with an in-house computer may also want to avail an outside service during periods of peak processing volume or major system breakdown. It may want to avail of all the specialised equipment programs or expertise offered by the bureau.

(ii) An organisation may want to start with the service bureaus with few

applications As the computer installation builds up to the point where the cost of computer time approaches the cost of renting a machine, it may establish its own computer department.

(iii) Even if a company can justify the use of its own computer it acquires the capacity that may not be utilised for quite some time. Usually, it takes at least 1 to 2 years to prepare and implement enough applications to ensure a reasonable utilisation for the machine after the arrival For this time, it makes little sense to pay for a loss of unused capability. Many users, therefore, find that the best way to start is with a service bureau.

However, there are also time sharing vendors who provide with a fee the usage of central computer and on line file storage to users who obtain access through remote terminals and tele-communication lines. This facility is analogous to that of gas or electricity supply. Such a facility is particulary useful for interactive problem solving which means that a user can quickly get a solution to a problem, structure another problem around this solution, get, another solution and thus iterate to a satisfactory solution. Time sharing vendors also generally offer a large number of specialised programs, many of which a user may find really helpful The cost of a time sharing service generally includes a fixed monthly charge for the terminals and other equipment and variable charges for terminal hook up, central processing and file storge used.

One should note however that by hiring rather than having in-house computer the user relinquishes control of vital business data and the in-house computer offers the possibility of strict security measures and stringent internal controls Also the users may find the generalised programs unsuited to their requirements and special programs rather too expensive. There could also be inordinate delay which is natural since the bureaus serve other clients

Consultants Vs. own staff

If the company has only a few applications to be computerised, there is no reason why it cannot entrust the job to a consultant, rather than employ a whole-time man to do the work. A member of the staff must, however, understand computers and liaise with consultants. He should have overall responsibility for the success of the computer effort and, preferably this should be his only responsibility.

One cause of many an unprofitable computer effort is the inadequate appreciation of the fact that systems analysis and the choice of areas of applications are a specialised professional job that requires a trained information technologist. Most of the systems that do not pay their way are ill conceived, poorly designed and probably should never have been computerised in the first place. At the same time there may be other areas in these very companies that could very profitably be computerised It takes an expert to identify and circumscribe the areas of maximum return.

By using a consulting organisation to initiate him into the mysteries of EDP, the user can avoid a number of inital pitfalls. The consultants will usually have experience and expertise in areas he wishes to computerize and they will

invariably be able to give him a more capable and more economical system than his staff would be able to design. Error checks and system control will be introduced as a matter of course by the professional. These can make all the difference between system success and failure. Moreover, the consultants will be able to assist the user in formulating an overall plan for the computer effort. This is *the* area where they can make the most significant contribution.

As discussed above, though consultants are greatly helpful in the initial development of MIS, there is an advantage in building up one's own expertise in data processing. Larger organisations can afford own data processing staff and that will help them to adapt the MIS in the face of ever-changing business objectives and continually triggered projects during the systems audit phase Consultants would take far too much time to understand the nuances and would therefore be an expensive proposition. The organisation that can build up its own computer expertise and became self-sufficient will find it quite profitable in the shape of prompt and tailormade system; after all, there is an advantage, say for a college, in building up its own facility rather than depend on outsiders for lectures.

Computer Industry The buyer must obviously be well familiar with the computer industry which is not just a few leading manufactures like IBM or ICL but a host of other agencies The computer industry can broadly be categorised into three major groups hardware suppliers, software suppliers and EDP services suppliers. Some of the major firms within these broad grouping are listed below :

1. **Hardware suppliers**

    Computer manufactures
    Independent peripherals manufacturers
    Data processing supplies companies
    Computer leasing companies
    Used computer equipment companies

2 **Software suppliers**

    Computer manufacturers
    Independent software companies
    User developed software suppliers

3. **EDP services suppliers**

    Computer manufacturers
    EDP service bureaus
    Time sharing service companies
    Facilities management companies
    Other EDP service suppliers as listed below.
    (a) Computer time rental
    (b) System design services
    (c) Contract programming
    (d) EDP selection
    (e) EDP education
    (f) Hardware maintenance.

Hardware Suppliers. Obviously, computer manufacturers are the leading

hardware suppliers. They manufacture and market CPU's, peripherals and data processing supplies Likewise, the small computer manufacturers produce and market mini-computers, visible record computers, special-purpose computers, and their peripherals. However, since recently, there has been a growth of independent peripherals manufacturers who specialise in the production of peripherals which they market usually at lower rates than those of the computer manufacturers. This is also the case with companies producing data processing supplies alone and marketing them at lower rates than those of the computuer manufacturers. Of late third party companies have been growing, these purchase computers from computer manufacturers and lease them to computer users at rates that may be 10 to 20 percent lower than the computer rental price.

Software suppliers If a company chooses not to develop its own software it can go in for software packages in the market The computer manufacturers, again, are the largest software suppliers though now a days many independent consultancy organisations who have package programs or can write special programs are proliferating Also, other computer users whose programs could be of use may be contacted in acquisition of program packages Such users may either give these packages free or at very low cost

Selecting the Computer Systems.

In case of procuring such machinary as machine tools, transporation equipment, air conditioning equipment, etc the management can normally rely on the time tested selection techniques and the objective selection criteria can be delegated to the technical specialist But computer is not just another machine This is not only because of its highly complex internal structure for which myriads of performance measures can be devised, though none of which satisfactorily describes the power and the capabilities of the computer, but also because of its open-endeness in that it can admit connection of a large number of peripherals with deffering capabilities in the near future and later Also, computer has a profound and long range effect on company's operations The user depends upon the buyer for support services systems design, education and training etc., and expansion of the computer installation for almost an indefinite period, therefore, this is not just buying the machine and paying the vendor for it but it amounts to to a marriage with the supplier Also, to make a proper choice of the computer and its peripherals, it is important that the job mix for which it is intended is precisely defined at the outset and this is rarely possible, not to speak of the fact that the job mix itself is liable to under-go considerable change with the changing objectives of the company

All computer systems offered in the market today have good hardware, competent software and roughly similar facilities Due to the rapid development of computer technology, the more recent the computer the better its performance and the lower its cost Therefore, as far possible, the latest possible technology should be acquired As long as this general principal is adhered to, the choice of a particular computer will not significantly effect the success of the computer effort.

Commercial data processing consists mainly of reading-in data, printing out data and storing and retrieving information from magnetic media. Thus, computer

performance for commercial work is mainly determined by the speeds and capabilities of input/output and storage peripherals. Scientific, engineering and operations research problems require good computational facilities Thus, the efficiency of a computer in handling such problems will depend on the core store available and the instruction execution speed, and *repertoire* A comparsion along these lines can be made quickly and quite effectively Detailed consideration of the specialities of each machine is rarely worthwhile as these minor features make little difference to the overall performance, What may be important are differences in facilities that affect the suitability of the computer for the particular applications in mind Thus one manufacturer may provide a printer that can print a line of 160 characters while another may provide only 132 characters If one of the applications is customer billing, this may make the difference between printing three instead of two bills side by side at the same time. Since time can be saved by printing three bills, the 160 character printer may lead to a much cheaper system Similarly, IBM was for some time the only manufacturer offering a large exchangeable disc facility. This gave its equipment an advantage in the case of users who wanted large random access files.

In addition, the software supplied by the manufacturer may make a significant difference if it contains a package of special applicability to the jobs envisaged. Experts maintain that since hardware speed and facilities are uniformly good today the selection of a computer should be made on software considerations While all manufacturers supply general software packages for linear programming, PERT etc, and packages for such ubiquitous applications as inventory control and production planning, a few manufacturers also offer packages specially designed for a particular industry such as IBM's "IPARS" packages for airline reservations and packages for applications in banking and insurance Obviously a manufacturer who has a package for a user's special needs will enjoy a significant advantage. In general most manufacturer s packages are good There are, of course, individual differences but these are usually not too important The ICL package for PERT is *said to be superior to all others, but the superiority may not be strong enough to* a computer acquisition decision.

Modern computers are marketed as series of compatible machines with increasingly powerful central processors and interchangeable peripherals This is very useful when, after a few years, the work expands to fill the existing computer storage and a bigger machine is required. A more capable machine of the same series will allow the existing programs to be carried over so that the large investment in programming may be preserved In case of breakdown, compatibility allows a wider range of machines to be used as back-up Thus the choice of a computer really becomes the choice of a model within a series, based on a long range plan of expansion This however, means that one is married to the manufacturer and subject to his whims and fancies. But marriage is preferable to reprogramming which can be prohibitively expensive.

The selection of a computer does not end with the choice of a manufacturer and a model. It continues to the selection of a configuration and a plan for its

gradual expansion. This can be as important as the choice of manufacturer and properly considered, can save a great deal of money.

Outside assistance in computer selection can be had from the computer manufacturers, independent consultants specialising in computers, or management consulting firms The distinction between vendor selection and machine selection is to be carefully noted at this stage. Choosing the vendor is essentially a matter of business judgment and prerogative of exercising it must be retained in-house. Once, however, the vendor is selected, he can be counted upon to provide useful guidance for machine selection also.

In vendor selection, the management has the choice between a single source and the multiple sources In the former case, the vendor is pre-selected on the basis of his reputation etc or because of standards consideration in multiple plant operations i e , a company using one brand of computers in its various plants may want to acquire the same for another plant. Also, a company with an existing computer may not want to switch brands and may wish to acquire the computer of a bigger size from the same source in order to avoid the need of reprogramming, file reorganisation and aclimatisation to a strange machine Not only the cost and capacity of a vendor's machine are to be considered but also his reputation, his research and development activities, his company's financial viability, software support, maintenance services, commitment to application development, education and training facilities, and so on

Validation of vendor's Proposals

This process consists of evaluating and ranking the proposals submitted by vendors and is quite difficult, expensive and time consuming, but in any case it has to be gone through. This problem is made difficult by the fact that vendors would be offering a variety of configurations The following matters have to be considered towards rigorous evalutaion ·

Criteria. Mandatory requirements would constitute over-riding criteria in that, if a vendor fails to meet them, he would be screened out without any further consideration. The desirable characteristics would surely be more difficult to evaluate because the vendors may ignore them or offer several alternatives The criteria may be listed in a descending order by importance as below .

(i) Software performance

(ii) Hardware performance

(iii) Support services

(iv) Compatibility

(v) Expansion capability

(vi) Delivery

(vii) Cost.

These criteria may have to be further sub-divided particularly for hardware performance and support services.

Methods Having established and ranked the criteria comes the question of validating the vendor's proposals against these The method selected may be a simple or a sophisticated one. Larger organisations would naturally tend to adopt

a sophisticated and objective approach. Following are some of the validation methods.

(1) Check lists. It is the most simple and rather a subjective method for validation and evaluation. The various criteria are put in the check list in the form of suitable questions against which the responses of the various vendors are validated Obviously, then, the vendor who has most cleverly and rhetorically worded his response is likely to get the award. Below we give an example on a software checklist and a support services checklist.

Example of a software validation checklist

(i) What is the package designed to do ?
(ii) Who developed the software package ?
(iii) How is the package organised ?
(iv) Is the package operable ?
(v) Can the package operate on our hardware configuration ?
(vi) Will the package require modifications ?
(vii) Who will modity the package ?
(viii) Can the package be modified if necessary ?
(ix) What are the overall costs ?
(x) Is comprehensive documentation available ?
(xi) Who will maintain the package ?
(xii) What are the package constraints ?
(xiii) Where is the package currently utilised ?
(xiv) What is the primary language ?
(xv) What input/output techniques are utilised ?
(xvi) What are the required input/output formats ?
(xvii) How must input be organised ?
(xviii) What controls are included ?
(xix) What user training is provided ?

Support Services Check lists

(i) *Performance :* What has been the vendor's past performance in terms of his past promises ?

(ii) *System development* : Are systems analysts and programming consultants available ? What are their qualities and cost ?

(iii) *Maintenance* · Is equipment maintenance provided ? What is the quality and cost ?

(iv) *Conversion :* What systems development, programing and hardware installation service will they provide during the conversion period ?

(v) *Training* Are the necessary training of personnel provided ? What is its quality and cost ?

(vi) *Documentation.* Are the necessary hardware, software and application manuals available ?

(vii) *Back up* Are several similar computer facilities available for emergency back up purposes ?

(viii) *Proximity* Does the vendor have a local office ? Are sales, systems

development, programming, and hardware maintenance services provided from the office ?

(ix) *Hardware.* Do they have a wide selection of a compatible hardware ?

(x) *Software* Do they have a wide variety of useful systems software application programs ?

Similar check lists may be developed for hardware compatibility etc. This method, however, is generally applied only to minor proposals

Criteria Tables The various criteria are listed in the descending order of their importance The importance of the various criteria are assigned suitable values. The responses of the various vendors are then assessed against these criteria by assigning response value against each criteria. The criteria value multiplied by the response value furnishes the response score which is accumulated. An example is given below.

Sample extract of criteria table with weighted values for a software comparison.

		Vendor A		Vendor B	
Desirable Characteristics	Criteria value	Response value	Response Score	Response value	Response Score
Availability	6	6	36	12	72
Cost	5	6	30	4	20
Reliability	4	6	24	7	28
Documentation	3	5	15	9	27
Maintenance	3	7	21	3	9
Training	2	7	14	6	12
Data Management	5	8	40	6	30
Total	28		180		198

The values assigned to the various responses remain subjective to quite an extent and this method is only slightly better than the check list method.

(iii) **Public evaluation reports** Several consultancy agencies compare and contrast the hardware and software performance for various manufacturers and publish their reports in this regard This method has been frequently and usefully employed by several buyers in the past For those criteria, however, where published reports are not available resort would have to be made to other methods of validation This method is particularly useful where the buying staff has inadequate knowledge of computer facts.

Ratings In acquiring a computer the manager wants to know how much "work" it can do in contrast to its "cost" to arrive at a measure of performance. The difficulty is that it is practically impossible to come to a reliable estimate of the "work" capacity or the "throughput" as it is sometimes called from a study of the specifications Indeed, a good estimate can only be obtained by testing its performance on standard commercial and scientific jobs. For this

resaon, a number of people have suggested simultation studies in which typical jobs are programmed on the computers to be evaluated and the time taken to execute them used as a measure of performance The earliest such test was the Gibson mix rating, named after the IBM employee who devised it. In this a special program containing the instruction mix of a typical program was created and run on different machines Such a test, however, grades only the efficiency of the central processor which is relatively unimportant for commercial purposes. A more basic defect, and one that plagues all instruction-mix tests, is the fact that, since jobs are programmed differently pn different computers, the typical instruction mix differs from machine to machine

A somewhat better yardstick was devised by the British General Post Office. A "work unit" was defined as the smallest amount of work that retains all the properties of a typical application for timing purposes Since the work of the Post office is mainly commercial, a commercial work unit was devised in 1962 and improved in 1967 Ratings of most computers are available in terms of the time taken for a Post Office unit II (POWU II) and these are more or less reliable guides to commercial performance.

In 1968, Dr K E Knight published a rating of available computers on the basis of performance over a representative-mix of scientific and commercial problems. The most valuable feature of this study is that it not only rates commercial and scientific performance separately for each machine but also attempts to establish a performance-per-dollar criterion

In general, performance rating is a difficult problem as each test has its particular limitations and biases and none truly represents the user's job mix. Published figures are available but should be used with care Indeed, the very concept of performance rating can be questioned as it implies that the computer is to be acquired for its general suitability rather than its suitability for the particular jobs at hand Thus while performance measures can be taken as broad guidelines, detailed evaluation of the proposed application system is a much better criterion for computer selection

Bench mark problems Bench mark problems are oriented towards testing whether a computer meets the requirements of the job on hand i e, they are required to be representative of the job mix Obviously bench mark problems can be applied only if job mix has been clearly specified The bench mark problems would then comprise *long jobs short jobs, tape jobs, disc jobs, mathematical problems, input/output loads* etc., in proportions typical of the job mix If the job is truly represented by the selected bench mark problems this approach can provide a realistic and tangible basis for comparing all vendors' proposals Tests should enable the organisation to effectively evaluate gross performance of the system in terms of hardware performance (CPU and input/output units), compiler language and operating system capability, diagnostic messages, ability to deal with certain types of data structures and effectiveness of software utilities It may also be possible to test a particular configuration. Bench mark problems approach

suffers from a couple of disadvantages. It takes considerable ingenuity to select problems representative of the job mix which itself must be precisely defined and requires the existence of operational hardware, software, services or systems. Nevertheless, the approach is very popular because it is in the nature on acid test of the functioning of a vendor's proposal. The manager can extrapolate in the light of the results of the bench mark problems, the performance of the proposals on the entire job mix.

Test problems Test problems disregard the actual job mix and are devised to test the true capabilities of the hardware, software or system. For example, test problems may be developed to evaluate the time required to translate the source code (program in an assembly or a high level language) into the object code (machine language), response time for two or more jobs in a multi-programme environment, overhead requirements of the operating system in executing a user program, length of time required to execute an instruction, etc The results, achieved by the machine can be compared and price performance jugdment can be made. It must be borne in mind, however that various capabilities to be tested would have to be assigned relative weightages

Growth and Compatibility The buyer should also keep in mind the inevitable growth of computer applications. Growth can occur in one or more of the following three lines. Demand of company's products increases or the product line expands resulting thereby into large volume of transactions. However, such an expansion poses a rather simple problem that can be solved by merely adding more input output units, storage devices and building blocks of memory. Possibility of expanding the memory this way and enhancing versatility of computer configuration is known as *Modularity of hardware* The other direction in which growth can occur is the increase in the number applications which may or may not be independent of the existing application. If the applications are independent of the exisiting applications the problems is identical with the one discussed for increase in the volume of transactions. In the case of other applications which are not independent of existing ones, some modifications of the existing programs would be essential A more severe direction that expansion can take consists of sophistication of the existing applications. For example an inventory control program, hihterto using a simple exponential smoothing sub-routine may be desired to be changed to a more sophisticated one incorporating tracking signals and adaptive response (Ref, O R. Study II). Or, the files may be desired to be fused into a data base This would call for drastic changes in both the files and the existing programs and even require more advanced peripherals. The buyer must therefore, keep in mind these directions of expansion.

The inevitable consequence of expansion in any of the directions discussed above is the need to step up the capacity of the central processing unit by acquiring a largen model This, in modern computers, is permitted by what is known as compatibility. Computer manufacturers offer a series or families of computers that are compatible in the sense that they possess the same ergonomic design or architecture, use the same set of instructions and have the same basic hardware design logic. Apparently, the implication of this is that the existing computer of the user can simply be upgraded to the next higher model in the family without any need for re programming i.e., upward compatibility.

This is true to some extent and there is a problem to it discussed in the following paragraph. But downward compatibility may or may not exist depending upon the size of the program A large program cannot be used as such if it requires more space than the contemplated reduced memory. For example, an inventory control program, at present handling (i) customer's orders, for stock allocation and placement of replenishment orders and (ii) receipts may have to be split into two programs to be run in two set ups. Other programs which, however, can be accommodated in the reduced memory can be used without any alterations whatever.

Upward compatibility poses a problem of its own. No doubt the existing programs can be used on buttressed memory yielding identical outputs but this may be inefficient. It is not at all unlikely that existing programs were written with economising on the storage space as the leading objective If these programs are now loosened and files unbundled, more storage space would be needed, which is now available anyway, but this would lead to substantial enhancement of the processing efficiency i e, less processing time would be needed to execute a loosened program on unbundled files Also, more powerful and versatile compilers and operating systems can be used on buttressed memory Thus program modifications would be necessary to tilt the emphasis from economising on storage space in favour of processing efficiency Now, the computer systems tend to expand in these directions very rapidly and this would indeed tie up the entire programming staff doing program modification in favour of processing To alleviate this situation, to some extent, the ABC principle of inventory control can be profitably employed here Most often used programs can be modified as and when the expansion takes place whilst rarely used programs may be retained as such without any modification

Getting only what you need A great deal of money can be saved by realizing that computer utilization is built up only gradually, and that the configuration can be strengthened in stages as the applications increase This can be viewed as a short range version of the "moving up" discussed earlier, but substantial economics are still possible.

The case of an Indian company that started with an IBM 1401 computer with the maximum possible core memory, six fast tape drives and disc units, is typical. For six months the discs were completly idle After that a disc based material planning and production control system was started This limped along, using about 10 hours of disc time a month for a year, after which it was discarded as being ill-conceived. Meanwhile, some other disc-oriented systems were hastily devised and at the end of two years the disc usage was around 20 hours a month.

Obviously, the discs could either have been dispensed with, the disc-based systems being implemented somewhat more crudely on tape, or disc facilities could have been rented to begin with, on another computer. The failure of the initial disc-based production planning and control system was due to inexperience and inadequate systems study before implementation The disc rental was Rs. 10,000

per month and at least two years' rental or about Rs. 240,000 can be said to have been wasted.

Similarly, it was unnecessary to start with six fast tapes. Four slow tapes could have been acquired in the beginning and later changed as the usage increased. If the argument that this would lead to more limited systems was strong, six slow tapes could have been acquired and later replaced by faster ones. The difference in rental would have been almost Rs 12,000 per month.

A small configuration can be enhanced in three complementary ways :

1. Speeds of peripherals can be increased.
2. Core memory can be increased.
3. The number, and capability of peripherals can be increased.

The easiest and most painless form of capacity enhancement is when a line printer, card reader, tape drive or other peripheral or even a central processor is *replaced by a faster unit with identical characteristics, i e that does the same* job, in the same way, in less time. This is, of course, difficult with purchased equipment, although the manufacturer may sometimes allow the slower unit to be "traded in" in part payment In U S A. and Europe, companies dealing in used computer equipment have sprung up These will buy the old unit at a half to a third of its original price and offer these to customers at a substantial reduction.

With a rented configuration, the only loss may be a part of the one time charge and this may be well worth the differential saving in a year or two's rental between the faster and the slower peripheral.

Increasing core memory, the number of tape drives, or the disc storage capacity has slightly more complex repercussions Existing systems may have to be re-programmed to some extent to take full advantage of the more powerful configuration. But the reprogramming required is usually not extensive and can be minimised by a carefully planned sequence of system implementation.

The simpler system should be implemented first. These usually take a few basic peripherals. Later they can be extended in scope, complexity and sophistication. The programming staff can often do well by getting started on simpler magnetic-tape systems graduating later to disc systems and perhaps much later to visual displays and other more sophisticated equipment. In fact, the enhanced configuration usually provides better ways of implementing systems—the more complex the system the more the improvements possible.

Purchase or rent : After deciding which computer to acquire, the basic configuration. and a general plan for its expansion, a further decision has to be taken as to whether the computer should be purchased or rented Until a few years ago, most computers were rented on the assumption that technological progress would soon make them obsolete. The rapidity with which new computers are being introduced has, however, slowed down. Also, managers have begun to realize the enormity of the problems associated with changing computers. A purchased computer that has been performing satisfactorily for, say, three years has already paid

for most of its cost and there would seem to be little reason for changing over. The operating costs of computers are small compared to their purchase price or rental and there can be little justification for additional expenditure just to reduce running costs. Even if new computers are not introduced, the computational capability still exists, and one can carry on the work for many years until the requirements substantially expand or alter. Besides, the cost and retraining involved in a change-over are avoided In very special cases the new equipment may provide the possibility of radically different and more effective systems and this may justify replacement.

The purchase price of a computer is usually calculated at about four years' rental. Since additional maintenance charges have to be paid on purchase equipment the break-even period is around five years The useful life of the equipment is, however, much longer

A plan for gradually increasing the power of the configuration, however, discourages purchase. Rented equipment is generally much easier to replace Companies dealing in used computer equipment will, however, buy used peripherals and replace them by (used) faster ones at very good terms Another solution is to purchase the part of configuration that is supposed to be usable for, say, three or four years like the central processor, and rent the rest that is expected to be replaced sooner.

Purchasing rather than renting equipment can also carry some financial benefits In India, a development rebate is allowed in the form of a concession on purchased equipment provided some other conditions are met Also, import licences and investment grants, in the form of Government loans at low rates, make purchase look more attractive

Expenditure on data processing

Expenditure on data processing and the distribution of expenditure varies widely with the nature of the company's business and the applications it decides to implement. It also varies with the number of years of computer experience.

A survey conducted by Booz, Allen and Hamilton[1] in 1967, analysed the average expenditure on data processing of 108 U S companies. The average expenditure over all companies was found to be 0 57% of sales With this standard, an Indian company with a turnover Rs 1 crore should spend Rs 57,000 per year on data processing It must be realized that in the U.S a large fraction of the expenditure is for applications that replace clerical labour. In India, these applications are not so important due to the lower manpower costs. Thus, even if computers were applied in India with the sophistication of the West, the expenditure should be considerably lower.

This study also showed that the percentage of sales spent on EDP increased with the number of years of experience the company had with EDP. This is not surprising if viewed from an organisational point of view. The computer department, once established, tends to proliferate and expand to more sophisticated and more

1. Neal J. Dean, 'The Computer Comes of Age', Harview Business Review. January, February, 1968.

expensive hardware, larger and costlier staff and multifarious applications. Payoffs, however, usually do not keep pace with increasing expenditure. An indepth study of thirty-six computer users by McKinsey and Company revealed *no correlation between the expenditure on computers and the benefits obtained.* Some companies are spending heavily for a very dubious payoff, others with less ambitious programs are reaping major rewards".

The average for the various types of industries tends to indicate the potential for computer applications. Industries using fabrication and assembly operations spend more on EDP as they are inherentty more complex and difficult to co-codinate than slow process. Also, industrial products require more expenditure on EDP than consumer products as they tend to be large and more complex and sophisticated in their manufacture and present greater opportunities for computer control and optimisation.

Analysis of computer costs by function are shown Table 1 for both the McKinsey and the Booz Allen Study. For every dollar that the American user spends on hardware he spends almost two on staff costs In India, this ratio would be somewhat less as labour costs are lower.

TABLE 1

ANALYSIS OF COMPUTER COSTS

		McKinsey Study	Booz, Allen Study
Rental and Equipment Costs		35%	38%
Computer Operations		30%	33%
Systems Analysis and Programming		35%	29%
Program Maintenance	15%		
New Programs	20%		

The McKinsey study shows that as much as 15% of the total costs is on program maintenance. This may come as a surprise to many who consider that once a set of programs is operational. they go on running for years without much additional effort. Good systems are, however, always improving in scope. Some 20% of the total cost is on new programs, often the only cost that is directly controllable and the first to be cut in the face of criticism and dissatisfaction with computer performance Yet this expenditure, less than the total hardware cost, holds the key to future profits through new developments.

The Booz Allen Study (1968) also analysed the utilization of computer effect among the various areas of the company. They also asked company personnel about the future systems they were planning. Table 2 summarizes their findings and indicates that the emphasis from routine applications in finance and administration is going to shifit to other areas.

TABLE 2

COMPUTER UTILIZATION IN THE FUTURE

Application Areas	Development of Computer Effort Now	In 3 to 5 years
Finance and Administration	44%	29%
Planning and Control	3%	7%
Research, Development and Engineering	11%	10%
Operating Functions		
Production	19%	22%
Distribution	10%	12%
Marketing	13%	15%

In the U S, most companies have already reaped the quick returns available from computerizing the routine, labour-intensive applications Now they are going into the more complex functional areas and into planning. These applications provide much higher payoffs, but they take longer to implement and their success is more critically dependent on how well the application is implemented.

Self-Examination Questions

A. You are selecting a computer for a commercial installation Rank the following features of the computer in order of their importance to your decision ·

1 Memory access speed.
2. Card reading speed.
3. Tape speed.
4 Tape capacity (i.e., how much information it can hold)
5. Disc speed
6. Disc capacity
7. The format of storing information on disc.
8 Printer speed and line size
9 PERT and Linear programming Software,
10 A COBOL compiler.
11. A FORTRAN compiler.
12 A payroll program
13. An inventory control program

B. If you can rent a computer for Rs 1,00,000 a month or buy it for Rs. 40,000,000, what would you do ? What other information would you need for your decision ? Describe your decision process.

C. How does the application plan assist you in "getting only what you need" ?

APPENDIX

Unit Record Equipment (Syn. Punched Card Machines)

Unit Record Equipment comprises the following eight electromechanical machines which operate on *only punched card* files :—

1. Keypunch
2. Card Verifier
3. Card sorter

These have been discussed in the context of the computer earlier

4. Reproducer
5. Interpreter
6. Collator
7. Calculator
8. Accounting Machine

These will be discussed below.

The nomenclature "Unit Record" arises because each punched card must carry one record unlike the computerised systems in which one card may carry more or less than one record. Unit Record Equipment was used extensively in business situations prior to the advent of the computer; but now-a-days, it is giving way to the computer. In fact, even the second generation computers had beaten it on cost benefit grounds.

Below, we discuss machines, 4 to 8 listed above.

4. *Reproducer* (*Syn. Reproducing Punch*) is used to copy all or part of a deck of punched cards into another deck of cards. It can reproduce in a straightforward way i.e column by column duplication of a deck of cards. It can also reproduce with offset, shifting fields from one position on the original deck of cards to different position on the cards in the new deck. Finally, it can do gang punching which means punching of all, or a selected part of the data on the master card into all cards which follow it. These 3 modes of reproduction are shown on the next page by figs. 1 to 3,

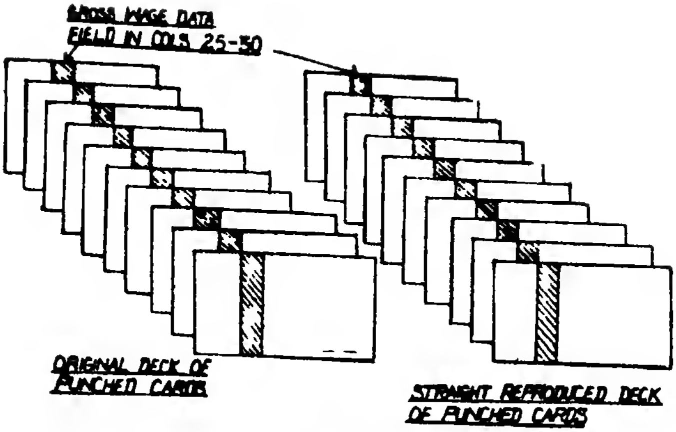


Fig. 1

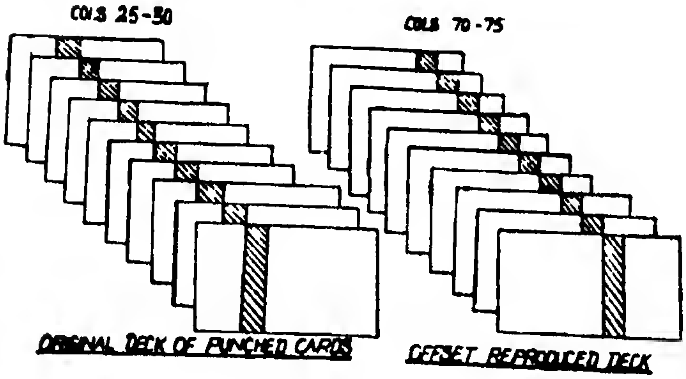
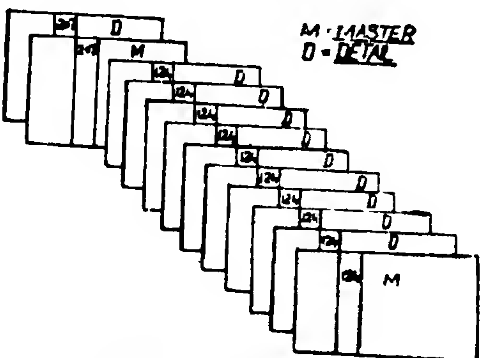


Fig. 2



GANGPUNCHING (STRAIGHT)

Fig. 3

5. *Interpreter*

Interpreting is a means by which punched cards which do not contain printing at the top can be read, and the information read printed on the top of the card. Depending upon the particular model of interpreter used, printing will take place on one or two lines throughout the body of the card. Alphabetic or numerical information both can be printed by the interpreter.

It may be noted that some of the keypunch models not only punch the card but also do interpretation at their tops at the same time.

6. *Collator*

A collator is basically used to merge two different files of cards It has two input feed mechanisms (Figure 4) and can drop cards into as many as 5 output hoppers.

Merging is the process by which two decks of cards arranged in some predetermined sequence, are combined to produce one large deck of cards in the same sequence. Fig 5 shows, for example, the merging of two files : Customer Order File and Customer Name/Address File.

Other capabilities of the collator are discussed below.

Select specific cards from a file. Fig. 6 shows, for example, the selection of master cards from a file. In a way, selection is decollation.

7. *Calculator*

It is a machine that can do addition, subtraction, multiplication and division. It can be put to 2 types of uses.

(i) Data calculated from a card is punched in same card, viz. net wage of a worker computed from his basic pay, allowances, deductions, etc is encoded in the last 5 blank column of the same card.

(ii) Data calculated from a set of cards is punched on a summary card.

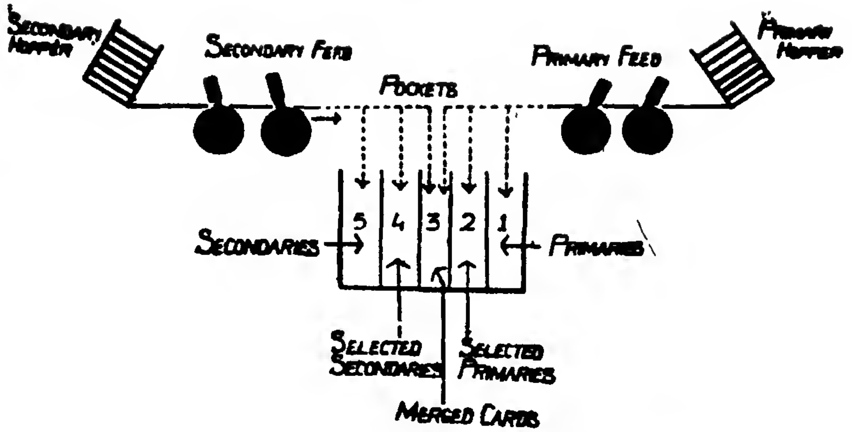


Fig. 4

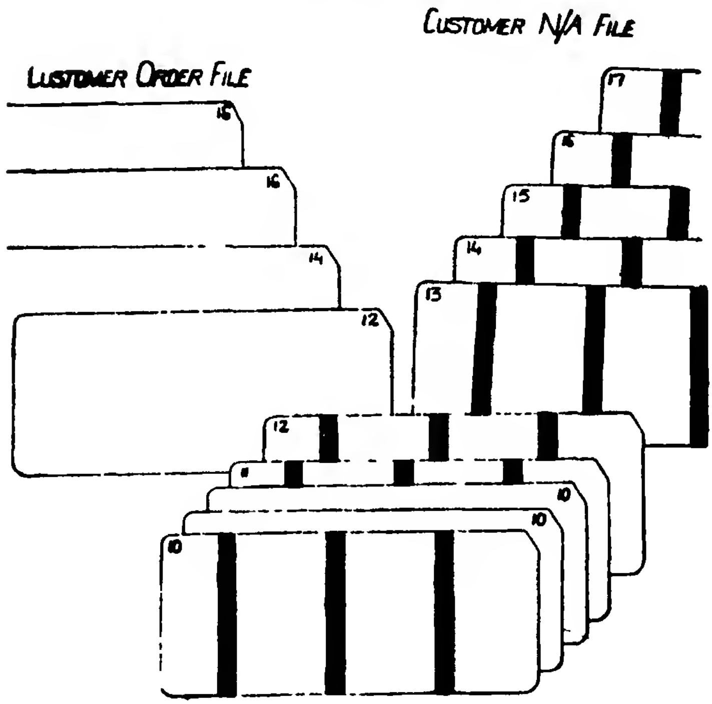


Fig. 5

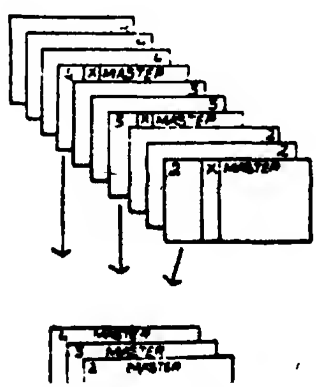


Fig. 6

Match merging : It means merging two files (fig. 7), selecting out all those cards that do not have a match in the other file.

Check the sequence of a deck of cards (fig. 8), routing any out of sequence cards to a reject hopper.

Simply select cards, from either of two files, which do not have a match in the other file (select all cheques for which there is no master file entry, for example).

8. *Accounting Machines* possess the following capabilities :—

1. Addition/substraction (in some models also multiplication/division) using a series of counters.

2. Printing the data punched in a card, as well as printing the contents of its counters.

3. Punching the contents of its counters into a card through a reproducing punch; and

4. Printing reports.

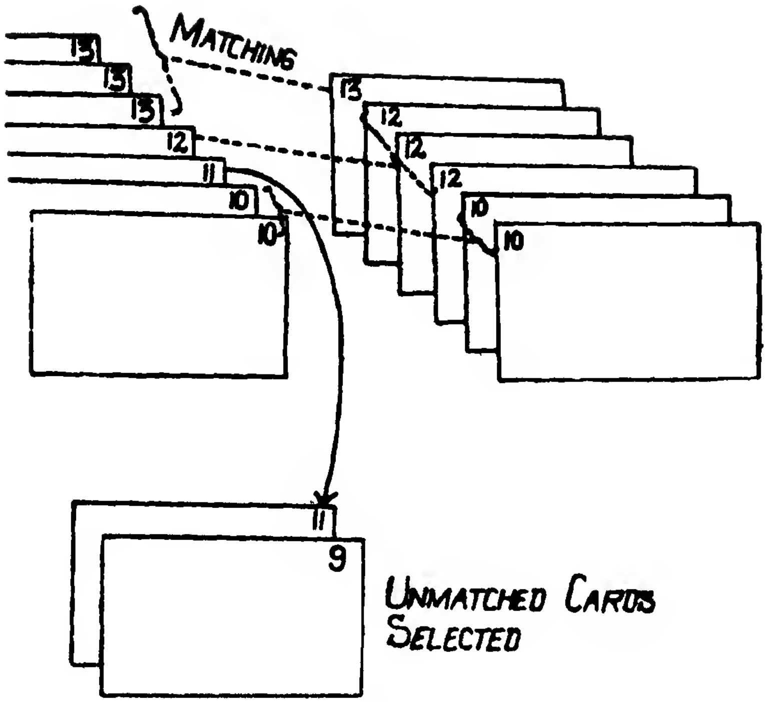

MATCHING
13
13
13
12
11
10
10
13
12
12
12
10
10
11
9
UNMATCHED CARDS
SELECTED

Fig. 7

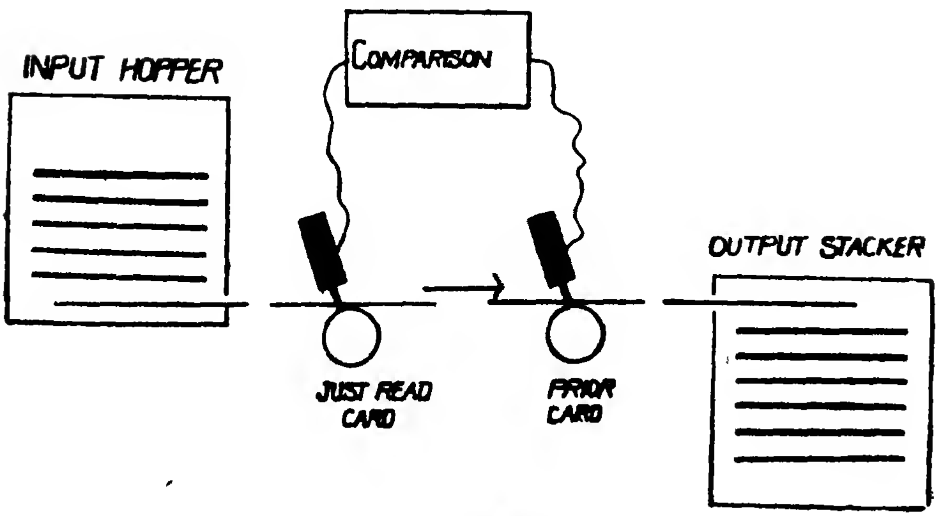

COMPARISON
INPUT HOPPER
OUTPUT STACKER
JUST READ
CARD
PRIOR
CARD

Fig. 8

A typical Stock Application on Unit Record Equipment

(See fig 9)

Step A. 1 Sales orders are assembled in batches.

A. 2. A card is punched for each line on the Sales order. The deck of the punched cards is then verified, going the verified deck A 3.

A. 4. The verified deck of punched cards is sorted by the stock item code on a card sorter, going the sorted depletions deck A 5.

A. 6. From the sorted depletions deck fed into the accounting machine a detailed sales listing in the following specimen format is printed·

Item Code	*Qty. demanded*	
1141	14	
1141	13	
1141	27	(designate the summary)
1142	10	

A. 8 Also produced by the reproducing punch connected to the accounting machines is a deck of punched cards, containing the total of all depletions for each stockitems to a card, in proper sequence

Steps B 1 to B 8 may be interpretted likewise for the receipt transactions

Steps 2. The depletion and addition deck (A 8 and B 8) are merged, giving the merged deck (3), on the collator.

Step 5. The master file (4) and the additions depletion deck (3), are match merged, into the combined file (7) the rejected cards thrown out as errors (6)

Step 11 The combined file (the master card followed by its summary depletion addition cards) is updated on the accounting machine, giving the updated master file (12) via the reproducing punch and the summary listing (13)in the specimen format below —

Stock item No.	Description	Old balance	Additions	Depletions	New balance

Step 8. In the step "the things are put back in the order", on a collator From the combined file (7) the old master and transactions are obtained after decollation.

A TYPICAL STOCK APPLICATION ON UNIT RECORD EQUIPMENT

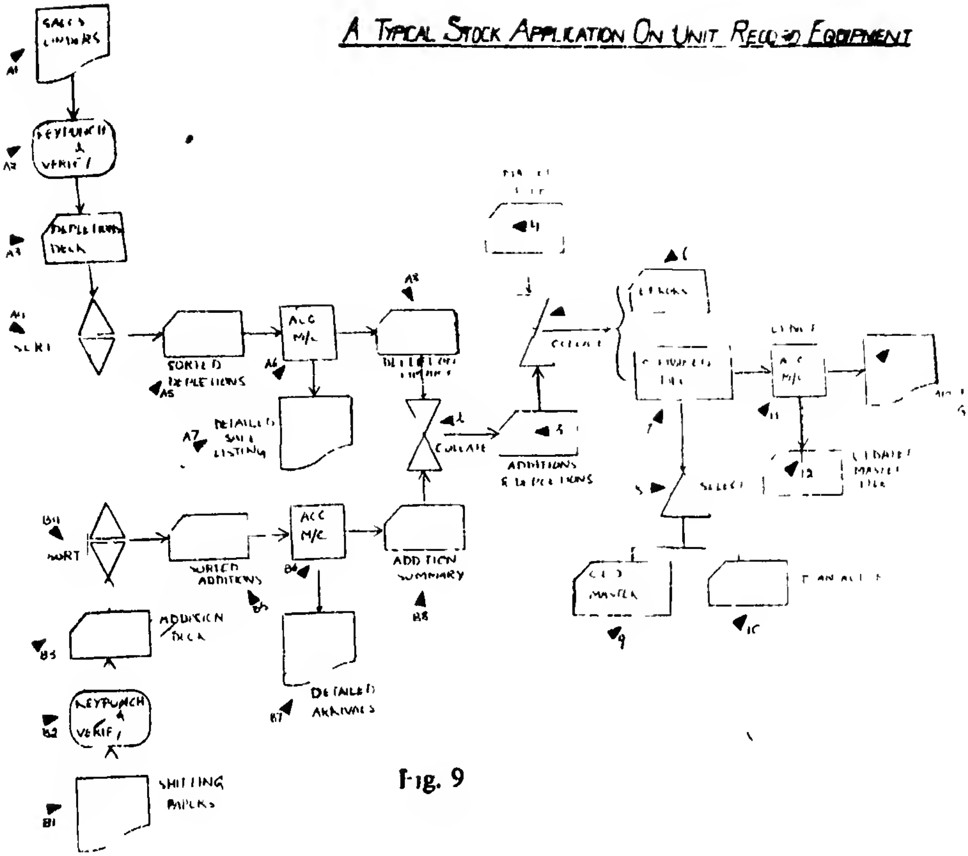


Fig. 9

Limitations of Unit Record Equipment

The unit record equipment has almost been completely overtaken by third generation computers The reasons should be clear from its limitations relative to electronic computers discused below.

Punched card machines (or unit record equipment) have electronic computer as a serious contender and their limitations have to be appraised particularly in relation to it Electronic computer is much faster, accurate, capable and versatile than punched card machines Each successive generation of the electronic computer has brought about a tremendous decrease in cost (and size) permitting corresponding increase in capablity in handling complexities This has eroded whatever little cost advantage the unit record equipment had over the second generation computers The principal limitations of the unit record equipment are stated below

1. It is widely, and often wrongly, believed that computers are very expensive and a company should be large and prosperous to afford the luxury of computer processing One should remember that the hardware costs have been steadily falling over the last 20 years and the trend appears to continue at an acclerated rate. Further, a few hours of computer time can be hired at low cost for selected applications; properly conceived and implemented, the effort can be very profitable A recent development is visible record computers, which have yet to find their way into India, are inexpensive and can be installed in house for smaller organisation also

2. The most significant limitations of punched card processing are the lack of memory and rudimentary processing capability There is virtually no memory except for the card currently being processed and the facility to take a few totals 'and sub totals Hence it is virtually impossible for tasks involving some degree of computational complexity This has limited the use of punched card machines merely to transaction processing (viz such applications as inventory accounting or payroll)

3 Even at the transaction processing level, these machines have a serious limitation Each record has to be accommodated on one (and only one) punched card , therefore, applications involving longer records e g., customer billing entailing customer name and address) are hard to mechanise.

4 Data files on cards are bulky and therefore, difficult to store and handle, there being ever present a hazard of dropping a pack of punched cards. The punched cards have to be run repeatedly through the punched card machine After a few passes they wear out and must be duplicated Direct access is virtually impossible, thereby forbidding not only inquiry handling (except for manual access which is error prone) but also to make the most use of every piece of information and update a number of files in a stroke Since parts and components of these machines keep on moving, they soon wear out and give rise to mistakes not apparently detectable

5. Retrieval of data is another problem that we come across in the use of punch card machines Data are gathered in numerous batches and transaction cards are kept in bunches Though not impossible but it us definitely difficult to trace some old data from these cards There is also the attending possiblity of these being misplaced in the prccess.

6. The cards grow in number with every transaction and in a short span of time may assume a volume that is difficult to handle. Also without adequate control measures, keeping track of the cards will be impossible.

7. Mechnical devices are inherently slower than the electronic components in the computer Typically, input and output speeds are much slower on unit record equipment and these limit the amount of work that can be done in a given time.

8. Programming is cumbersome and is done by inserting contacts in a plugboard or pannel.

The biggest problem in starting with unit record equipment and later moving on to a computer is that unit record processing embodies completely different concept from the computer Thus, the systems requirements with punched card equipment are very different from those with a computer Retention of these principles can be very wasteful and inefficient on the computer One of the most important principles in data processing is to make the best use of every piece of information. This is usually not practicable with unit record equipment, where systems are designed to produce specific reports Similarly, using a run to update a number of files is foreign to unit-record-trained personnel

It has been the experience in many installations that personnel trained on purched card equipment find it very difficult to adapt their thinking to computer techniques This can lead to poorly designed computer systems The management also gets a restricted view of data processing and this colours their subsequent decisions, often crippling the data processing effort.

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THE INSTITUTE OF CHARTERED ACCOUNTANTS OF INDIA

STUDY MATERIAL

F.S.P. S.A. & D.P.—6
COMBINATION—B

FINAL COURSE (N)

SYSTEMS ANALYSIS & DATA PROCESSING

CONTROLS AND AUDITING

STUDY—VI

Contents :

Prescribed Readings : An Audit Approach to Computers *by A. Pinkley.* The General Educational Trust of the Institute of Chartered Accountants of England and Wales.

Hardware and Software Controls

Most of the controls are based on the concept of redundancy. Extra information is carried for verification of processing. Alternatively, computations may be performed twice. In manual systems the totals are frequently cross-checked (e.g. in the Chi-square tabulation). This may be extended for control over computations and processing in computerised system also However, carrying extra information or doing extra computations is expensive by way of additional storage and computational requirements. Thus, although in theory, it may be possible to keep strictest control over errors, in practice, however, this should not be carried to the extreme. Obviously the right extent of such controls is established by the minimum total cost comprising two components : (i) the storage and computational cost and (ii) loss arising out of errors, if any. Where possible, such a cost analysis should be carried out before embedding extra checks and controls in the program or in the hardware Regarding the latter, *i.e.*, computer hardware, it has proved itself over the years to be remarkably accurate. Rarely, therefore, the user of a computer would want to go beyond the controls and checks provided in the hardware by the manufacturer, at least, in business applications.

I.I. Hardware Controls

Parity Check , (Please see, study I),

Echo Checking , It is another hardware check Consider the read/write heads of a magnetic tape as an example. What is being written may also be read immediately after and compared. Another such echo-check can be used for the printer that is actuated at the position desired by an instruction and the actual actuation is sensed and compared with what was intended An error signal will be given in case of discrepancy.

1.2. Software Checks : (Please see Study III for their details)

Valid Code Check is being discussed here again to clarify an incidental point of importance. It is used for testing the integrity of the Input data Consider an inventory file as an example. The transaction file for updating the inventory file consists of the following transactions :

Transaction	*Code*	*Frequency*
Receipt	R	30%
Issue	I	50%
Coustomer order	S	20%

The flowchart to be translated into program instructions subsequently should be as per Fig 2 rather than being implicit as shown in Fig 1.

Incidentally, there is another matter of importance in this connection Which of the three questions should be asked first ? The answer to this is to be found in the frequency of each type of transaction. Suppose that the frequency of each type of transaction for a particular situation is as given in the table above. Issues are the most frequent. "Is it I" should be the first. Compare the flowcharts of Figs. 3 and 4. In the former, the program would ask 2nd and 3rd questions 50% of the time whereas the same question in the other case would be asked 100% of time. The former, therefore, requires less processing time.

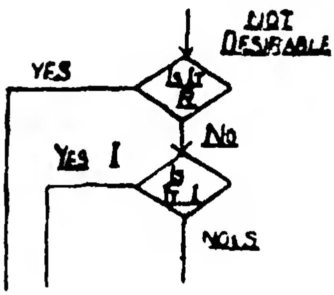


Fig. I

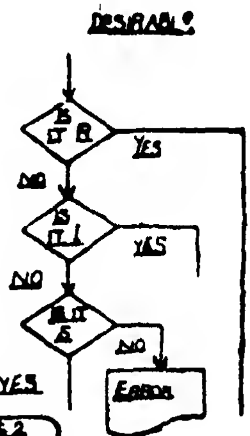


Fig. 2

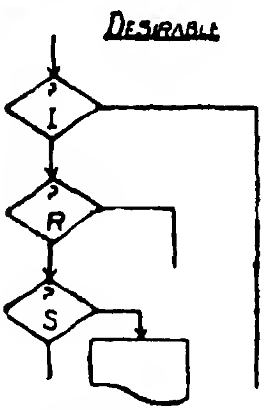


Fig. 3

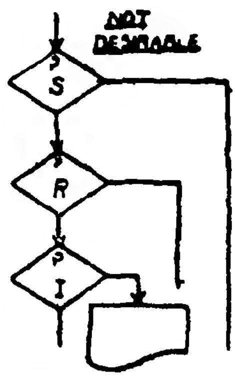


Fig. 4

1.2.1. **Control totals** are used in batch processing. A control figure is developed for a field prior to processing by the computer. For example, the wages of all employees' transactions in a payroll batch may be added Subsequently the

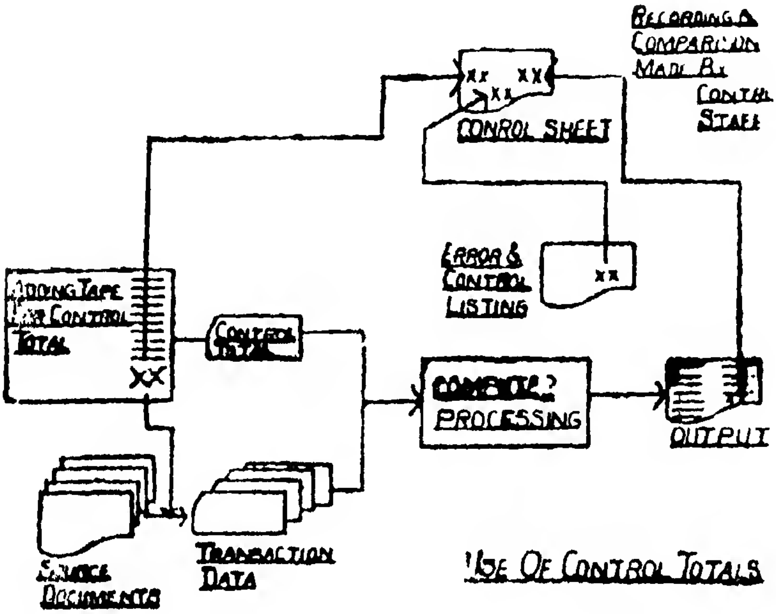


Fig 5

program would also compute the total and compare it with the one derived manually on adding machine etc. Instead of the financial totals, like wages an awkward data field which is normally not totalled may be picked up. Its total over the batch provides the control total and is known as the hash total. The total of item codes in a batch would costitute a hash total. Occasionally, just the total number of transactions are reoreded as a control total. The computer must print out message confirming the comparison even though the comparison disclosed no discrepancy.

1.3. Input Controls

Input data may be likened to raw materials in a production process Until raw data is captured and input into the computer in suitable form no sensible processing can be performed. The quality of the output is bound to be impaired and even the files are liable to be corrupted. From capture of data at source to its input into the computer there are several intermediate steps and stringent controls have to be established at the various stages to minimise errors and loss of documents during transmission. Good system design would include provisions for insuring accurate and complete input into the system. The auditor, in turn, should be familiar with the sources of errors, techniques of their prevention and handling

the erreoneous transactions Input errors can occur during the following four stages :

(i) *Incorrect capture or recording of the input data.* The goods receiving section, for example, may put a lot into inventory without the preparation of a receiving document. The stock control clerk, as another example, may write the incorrect item code on an issue note.

(ii) *Incorrect transcription.* Even experienced keypunch operators make errors A study of keypunching error rates of experienced operators indicated an error rate of 1 in every 1600 key-strokes to 1 in every 4000 key-strokes. This is likely to be further compounded if the source documents are damaged, illegible, not properly filed etc.

(iii) *Loss of documents during transmission.* A document may be fraudulently removed from a batch during transmission or it may drop or two documents may stick together so that the one behind is overlooked by the keypunch operator

(iv) *Incorrect processing by the computer equipment.* The input peripherals, as discussed earlier, are liable to malfunction In the direct input devices, the data is not only liable to be erroneously keyed in but also subject to errors during transmission. In the former contingency appropriate procedures have to be embedded in the program to detect errors. The errors during transmission have to be detected by the transmission equipment controls.

Also, a complete record may not be read because of equipment malfunctioning or some programming defects.

Wrong files or wrong programs may be inadvertently put on by the computer operator.

It transpires then that controls have to be placed (i) at the points of origination of source documents and their transcription into machine readable from, (ii) at the points where data is handled or transmitted and (iii) at the points where data enters the computer (iv) also important is to lay down clear procedure for handling detected errors i e , rectifying them and resubmitting them for processing. These procedures themselves could be another source of error.

Below we discuss the controls at these points :

(i) Controls over creation and conversion of input data.

(a) provide exact number of spaces for the length of various data fields so that if the actual length exceeds or falls short of the spaces provided the fact can be spotted by the clerk who makes the entries. Also different kinds of documents may be coloured differently.

(b) Documents may also be subsequently vetted by another person for correctness, legibility, documents being undamaged and of proper colour.

(c) Important financial transactions may be transcribed in duplicate on two cards by different persons, to be subsequently compared by the program.

(d) During transcription of the data on punched cards there is a scope for keypunch verification. It is, however, to be noted that mechanical verification doubles the keypunching expenses and it may therefore be desired to verify only the critical data fields. For example, the customer's address, even if erroneous to some extent, serves the purpose but erroneous account number would occur into a wrong transaction. Therefore, such critical fields as financial values and item codes etc., must be verified whereas others like customer's address may not be verified. Also, cards, pre-punched for the permanent data fields may be used. For example, for a stock item a card may be punched for the permanent data field, verified and reproduced into a large number of cards. Such sets of cards for various stock items may then be stocked and cards out of them pulled out as and when receipt/issue transactions take place so that only variable data fields need be punched and verified latter. This would considerably cut down the verification expenses Use of turnarond documents (viz., cards pre-punched for permanent data fields and distributed amongst workers for punching the quantity finished off an operation, time taken etc.) should be encouraged for the same reasons.

Statistical quality control techniques (Ref : Study VII) may also be employed to control accuracy with which the various operators punch the data. Each operator's work is checked on sample basis. If the error rate is acceptable no verification is made ; if it is not verification is made.

The punched cards may also be visually inspected if there is a provision for interpretation at the top. Likewise, the typewriters may produce punched cards or paper tape as output, for example, for an invoice being typed, the typed material may then be visually inspected.

In key-to-tape devices blocks of data are written twice for purposes of verification. Any error the operator notices may be rectified there and then.

(e) Use may be made of check digits (Ref : Study III) for such codes as customer's account numbers, stockitem numbers etc.

1.4 Control Over input data reed into the computer

These controls can be exercised by embedding header and trailer control labels in the files discussed later and sofeware controls in the programs discussed in study III.

Besides, the batch totals computed prior to processing and tallied with the ones computed and printed by the computer after each processing run ensures that all items have been processed in each run Obviously, the adjustments for erroneous transactions would be made in the batch totals computed by the computer in each run. Thus batch totals provide run-to-run control.

Control over handling of input data

Batch control tickets, control totals and external file labels, discussed elsewhere, provide control over handling of input data.

Control over error handling routines

It is highly desirable to lay down clear procedures for handling errors since they themselves could be a source of further errors However, the operator is seldom permitted to rectify errors

The error listing containing reasons for each rejects should be compiled on the computer during each processing run The list is then forwarded to the originating department for investigations, rectification and resubmission. The originating department may be followed up by the computer data processing control section or resubmission may be the responsibility of the originating department itself Anyhow, the responsibility should be clearly laid down The transmission of listings to the originating department should be logged by the control section. Also, the originating department should be instructed regarding error handing procedures When erroneous master records are discovered they may be put on a suspense file for later investigations.

Amendments to master files and programs should be checked for proper authorisation A register should be maintained for all the changes incorporated for review by the originating department.

1.5. Output Control

Output control is essential to ensure that those and only those authorised to receive the output get them. Incidentally, these recipients constitute a feedback loop i.e., they can detect errors and report them to the computer data processing section who, however, must vet the output reports for completeness etc. before forwarding them to the concerned.

Distribution controls specify the number of copies prepared by the computer operator. The computer operator may have to remove carbons i.e., decollate the

multiple copy sets or he may have to burst forms on continuous stationary. A included is such a control sheet for report distribution as below :

Report Distribution Control

Job No.. .... Job Title.........

Report Title Copies Destination Deleave Burst Bin

Report Title	Copies	Destination	Deleave	Burst	Bin

Output data fields may also be subjected to such program checks as rea ableness, completeness etc.

1.6. Organisational Controls

In the chart depicted in Fig. 6 it is to be seen that the control section c stitutes the hub of activity in the computer installation. It acts like a bufier betw the user departments and the computer room and associated activities.

The original documents are received in batches in the control section, ve for control totals and finally passed on to data preparation section for transcript on punched cards or paper tape. The punched cards or tape are received back control section from the data preparation section, are vetted for control totals sent to the computer room, Output documents from the computer are also

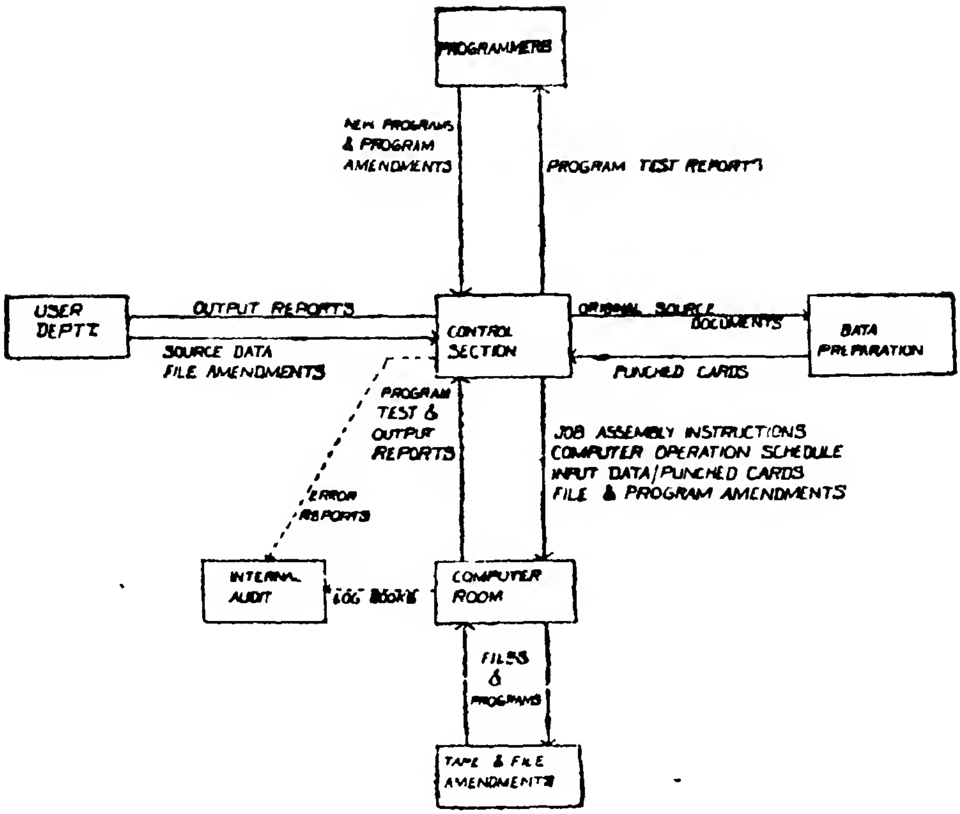


Fig. 6

checked before being forwarded to the user departments. The control section also maintains a timing schedule on the input, output times of all jobs and a document register. The compilation of the schedule and follow-up against it is particularly facilitated in case of sequential files, because of batching. The librarian is also kept posted with the specifications of programs and files to be wanted in the computer room against the schedule. The programmers, too, send their programs to the operator via the control section and likewise the test reports reach them via the control section.

For file amendments the master tapes are frozen until a computer print-out of the amendments is agreed to by the user department with control section against the original file amendment sheet. The amendments may be dichotomised as:

(i) Such trivial changes as the change in the address of a supplier.
(ii) Such changes of consequence as the new discount rate by a supplier.

Obviously the latter category deserves more vigilance from the control section.

It can be seen that each section in data processing department is separate from the other and each group has its own supervisor. This division of responsibi-

lity meets the axioms on which conventional auditing is based. This should dismiss any apprehensions that concentration of files at one place in a computer installation is susceptible to fraud. Although those intent upon fraud can beat any system, the computerised systems are no worse than the manual systems. In manual systems the principle of division of responsibility is applied within the departments whereas in computerised systems the emphasis shifts to division within the data processing department. Certainly, it is highly desirable to infuse team spirit amongst the data processing personnel in the early stages of development of a computerised system but as the system matures it is necessary to discourage the informal structure. Admittedly, such segregation of duties may not be possible in a small organisation, but then the data processing department would not have much importance.

It is also essential to maintain such safeguards to see that ;

(i) the operators are not permitted to amend any input data ;
(ii) programmers do not have any access to the program decks and files :
(iii) no one in the d p department has access to any of the financial records in the company ;
(iv) access to the computer room is restricted,
(v) a librarian is the sole incharge of device and program storage.

DAILY LOG SHEET

N B : Times recorded are to run consecutively. Non-productive time should be indicated in the right hand column.

Date............ ....

Job	Program No	Time			Operator's initials	Notes
		On	Off	Duration		

Log Sheet Inspected by Date...........

A Specimen Daily Log Sheet

The log book is required to be filled in by the computer operator everyday. In it are to be entered the jobs processed during the day and their time, any interruptions or breakdown etc. To the control section it gives an idea of set-up and other non productive computer time and enables them to do more realistic scheduling To the auditor it gives an idea of the condition of the machine or any unauthorised use of the equipment Also, the auditor can establish the types and frequencies of errors A specimen log sheet is shown on the previous page.

Conosle log book

It is a typewriter so connected with the computer that the manipulation of the computer switches by the operator, and data entered through the console, and the error routines are automatically typed It is also possible to list the jobs and their times, therefore, dispensing off with the log-sheets to a large extent This attachment is however not available with all computers

Organisation of the Data Processing Department

A typical organisation chart is depeicted below :

- Data Processing Manager
  - Development Section
    - Systems Analysis
    - Programmers
  - Control Section
  - Operations Section
    - Computer operators
    - Input preparation equipment operators
    - Librarian

In view of the nature and importance of the control section its incharge should be a person of a fairly high rank. The duties of all in the control section should be detailed in procedure manuals. It is highly desirable that the work of this section is evidenced, say, in a register maintained by the section Aside from the control section there may be control group as such or someone incharge of control work on part-time in each user department. This is desirable in view of the fact that the data to start with is originated and the final output consumed by the user departments.

For details of duties of computer operators the student is referred to Study IV.

1.7. File Control is obviously important since even in a small computer installation the number of files may run into scores. The tape reels and other removable devices viz, exchangeable discs should be assigned a unique code number. A paper label bearing this code may be affixed on the device. For the

magnetic tape reels there are available protection rings (Fig 7). The file may or may not be written depending upon the ring is inserted in the reel or not. The reels should be stored in a systematic way. The tapes that are used for merely program debugging and testing and assist in application program execution viz., for sorting operations should have reels of different colour. Such tape reels are known as scratch devices. When the number of file storage devices is sizable creation of a library exclusively under a person should be considered. Amongst other things such as noting the locations of the devices the librarian should keep a log of each tape. The log should show the number of occasions each tape was put to use and any errors with complete details that occurred during any of processing.

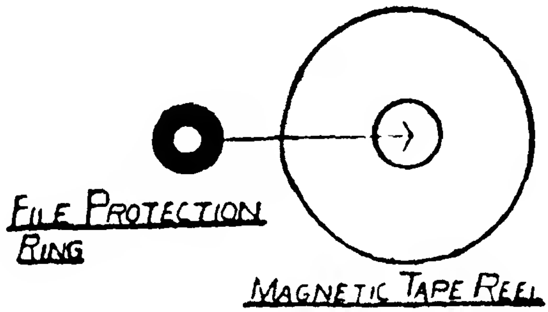
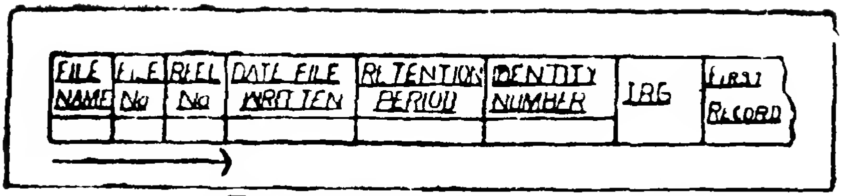


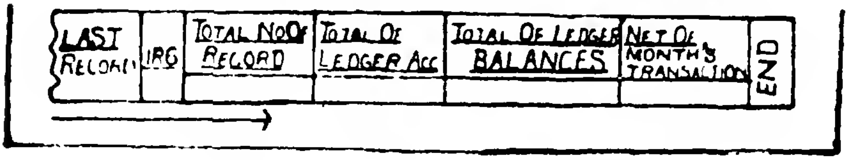
Fig. 7

The controls can also be embedded magnetically in the magnetic tapes. These are known as header and trailer labels that are put at the start and end of each file A typical layout of these labels is shown in Fig 8. It is usual to incorporate instructions in the application programs for containing the header data of the file and comparing it with that actually on the file that is brought into the CPU for comparison The header data may also be printed on continuous stationary for visual check by the operator It can also be ascertained if the file is due for updating. For example, consider a payroll file that is updated monthly, on first of every month. If the file is updated on 1-9-82 the date 1-9-82 may be stored in the tape. Subsequently if, for example, the tape is sought to be processed on 30-8-82 the program would disclose the right date 1-9-82 by printing it on continuous stationary for the operator's information. The trailer label comprises the various control totals with which the student should be familiar by now.

Amendment to Master Files : For amendments to master files the persons authorised to make amendments should be specified preferably in the procedure manual. The amendment sheets are usually available in pre-numbered forms to control amendment initiations. The control section ought to vet and verify these for authenticity. Also, the control section should schedule for the amendments on computer room and vet the computer print-outs as obtained from the computer room. Subsequently, these print-outs are to be reconciled with the original amendment sheets by the control section alongside the user departments The control totals in trailer label of the magnetic tapes must be updated by the program even for amendments The control totals may be printed for vetting by the control section and verification by the accounts department. Since there are no control totals for magnetic disc files unless they are specifically used for sequential access it is necessary to review their contents periodically.



A Typical Layout Of A Header Label



A Typical Layout Of A Trailer Label

Fig 8

The contents of the files, anyway, whatever the media, need to be verified independently on periodic basis Some of the contents would be automatically reviewed by the accounts department *viz.*, for such outputs as invoices to the customer. The remaining contents may have to be specfically printed on cyclic basis in a specified interval time.

Audit Control Considerations

In appraising the soundness of the organisational controls, the auditor may go by the following matters.

(i) Does division of responsibility exit ?

(ii) Does an organisation chart supporting the division exist ?

(iii) Are the duties of various personnel clearly spelled out in procedure manuals ?

(iv) Maintenance of the various details in the computer room log book.

(v) Is the log book subsequently examined by someone ?

(vi) Are operation instructions for each program available to the computer operator ?

(vii) Regarding file storage are arrangements for storage, security and back up files satisfactory ?

(viii) Maintenance of logs of files.

(ix) Are scratch files suitably distinguished from other files ?

(x) General discipline in the library.

(xi) Are the control records being maintained in the control section sufficiently detailed and meticulous ?

(xii) Are the input documents and output documents verified for accuracy ?

(xiii) Is the access to the computer room restricted ?

(xiv) Is the access to the library restricted ?

(xv) Do the standby arrangements in case of machine breakdown exist ?

1.8 Procedural Control

Procedural control pertains to various applications *e g*., accounts payable. Such controls should be comprehensive *i e*, they should be applied at each stage starting with the originating of input data extending up to the destination of the output documents. Control totals constitute the backbone of the procedure controls and, as the name suggests, are applicable to each batch Three types of control totals can be distinguished : count of the documents, financial total and hash total.

It is to be noted that in rudimentary E.D.P. systems, the computer and therefore, the control totals would be ignored. As the system grows such checks and controls are necessary to be introduced in the data processing department. Also, many of the checks are programmed for so that the computer capabilities are brought to bear on the data As we shall see shortly there are specific programs that only edit the data for the application program Once again, it is to be recommended that the checks and controls are embedded at the time of systems development.

Cost-benefit analysis must be made for each check. For example, noting a customer address wrongly would not spoil the system or lead to any loss whereas a financial data item may have serious repercussion. It is, therefore, futile to be fussy about the former, customer address. In contrast, for some of financial data inputs two sets of punched-cards by different operators may be prepared and compared by the computer for any discrepancies Regarding transaction files and the permanent fields in the master file there have to be different standards. An error in the transaction file *e g*, punching 32 hours put by an operator in place of 23 hours normally leads to one-time error only and is unlikely to be repeated again. But an error in the master file e.*g*, Rs. 11/*hour* wage rate against the correct one Rs 10/*hour*, is a perpetual source of errors.

For each transaction a punched card is prepared. The deck of transaction cards is headed by a card containing the controls totals for the batch The totals may be computed manually with the aid of simple adding machine or some sort of punch card equipment may be used to perform punching and adding simultaneously. The control totals are later compared with those computed by the computer during processing of the batch.

Fig 9 shows, however, the various stages for procedure controls in more detail. The user department should have definite rules to follow and checks to perform. Preferably these should be laid down in the procedure manual in detail. To start with the user department would vet the source documents for any apparent discrepancies *viz*, the goods receiving may look for the right purchase order number on the advice notes sent by the vendors, the accounts department may likewise vet the invoices. For important documents they must verify if the initials of the authorised official are authentic. There are frequently pre-numbered challans, material requisitions, etc. For each batch the user department must ensure that they can be accounted for. The hash and financial totals and counts of transactions are to be performed by the user department for each batch. The transaction codes are to be entered and verified by the user department. Finally, the user department should forward these to the control section of the data processing department

according to their schedule or time table.

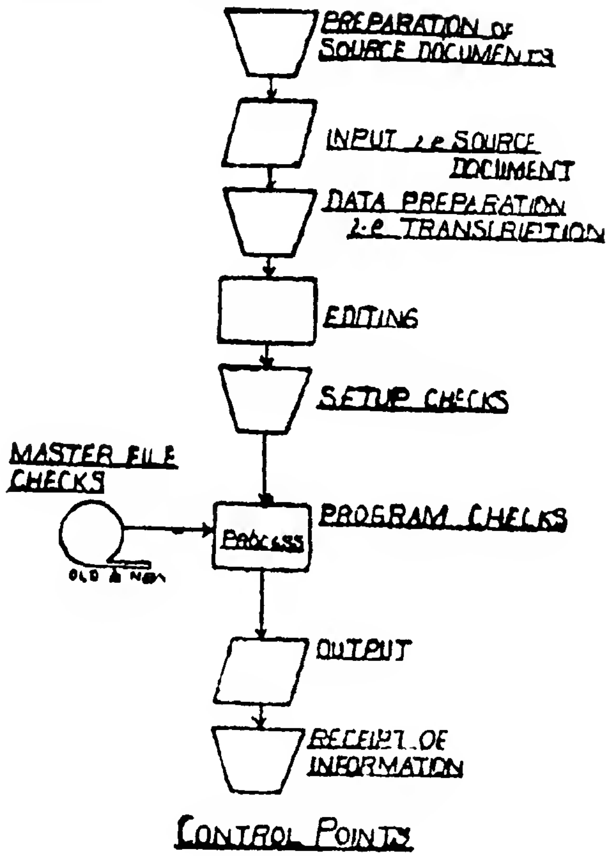


Fig. 9

In systems employing remote terminals the access to terminals should be restricted since advantage of batches and the associated controls cannot be had.

Control section, upon receipt of the batch of documents, scrutinise the documents if they are properly completed and authorised. Control section should maintain a register wherein they note down the particulars of the batches and the control totals. They may have occasionally to follow up the user departments for sending in the documents in time if they happen to be late. Control section should foward the documents to the data preparation and arrange for the input media (*viz.*, magnetic tapes) in the computer room.

In the data processing section punching and verifying should be the responsibilities of different persons. The procedures for re-transcription upon errors should be clearly laid down. It is also necessary to keep a record of the errors that

were rectified. Some inputs, mostly financial, may call for highest degree of accuracy. Two sets of the same transactions may be prepared by different persons. Subsequently, the computer would compare the two and compile list of errors, if any. On the other hand, some transactions or data fields may not call for a high degree of accuracy. No verification needs to be carried out for such cases. The transcribed data (*i.e.* deck of punched cards or paper tape) is then forwarded to the control section who would ascertain the count of the documents in the batch and the transactions prepared. Also, they ought to ensure the data preparation section works to schedule, and no batch is lost.

Editing : The input data are subjected to intensive editing by human beings or computer or both Such editing ensures the integrity of the master files, engenders trust in the output information and reduces the time required for re-processing the erroneous cases. The various stages of editing are supposed to be scheduled by the control section who would also review the results of each editing run and follow up the concerned for action on erroneous cases, which are discussed below.

Precomputer Editing may either be manual or by means of some punched card equipment. The number of cards may be counted, their sequence ascertained; their contents printed for verification and financial and hash totals verified.

Computer Editing : In computer installations employing magnetic media the transaction data on punched cards or paper tape is transferred on some magnetic media. Advantage may be taken of this to perform limit tests, format checks, checks for valid transaction codes, verification of financial and hash totals, appropriateness of check digits etc Furthermore, the transaction data can be checked against the master file which should be used for verification only and no updating at this stage. These checks may include, for example, comparision of some permanent fields occuring both on a transaction record and on its counterpart in the master file. The error list is invariably compiled for this run and forwarded to the control section.

Set-up Checks are performed by the computer operator who would ascertain that the right program and files are being used Towards this purpose there may be provision in the program to print the header label (Fig 8) of the file and the name of the program.

Master File Controls : An error in a transaction file is not so serious as that in the master file since it is not going to be repeated whereas the latter could be a perpetual source of errors. To ensure the integrity of the master file is therefore highly important.

Any amendments desired in the master file must be approved by the highest authority on the data proposed to be amended. There should be pre-numbered forms for amendments which must be verified in the control section with regard to initials of the concerned. The amendments should be scheduled for the computer room by the control section. The print-outs of the amendments ought to be scrutinised both by the control section and the concerned user department. To ensure the integrity of the master file, which is liable to be corrupted by an error in a transaction, the contents of the master file should be printed and verified periodically. During processing by the program the record count. financial and hash totals can be obtained and maintained in the trailer label. The print-outs of these controls may subsequently be verified by the control section and the user department.

Output Control The computer room forwards the print-outs to the control section who check these for accuracy, completeness and control totals Control section carries out follow up on error lists The output documents are finally forwarded to the user departments The control section should also exercise control over confidential information *viz* , salaries.

Applications in any computerised organisation are numerous The auditor would have to review the procedure controls for each application. Towards this purpose he may study the system flowcharts, program specifications, file layouts, input and output data fields, methods of input, list of program checks etc Obviously, this amounts to a 'considerable task for the auditor. The nature of controls may also change with time This further complicates the auditor's task. It is, therefore, desirable that the auditor drafts a check list or control sheet so that no aspect of procedure controls is overlooked The check list should include all important fields and the auditor should note at what stage controls for these are established and what stage these are verified With regard to checks and verification the computer capabilities are often ignored and the management tend to rely upon clerical verifications This mistrust about the computer or ignorance about the capabilities is, however, not justified. Much of checking can be performed by the computer more efficiently The auditor should, therefore, ascertain the extent to which computer is employed towards this purpose

A specimen check list is shown below :

Control field	Types of control	Control establish by				Control verifications				
		User Deptt. Control Section	D.P., Deptt Control Section	Input preparation section	Computer	D P Control Section	Input	Intermediate Runs	Output	After Computer
Item lines on goods receipt note	Record count									

2. Audit Trail

In conventional auditing of non-automatic systems the auditor enjoys certain facilities with regard to audit trail *i e.*, correlating inputs with outputs He starts at the general ledger by reference to specific journal entries, traces back to

selected source documents which started the general ledger postings. Likewise, he may start with a purchase order and review the goods receipt notes for delivery against it, in ·e·tion reports, stock records, accounts in nominal and purchase ledgers, the invoices and ultimately payment. This mode of examining the soundness of the system has been the cornerstone of conventional auditing. The facilities the auditor enjoys with regard to tracing audit trail in manual systems are summarised below (Ref: Fig. 10):

Source documentation are records of originating transactions which are used for posting to the journal and the ledger. These aid the auditor in exploring the underlying details of the accounts and in determining the effectiveness of internal controls.

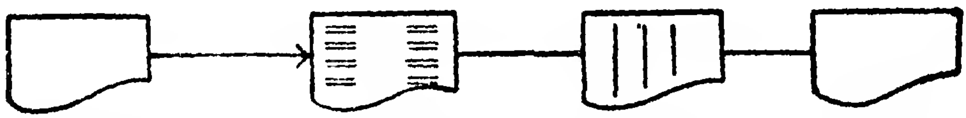


Fig. 10

Detailed chronological journal is the record of all events in a dairy form.

Ledger Summary provides the detailed information about how a balance accumulated to what it is to date.

Furthermore, all the records are visible, easily accessible and the auditor can observe the transactions being posted, ledgers being updated, summaries being prepared and inquiries being handled. He can thus form an overall and general impression about these activities

Computerised systems, however, do pose some problems in this regard which though have been exaggerated in the past. These are enumerated below :

(i) Source documents once transcribed on to a machine readable medium are not retained in a manner that permits their subsequent access.

(ii) In case of CRT's there may be no record whatever of input transactions.

(iii) Ledger summaries may be replaced by master files which do not show the details of the build up of the balance or summarised values.

(iv) Transaction listing is often not provided.

(v) It is usually not necessary to prepare detailed reports and the management may be content with merely reports on exception basis.

(vi) If the records are maintained on magnetic media they would not be visible and the auditor would not be in a position to observe the various data processing activities and form opinion about them easily.

Because of these factors the audit trail is liable to be rendered sketchy. Nevertheless a few additional steps may easily and inexpensively complete the audit trail. This we shall discuss separately for the magnetic tape and magnetic disc installations. Punch card files can be treated in the same manner as the magnetic tape files.

Audit Trail/Magnetic tape files

In the case of magnetic tape installations, the transaction listing, though not absolutely essential for the management, can nevertheless be pritned out for the auditor and even the management The master files, ordinarily, do not carry the details of the build up of the summary figures but these details can also be incorporated in the files to complete the audit trail [However, for the individual records it would be expensive to show the details of each and every transaction in the magnetic tape files] Also, if the older files are retained on the father-son concept, it is possible to reconstruct the latest file from the pervious week's master file copy. These measures can ensure the completeness of the audit trail without much additional expense and effort.

Audit Trail/Magnetic Disc Files

Sorting of the transactions is not needed and the transactions can be (and are) processed in any order, though magnetic disc files may be updated in the manner of magnetic tape files. i e, with batched and sorted transactions. In the latter case, the problem of completing the audit trail can be handled exactly as discussed above in the context of magnetic tape files. Now, in the former case, when transactions are not sorted and are processed in any order, the transaction list can still be printed in different headings representing each kind of transaction for ease in subsequent reference and connection In the magnetic disc, unlike the magnetic tape, data are updated by overlaying ; therefore, the contents of the older master files have to be periodically dumped on magnetic tapes to enable the auditor to trace any new balance figures to its source documents It is also not unusual that the input transactions for the magnetic disc files may be sorted in an order to minimise seeking time for the file device but it may not be useful for reference purposes , therefore, it may be desired to sort the transaction documents by same convenient key after their transcription and processing

Audit Trail/Advanced System. In advanced magnetic disc installations the transactions may be directly input via remote terminals, eliminating thereby the need for source documents. However, management would usually want that the system maintains transaction file records of the transaction input The record may also contain reference of the individual who input the transactions This file than provides a transaction list which may be sorted by the desired key and its contents printed However, although, the provision of such transaction list discussed above is a way towards completing the audit trail, because of the fact that several files may be updated with a single transaction the audit trail may seldom be complete in advanced systems.

The appropriate provisions to complete the audit trail in computerised system have to be made right during the systems design stage. The auditor should therefore be involved at the earliest. Below, we give three methods by which the complete audit trail can be had though other alternative methods can be conceived by the systems designer.

(i) As depicted in figure 11 the printout of the filed record not only gives the current balance but also all the change references. With the later the summary figures can be traced back to the transaction list and finally, to the source documents.

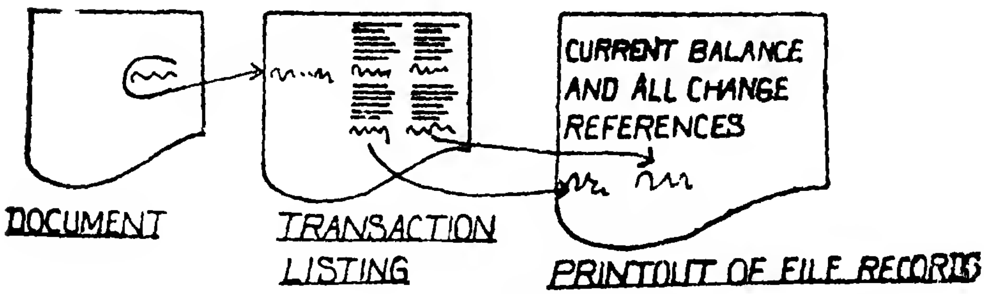
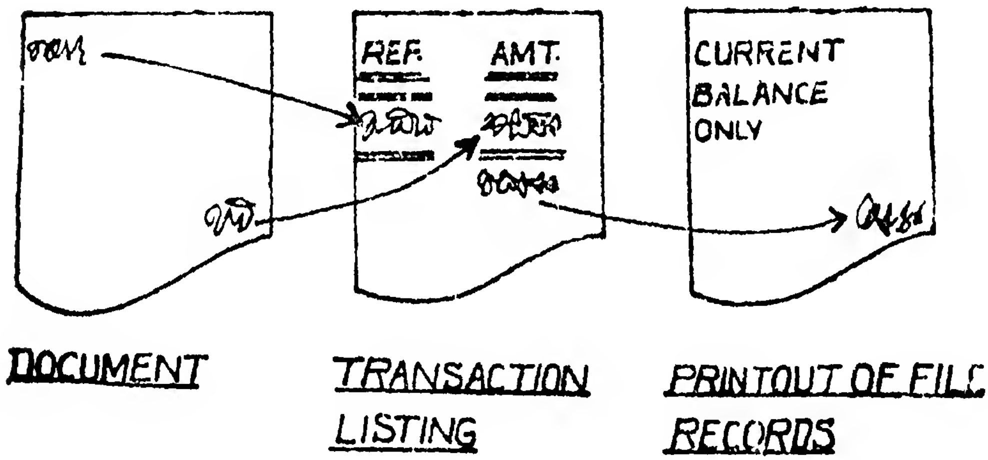


Fig 11



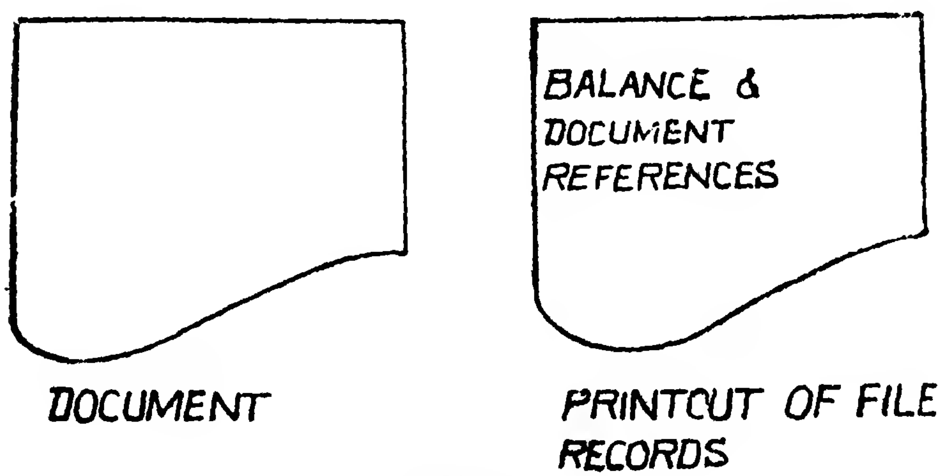


Fig. 13

(ii) The printout of the file records (Fig. 12) gives just the current balance and no change references. Therefore, the transaction listings have to be searched and correlated to arrive at those balances for audit purposes.

(iii) The summary printout (Fig. 13) not only gives the balance figures but also the references of transactions that caused the change in the balance so that the audit trail is obvious. The design, however, would be suited to situations in which the transactions are few and far between.

3. General Audit Considerations

Computer is not subject to fatigue and such other human failings. Its error freeness is unprecedented. But the programs it executes, the files it updates, the data that is put into it are prepared by human beings Therefore, although the computer itself is virtually free from error this is not so with the data processing system as a whole.

On the other end of the spectrum, perhaps because of its mysterious invisible data storage, the computer appears to be imposing an extra burden on the auditor This, however, is not true. It only shifts the emphasis from vouching individual records to checking the system as a whole Also the controls have to be embedded during the systems development stages. These cannot be grafted upon an existing system.

There is an argument that since the unit record equipment has been ignored in auditing practices, the computer too should be ignored. But there is no reason (or excuse) for having ignored the unit record equipment. It is surely a significant centre of data processing though the computer is far more powerful and flexible a tool than the unit record equipment controlled by means of a trivial and rigid program. This is supported by the following facts ·

1. It does away with departmentalisation to quite an extent, and assumes the role of the nerve-centre of an information processing system For example the input of the particulars of a customer order could be comprehensively processed in the computer room i.e , the accounts receivable file may be updated, invoices to customers and despatch notes to the despatching department may be prepared, repenishment order may by placed on purchasing department, or even on the suppliers and stock records may be updated.

2. The trend now-a-days is towards an ultimate of paperless information systems *viz.*, the particulars of customer orders processed in the manner of I above do not necessitate printing of such intermediate information as updated stock balances. In systems employing remote terminals, even the customer's order particulars may be keyed in via CRT's. Although entirely paperless information systems may not be possible in the near future or it may even be a myth, yet it is a fact that several printouts as were prepared in manual systems have been curtailed. It becomes difficult therefore to trace an output back to inputs or *vice versa.* In other words computer renders the audit trail incomplete.

3. Problems of invisible data stored in such magnetic media as tape and disc.

4. Although human beings are prone to errors they are also good enough in detecting these The computer does not possess this capability. Checks, therefore, have to be provided for many contingencies. And yet there is a possibility of bugs stikcing on in the program or creeping during file conversion.

5. Quite a few frauds, unique to the computer, have been reported.

   (A) A programmer increased his overdraft limit from 200 to 2000.

   (B) By means of a fraudulent program, a programmer gleaned a half cent from everyone's wage/salary in his organisation rather than rounding it off for the individual.

   (C) Invoices for fictitious goods from fictious companies were settled through a computerised system.

   (D) Unauthorised sale of computer time was made to outsiders.

   (E) Information in the output documents was given to company's competitors.

3.1 Audit approach. *It would depend upon computer configuration, the stages* of systems development and soundness of internal controls. The following three approaches, in practice, can be distinguished :

1. Systems that do not employ magnetic auxiliary storage are usually simple and print outs of information are generally available to the same extent as in manual system. However, not all systems employing magnetic tapes or discs would be complex and some of these too may be as simple as those with non-magnetic auxiliary storage. Obviously, the question of incomplete audit trail does not exist in such simple systems. The auditor may therefore give only cursory attention to the existence of the computer.

2. In magnetic file installations that have not yet switched on to operation by remote terminals but are sufficiently complex to merit review of internal control the D.P. department is to be audited and management should also make arrangements for completing the audit trail to enable the auditor to trace the system by means of transactions as in conventional auditing.

3. Where the internal control in both the data processing and user departments is sufficiently reliable the stipulation on audit trail may be dropped. Instead the auditor may employ special techniques.

It is to be noted that the batch control techniques are a prominent safeguard in sequential processing—so much that often even magnetic disc files are handled on a batch mode simply to derive benefit of the batch control technique. With magnetic disc files, especially used in conjunction with remote terminals, batch control is lost. With such systems and sometimes also advanced systems around the magnetic tape, provision of audit trail becomes quite expensive and an unnecessary extra burden to the management. Under such circumstances, he may want to make use of test packs and other special techniques which, basically, depend upon the computer capabilities themselves for performing audit. But the special techniques also are fairly expensive. Therefore, it must be considered whether special print-outs to complete the audit trail are cheaper than the special techniques. If so, the auditor should

endeavour to convince the client regarding producing special print-out for purpose of auditing. It is particularly useful if the management goes in for auditor's requirements with regard to selective print-outs at an early stage of development of the system. This would obviate the need for special print-outs at a later stage.

Computerisation should normally strengthen internal control One reason, and a leading one at that, is that the systems analysis is carried out. Several systems have reported improvements in this and other directions merely because of the review and the overhaul of the system rather than because of the computer itself Also, computer concentrates data processing at one place and it is therefore, easy to apply various controls. Computer's error-freeness, of course, is another reason for better internal control

The auditor should also be concerned with the information system, *i e*, see to what extent management's information requirements are being met and in time. Unlike manual systems it would usually be difficult for the auditor to form a right opinion in this regard This is because there would be few print-outs. We shall discuss matters pertaining to internal control in the user as well as the data processing departments in the following pages

Regarding liasion between management and the auditor it is to be desired that rapport is built right during the development of the computer system so that the auditor advises the client about print-outs and controls etc, at a time when it is so easy to make arrangements for these in the programs. Also, development of the systems as a whole should be the auditor's concern Regarding introduction of programs he would want to know the results of tests on programs ; who approves the programs and who looks after the programs and other documentation. Not only new programs but also amendments of these have to be under auditor's surveillance, *i.e.*, who authorise amendments, who tests the modified programs and what are test results ? The auditors should also see that the software library is well maintained.

3 2 The Audit Process

The audit process may be described by means of the flow-chart given below. It consists of the following ten steps :

1. The auditor gains an overview of the system by acquainting himself with the hardware, software, brainware and the interaction of the user with the computer. He also studies the existing and proposed organisation charts for the computer data processing department as well as the entire organisation. He may want to go down to such detail as the lines of authorities, job titles and, descriptions, number of employees and categories, vacancies etc. He gathers or compiles a list and layout of the computer hardware *i.e.*, its configuration, including location of communication channels and terminals, capabilities of the CPU and the peripherals and the auxiliary equipment. It should also be ascertained if the equipment is purchased, rented or leased and how many shifts it is operated and what is the daily meterage etc. Regarding software, the languages in use should be ascertained and systems development, program and operating documentations should be collected Such documentation would assist the auditor to understand the organisational objectives and to see to what extent these are being met by the computerised system.

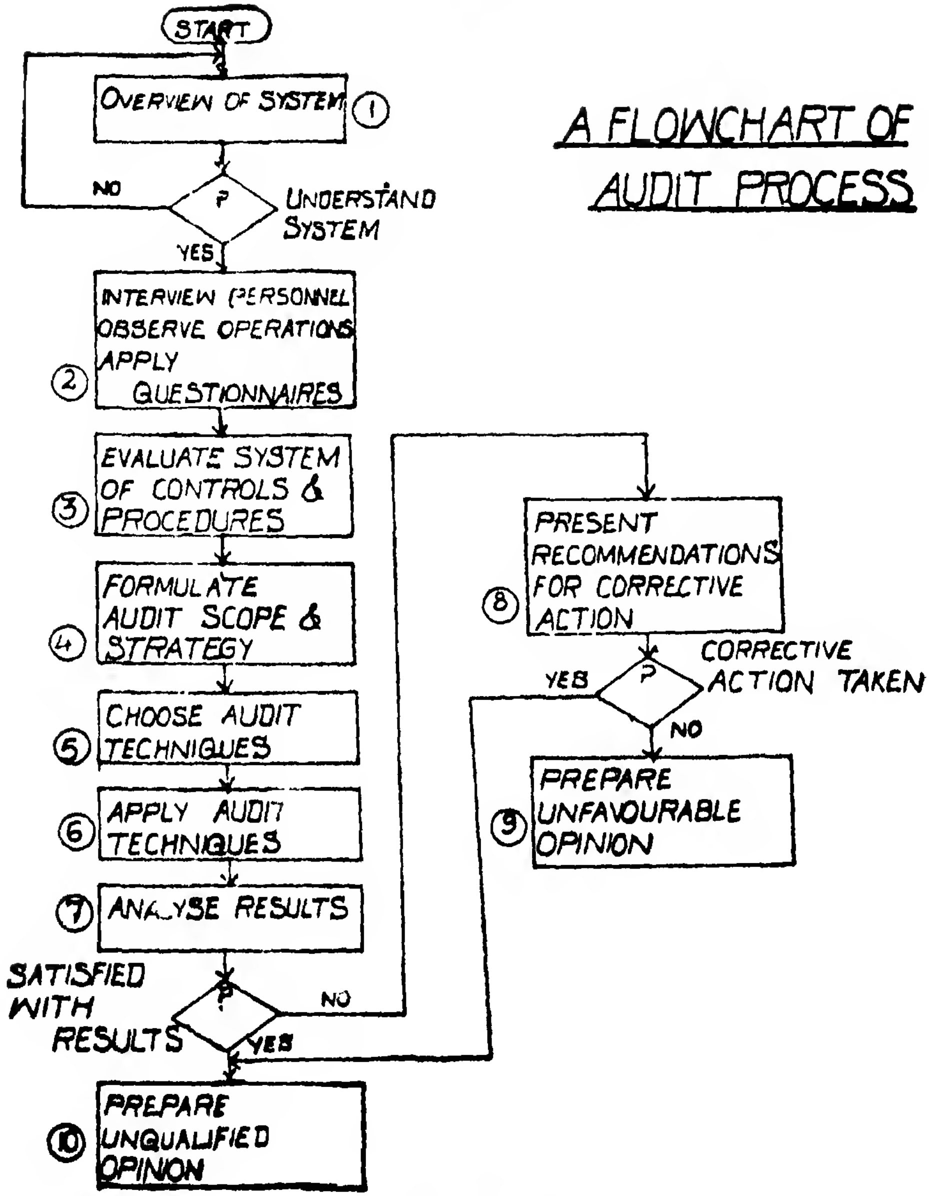


Fig. 14

2. The auditor may want to interview the personnel (users) across all the departments in the entire organisation to obtain evidence about worthwhile of the computerised system that is to what extent the system is meeting its objectives, realising the anticipated payoffs, how the users are interfacing with the system for inputs and to what extent they are satisfied over completeness and accuracy of the outputs and, above all

how satisfactory and comprehensive are the controls. Regarding the controls, he may have to interview and observe the computer operating personnel Observing the system closely is a sound technique for uncovering its weaknesses and errors and determining the direction and intensity of further audit. There is no substitute for observing. The auditor would also want to use questionnaries which he should preserve in his working papers because they provide evidential information required for expression of an opinion.

3. Controls are embedded in the system to enhance its efficiency, integrity and accuracy and prevent fraud and errors. Obviously, the auditor should have acquired sufficient understanding of good standards, operating procedures and controls as discussed in earlier sections and studies. The auditor's primary purpose in reviewing the internal controls may be summarised as below.

"There is to be appropriate study and valuation of the existing internal control as a basis for reliance thereon and for the determination of the result in the extent of the test to which procedures are to be audited." As a secondry purpose, the auditor may also want to make constructive suggestion to the client regarding improvement in the control though this is entirely his discretion.

The auditor would have to investigate the controls that are applied to computer data processing organisation as a whole and those associated with each individual application. These are listed below and have been discussed in detail elsewhere :

General System Control	*Specific Application Controls*
Organisation	Input and output controls
Documentation	Processing controls
Hardware controls	Audit trail
File safeguards	

The principal tool for reviewing both categories of controls is the questionnaire a sample of which on documentation controls alone is shown in Appendix A to this study note. The other source for reviewing the application controls only is the error lists which, however, provide only negative evidence about the controls which are operating and does not give any idea of the controls which are either wrongly operating or are missing.

4. However, questionnaries and review of error lists can lead only to preliminary evaluation which can only be finalised upon compliance tests which for example, would consist of ascertaining if the cards under verification are being notched, if the computer log book is properly maintained, if programs are properly authorised, if the control totals are being reconciled etc. The program controls would be tested either by error lists or by special auditing tecniques to be discussed later.

The questionnaire may for example, inform the auditor that files are being retained on the grandfather-father son principle but he would have to apply the compliance tests to see if this is really so. Towards

this purpose, he may take possession of the current, say, accounts payable file and then ask the EDP staff to recreate it from the previous period's files and then subsequently compare the actual with recreated version. As a more comprehensive and aggressive test of this he may cease all the computer operations, ask the EDP staff to bring the back up system into operation by reproducing the current files from the grandfather-father files and switching to back-up facilities and contingency procedures. Any failure or lethargy to switch over to the back up system and back to the original would be noted, investigated and corrective measures formulated. The compliance tests would determine the direction and the depth of the audit and the mix of the techniques to be employed towards substantiative tests.

5. The requirements of auditing in computerised systems may be summarised as below ;

   (a) Evaluate the system of controls

   (b) Trace and test transactions

   (c) Verify the accuracy of the files of the data base

   (d) Form an opinion and recommendations about reliability and integrity of the outputs from the system, and communicate these in the form of reports. To execute these the auditor must :

      (i) observe and review.
      (ii) inquire,
      (iii) inspect,
      (iv) test,
      (v) sample,
      (vi) confirm and
      (vii) compare

   These tasks can be performed by the auditing techniques discussed below the mix of which the auditor would have to determine on the basis of the evaluation of controls by means of compliance tests.

6. Apply audit techniques as discussed in the following section.

7. to (10). The student should be in a position to elaborate these steps on the basis of his auditing studies,

4. Audit Techniques

4 1 Auditing Around the Computer

The approach is also known as the black box approach. Black box approach symbolises auditor's attempt to find audit satisfaction with regard to the accounts maintained in computer by tracing, correlating and vouching data from output to source documents and *vice versa* by ignoring the process of transformation that takes place inside the black box i e. the computer.

In respect of the computer, this approach would generally be employed to study the behaviour of systems whose mechanism is too complex to comprehend. Without going into the transformation taking place in the systems the analyst

manipulates the input and studies the effects on output. In the context of auditing computerised accounts, the auditor employing this approach does the same thing. He is not concerned with the program or mechanism that causes the transformation of the input data into output data. He would select test transactions processed by the computer and ascertain that the computer outputs corresponding to them give the right results.

However, with growing use of computer in business situations it is high time when the auditors shed their aloofness to computer and get more intimately involved with it so that they understand the machine and its working and thereby make effective use of this knowlege in auditing.

The advantages and disadvantages of this approach of ignoring the computer for auditing are given below :

Advantages

(i) The auditor does not require any knowledge of the computer and data processing.

(ii) The requirements of the auditor are not inconsistent with those of the systems analyst It is only possible, of course, if the system is to be designed around the magnetic tape

(iii) It is simple, straightforward and easily understood by everyone.

(iv) It is cheap in terms of audit resources to be used.

Disadvantages

(i) The auditor is not utilising the potential of the computer to assist him in his burdensome task and where the data is voluminous, it would be impractical to do manual test.

(ii) This approach ignores a positive test of the program controls though, in fairness, it must be stated that the error listings provide quite a good, though not complete, view of this

(iii) It amounts to auditing in nature of post mortem rather than preventive auditing.

(iv) The insistence on maintaining the trail for this approach would inhibit the system designer to think in terms of sophisticated systems. Also, this may tie up the printer because of enormous print out requirements of the auditor.

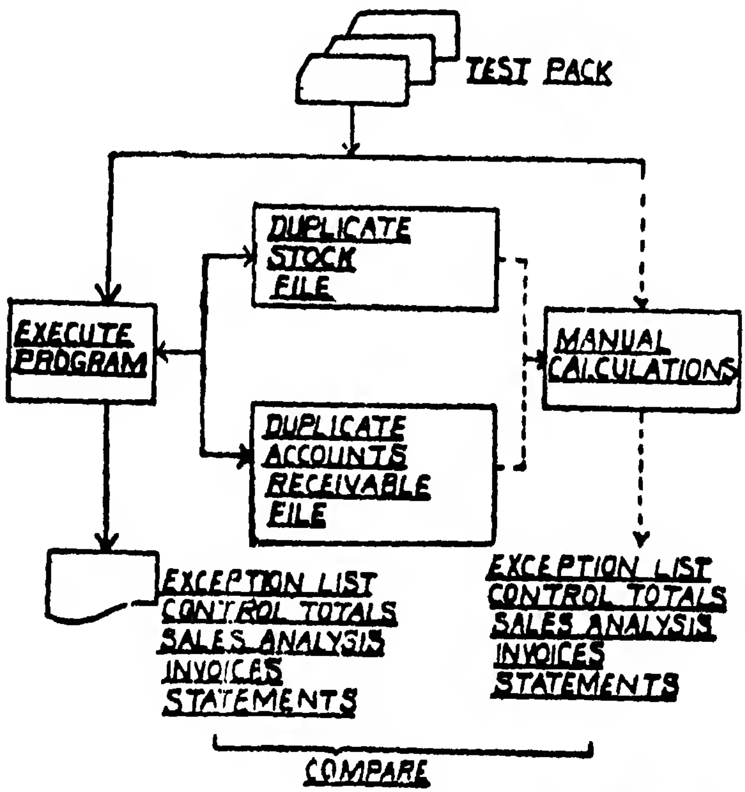
4.2 Test Packs

Test packs are a set of data in machine readable form i.e., on punch cards, paper tape, magnetic tape, etc. This data is devised by the auditor himself to test the various alternative branches of the program and the program controls for both valid and invalid conditions. This test data is then run against selected master records to judge the robustness of the computer programs. The test data approach is particularly useful when (i) a significant part of the system of internal control is embodied in the computer program (ii) the audit trail is sketchy and it is difficult, impractical and expensive indeed to trace inputs to outputs and *vice versa*, and (iii) the volume of records is so large that it is more economic to use the test packs.

In this method, the auditor has to satisfy himself that the program he tested is really being used by the client. This he can do in one of the following three ways : (i) the auditor tests the program with his test data and then supervises the client's normal processing run with the tested program, (ii) the auditor repeatedly uses the test data during the auditing period which, however is feasible only if he has access to the computer via remote terminals (iii) he satisfies himself over the internal controls of the client over programs and program amendments and infers that the program tested is representative of the program used during the audit period.

The test data may not necessarily be put on such machine readable file media as punched cards, paper tape or magnetic tape but it may also be put on MICR or OCR documents so that these are both machine and human-readable, or he may put it on plain paper especially when he intends not only to test the programs but also transcription.

USE OF A TEST PACK



The following steps are involved in developing and using the test data :

(i) selection of the master records.

(ii) selection of the transactions.

(iii) developing the approriate working papers.

(iv) pre-computing test results in long hand for the selected master records and transactions.

(v) investigation of the effects of transactions on the master records in case current live master records are being used This is nessessary to reverse the updated master records back to their original status.

Selection of marter records : The following three possibilities exist for selection of the master records (i) current live master records (ii) special audit records put in the master file and (iii) obsolete master records or copies of the current live master records.

In cases (i) and (ii) the master records are included in the current master file and, therefore, are processed for testing by the auditor during the course of the normal processing run but in case (iii) the auditor specially sets up the computer for test purposes. The pros and cons of each method are discussed below :

Current Master Records. The advantages of these records are that the auditor can do his testing on a surprise basis. He need not set up the computer specially for testing and since the tests are being performed on the current records any omission or discrepancies in them can be brought out. However, the dangers of distrupting the normal processing runs are so great that this method should not be attempted by an auditor lacking any way in computer expertise. The auditor is also liable to be criticised for any complications in the normal processing run. Above all, the up-dated records have to be brought back to their original status.

Special Audit Records. The auditor devies these records and puts them at the end of the client's master file. These dummy records are then up-dated by the test transactions during the normal process run.

The major advantage of this approach is that the current master records are not updated ; with the result their contents have not to be reversed back to the original status but the output reports are liable to contain the results of the up-dating of the dummy records making clerical adjustments of these reports necessary. Since the current master records are not updated their weaknesses woulud not be revealed. The dummy records are also liable to clutter up the master files. This approach also requires a great deal of preplanning on the part of the auditor and participation by the client The dummy records are also liable to be used towards fraudulent ends by client's staff.

Obsolete Master Records. These records may come from the previous grandfather or great grandfather files or may be copied from the current master files. Incidentally, since the magnetic disc files are updated by overlaying their older versions would not normally be available and it would be necessary to dump their contents periodically on magnetic tapes. Computer has to be specially set up for running the test packs which is expensive though this obviates any complications in the normal processing run and reversal of the status of the master records to their current status.

Types of Transactions

In order to devise appropriate test transactions the auditor would have to study the program documentation viz., decision tables to mark out the various

program branches for testing. He may also want to test the various program contorls for valid and invalid transactions. He may however, find it expedient to use the packs compiled by the programmers during program debugging. This incidentally can reveal the outdated or missing program controls and the auditor can advise the client accordingly, of course on discretionary basis. Also with the advancement in computerised systems the client's programs are likely to be refined so that the auditor can use them with more reliance.

Working Papers. For development and review purposes, the tests should be documented in working papers which may include the following schedules.

(i) *Test data controls.* This schedule describes the conditions that are to be tested, the precomputed results and later the actual results. The last item, the actual results should also be accompanied by the output report in which it appears.

Transaction data. This worksheet includes all transactions which are to be processed for both valid and invalid transactions.

Master file data includes the contents and format of the selected master records.

The documentation may also include the decision tables in which the conditions to be tested and the actions to be taken by the computer are put in a tabular decision format, and the computer print-outs of the actual test results.

Advantages and disadvantages of test packs

Advantages :

(i) The auditor need not possess detailed knowledge of the computer data processing.

(ii) Verification of the computer output with per-calculated results can be readily accomplished.

(iii) The necessity to maintain the conventional audit trail is diminished.

(iv) The test pack approach is particularly useful where permutations and combinations of contingencies are limited.

(v) It is a highly objective approach.

Test packs can be applied on a surprise basis to discourage the unauthorised modifications of programs.

Disadvantages :

(i) If the file structure is changed or changes are incorporated in the programs the test packs would be invalidated and new ones may have to be compiled and this happens all too often.

(ii) Since the auditor may be dealing with organisations of diverse backgrounds he would have to compile such test pack for each of them and that may turn out to be quite brudensome a task.

(iii) If a minor deviation is detected by the auditor he would not be in a position to assess the extent of its seriousness.

(iv) In complex systems where there exist myriads of permutations and combinations of contingencies it would be unwiledy.

(v) The auditor would have to familiarise himself sufficienlty with the program logic.

(vi) Despite full efforts to test potential errors some errors may still exist. This is in view of tens of hundreds of different paths that may exist in a program.

(vii) It is not unlikely that the test packs leave undetected some fraud motivated instructions in the program since test pack design does not take them into consideration.

4.3 Using the computer for Auditing

Doubtless that computer obviates several intermediate printouts and the hidden records in the magnetic media could be problems for the auditor not adequately trained in computer data processing ; but the very fact that records are in a machine readable form suggests possibility of using the capabilities of the computer to assist the auditor in his burdensome task This, however, requires that the auditor is sufficiently proficient in computer data processing and programming. Computer, if the appropriate instructions are written and fed into it as the program, can perform following computational or comparison tasks :

(i) The auditor can test extensions and foot totals on a sample or entire population of items.

(ii) The computer can select the accounts for confirmation on the basis of some quantifiable criteria (e.g, accounts exceeding a certain limit or accounts at nil value) and any sampling plan. The format of the request can be designed to facilitiate mailing and audit follow up. In one set up computer can print out the confirmation request and its envelope and the envelope for the reply and even a reminder. Subsequently these confirmation requests can be mailed to the customers and the vendors for their confirmation

(iii) The computer can examine records for completeness, consistency, valid conditions etc For example, customer file records might be examined to determine those for which no credit limit is specified etc.

(iv) The data can be summarised in various ways and several analyses can be performed on it. The examples would include stock valuation reports and aged accounts receivables.

(v) The computer may be programmed to select audit samples by the use of random numbers by systematic selection techniques and they can be printed for the auditor The examples would include printing the details of those customer accounts which are beyond a certain rupee amount and those below this amount on a sampling basis.

(vi) The data fields which appear in more than one file can be compared by a computer program.

(vii) The auditor can usefully employ the sequential sampling plans of the statistical quality control techniques. Let us first explain what these plans are in the context of inspection of lots of items received from outside vendors. A sample from a lot is taken and each item in it is inspected. Two numbers are derived statistically. Call these a and b where $a \angle b$. If the number of rejects are less than a the lot is accepted. If the number of rejects are more than or equal to b the lot is rejected.

If the number of rejects lie within *a* and *b* another sample is taken from the lot. This process of three alternatives upon the results of each successive sample is continued until the lot is either accepted or rejected. Now in the context of auditing, the auditor, over the years would have formed the opinion as to the number of permissible errors, say in a payroll application in a sample of a given size. Based on the actual results he may accept or reject or proceed with the next sample.

(viii) Audit data can be compared with company records. For example, the actual inventory counts can be compared with those on the records by means of the computer For this procedure the audit data must be converted to machine readable form. It is to be noted that the aforesaid computations and comparisons would also be performed by the client and it is expected that he would have the necessary programs written for these. The auditor may want to use these programs after satisfying himself that they are satisfactory. He may assure himself in this regard by appraising the internal controls such as documentation, separation of duties, change procedures and tape library procedures.

Generalised Computer Audit Programs

There are many audit functions which change very little from client to client. This would suggest the possibility of using the generalised audit programs which may, however, have to be modified slightly for each client. The difficulties and the favorable points for using the generalised audit programs are discussed below :

(i) Computers of different makes would make it extremly difficult to write generalised audit programs. In the past, even in a manufacturer's line there has not been compatibility. Nevertheless, now a days, the computer manufacturers are marketing families of compatible computers. Also, if the client switches over to another brand the computer manufacturers would provide compatibility through simulators and emulators (Ref : Study V) The fact that a few computers dominate the market makes writings of generalised auditing programs easy. Furthermore, the increasing use of the high level language, COBOL is a favourable trend in this regard.

(ii) For the same model of computer also, numerous configurations are possible. Therefore, the generalised audit program would have to cover all these configurations. This would make the programs very bulky and difficult and expensive to write since, for example, records maintained on a direct access file are referenced and accessed differently from records on a magnetic tape.

(iii) COBOL compliers may not be available with smaller computers. Also, the environment and data divisions of the COBOL programs may differ from client to client.

(iv) Clients would be maintaining their files in different formats. This would necessitate conversion of these individual formats into a standardised format to suit the generalised audit programs.

4. Audit Review of Application Programs

We mentioned earlier that a set of fraudulent instructions stuffed into the program would be difficult to detect by the test pack or special audit program techniques. This should highlight the necessity for the auditor scrutinising the program code itself Surely, this would require him to be an expert in programming in the language in use Also, he would have to work hard since the programs written by others are notoriously difficult to be deciphered But the auditor would be coming into grip with the program which is the most sensitive spot in computerised system for exposure to fraud and errors Towards this purpose, he may run typical data in the program logic or its flowcharts manually through the various paths to satisfy himself with its logic and may detect extra fraudulent instructions i e, devious little modules He may also review the program for completeness and inclusion of adequate controls

The fraudulent programmer can build trap doors in the program which *open* to operate when a particular event or sequence of instructions happens These trapdoors comprise illegal sets of instructions For example these instructions may be activated in favour of a particular account number. Now if the auditor rigorously reviews the programs coupled with their testing by the test pack the possibility of undetected errors is minimised Reviewing the programs is the strongest auditing technique Human beings, even with standardised procedures, tend to act differently and inconsistently at different occasions and this is not so with the computer which, fed with the same program and run repeatedly, would produce the same outputs

Thus, if the auditor has reviewed the programs and satisfied himself he can place a great deal of reliance in the system The drawbacks of this approach are that it is time consuming and auditors ought to know the language in use If it is other than COBOL it would be very difficult to learn Nevertheless, this technique is the most robust and should be applied to test the most important and frequently used programs The following are the reasons which may prompt an auditor to go in for the review of the programs

(i) To detect and dicourage errors and fraud.

(ii) To appraise the extent to which good program practices have been put to use i e. if the standard data names and the meaningful paragraphs are being used instead of programmer's relatives' pet names

(iii) To satisfy himself that sound accounting practices have been followed and doubtful ones uncovered. The systems designers, for example, may fraudulently change over to different inventory valuing methods, which can be revealed in the program review.

(iv) To ascertain if the program documentation is current. The review may uncover, for example, several codes which are not to be found in the documentation It is also easy for the programmer to trace any subsequent modifications to the program easily if he has reviewed it once.

(v) Incidentally, such reviews can provide a great deal of expertise and experience to the auditor which he may use profitably in other organisations.

(vi) This approach would naturally impress the data processing staff, and discourage fraud amongst them.

If the number of rejects lie within *a* and *b* another sample is taken from the lot. This process of three alternatives upon the results of each successive sample is continued until the lot is either accepted or rejected. Now in the context of auditing, the auditor, over the years would have formed the opinion as to the number of permissible errors, say in a payroll application in a sample of a given size. Based on the actual results he may accept or reject or proceed with the next sample.

(viii) Audit data can be compared with company records. For example, the actual inventory counts can be compared with those on the records by means of the computer For this procedure the audit data must be converted to machine readable form. It is to be noted that the aforesaid computations and comparisons would also be performed by the client and it is expected that he would have the necessary programs written for these. The auditor may want to use these programs after satisfying himself that they are satisfactory. He may assure himself in this regard by appraising the internal controls such as documentation, separation of duties, change procedures and tape library procedures.

Generalised Computer Audit Programs

There are many audit functions which change very little from client to client. This would suggest the possibility of using the generalised audit programs which may, however, have to be modified slightly for each client. The difficulties and the favorable points for using the generalised audit programs are discussed below :

(i) Computers of different makes would make it extremly difficult to write generalised audit programs. In the past, even in a manufacturer's line there has not been compatibility. Nevertheless, now a days, the computer manufacturers are marketing families of compatible computers. Also, if the client switches over to another brand the computer manufacturers would provide compatibility through simulators and emulators (Ref : Study V) The fact that a few computers dominate the market makes writings of generalised auditing programs easy. Furthermore, the increasing use of the high level language, COBOL is a favourable trend in this regard.

(ii) For the same model of computer also, numerous configurations are possible. Therefore, the generalised audit program would have to cover all these configurations. This would make the programs very bulky and difficult and expensive to write since, for example, records maintained on a direct access file are referenced and accessed differently from records on a magnetic tape.

(iii) COBOL compliers may not be available with smaller computers. Also, the environment and data divisions of the COBOL programs may differ from client to client.

(iv) Clients would be maintaining their files in different formats. This would necessitate conversion of these individual formats into a standardised format to suit the generalised audit programs.

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(vi) This approach would naturally impress the data processing staff, and discourage fraud amongst them.

The auditor would have to go through the following steps for review of the programs :

(i) Review programming and documentation standards of the organisation This not only gives the idea to the auditor of the quality of programs but also provides a very useful frame of reference

(ii) Selection of the program for review. This program could be the most sensitive one , but to start with the auditor may want to pick up a simple program especially when several programs have been written by one programmer in order to familiarise himself with style of that programmer In this way, the auditor would be in a position to understand his more complex programs easily later

(iii) Get the copy of source code As we mentioned elsewhere, a program of some scope may require as many as 20 rounds of debugging. Add to this several amendments upon routine application Therefore, there could be numerous program listings and the auditor would not be sure if the one given to him represents the current program It is highly desirable therefore that the auditor obtains the program litsing for the program in use on an especial compilation run

(iv) Get program documentation More often than not there may be no documentation at all and the auditor may have to request the client's EDP personnel to prepare a set for him Even if it is available it is not at all unlikely that it is out of date in which case the auditor may want that it is up-dated by the client's EDP staff.

(v) Determine the files used by the programmer This can be readily done by referring to SELECT clauses in the ENVIRONMENT DIVISION of a program written in COBOL where each file is referenced to the specific hardware device (s) The auditor should next refer to the DATA DIVISION and correlate its contents with the documentation. Any data field encountered in the DATA DIVISION but missing in the documentation should suggest the either the documentation has not been kept up-to-date or there have been unauthorised amendments to program The later situation may lead to the discovery of a major fraud.

(vi) Gain understanding of each of the file by dumping two or three of its pages and scrutinising each and every field layout. This is one of the most crucial part of the review and the auditor should not hesitate in critically questionning the EDP staff at slightest doubt.. In fact, he should be on the guard against clever and evasive explanations.

(vii) Once the auditor has reviewed few pages of each of the file he can concentrate on the program logic and IF statements require his particular attention No doubt, it would require a great deal of labour to trace the various paths of the program via these statements especially when these run into myriads in any program but they are of great importance and it can be asserted that most of the frauds would be committed via these Next in importance are GO TO statements which are used to transfer the program control to some external modules which should be carefully reviewed in regard to the transfer of control and why. Each and every GO TO statement should be carefully verified Incidentally, many of the GO TO statements would be part of the IF statements. These two statements between them would have provided quite a good deal of insight into the program logic to the auditor. The review of these may be followed by scrutity of such verbs as CALL, FILL,

ACCEPT, RETURN. All these statements signify that some external influence is being applied to the program which needs particular attention of the auditor.

5. Audit of Advanced Systems

The on line, real time, integrated systems do pose problems *with regard to* audit trail and auditing. A transaction can be input at random as it occurs via remote trminals A transaction may up date several files at a stroke and intermediate print outs may or may not be printed which endangers the audit trail i e, it can be eliminated. The various problems are discussed below.

Source documents The transactions are input at random without batching via remote terminals. However, to keep the management audit trail intact at least it is usually necessary to get a printed copy of the transactions from the terminals Also, the transactions may by logged at the compüter centre In some applications the terminals may prepare locked log of all transactions entered through them. The log at the computer centre can then be utilised for file reconstruction in the event of any damage to the existing files

Authorisation

(i) Each user is supplied with a password which identifies him as the authorised user and he has to enter this password *prior to* entering the transaction details.

(ii) The computer performs the various program checks on the transaction to ascertain the validity of the transactions.

(iii) The listing of the various transactions input may be prepared and sent to the control section

(iv) The printout of the transactions produced by the computer serves as approval of the input

Furthermore, the programmed decision rules built into the program may be reviewed by the concerned managers as authorisation for their automatic execution by the computer. Examples of such rules would include determination of the economic order quantities reorder levels, forecasts etc

Audit Trail As mentioned earlier most of the intermediate printouts for processing a transaction comprehensively in the integrated systems need not be produced but the requirements of the management, governments and other agencies may constrain the systems designer to produce these to maintain the audit trial.

Control totals seemingly are not possible with such advanced systems. Nevertheless they can be accumulated for each type of transaction entered via the terminals and later compared with their counterparts at the originating points in the organisation Also control totals may be prepared for each remote terminal

The audit techniques described earlier may also be applied to integrated systems.

It is to be noted here that the auditor's burden will sharply increase since programs are inherently very complex and the auditor must be highly proficient in computerised systems. In addition to these techniques the auditors may also want to continuously monitor the realtime integrated systems by incorporating an audit program in the operating system. This program would sample the transactions entered into the system at random intervals and write them on an audit tape for purposes of testing. Also, the auditor may make use of the terminals to introduce random test transactions.

APPENDIX A
DOCUMENTATION QUESTIONNAIRE

	Answer		*Answer Based on*	
	Yes	*No*	*Observation*	*Test*
1. Does the current documentation for each computer program contain :				
(a) a narrative description ?				
(b) flowchart, and decision tables ?				
(c) program description ?				
(d) description of programming aids ?				
(e) controls used ?				
(f) description of input/output ?				
(g) terting procedures used ?				
(h) console run book ?				
(i) approval and change sheet ?				
2. Does the console run book contain :				
(a) flowchart of specific applications ?				
(b) set-up instructions ?				
(c) description of input/output layouts ?				
(d) console switch setting ?				
(e) operating messages (error halts, (etc.) ?				
(f) procedures for labelling and disposition of input ?				
3. Are there written procedures for program maintenance which provide for ;				
(a) a formal request for a program change ?				
(b) approval by a responsible individual for a change ?				
(c) a test of the change prior to acceptance ?				
4. Are there duplicate copies of either the source decks, object decks or from the EDP premises as protection against loss ?				
5. Is there a definable system development methodology ?				
6. If so, describe it below.				

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reflux temperature of the system resulted in complete solution and colour fade to a light yellow.

2. $MoCl_3$ *in cold* 96% H_2SO_4. To 2.0 g $MoCl_3$ were added 55.1 g 96.4% H_2SO_4. Mixing was carried out at ambient temperatures for 24 hr in a stoppered flask. Centrifuging the 3.5 per cent mixture revealed a weakly coloured blue solution. The solid residue contained most of the original $MoCl_3$.

3. $MoCl_3$ *in fuming* H_2SO_4, 15–18 *and* 30–33% SO_3. A 4.8 per cent solution of $MoCl_3$ in 15–18 per cent SO_3 fuming H_2SO_4 was prepared by refluxing for 2 hr at 170–180°. The solution was dark blue-green. Further refluxing at 180° for 4 hr showed no further colour change. No residue remained.

A 6.5 per cent solution of $MoCl_3$ in 30–33 per cent SO_3 fuming H_2SO_4 was prepared in a similar manner by refluxing at 120–125°. The final colour of the solution was bright blue, and the time required for complete dissolution was less than 1 hr.

4. K_3MoCl_6 *in hot* 96% H_2SO_4. A 4.94 per cent solution of K_3MoCl_6 in 96 per cent H_2SO_4 was prepared by slow dropwise addition of the acid to the solid in a 50 ml round bottom flask fitted with a condenser. Fumes of HCl were released rapidly during the addition. After the major part of the evolution ceased, the mixture was heated to accelerate solution. At 190°, the solution was dark green, with little or no solids remaining.

5. K_3MoCl_6 *in cold* 96% H_2SO_4. A 3.9 per cent mixture was prepared in the cold as above. Centrifugation of the suspension yielded a red-brown solution.

6. K_3MoCl_6 *in fuming* H_2SO_4, 15–18 *and* 30–33% SO_3. Two 7.0 per cent solutions were prepared by slow dropwise addition of the respective fuming acids onto K_3MoCl_6. HCl was given off voluminously. Heating to the reflux temperatures (170–180° and 120–125°) yielded dark blue solutions within 2 hr. The colour development in the 30–33 per cent acid, however, occurred more rapidly than in the 15–18% SO_3 acid. No residue remained.

Preparation of sulphur trioxide adducts

Apparatus. A complete description is found in Reference 1. The following summarizes the essential features.

Fuming H_2SO_4 (30–33% SO_3) was used as source for SO_3. About 50 ml of the acid were placed in a 100 ml round flask which contained a well for thermometer placement. Commercially available dry nitrogen, which was used as carrier gas, was passed through a hot tube filled with copper turnings to remove traces of oxygen, and then bubbled through the heated acid via a capillary bleed tube, the tip of which was positioned about ½ in. below the surface of the acid. Temperatures for efficient SO_3 evolution ranged from 115 to 125°. The SO_3–N_2 mixture was passed through a fine porosity glass fritted filter tube to remove brownish-black decomposition products formed during the heating of the fuming acid. The filter was placed well within the reaction tube to prevent liquefaction of the SO_3. Temperatures were measured by placing a thermometer at a fixed position over the reaction boat and calibrating against materials of known melting point placed in the boat.

Procedure. Two to five grammes of sample were weighed into a Vitreosil silica boat and inserted into the reaction chamber maintained at a pre-chosen temperature in an electric tube furnace. Pretreatment of K_3MoCl_6 was necessary to remove small amounts of moisture; the solid was heated in the reaction tube under nitrogen at 110–120° for 2 hr. The dry nitrogen under a slight excess pressure passed through the heated fuming acid and carried the evolved SO_3 through the filter tube into the reaction chamber.

1. $MoCl_3$–SO_3. Reaction began at 80° with the formation of a grayish black coating. With the exception of increased coverage of the surface, no other changes were observed as the temperature was raised. Physical examination of the final product indicated surface reaction only. Changing the sample size, $MoCl_3$ particle size, SO_3 evolution rate, etc. did not effect further reaction. Decomposition occurred without melting under the SO_3–N_2 atmosphere at 200–220°, yielding varied coloured products. In a control run, $MoCl_3$ under dry nitrogen was stable up to 260°.

2. K_3MoCl_6–SO_3. In a similar control run under dry nitrogen, K_3MoCl_6 was stable up to 250°. In contrast to the $MoCl_3$–SO_3 system, when SO_3 was passed over the K_3MoCl_6, reaction occurred readily at 80°, and continued up to 190° before decomposition became obvious at 200–210°. Equilibrium reactions from 64° to 180° yielded a liquid phase having colours ranging from dark blue at the low range (64–80°) to bluish green at the intermediate range, 80–165°, and dark green at the high range (165–180°).

Analysis

Molybdenum was determined gravimetrically using 8-quinolonol. Chloride was determined volumetrically with standard mercuric nitrate using sodium nitroprusside as indicator.

The oxidation state of molybdenum was determined as follows: excess standard potassium dichromate–concentrated sulphuric acid solution was used to oxidize the molybdenum in solution and in the SO_3 adducts to the maximum valence of six. The unreacted dichromate was found by titration

with standard aqueous ferrous sulphate.[1] The figures obtained are effective oxidation numbers, and therefore are only average values.

The sulphur trioxide values were calculated on the basis that increase in weight of the adduct is attributable to the addition of SO_3 to K_3MoCl_6. See discussion for justification of this assumption.

RESULTS AND DISCUSSION

Table 1 summarizes the solubility data and oxidation numbers for $MoCl_3$ and K_3MoCl_6 in concentrated H_2SO_4. The valence states show oxidation to about the same levels for the heated solutions, and are probably mixtures of molybdenum 3

TABLE 1. SOLUBILITY AND EFFECTIVE OXIDATION NUMBER FOR MOLYBDENUM IONS IN CONCENTRATED SULPHURIC ACID

System	Temperature	% Mo in solution	Calculated % solubility of salt	Colour	Effective oxidation no.
$MoCl_3$–H_2SO_4	Ambient	0.0828	5.1	Light blue	3.02
$MoCl_3$–H_2SO_4	180°	1.99	100	Olive green	4.8
K_3MoCl_6–H_2SO_4	Ambient	0.768	87	Red brown	2.98
K_3MoCl_6–H_2SO_4	180°	1.11	100	Dark green	4.5

through 6. The 180° systems show essentially complete dissolution. Table 2 summarizes the adducts of SO_3 with K_3MoCl_6 formed reversibly at several temperatures. Evidence for reversibility in the 64° to 180° range was shown by constancy of the six to one Cl/Mo ratio, the molybdenum oxidation state, and the ease by which SO_3 could be

TABLE 2. SULPHUR TRIOXIDE ADDUCTS OF K_3MoCl_6 AT VARIOUS TEMPERATURES

Temperature	No. equivalent SO_3 molecules	Colour	Effective oxidation no.
64	10	Dark blue	3.0
137	8	Dark blue green	3.0
155	6	Green dark blue	3.0
165	5	Green blue	3.0
180	4	Green	3.0
194	–	Dark brownish green	4.0

added to or removed from a given sample by adjustment of the reaction temperatures.[1]

In the presence of the SO_3 atmosphere, the weight of an adduct invariably increased as the temperature was lowered; the weight decreased in a similar way as the temperature was raised. For a given temperature, the per cent increase over the original K_3MoCl_6 weight was constant irrespective of the direction from which this temperature was attained. As a check of the constancy of the six to one Cl/Mo ratio in the adduct, the reaction chamber effluent was analysed for Cl; none was found, thus affirming that addition reactions indeed had occurred.

The 4.0 oxidation number for the adduct formed at 194° is consistent with the colour change and gas evolution which occurred within the decomposing liquid at that temperature, and is included as an approximate upper limit of reversibility. The Cl/Mo ratio for this adduct was 5:1 indicating loss of Cl during the decomposition. Direct determination of the oxidation states in the fuming acids were impractical and at best would have yielded questionable values. Therefore, by analogy with the colours

developed in the several solvents, the oxidation states were deduced as follows. Striking colour similarities were apparent for the fuming acid and SO_3-adduct systems. At higher SO_3 contents, e.g. 30–33% SO_3 fuming H_2SO_4 and K_3MoCl_6 adducts containing $10\,SO_3$, the systems were dark blue; at lower SO_3 contents, e.g. 15–18% SO_3 fuming H_2SO_4 and adducts containing 5–8 SO_3, the systems were green blue to dark blue green. Based on the 3.0 oxidation numbers for the 4–10 SO_3 adducts at 180° –64°, respectively, and the fact that continued refluxing of the fuming acid systems showed no colour changes, it was assumed that the oxidation state of the molybdenum ions in the fuming acid was also 3.0. Since the available free SO_3 in C.P. concentrated H_2SO_4, compared to fuming H_2SO_4, is low, and may be available only through dissociations such as

$$2H_2SO_4 \rightleftharpoons SO_3 + OH_3^+ + HSO_4^-\ ^{(1)}$$

$$SO_3 + 2H_2SO_4 \rightleftharpoons H_3SO_4^+ + HS_2O_7^-$$

the colours for the concentrated H_2SO_4 solutions at high temperatures which were dark or olive green probably resulted from oxidation which produced the effective states from 4.5 to 5 in the absence of excess SO_3. Another assumption supported by the data but which would be difficult to prove is that the SO_3 "molecules" in solution or in the liquid adduct phase act as protective surrounds or negative catalysts for the central molybdenum(III) ion, thus preventing oxidation even at temperatures as high as 180°. It is unfortunate that infra-red spectra could not be run for reasons which exclude such measurements from systems which involve free SO_3.

In the absence of additional data on the ambient temperature reactions of $MoCl_3$ and K_3MoCl_6 in concentrated H_2SO_4, explanations for the light blue and red brown coloured solutions are tentative. That the $MoCl_3$ dissolves at all may result from the SO_3 produced by above equilibria. The red K_3MoCl_6 solution would then contain a blue component as well as red, indicative of different compound or complex formation.

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RUTHENIUM (VIII) IN NITRIC ACID

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Abstract Absorption spectra of solutions of RuO_4 in nitric acid have been measured and electromigration experiments performed From these experiments it has been concluded that no charged species are formed The distribution coefficient of RuO_4 between nitric acid and CCl_4 and kerosene have been measured, but attempts to explain quantitatively the salting behaviour by applying McDevit and Long's theory have been unsuccessful Approximate values of the thermodynamic quantities for extraction into CCl_4 have been calculated, and activity coefficients have been obtained from solubility data A method for selective extraction of ruthenium from mixtures such as fission products by oxidation with sodium hyprochlorite and extraction into kerosene is indicated

RUTHENIUM has been described e g by CLAUS[1] as existing in nitric acid solution as its tetroxide RuO_4 together with its trivalent and tetravalent oxidation states.[2] The tetroxide is easily extracted with non-polar solvents such as CCl_4[3] and kerosene The distribution coefficient decreases with increasing nitric acid concentration The fission product ruthenium accompanies uranium and plutonium during most extraction processes and is one of the major sources of γ-activity of the purified products In the present work a specific method for the removal of ruthenium from mixtures is developed The ruthenium in its tri- or tetravalent state is oxidized by sodium hypochlorite and extracted with kerosene

Recently MARTIN[3] has studied the extraction of RuO_4 from nitric acid solutions into CCl_4 In order to account for the decrease of the distribution coefficient with increasing acidity he assumed the equilibrium

$$RuO_4 + H_2O \rightleftharpoons H_2RuO_5 \rightleftharpoons HRuO_5^- + H^+ \tag{1}$$

to play a major role along with salting in, but he did not provide positive evidence for these reactions For the dissociation equilibrium he found from distribution experiments

$$K_h' = \frac{K_d(D_0 - D)}{[H^+]D} = 5.7 \times 10^{-15}\ l\ mole^{-1} \tag{2}$$

where D = distribution coefficient at hydrogen-ion concentration H^+ and D_0 = distribution coefficient in pure water If the equilibrium is written in the form

$$RuO_4 + H^+ \rightleftharpoons HRuO_4^+ \tag{3}$$

the expression

$$K_h = \frac{[HRuO_4^+]}{[H^+][RuO_4]} = 0.57\ l\ mole^{-1} \tag{4}$$

* Part of a thesis submitted to the Technion Israel Institute of Technology Haifa in partial fulfillment of the requirements of a M Sc degree

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